Cross-Layer Energy and Delay Optimization in Small-Scale Sensor Networks

Shuguang Cui, Ritesh Madan, Andrea J. Goldsmith, and Sanjay Lall

Abstract— The general joint design of the physical, MAC, and routing layers to minimize network energy consumption is complex and hard to solve. Heuristics to compute approximate solutions and high-complexity algorithms to compute exact solutions have been previously proposed. In this paper, we focus on synchronous small-scale networks with interference-free link scheduling and practical MQAM link transmission schemes. We show that the cross-layer optimization problems can be closely approximated by convex optimization problems that can be efficiently solved. There are two main contributions of this paper. First of all, we minimize the total network energy that includes both transmission and circuit energy consumptions, where we explore the tradeoff between the two energy elements. Specifically, we use interference-free TDMA as the medium access control scheme. We optimize the routing flow, TDMA slot assignment, and MQAM modulation rate and power on each link. The results demonstrate that the minimum energy transmission scheme is a combination of multihop and single-hop transmissions for general networks; including circuit energy favors transmission schemes with fewer hops. Secondly, based on the solved optimal transmission scheme, we quantify the best trade-off curve between delay and energy consumption, where we derive a scheduling algorithm to minimize the worst-case packet delay.

Index Terms— Cross-layer, energy efficiency, routing, link scheduling, link adaptation, convex programming, minimum delay, TDMA.

I. INTRODUCTION

IN a typical sensor network, sensors are powered by small batteries that cannot be replaced. Hence, sensor nodes can only transmit a finite number of bits until they run out of energy. Consequently, reducing the energy consumption per bit for end-to-end transmissions is an important design objective for such networks. Since all layers of the protocol stack affect the energy consumption per bit for data transmission from source to destination, energy minimization requires a joint design across all layers [1]. Moreover, modeling the energy consumption in the circuit is important since a significant amount of energy overhead can be dissipated in the circuit [2].

Much cross-layer optimization work has focused on throughput maximization. For example, joint routing, power control, and scheduling for Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) networks are considered in [3]-[5] to maximize the network throughput. Although energy minimization is not the main objective of these papers, similar cross-layer methodology can be applied to find energy efficient solutions. Joint scheduling and power control to reduce energy consumption and increase single-hop throughput are considered in [6]. Cross-layer design based on computation of optimal power control, link schedule, and routing flow is described in [7]. The aim of this paper is to minimize the average transmission power over an infinite horizon. Energy efficient power control and scheduling, with no rate adaptation on links, for QoS provisioning are considered in [8]. However, it is shown in these papers that the general cross-layer problem formulation of optimizing the physical, medium access control, and routing layers to minimize total energy consumption is extremely complex and hard to solve. Heuristics can be used to compute approximate solutions (see, for example, [9]), while high-complexity algorithms (for example, column generation in [10]) are required to compute exact solution or solutions with bounded accuracy.

In addition, hardware power consumption is usually neglected in the traditional cross-layer optimization framework. This is a reasonable approximation for long range applications where the average transmission distance is usually on the order of several hundred meters such that the transmission power dominates the circuit processing power. However, in sensor networks, the average distance can be as short as several meters [2]. As a result, the circuit processing power becomes comparable to or even dominates the transmission power [2]. Therefore, for an energy efficient network design, the transmission power and the circuit processing power need to be jointly considered in the cross-layer optimization model. Such joint optimization considering hardware power consumption is investigated in [2] and [11]-[14], where the authors consider a joint design between the link layer and the silicon layer. By modeling power consumption in the underlying circuits, optimal modulation schemes are derived to minimize the total energy consumption. However, these results do not optimize the medium access control (MAC) or routing layers. In [15], the delay-energy tradeoff is analyzed for a data collecting

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sensor network, where the energy consumption includes both the transmission energy and the circuit processing energy. However, the proposed optimization model only applies to tree topologies where the SINK node is the root.

In this paper, we formulate and solve a cross-layer energy minimization problem. Our problem formulation is restricted only in the sense that we consider interference-free scheduling in a small-scale network where synchronization among nodes is achievable. Specifically, we focus on the joint design of the routing, MAC, and physical layers to minimize the overall network energy consumption that includes both the transmission and the circuit processing energy consumption. Instead of seeking heuristics or high complexity algorithms for cross-layer design of arbitrary networks, we focus on small-scale sensor networks that only involve several dozens of nodes, which are closely located. For such networks, interference-free TDMA is a good choice for the MAC layer because it is feasible in a relatively small network and is energy efficient given that nodes can be perfectly scheduled to be in sleep and active modes to save energy, where we assume a central controller (the SINK node) and perfect synchronization among nodes [1]. Due to the simplification at the MAC layer, the cross-layer design problem can be relaxed to a convex one, for which we have known techniques to solve efficiently [16]. The relative performance degradation caused by the relaxation is low. In addition, we propose an algorithm that schedules transmissions to minimize the delay for a given routing, MAC, and physical layer transmission scheme, and we then characterize the delay-energy tradeoff using convex optimization techniques. In comparison to the model proposed in [15], which only applies to tree topologies, the model proposed in this paper applies to general topologies with one SINK node and no loops[2] (for which the tree topology is a special case). In addition, the model proposed in this paper is based on convex optimization techniques, which approaches the solution from a different angle.

