Microeconomics Problem Set 1

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Question 1

A consumer has preferences over bundles of two goods with indifference curves defined by $x_2 = \frac{k}{x_1^2}$ for $k > 0$.

a) Utility function (UF) representing the same preferences would be: $x_1^2 x_2 = U(x_1, x_2)$, solving for k.

b) UF representing non-monotone preferences with the same IC would be: $\frac{1}{x_1^2 x_2} = U(x_1, x_2)$. We see if both x_1 and/or x_2 increased, the utility would decrease; not monotone preferences.

c) We get the UF $U(x_1, x_2) = x_2 - x_1^2$. But if we check $(0,0)$ and $(1,1)$, both give utility k, but $(1,1)$ should be strictly preferred ("more is better") in the face of monotone preferences.

Question 2

A worker consumes one good and has a preference for leisure. She maximizes the UF $U(x, L) = xL$. This worker can choose any $L \in [0,1]$, and receives income $w(1-L)$; w is the wage rate, and p is the price of the consumption good. In addition to her wage income, the worker has a fixed income of $y \geq 0$.

a) The utility maximization problem for this consumer with $x \geq 0$ and $L \in [0,1]$ is:

$$\max_{x,L} U(x, L) = xL$$
$$subject\ to\ y + w \geq xp + wL$$

b) Moving forward, under the assumption of monotone preferences, I will assume no interior solution, so the budget constraint used is binding =. Since the preferences are Cobb-Douglas, there will be optimal solution under convexity. The Marshallian demands for $x(p, w, y)$ and $L(p, w, y)$ can be solved using the Lagrangian function:

$$Lagrangian\ function:$$
$$\mathcal{L} = xL + \lambda[y + w - xp - wL] \quad (1)$$
$$FOC\ (assuming\ no\ corner\ solutions):$$
$$\frac{\partial}{\partial x} = L - \lambda p = 0 \quad (2)$$
$$\frac{\partial}{\partial L} = L - \lambda w = 0 \quad (3)$$
$$\frac{\partial}{\partial \lambda} = y + w - xp - wL = 0 \quad (4)$$

From (2) and (3), $\frac{L}{x} = \frac{\lambda p}{\lambda w}$ gives $\frac{L}{x} = \frac{p}{w}$.

Which gives $wL = px$. Substituting this into (4), we can find the Marshallian demand as:

$$0 = y + w - wL - wL$$
$$0 = y + w - 2wL$$

Solving, we will get

$$x(p, w, y) = \frac{y + w}{2p}$$

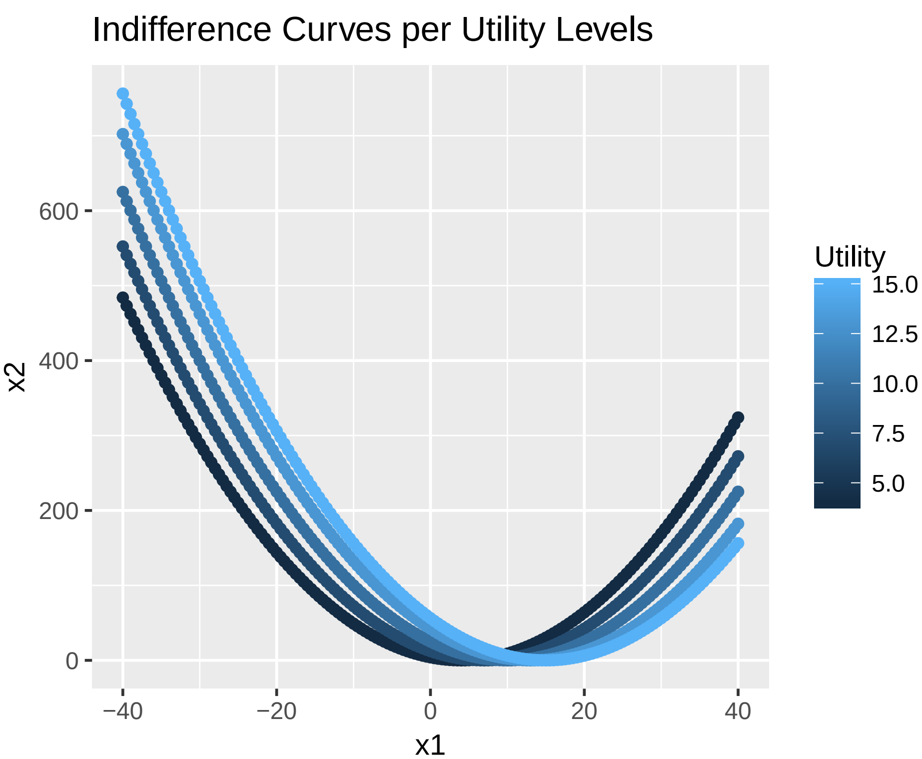
$$L(p, w, y) = \frac{y + w}{2w}$$

c) The indirect utility V(p,w,y) can be found by substituting $x(p, w, y) = \frac{y+w}{2p}$ as x and $L(p, w, y) = \frac{y+w}{2w}$ as L in the original $U(x, L) = xL$ function:

$$V(p, w, y) = (\frac{y + w}{2p})(\frac{y + w}{2w}) = \frac{(y + w)^2}{4wp}$$

Question 3

A consumer has preferences over two goods represented by $U(x_1, x_2) = x_1 + 2\sqrt{x_2}$ a) To sketch this consumer's IC, I will use the package *ggplot2*. Solving for $x_2 = (\frac{u-x_1}{2})^2$, I will solve for different x_2 values given $u = 4, 7, 10, 13, 15$. The following is the consumer's indifference curves:



The graphs seem to overlap, but in the x_1, x_2-space they are forming a convex (bowl) shape.

b) Find the Marshallian demand function $x(p, w)$:

We first set up the utility maximization problem

$$\max_{x_1,x_2} x_1 + 2\sqrt{x_2}$$
$$subject\,to\,p_1x_1 + p_2x_2 \leq w$$

Then we use the lagrangian:

$$
\begin{aligned}
&Lagrangian\,function: \\
&\mathcal{L} = x_1 + 2\sqrt{x_2} + \lambda[w - p_1x_1 - p_2x_1] \quad (1) \\
&FOC\,(assuming\,no\,corner\,solutions): \\
&\frac{\partial}{\partial x_1} = 1 - \lambda p_1 = 0 \quad (2) \\
&\frac{\partial}{\partial x_2} = \frac{2}{2\sqrt{x_2}} - \lambda p_2 = 0 \quad (3) \\
&\frac{\partial}{\partial \lambda} = w - p_1x_1 - p_2x_2 = 0 \quad (4)
\end{aligned}
$$

From (2) and (3), we get $\sqrt{x_2} = \frac{p_1}{p_2}$ and $x_2 = \frac{p_1^2}{p_2^2}$. When we plug into our original budget constraint, we get

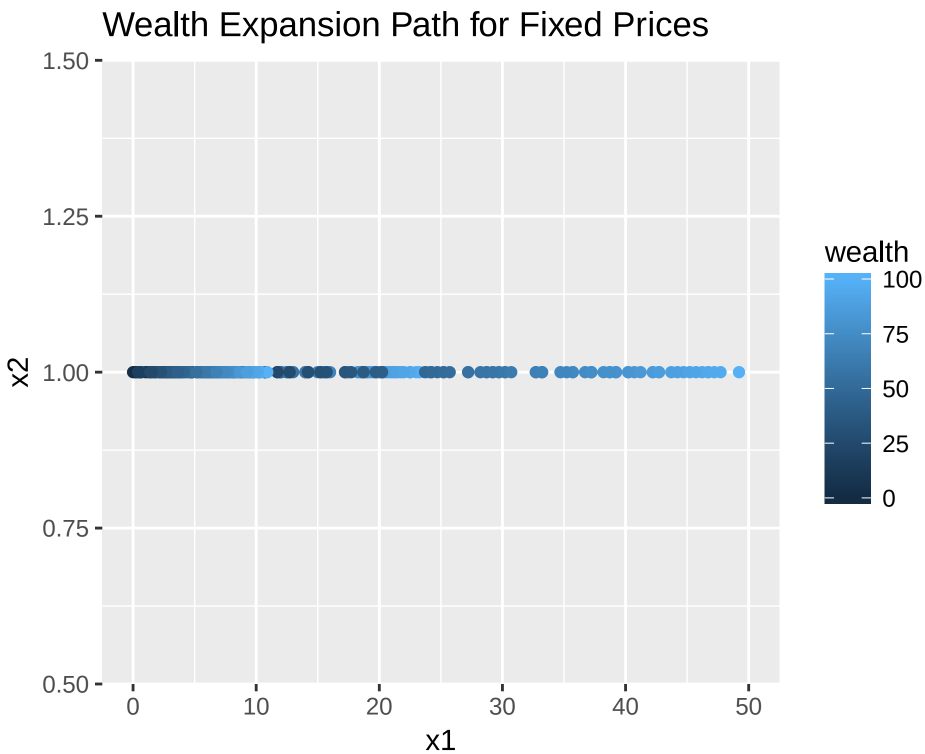
$$w = p_1x_1 + p_2(\frac{p_1^2}{p_2^2}) = p_1x_1 + \frac{p_1^2}{p_2}.$$

