Microeconomics Problem Set 1

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Question 1

A consumer has preferences over bundles of two goods with indifference curves defined by $x_2 = \frac{k}{x_1^2}$ for $k > 0$.

a) Utility function (UF) representing the same preferences would be: $x_1^2 x_2 = U(x_1, x_2)$, solving for k.

b) UF representing non-monotone preferences with the same IC would be: $\frac{1}{x_1^2 x_2} = U(x_1, x_2)$. We see if both x_1 and/or x_2 increased, the utility would decrease; not monotone preferences.

c) We get the UF $U(x_1, x_2) = x_2 - x_1^2$. But if we check $(0, 0)$ and $(1, 1)$, both give utility k, but $(1, 1)$ should be strictly preferred ("more is better") in the face of monotone preferences.

Question 2

A worker consumes one good and has a preference for leisure. She maximizes the UF $U(x, L) = xL$. This worker can choose any $L \in [0, 1]$, and receives income $w(1 - L)$; w is the wage rate, and p is the price of the consumption good. In addition to her wage income, the worker has a fixed income of $y \geq 0$.

a) The utility maximization problem for this consumer with $x \geq 0$ and $L \in [0, 1]$ is:

$$\max_{x,L} U(x, L) = xL$$

$$subject\ to\ y + w \geq xp + wL$$

b) Moving forward, under the assumption of monotone preferences, I will assume no interior solution, so the budget constraint used is binding $=$. Since the preferences are Cobb-Douglas, there will be optimal solution under convexity. The Marshallian demands for $x(p, w, y)$ and $L(p, w, y)$ can be solved using the Lagrangian function:

$$Lagrangian\ function:$$
$$\mathcal{L} = xL + \lambda[y + w - xp - wL] \qquad (1)$$
$$FOC\ (assuming\ no\ corner\ solutions):$$
$$\frac{\partial}{\partial x} = L - \lambda p = 0 \qquad (2)$$
$$\frac{\partial}{\partial L} = L - \lambda w = 0 \qquad (3)$$
$$\frac{\partial}{\partial \lambda} = y + w - xp - wL = 0 \qquad (4)$$

From (2) and (3), $\frac{L}{x} = \frac{\lambda p}{\lambda w}$ gives $\frac{L}{x} = \frac{p}{w}$.

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Which gives $wL = px$. Substituting this into (4), we can find the Marshallian demand as:

$$0 = y + w - wL - wL$$
$$0 = y + w - 2wL$$

Solving, we will get

$$x(p, w, y) = \frac{y + w}{2p}$$

$$L(p, w, y) = \frac{y + w}{2w}$$

c) The indirect utility V(p,w,y) can be found by substituting $x(p, w, y) = \frac{y+w}{2p}$ as x and $L(p, w, y) = \frac{y+w}{2w}$ as L in the original $U(x, L) = xL$ function:
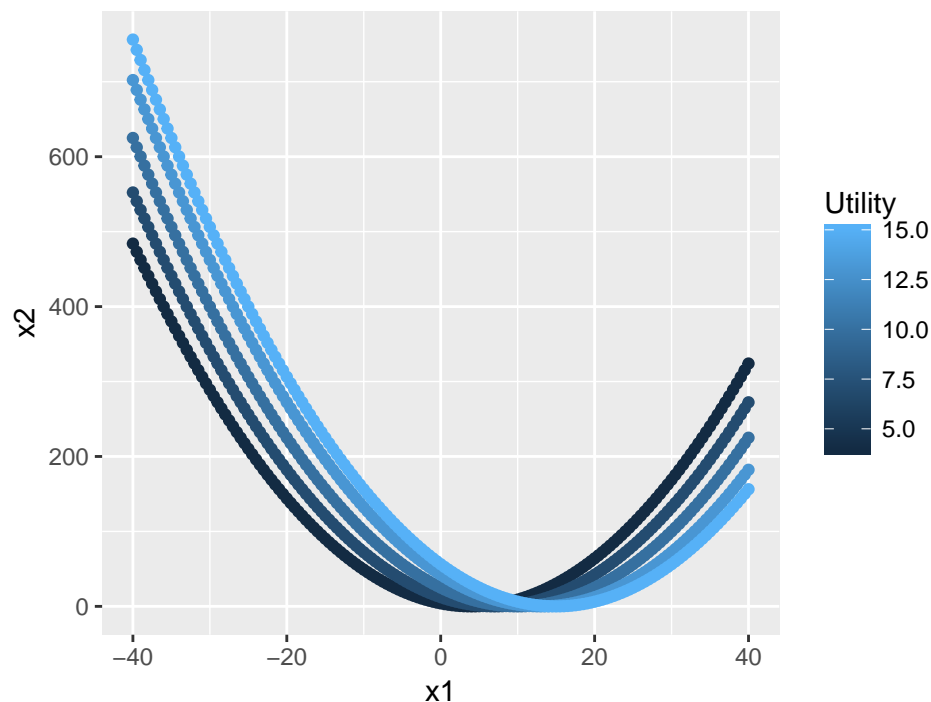
$$V(p, w, y) = \left(\frac{y + w}{2p}\right)\left(\frac{y + w}{2w}\right) = \frac{(y + w)^2}{4wp}$$