In summary, there are two main contributions of this paper. First of all, we propose a convex optimization framework based on practical MQAM modulation schemes to minimize the total network energy that includes both transmission and circuit energy consumption across the network, where we explore the tradeoff between the two energy elements. Secondly, based on the solved optimal transmission scheme, we quantify the best trade-off curve between delay and energy consumption, where we derive a scheduling algorithm to minimize the worst-case packet delay in the network.

The rest of the paper is organized as follows. In Section II, we describe the system model, formulate the optimization problem and give an overview of the computational algorithms that can be used to solve these problems. Section III considers several different optimization problems that are special cases of the generic problem and Section IV gives numerical results. Section V describes an algorithm to schedule transmissions to minimize the delay and characterizes the delay-energy tradeoff. Section VI summarizes our conclusions.

II. CROSS-LAYER OPTIMIZATION

A. System Model

Network

We consider a stationary sensor network or a network that varies slowly such that we can ignore the dynamics of node movement. We denote the distance between node i and node j as d_{ij}. We assume an additive white Gaussian noise (AWGN) channel with a κth- power path-loss for each link $i \to j$. The received power $P_r(i, j)$ at node j is thus given by $P_r(i, j) = P_t(i, j)/(G_0 d_{ij}^\kappa)$, where $P_t(i, j)$ is the transmit power and G_0 is the loss factor at $d = 1$ m. Note that G_0 depends on the antenna gain, carrier frequency, link margin and other system parameters [2].

Physical Layer

We assume that uncoded M-ary Quadrature Amplitude Modulation (MQAM) is used. The constellation size M_{ij} assigned to link $i \to j$ is given as $M_{ij} = 2^{b_{ij}}$, where b_{ij} is the number of bits per symbol at which node i transmits to node j. We assume that Nyquist pulses are used and hence the MQAM symbol rate is approximately equal to the transmission bandwidth B (see, for example, [13]). Also, a bit error rate (BER) of P_b is maintained.

MAC Layer

We consider a slotted synchronous TDMA MAC scheme, where each frame of length T is divided into multiple slots of length Δ. Thus the link schedule is periodic, i.e., if link $i \to j$ transmits in slot n, it also transmits in slot $n+T/\Delta$. Also, only one link is assumed to transmit in each slot. Thus, within each TDMA frame of length T, if link $i \to j$ is allocated n_{ij} slots it transmits for time $t_{ij} = n_{ij}\Delta$. Note that each link may be allocated a different number of time slots. Obviously, we have $\sum_i \sum_j t_{ij} \leq T$. Again, the feasibility of such a synchronous TDMA scheme is enabled by our assumption on small-scale networks.

Traffic Flow

In this paper, to keep the notation simple, we consider a single commodity flow where each node can generate information that needs to be communicated to a single SINK node. The corresponding scenario is illustrated in Fig. 1. We note here that our methods are general and can be extended for multi-commodity flow problems (*i.e.*, with multiple SINK nodes) as well. There are N sensor nodes in the network; without loss of generality, we denote the SINK node as the Nth node. Each node generates data at the rate of R_i bits per second (bps), $i = 1, \cdots, N$. Also, each node can relay the data of other nodes. For the SINK node, we have $R_N = -\sum_{i=1}^{N-1} R_i$, where the negative sign means that the SINK node has only incoming traffic.

We assume that the network information flow is scheduled in a deterministic way. As such, no random delay due to random packet transmission is considered in this paper. Specifically, the bits are available for transmission at each node at

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[1]It is worth noting that most of currently operating sensor networks are of small scales (with usually less than one hundred nodes), where centralized synchronous TDMA is the most widely adopted medium access protocol. Such application examples include temperature monitoring networks in large power plants, for which the readers can obtain more information from one of the largest commercial sensor network companies: Dust Networks, Inc. (http://www.dustnetworks.com/), which is a spin-off from the Berkeley smart-dust project.

[2]A loop means a transmission route that originates and terminates at the same node. A network without such routes is called a loop-free network.
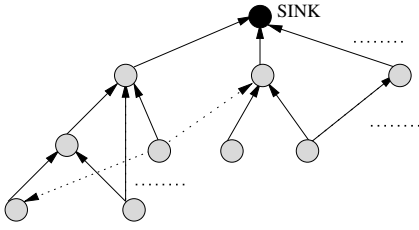
Fig. 1. General topology for data collection in a sensor network.

times $T, 2T, \ldots$, i.e., each node i has $R_i T$ bits available for transmission at the end of each frame of length T. If data is transmitted at rate b_{ij} bits per symbol over link $i \to j$ for time t_{ij}, then the number of bits transmitted from i to j is $W_{ij} = B b_{ij} t_{ij}$. We will allow W_{ij} to take real values. After we obtain the optimal values (which are likely to be of real values), we could round them to the nearest integers if no bit splitting (fractional bit) is allowed in the routing protocol. This is a good approximation if the symbol time is much smaller than the slot length Δ. The approximation is very standard and has been used in, for example, [9], [17], [18]. The flow conservation equations are satisfied in every frame, i.e., the difference between the number of outgoing bits and the number of incoming bits is equal to the number of bits generated by the node locally.

$$\sum_{j=1}^{N} (W_{ij} - W_{ji}) = R_i T. \qquad (1)$$

Energy Consumption
For each link, we use the same circuit model as in [2] for the transmit and receive signal paths. We neglect the energy consumption of baseband signal processing blocks (*e.g.*, source coding, pulse-shaping, and digital modulation).