If we solve for x_1 and x_2, we get the Marshallian Demands as

$$x_1(p,w) = \frac{w - \frac{p_1^2}{p_2}}{p_1} = \frac{p_2w - p_1^2}{p_1p_2}$$

$$x_2(p,w) = \frac{p_1^2}{p_2^2}$$

c) The wealth expansion path is sketched with fixed prices $\bar{p}$. It is a horizontal line centered at x_2.



d) To find the indirect UF, substitute x_1 and x_2 from (b) in the original UF: $V(x_1(p,w), x_2(p,w)) = \frac{p_2 w - p_1^2}{p_1 p_2} + \frac{2p_1}{p_2} = \frac{p_2 w + p_1^2}{p_1 p_2}$

e) Using my answer from (d), I can find the Marshallian Demands by using Roy's Identity. To find $x_1(p, w,)$ use

$$\begin{aligned}
x_1(p,w) &= \frac{\frac{-\partial V}{\partial p_1}}{\frac{\partial V}{\partial w}} \\
\frac{\partial V}{\partial p_1} &= \frac{2p_1}{p_1 p_2} - \frac{p_2 w + p_1^2}{(p_1 p_2)^2} = \frac{2p_1^2 p_2 - p_2^2 w - p_1^2 p_2}{p_1^2 p_2^2} \\
\frac{\partial V}{\partial w} &= \frac{p_2}{p_1 p_2} = \frac{1}{p_1} \\
\frac{\frac{-\partial V}{\partial p_1}}{\frac{\partial V}{\partial w}} &= -\frac{\frac{2p_1^2 p_2 - p_2^2 w - p_1^2 p_2}{p_1^2 p_2^2}}{\frac{1}{p_1}} = -\frac{p_1^2 p_2 - p_2^2 w}{p_1^2 p_2^2} \\
&= -\frac{p_1^2 - p_2 w}{p_1 p_2} = \frac{p_2 w - p_1^2}{p_1 p_2}
\end{aligned}$$

To find $x_2(p,w)$ in a similar manner, $x_2(p,w) = \frac{\frac{-\partial V}{\partial p_2}}{\frac{\partial V}{\partial w}}$

$$\begin{aligned}
x_2(p,w) =& \frac{\frac{-\partial V}{\partial p_2}}{\frac{\partial V}{\partial w}} \\
\frac{\partial V}{\partial p_2} =& \frac{1}{p_1} * [\frac{w}{p_2} - \frac{(p_2 w + p_1^2)}{p_2^2}] = \frac{w p_2^2 - p_2 w - p_1^2}{p_1 p_2^2} = -\frac{p_1}{p_2^2} \\
\frac{\partial V}{\partial w} =& \frac{p_2}{p_1 p_2} = \frac{1}{p_1} \\
\frac{\frac{-\partial V}{\partial p_2}}{\frac{\partial V}{\partial w}} =& - \frac{-\frac{p_1}{p_2^2}}{\frac{1}{p_1}} = \frac{p_1^2}{p_2^2}
\end{aligned}$$

So we get $x_1(p,w) = \frac{p_2 w - p_1^2}{p_1 p_2}$ and $x_2 = \frac{p_1^2}{p_2^2}$, same as from (b).