Question 3

A consumer has preferences over two goods represented by $U(x_1, x_2) = x_1 + 2\sqrt{x_2}$

a) To sketch this consumer's IC, I will use the package *ggplot2*. Solving for $x_2 = \left(\frac{u - x_1}{2}\right)^2$, I will solve for different x_2 values given $u = 4, 7, 10, 13, 15$. The following is the consumer's indifference curves:

The graphs seem to overlap, but in the x_1, x_2-space they are forming a convex (bowl) shape.

b) Find the Marshallian demand function $x(p, w)$:

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We first set up the utility maximization problem

$$\max_{x_1, x_2} x_1 + 2\sqrt{x_2}$$

$$subject\ to\ p_1 x_1 + p_2 x_2 \leq w$$

Then we use the lagrangian:

$$Lagrangian\ function:$$
$$\mathcal{L} = x_1 + 2\sqrt{x_2} + \lambda[w - p_1 x_1 - p_2 x_1] \qquad (1)$$

$$FOC\ (assuming\ no\ corner\ solutions):$$

$$\frac{\partial}{\partial x_1} = 1 - \lambda p_1 = 0 \qquad (2)$$

$$\frac{\partial}{\partial x_2} = \frac{2}{2\sqrt{x_2}} - \lambda p_2 = 0 \qquad (3)$$

$$\frac{\partial}{\partial \lambda} = w - p_1 x_1 - p_2 x_2 = 0 \qquad (4)$$

From (2) and (3), we get $\sqrt{x_2} = \frac{p_1}{p_2}$ and $x_2 = \frac{p_1^2}{p_2^2}$. When we plug into our original budget constraint, we get
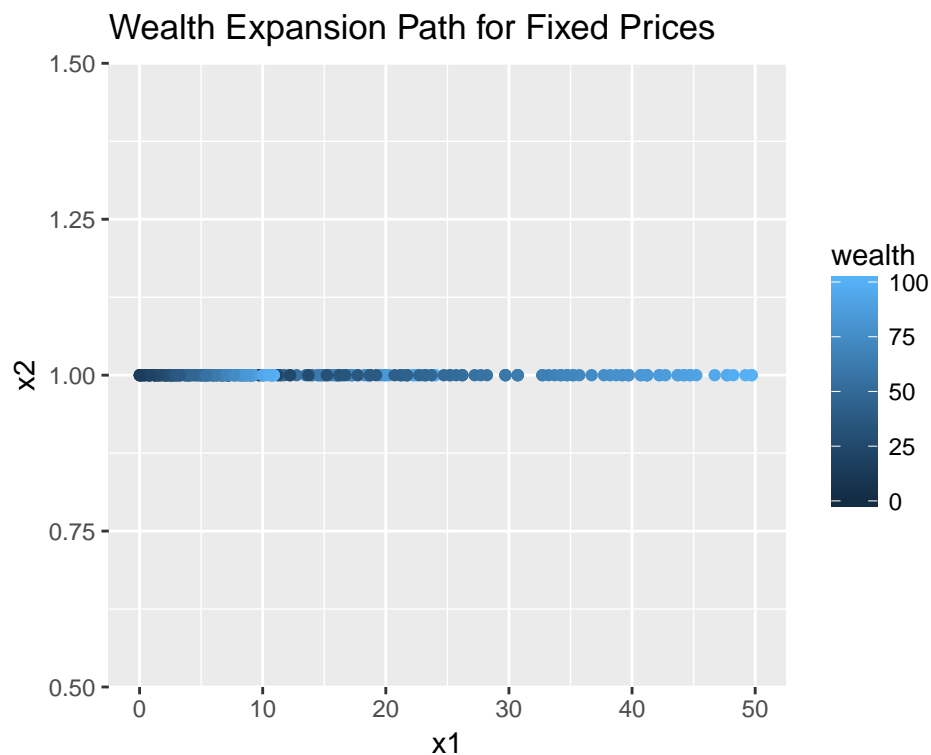
$$w = p_1 x_1 + p_2 \left(\frac{p_1^2}{p_2^2}\right) = p_1 x_1 + \frac{p_1^2}{p_2}.$$