We assume that each node operates in one of two modes: active mode and sleep mode. When a node transmits/receives data to/from another node, it is in the active mode. When a node is not transmitting or receiving data, it turns off all circuits to enter the sleep mode; this saves energy. When switching from the sleep mode to the active mode, there is a transient mode that is mainly caused by the recovery of the phase-lock loop. However, the transient mode duration is usually on the order of micro-seconds [2] while the slot time considered here is much larger. As such, the consumed energy in transient modes is relatively small. Each node can turn on the circuitry slightly before the start of the assigned time slot to compensate for the delay caused by the transient mode. Hence, we do not model the transient mode in this paper. In the sleep mode the power consumption is dominated by the leakage current of the switching transistors. Since for analog circuits the leakage power consumption is usually much smaller than the power consumption in the active mode, leakage power is neglected in the total energy consumption. We assume node synchronization; this allows a node to wake up only when it is transmitting or receiving data. This is more energy-efficient than systems where nodes are always in the active or listen modes, since listening circuitry may consume a significant amount of energy [19], [20].

For uncoded MQAM, as discussed in [2], the total energy consumption (that includes both the average transmission energy[3] and the circuit energy) to transmit W_{ij} bits on link $i \to j$ in time t_{ij}, with a target BER P_b, can be approximated (upper bounded) by the function [2]

$$\epsilon(W_{ij}, t_{ij}) = x_{ij} t_{ij} \left(2^{\frac{W_{ij}}{B t_{ij}}} - 1 \right) + y_{ij} t_{ij}, \qquad (2)$$

where the first term is the transmission energy and the second term is the total transceiver circuit processing energy. The coefficients x_{ij} and y_{ij} are system constants and are given by $x_{ij} = 2(1+\alpha) B N_f N_0 G_0 d_{ij}^\kappa \ln \frac{2}{P_b}$ and $y_{ij} = (P_{ct}^i + P_{cr}^j)$, where α is a constant defined by the power amplifier efficiency [2], N_f is the receiver noise figure, $\frac{N_0}{2}$ is the thermal noise spectral density, P_{ct}^i is the total transmitter circuit power consumption for node i in the active mode excluding the power consumed in the power amplifier, P_{cr}^j is the total receiver circuit power consumption for node j in the active mode, and B is the transmission bandwidth. The following lemma follows easily.

Lemma 2.1: The function $\epsilon(W, t)$ is convex in W, t for $W, t \geq 0$.

Proof: See Appendix A. ∎

It is well known [21] that Quadrature Phase Shift Keying (QPSK) requires the same transmission energy per bit as Binary Phase Shift Keying (BPSK) while satisfying the same BER requirement. However, to transmit a certain number of bits, QPSK requires only half the transmission time as BPSK so its circuit energy consumption can be reduced by half [2]. Therefore, the minimum candidate bit rate (per symbol) for MQAM can be predetermined as $b_{min} = 2$ in order to obtain an energy-efficient solution. Note that we neglected the energy required to maintain network synchronization, which is an interesting topic on its own for future work.

The maximum allowable bit rate is constrained by the maximum available power P_{max} at the transmitter, specifically

$$b_{ij} \leq C_{ij} = \log_2 \left(1 + \frac{P_{max} - P_{ct}^i}{x_{ij}} \right), \qquad (3)$$

where we assume that P_{max} is the same across all the nodes.

B. Cross-layer Design Among the Physical, MAC, and Routing Layers

Cross-layer design is a joint design optimization across all or several layers in the protocol stack under given resource constraints, and it overcomes the drawbacks of layered design to improve the network performance. Cross-layer design can include information exchange between different layers (not necessarily neighboring layers), adaptivity at each layer to this information, and diversity built into each layer to insure robustness [1], [13]. For example, the physical layer can deploy adaptive modulation and coding to exploit or compensate for the time-varying wireless channel. Then, this adaptivity at the physical layer can be used by higher layers to achieve better performance: The MAC layer can assign a longer channel usage time to links with low-rate modulation

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[3]The transmission energy stands for the average transmission energy throughout the paper.

schemes to meet the throughput or energy constraint; the routing layer can reroute traffic to links supporting high-rate modulation schemes to minimize congestion; and the application layer can use multi-description codes to leverage the diversity of different routes. Significant performance gain can be achieved by these interactions between different layers [1], [13]. However, to maximize the achievable gain, the dynamics among different layers should be analytically modeled, which is the main goal of this paper regarding a particular network application described as follows.

C. Optimization Problem

The goal of the proposed approach is to minimize the total average power consumption across all nodes[4]. This is equivalent to minimizing the total energy consumption at all nodes in a frame of duration T. Note that in sensor networks, where multiple sensors cooperate to perform the same task, per-node fairness is less important than that in general ad hoc networks. The problem of optimizing the different layers to minimize power consumption can be stated as the following optimization problem.

$$
\begin{aligned}
\text{min.} \quad & \sum_{i=1}^{N-1} \sum_{j=1}^{N} \epsilon(W_{ij}, t_{ij}) \\
\text{s. t.} \quad & \sum_{i=1}^{N-1} \sum_{j=1}^{N} t_{ij} \leq T \\
& \sum_{j=1}^{N} (W_{ij} - W_{ji}) = R_i T, \quad i = 1, \cdots, N \\
& 2B t_{ij} \leq W_{ij} \leq B C_{ij} t_{ij}, \quad t_{ij} \in \{0, \Delta, \ldots\} \\
& i = 1, \ldots, N-1, \quad j = 1, \ldots, N
\end{aligned} \quad (4)
$$

where the variables are t_{ij}'s and W_{ij}'s. The first constraint is the TDMA constraint, the second constraint is the flow conservation constraint, the first $\leq$ in the third constraint is the minimum rate constraint (due to the fact that QPSK is more efficient than BPSK), and the second $\leq$ in the third constraint is the maximum rate constraint (due to maximum power constraint). The fourth constraint models the fact each link is allocated an integer number of slots. Note that we obtain the constellation size on link $i \rightarrow j$ as $b_{ij} = \left\lceil \frac{W_{ij}}{B t_{ij}} \right\rceil$. For the examples computed in this paper, the increase in energy consumption by using integer b_{ij} compared to that for real $\frac{W_{ij}}{B t_{ij}}$ is less than 1%. In general, more accurate solutions can be computed using branch and bound methods (see, for example, [22]) to find the optimal b_{ij}.