(f) I will use the original indirect utility function $V(p_1, p_2, w) = \frac{p_1^2 + p_2 w}{p_1 p_2}$ and at the new provided prices, $V(2p_1, \frac{p_2}{2}, w) = \frac{(4p_1^2 + 0.5 p_2 w)}{p_1 p_2}$. I will test the claim that the consumer is worse off in terms of utility as the squared p_1 term in the numerator has a larger impact than the halved p_2 term in the denominator, so that: $V(p_1, p_2, w) > V(2p_1, 0.5p_2, w)$.

Proof of claim: I will test by contradiction, and assume the consumer is **better off**. Since the denominator is the same, I will compare the numerators:

$$\begin{aligned}
p_1^2 + p_2 w \leq& 4p_1^2 + 0.5 p_2 w \\
\frac{p_2 w}{2} \leq& 3p_1^2 \\
w \leq& \frac{6p_1^2}{p_2}
\end{aligned}$$

Since we are given $w > \frac{8p_1^2}{p_2}$, we see by contradiction $w \not\leq \frac{6p_1^2}{p_2}$, and the consumer is **worse off**.

(g) To solve for $e(p,u)$, fix the value of utility as constant and solve for w. Since $V = \bar{u} = \frac{p_2 w + p_1^2}{p_1 p_2}$

$$\begin{aligned}
\bar{u} p_1 p_2 =& p_2 w + p_1^2 \\
w =& \frac{\bar{u} p_1 p_2 - p_1^2}{p_2}
\end{aligned}$$

(h) To find the Hicksian demand function $h_1(p,u)$ and $h_2(p,u)$, we differentiate the expenditure function with respect to the associated prices:

$$\begin{aligned}
\frac{\partial e}{\partial p_1} =& \frac{1}{p_2} * (p_2 U - 2p_1) = \frac{p_2 U - 2p_1}{p_2} \\
\frac{\partial e}{\partial p_2} =& -\frac{1}{p_2^2} * (p_1 p_2 U - p_1^2) + \frac{1}{p_2} * (p_1 U) = \frac{p_1^2}{p_2^2}
\end{aligned}$$

Question 4

Let us check the definition of revealed preference, which is:

"A bundle x is revealed preferred to $(x' \neq x)$ if x is chosen at (p, w) such that $p * x' \leq w$, which means x' is affordable at (p, w) but not chosen."

a) Original budget constraint: $x_1(p_1 + t) + x_2 p_2 = w$

New budget constraint (lump sum tax): $x_1 p_1 + x_2 p_2 = w - tx$

I will show that bundle x is affordable when x' could have been bought, and thus x' RP x.

Since we are given $(p_1', p_2') = (p_1, p_2)$:

$$
\begin{aligned}
p_1' x_1 + t x_1 + p_2' x_2 \leq& w \\
And\ (p_1', p_2') =& (p_1, p_2) :\ plug\ in \\
p_1 x_1 + t x_1 + p_2 x_2 \leq& w \\
p_1 x_1 + p_2 x_2 \leq w - t x_1&
\end{aligned}
$$

But we are given that the consumer bought bundle x' at prices (p_1, p_2), so x' is revealed preferred to x.

b) We know under the assumption of monotonicity, both equations should be binding. If WARP is satisfied, it should alos mean that $x_1' > x_1$.

Before tax:

$$x_1 p_1 + x_1 t + x_2 p_2 \leq w \rightarrow x_1 p_1 + x_2 p_2 = w - x_1 t$$

After tax:

$$x_1' p_1 + x_2' p_2 = w - x_1 t$$

So then:

$$
\begin{aligned}
x_1' p_1 + x_2' p_2 &= x_1 p_1 + x_2 p_2 \\
x_1' p_1 - x_1 p_1 &= x_2 p_2 - x_2' p_2 \\
p_1(x_1' - x_1) &= p_2(x_2 - x_2') \\
\frac{p_1}{p_2}(x_1' - x_1) &= x_2 - x_2'
\end{aligned}
$$

We know $x_1' > x_1$ from $x_1' - x_1 > 0$ by our assumption, and . p_1 and p_2 are given as > 0. So $\frac{p_1}{p_2}(x_1' - x_1) = x_2 - x_2' > 0$. Then $x_2 - x_2' > 0$ gives $x_2 > x_2'$. Though, the change in consumption of x_2 is unknown for certain due to not knowing what type of good x_2 is.

My conclusion is that the consumer consumes more of x_1 and less of x_2 after the lump sum tax.