If we solve for x_1 and x_2, we get the Marshallian Demands as

$$x_1(p, w) = \frac{w - \frac{p_1^2}{p_2}}{p_1} = \frac{p_2 w - p_1^2}{p_1 p_2}$$

$$x_2(p, w) = \frac{p_1^2}{p_2^2}$$

c) The wealth expansion path is sketched with fixed prices $\bar{p}$. It is a horizontal line centered at x_2.

Wealth Expansion Path for Fixed Prices

d) To find the indirect UF, substitute x_1 and x_2 from (b) in the original UF: $V(x_1(p,w), x_2(p,w)) = \frac{p_2 w - p_1^2}{p_1 p_2} + \frac{2p_1}{p_2} = \frac{p_2 w + p_1^2}{p_1 p_2}$

e) Using my answer from (d), I can find the Marshallian Demands by using Roy's Identity. To find $x_1(p,w,)$ use

$$x_1(p,w) = \frac{-\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial w}}$$

$$\frac{\partial V}{\partial p_1} = \frac{2p_1}{p_1 p_2} - \frac{p_2 w + p_1^2}{(p_1 p_2)^2} = \frac{2p_1^2 p_2 - p_2^2 w - p_1^2 p_2}{p_1^2 p_2^2}$$

$$\frac{\partial V}{\partial w} = \frac{p_2}{p_1 p_2} = \frac{1}{p_1}$$

$$\frac{-\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial w}} = -\frac{\frac{2p_1^2 p_2 - p_2^2 w - p_1^2 p_2}{p_1^2 p_2^2}}{\frac{1}{p_1}} = -\frac{p_1^2 p_2 - p_2^2 w}{p_1^2 p_2^2}$$

$$= -\frac{p_1^2 - p_2 w}{p_1 p_2} = \frac{p_2 w - p_1^2}{p_1 p_2}$$

To find $x_2(p, w)$ in a similar manner, $x_2(p, w) = \frac{\frac{-\partial V}{\partial p_2}}{\frac{\partial V}{\partial w}}$

$$x_2(p, w) = \frac{\frac{-\partial V}{\partial p_2}}{\frac{\partial V}{\partial w}}$$

$$\frac{\partial V}{\partial p_2} = \frac{1}{p_1} * [\frac{w}{p_2} - \frac{(p_2 w + p_1^2)}{p_2^2}] = \frac{wp_2^2 - p_2 w - p_1^2}{p_1 p_2^2} = -\frac{p_1}{p_2^2}$$

$$\frac{\partial V}{\partial w} = \frac{p_2}{p_1 p_2} = \frac{1}{p_1}$$

$$\frac{\frac{-\partial V}{\partial p_2}}{\frac{\partial V}{\partial w}} = -\frac{-\frac{p_1}{p_2^2}}{\frac{1}{p_1}} = \frac{p_1^2}{p_2^2}$$

So we get $x_1(p, w) = \frac{p_2 w - p_1^2}{p_1 p_2}$ and $x_2 = \frac{p_1^2}{p_2^2}$, same as from (b).