Since the objective function is convex, if we relax the last constraint, the above optimization problem is a convex optimization problem. Hence, it can be solved efficiently using

interior point methods (see, for example, [16]). Specifically, we solve the following relaxed problem.

$$
\begin{aligned}
\text{min.} \quad & \sum_{i=1}^{N-1} \sum_{j=1}^{N} \epsilon(W_{ij}, t_{ij}) \\
\text{s. t.} \quad & \sum_{i=1}^{N-1} \sum_{j=1}^{N} t_{ij} \leq T - N(N-1)\Delta \\
& \sum_{j=1}^{N} (W_{ij} - W_{ji}) = R_i T, \quad i = 1, \cdots, N \\
& 2B t_{ij} \leq W_{ij} \leq B C_{ij} t_{ij} \\
& i = 1, \cdots, N-1, \quad j = 1, \cdots, N
\end{aligned} \quad (5)
$$

From the optimal solution $\{t_{ij}^*, W_{ij}^*\}$ (and accordingly $b_{ij}^* = \frac{W_{ij}^*}{B t_{ij}^*}$) for the above relaxed problem, we can obtain a feasible solution to problem (4) as follows.

(a) $t_{ij} = \Delta \left\lceil \frac{t_{ij}^*}{\Delta} \right\rceil$. Note that the TDMA constraint in the relaxed problem has been modified such that the t_{ij}'s thus obtained are feasible.

(b) $b_{ij} = \left\lceil b_{ij}^* \right\rceil$.

(c) $W_{ij} = t_{ij} b_{ij} B$.

For a fixed N, as $\frac{\Delta}{T} \rightarrow 0$, the energy consumption for the suboptimal solution computed using the relaxed problem converges to the energy consumption for the optimal solution to problem (4). In this paper, we will assume that $\frac{\Delta}{T}$ is very small and neglect the effect of slotting. This is only valid when the number of nodes is limited such that accurate synchronization is achievable across the network. Thus we solve the optimization problem (4) without the last constraint in the rest of the paper.

In addition, it is worth clarifying two general cases regarding the rate selection in the physical layer that we will investigate in the following sections. For the first case, we allow optimal rate selection for each link, which leads to strategies with the so-called "Link Adaptation". For the second case, we fix the transmission rate for each link (then only one quantity out of W_{ij} and t_{ij} remains as the design variable), which leads to strategies with the so-called "Fixed Links". Please be aware of these two terminologies.

III. APPLICATIONS AND SPECIAL CASES

A. MAC Optimization with Link Adaptation

Here, we consider single-hop transmissions where each node directly transmits data to the SINK node (*i.e.*, a star topology). In this case, the energy optimization problem can be simplified to

$$
\begin{aligned}
\text{min.} \quad & \sum_{i=1}^{N-1} \epsilon(W_{iN}, t_{iN}) \\
\text{s. t.} \quad & \sum_{i=1}^{N-1} t_{iN} \leq T \\
& 2B t_{ij} \leq W_{iN} \leq B C_{iN} t_{iN} \\
& i = 1, \cdots, N-1, \quad j = 1, \cdots, N
\end{aligned} \quad (6)
$$

where the variables are the t_{iN}'s.

In this case, we can obtain more tractable results in a special scenario, where we assume a uniform network, i.e., all nodes are identical: $C_{iN} = C$, $x_{iN} = x$, and $y_{iN} = y$ (see Eq. (2)) for all i's. This would be true if all nodes have identical circuits and are located at the same distance from the SINK node. We consider the following two scenarios.

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[4]Our methods can be easily modified to consider the maximization of network lifetime [9], if the network lifetime is defined as the time at which the first node in the network runs out of energy. Actually, if the network lifetime is defined as the average node lifetime that is defined as $\frac{1}{K} \sum_k \frac{E_0}{E\{P_k\}} = \frac{1}{K} \sum_k \frac{E_0 T}{E\{\epsilon_k\}}$ with E_0 the battery energy available to each sensor (we assume that E_0 is the same for all the sensors), $E\{P_k\}$ the average power consumed at node k, and $E\{\epsilon_k\}$ the energy consumed at node k in each period of T, minimizing the total energy consumption as we do in this paper is equivalent to maximizing the lower bound of the network lifetime. This can be proved from the fact that

$$
\frac{1}{K} \sum_k \frac{E_0}{E\{P_k\}} \geq \frac{E_0}{E\{\frac{1}{K} \sum_k P_k\}} = \frac{E_0 T}{E\{\frac{1}{K} \sum_k \epsilon_k\}},
$$

which is based on Jensen's inequality.

(a) *Minimization of the total transmission energy*: The corresponding optimization problem is as follows.

$$\text{min.} \quad \sum_{i=1}^{N-1} x t_{iN} \left(2^{\frac{W_{iN}}{B t_{iN}}} - 1 \right)$$

$$\text{s. t.} \quad \sum_{i=1}^{N-1} t_{iN} \leq T$$
$$2B t_{ij} \leq W_{iN} \leq BC t_{iN}, \quad i = 1, \cdots, N-1 \tag{7}$$

We have the following result about the optimal solution.

Lemma 3.1: If $2 \leq \frac{\sum_{i=1}^{N-1} W_{iN}}{BT} \leq C$, the optimal solution to problem (7) is given by

$$t_{iN}^* = \frac{W_{iN}}{\sum_{j=1}^{N-1} W_{jN}} T.$$

If $\frac{\sum_{i=1}^{N-1} W_{iN}}{BT} < 2$, then $t_{iN}^* = \frac{\sum_{i=1}^{N-1} W_{iN}}{2B}$. Also, note that if $\frac{\sum_{i=1}^{N-1} W_{iN}}{BT} > C$, the problem is infeasible.

Proof: See Appendix B. ∎

From the above result we see that in general the optimal time slot for each link to minimize the total energy is proportional to the relative magnitude of traffic load W_{ij}, which is intuitively correct.

(b) *Minimization of the total transmission and circuit energy:* Note that $x t_{iN} \left(2^{\frac{W_{iN}}{B t_{iN}}} - 1 \right) + y t_{iN}$ is not a monotonically decreasing function of t_{iN}. In this case, we have the following lemma.

Lemma 3.2: Let t^* be the optimal solution to the following problem in variable t.

$$\text{min.} \quad x t \left(2^{\frac{\sum_{i=1}^{N} W_{iN}}{Bt}} - 1 \right) + y t$$

$$\text{s. t.} \quad 2Bt \leq \sum_{i=1}^{N} W_{iN} \leq BCt \tag{8}$$

Then the optimal solution to the problem in (8) is given by $t_{iN}^* = \frac{t^* W_{iN}}{\sum_{j=1}^{N} W_{jN}}$.

Proof: See Appendix C. ∎

In this particular case, we see that the optimal time slot for each link is proportionally divided from a modified time budget t^*, which takes into account the effect of circuit energy.

B. Routing and MAC Optimization with Fixed Links

Instead of treating the link rate b_{ij} as a design variable as in the general formulation (5), we can also investigate a special case where we assume that each link uses the same modulation scheme, i.e., $b_{ij} = b$ for all links. Given the relation $W_{ij} = b B t_{ij}$, if we know W_{ij}, then we know t_{ij}, and vice versa. Therefore, we now only have one variable to define. According to Eq. (2), the total energy to transmit W_{ij} bits over link $i \rightarrow j$ is given by

$$\epsilon(W_{ij}) = \frac{x_{ij} W_{ij}}{Bb} \left(2^b - 1 \right) + \frac{y_{ij} W_{ij}}{Bb}$$

The problem of minimizing the total energy consumption can be written as

$$\text{min.} \quad \sum_{i=1}^{N-1} c_{ij} W_{ij}$$

$$\text{s. t.} \quad \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \frac{W_{ij}}{Bb} \leq T$$
$$\sum_{j=1}^{N} (W_{ij} - W_{ji}) = R_i T, \quad i = 1, \cdots, N \tag{9}$$

TABLE I
SYSTEM PARAMETERS

$f_c = 2.5$ GHz	$B = 10$ KHz
$\kappa = 3.5$	$P_b = 10^{-3}$
$P_{ct}^i = 98.2$ mW	$P_{cr}^i = 112.5$ mW
$P_{max} = 500$ mW	$T = 1$ s
$G_0 = 70$ dB	$\Delta = 0.001T$
$\alpha = 1.9$ (fixed rate)	$\alpha = 7.6$ (adaptive rate)
$N_0/2 = -134$ dBm/Hz	

where the variables are W_{ij}'s, and the constant $c_{ij} = \frac{x_{ij}(2^b - 1) + y_{ij}}{Bb}$. For this linear programming (LP) problem, the number of variables is $O(N^2)$ and the complexity of solving this problem is polynomial in N (see [23]). Note that this is the underlying system model in [9], [17], [24], [25] even though the problem considered in these papers is that of maximizing the time at which the first node in the network runs out of energy (which can again be written as a linear program).

IV. NUMERICAL RESULTS FOR CROSS-LAYER ENERGY MINIMIZATION

In this section by solving the cross-layer optimization problem in different scenarios, we obtain optimal sets of TDMA slot lengths for all the links, which minimize the total energy consumption. However, the optimal solution has no information on the ordering of these time slots within each frame, which will be discussed in the next section where we show that the ordering affects delay peformance.

A. MAC Optimization with Link Adaptation

For the single-hop scenario (the star topology), we now consider the general case where the d_{iN}'s are distinct, which needs to be numerically solved based on Eq. (6). For a 5-node network, the system parameters are specified in Table I and the optimal transmission scheme is shown in Table II. For reference, we also show the results for a uniform TDMA system, where the time allocated to each link is $t_{iN} = \frac{T}{(N-1)}$. For each item u/v in the table, u is the value for the optimized system and v is the value for the reference system. The total energy required for the optimized non-uniform TDMA is 30.9 mJ, while 39.3 mJ is required for the uniform TDMA. The energy saving is about 21%. We next show that optimizing the time slots may lead to another benefit, i.e., enabling more feasible solutions with fixed constraints on frame length and constellation size. Specifically, if we change the frame length to $T = 0.1$ s, the time slot length for each node becomes $\frac{T}{4} = 0.025$ s under the uniform TDMA scheme. This requires a minimum constellation size of $b = \frac{\nu W_{iN}}{0.025B} = 8$ to send W_{iN} bits from node i to the SINK in time T. Since the maximum allowable constellation size for node 4 is $C = 6$ (restricted by the maximum available power), the uniform TDMA scheme is infeasible. However, the optimized TDMA is feasible; it allocates more transmission time to node 4. The allocated transmission times are $\mathbf{t} = [0.014, \ 0.02, \ 0.025, \ 0.033]$ s.