(f) I will use the original indirect utility function $V(p_1, p_2, w) = \frac{p_1^2 + p_2 w}{p_1 p_2}$ and at the new provided prices, $V(2p_1, \frac{p_2}{2}, w) = \frac{(4p_1^2 + 0.5 p_2 w)}{p_1 p_2}$. I will test the claim that the consumer is worse off in terms of utility as the squared p_1 term in the numerator has a larger impact than the halved p_2 term in the denominator, so that: $V(p_1, p_2, w) > V(2p_1, 0.5 p_2, w)$.

Proof of claim: I will test by contradiction, and assume the consumer is **better off**. Since the denominator is the same, I will compare the numerators:

$$p_1^2 + p_2 w \leq 4p_1^2 + 0.5 p_2 w$$
$$\frac{p_2 w}{2} \leq 3p_1^2$$
$$w \leq \frac{6p_1^2}{p_2}$$

Since we are given $w > \frac{8p_1^2}{p_2}$, we see by contradiction $w \not\leq \frac{6p_1^2}{p_2}$, and the consumer is **worse off**.

(g) To solve for $e(p, u)$, fix the value of utility as constant and solve for w. Since $V = \bar{u} = \frac{p_2 w + p_1^2}{p_1 p_2}$

$$\bar{u} p_1 p_2 = p_2 w + p_1^2$$
$$w = \frac{\bar{u} p_1 p_2 - p_1^2}{p_2}$$

(h) To find the Hicksian demand function $h_1(p, u)$ and $h_2(p, u)$, we differentiate the expenditure function with respect to the associated prices:

$$\frac{\partial e}{\partial p_1} = \frac{1}{p_2} * (p_2 U - 2p_1) = \frac{p_2 U - 2p_1}{p_2}$$
$$\frac{\partial e}{\partial p_2} = -\frac{1}{p_2^2} * (p_1 p_2 U - p_1^2) + \frac{1}{p_2} * (p_1 U) = \frac{p_1^2}{p_2^2}$$

Question 4

Let us check the definition of revealed preference, which is:

"A bundle x is revealed preferred to $(x' \neq x)$ if x is chosen at (p, w) such that $p * x' \leq w$, which means x' is affordable at (p, w) but not chosen."

    a) Original budget constraint: $x_1(p_1 + t) + x_2p_2 = w$

New budget constraint (lump sum tax): $x_1p_1 + x_2p_2 = w - tx$

I will show that bundle x is affordable when x' could have been bought, and thus $x' \ RP \ x$.

Since we are given $(p'_1, p'_2) = (p_1, p_2)$:

$$
\begin{aligned}
p'_1 x_1 + tx_1 + p'_2 x_2 &\leq w \\
And \ (p'_1, p'_2) &= (p_1, p_2) : \ plug \ in \\
p_1 x_1 + tx_1 + p_2 x_2 &\leq w \\
p_1 x_1 + p_2 x_2 &\leq w - tx_1
\end{aligned}
$$

But we are given that the consumer bought bundle x' at prices (p_1, p_2), so x' is revealed preferred to x.

    b) We know under the assumption of monotonicity, both equations should be binding. If WARP is satisfied, it should alos mean that $x'_1 > x_1$.

Before tax:
$$x_1 p_1 + x_1 t + x_2 p_2 \leq w \rightarrow x_1 p_1 + x_2 p_2 = w - x_1 t$$

After tax:
$$x'_1 p_1 + x'_2 p_2 = w - x_1 t$$

So then:
$$
\begin{aligned}
x'_1 p_1 + x'_2 p_2 &= x_1 p_1 + x_2 p_2 \\
x'_1 p_1 - x_1 p_1 &= x_2 p_2 - x'_2 p_2 \\
p_1(x'_1 - x_1) &= p_2(x_2 - x'_2) \\
\frac{p_1}{p_2}(x'_1 - x_1) &= x_2 - x'_2
\end{aligned}
$$

We know $x'_1 > x_1$ from $x'_1 - x_1 > 0$ by our assumption, and . p_1 and p_2 are given as > 0. So $\frac{p_1}{p_2}(x'_1 - x_1) = x_2 - x'_2 > 0$. Then $x_2 - x'_2 > 0$ gives $x_2 > x'_2$. Though, the change in consumption of x_2 is unknown for certain due to not knowing what type of good x_2 is.

My conclusion is that the consumer consumes more of x_1 and less of x_2 after the lump sum tax.