TABLE II
OPTIMIZED TDMA (SAME W_{iN}'S) V.S. UNIFORM TDMA, $N = 5$

node	1	2	3	4
d_{iN} (m)	2	5	8	14
W_{iN} (bits)	2000	2000	2000	2000
b_{iN}	13/5	9/5	7/5	5/5
t_{iN} (s)	0.015/0.04	0.022/0.04	0.029/0.04	0.04/0.04
E_i (mJ)	3.8/8.4	5.9/8.6	8/9.1	13.2/13.2

(a) Minimizing transmission energy only, $R_2 = R_3 = 0$

(b) Minimizing total energy consumption, $R_2 = R_3 = 0$

(c) Minimizing transmission energy only, $R_2 = 8000$ bps, $R_3 = 2000$ bps

(d) Minimizing total energy consumption, $R_2 = 8000$ bps, $R_3 = 2000$ bps

Fig. 2. Optimal routing and MAC.

Fig. 3. Optimal routing and MAC for a random network of 50 Nodes with $R_1 = 6000$ bps.
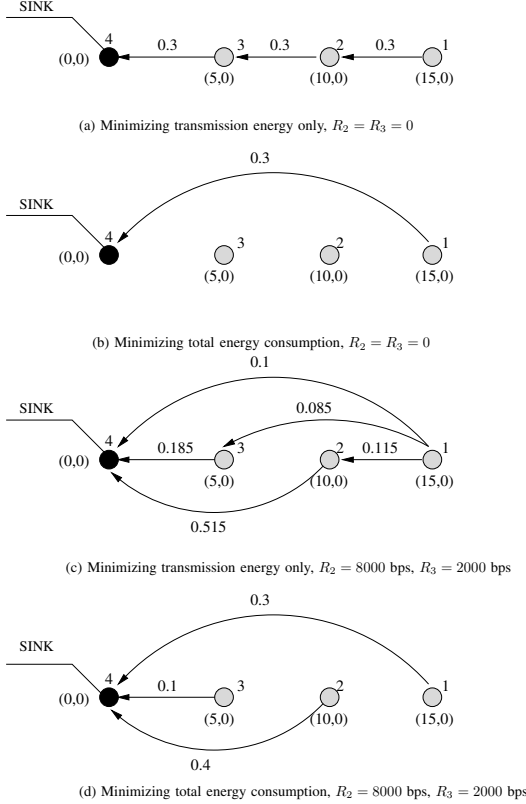
B. Routing and MAC Optimization with Fixed Links

We first consider a simple network with a string topology as shown in Fig. 2, where each node is labeled with its (x, y) location. The system parameters are again those shown in Table I. We first assume that node 2 and node 3 have no data to transmit, and node 1 generates data at a rate of $R_1 = 6$ Kbps. If we minimize only the transmission energy, the optimal transmission scheme is shown in Fig. 2(a). For this case, multihop routing is optimal. The time slots assigned to each link are labeled above the link. When we minimize the total transmission and circuit energy consumption, the optimal transmission scheme is shown in Fig. 2(b). Thus minimizing circuit energy consumption leads to an optimal scheme with fewer number of hops than that corresponding to minimizing just the transmission energy. This is because as the number of hops increases more energy is dissipated in the node circuits.

We now consider the case where node 2 and node 3 both have their own data to transmit; in particular $R_2 = 8$ Kbps and $R_3 = 2$ Kbps. The optimal transmission scheme which minimizes transmission energy is shown in Fig. 2(c). The optimal scheme consists of a combination of single-hop and multihop transmissions. This is because when a node uses multihop
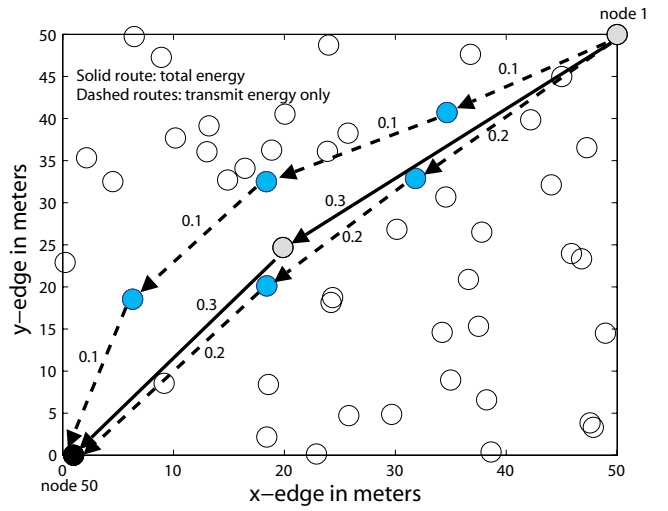
transmissions to send data to the SINK, some hops may be allocated a smaller amount of time than when a node sends data directly to the SINK. Hence, multihop transmissions may not be feasible (note that the constellation size is fixed) even when single-hop transmissions are. If we minimize the total energy consumption, the optimal scheme is shown in Fig. 2(d), where we see that the optimal strategy for each node is to transmit the information to the SINK with single hop. This is due to the fact that multi-hop transmissions cause extra circuit energy consumption as that in Fig. 2(b).

To illustrate the generality of the optimization model, we now solve the problem for a network with 50 nodes, where all the node are randomly deployed into a square region of size 50 m × 50 m, with the exception for two nodes: Node 1 is the source node located at $(50, 50)$ with $R_1 = 6000$ bps; and node 50 is the SINK node located at $(0, 0)$. The optimal routing and TDMA slot assignment are shown in Fig. 3, where the route in solid lines is for the case where we minimize the total of the transmission and circuit energy consumption, and the routes in dashed lines are for the case where we only minimize the transmission energy. For the latter case, the optimal routing strategy is to split the traffic into two sub-streams: The first one carries 2000 bps; and the second one carries 4000 bps. For the result regarding the number of hops, we see the same phenomena as in the simple string topology: Minimizing the total energy consumption leads to routes with less hops. This conclusion is true in general due to the circuit energy overhead in relay nodes.

C. Routing and MAC optimization with Link Adaptation

We now consider the general case where we jointly optimize routing, MAC, and link adaptation. The optimization model is given by Eq. (4). To just illustrate the key points, we again consider the simple string topology as above with $R_1 = 6$ Kbps, $R_2 = 8$ Kbps, and $R_3 = 2$ Kbps. When the circuit energy consumption is included, the optimal routing, scheduling, and modulation constellation size b are as listed in Fig. 4. The number above each link is the time slot length
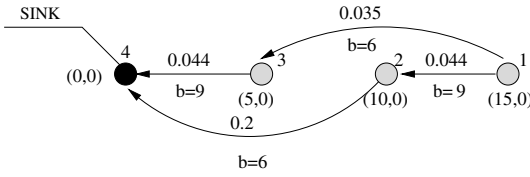
Fig. 4. Minimizing total energy consumption, $R_1 = 6000$ bps, $R_2 = 8000$ bps, $R_3 = 2000$ bps.

Fig. 5. Two different schedules.

assigned to that link and the number below each link is the optimal constellation size used for that link. The network energy consumed within each period T is 0.022 J, while the network energy consumed without link adaptation (as shown in Fig. 2(d)) is 0.081 J. We see that link adaptation leads to a 73% lower energy consumption than that without link adaptation. Also, direct transmissions combined with multi-hop routing are shown to be optimal in this case compared with pure single-hop transmissions as shown in Fig. 2(d). The reason why multihop routing becomes more efficient in this particular example is that link adaptation reduces the transmission time for each hop by using higher constellation sizes such that the extra circuit power consumption in the relay nodes is reduced. Finally, it is worth noting that for the presented numerical results in this section, the relative error caused by convex approximation is negligible, which is below one percent.

V. DELAY ANALYSIS

In this section, we show that the ordering of time slots affects the delay performance. Since we adopted a deterministic scheduling scheme based on TDMA, no random packet delay is considered. The delay of interests here includes deterministic queuing delay at intermediate nodes and transmission delay over each hop. Note that for our traffic model, a packet of $R_i T$ bits (all the bits in the transmission buffer) is available at node i for transmission at times $T, 2T, \ldots$. Although the flow conservation equations are satisfied over every frame of length T, some packets may experience source-to-destination delays that are larger than T. We give a constructive proof to show that a worst-case delay of T is achievable by ordering the transmissions on the links appropriately. We then characterize the optimal delay-energy tradeoff.

A. Minimum Delay Scheduling Algorithm

We define the delay D_i that a packet experiences over hop i as the sum of two delay components. The first component is the queuing delay that is the time for which a packet waits at a node, starting from its arrival until the outgoing link that carries this packet is scheduled to start transmission. The second component is the transmission delay that is equal to the time slot length assigned to the outgoing link. Note that we neglect the propagation delay because the node-to-node transmission distance is usually short in sensor networks. The values of the D_i's are determined by the slot lengths assigned to each link as well as the order in which the links are scheduled. Thus, the total end-to-end delay a packet experiences is given by $\sum_{i=1}^{K} D_i$ where K is the number of hops the packet traverses.
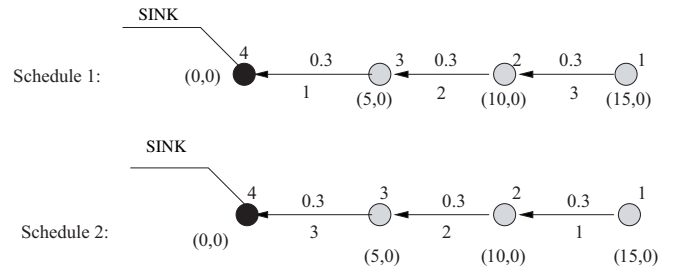
Consider a non-uniform TDMA scheme (where different links may be assigned with different slot lengths) with a known slot length for each link. If there are M active links, there are $M!$ such TDMA schedules for the given slot length assignment. Each of these $M!$ different schedules can correspond to different values of packet delays, although the total energy consumption is independent of the schedule. To illustrate this, we consider the example shown in Fig. 5. We assume that the time slot length for each link is 0.3. Based on this assignment, two different schedules are shown in Fig. 5 where the scheduling index is labeled below each link. For schedule 1, the transmission order within each period T is link $3 \rightarrow 4$, then link $2 \rightarrow 3$, and finally link $1 \rightarrow 2$. Thus, nodes 2 and 3 can only forward the data they collected in the last period. Consequently, each bit originating from node 1 only traverses one hop (to the next node) within each period T. The total delay is thus $3T$ and this is the worst-case delay among all possible schedules. For schedule 2, with the reversed transmission order compared to schedule 1, it can be easily shown that all the information bits from node 1 arrive at node 4 within one period T. This is a schedule with the minimum delay.

Generally speaking, if we schedule the outgoing links before the incoming links for node i, the data forwarded by node i is the data it collected in the last period. Thus, the corresponding D_i can be up to T. Consequently, the worst-case delay that each packet experiences from the source to the destination can be as large as KT for a path of K hops. For a simple route such as the one shown in Fig. 5, it is easy to find the best schedule to achieve the minimum delay. Specifically, we can schedule the first hop the earliest, then the second hop, and so on. However, for a complex network (such as the one shown in Fig. 1) with arbitrary routes, finding the best schedule to achieve the minimum delay is not as easy. We now discuss how to find the minimum delay schedule for general cases.

We define the *delay of a schedule* to be the maximum delay encountered by any packet in the network. As discussed above, the minimum possible delay of a schedule is $T = \sum_{i=1}^{N-1} \sum_{j \in \mathcal{M}_i} t_{ij}$. We have the following lemma stating a sufficient condition for a schedule to have the minimum possible delay T. The proof of this lemma is given in Appendix D.

Lemma 5.1: Consider a TDMA schedule over a time period T. It is sufficient for every node to schedule its outgoing links after its incoming links to achieve the minimum possible delay of T.

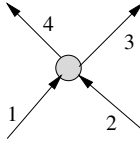Note that the condition specified in the lemma is sufficient

Fig. 6. A counter example.

to achieve the minimum delay, but not necessary. This can be illustrated by the following example. As shown in Fig. 6, we assume a node that has two incoming links (1, 2) and two outgoing links (3, 4). If we schedule links (1, 2) before links (3, 4), all the information collected by the node is sent out within the same period since the sufficient condition (given by the lemma) holds for this node. However, if we further assume that the data rate and time slot length are the same among all the four links, the scheduling order (1, 3, 2, 4) can guarantee the data collected by the node to be sent out within the same period although the sufficient condition no longer holds for this node.

We now describe an algorithm to find a schedule with the minimum delay for a given assignment of slot lengths to links. We first consider a tree network topology and then generalize the algorithm to an arbitrary topology with no loops. For a tree topology, shown in Fig. 1 if we neglect all the dotted links, the root of the tree is the SINK node. For this simple topology, the algorithm can be directly obtained from the proof of Lemma 5.1. It is easy to find a schedule to guarantee that at each node all the outgoing links are scheduled after all the incoming links. We can classify the links into levels according to the distance, in number of hops, from the root. This is illustrated in Fig. 1 (after removing all the dotted links). For each TDMA frame of length T, we construct the schedule in reverse order, *i.e.*, the slots are aligned from the end point of the frame. Let us denote the set of links to be scheduled at each step i as $\mathcal{L}_i$. At step one, $\mathcal{L}_1$ includes all the links on level 1; at step 2, $\mathcal{L}_2$ includes all the links on level 2 and so on. At the last step d, $\mathcal{L}_d$ includes all the links on the last level d (where d is the depth of the tree). The scheduling order within each $\mathcal{L}_i$ can be arbitrary. According to this scheduling algorithm, links in $\mathcal{L}_d$ will be activated first and links in $\mathcal{L}_1$ will be activated last. Constructing the schedule in reverse order is motivated by the recursive proof for the lemma, which is also a key in deriving the scheduling algorithm for general topologies.

An arbitrary loop-free topology with one SINK node is shown in Fig. 1. Such a topology can always be represented by a regular tree with extra branches. These branches can be between nodes in layers that are separated by more than one level, and in either direction. These extra branches are shown by dotted lines in Fig. 1. Since there may be cross-level routes, the scheduling algorithm proposed for the regular tree topology cannot be applied directly to the general case.

For the general case, the algorithm is described below. Again the link schedule is constructed in reverse order starting from the end of the frame of length T. Accordingly, we start with node N (the SINK node, which has no outgoing links) and schedule its incoming links first, which are the links from nodes in set $\mathcal{N}_N$.

Algorithm:
1) *Initialization*:
   $\mathcal{Q} = \mathcal{N}_N$, $\mathcal{L} = \{\}$; For all $i \in \mathcal{Q}$, $\mathcal{L} = \mathcal{L} + \{i \to N\}$; Scheduling all links in $\mathcal{L}$; $\mathcal{L}_{all} = \mathcal{L}$;
2) *If any nodes in $\mathcal{Q}$ are not leaf nodes, repeat*:
   $\mathcal{Q}_0 = \{\}$, $\mathcal{L} = \{\}$; For each $i \in \mathcal{Q}$ that is not leaf node, for all $j \in \mathcal{N}_i$:
   If all the outgoing links of node i are in $\mathcal{L}_{all}$, then $\mathcal{Q}_0 = \mathcal{Q}_0 + j$ and $\mathcal{L} = \mathcal{L} + \{j \to i\}$;
   Scheduling all links in $\mathcal{L}$; $\mathcal{L}_{all} = \mathcal{L}_{all} + \mathcal{L}$, $\mathcal{Q} = \mathcal{Q}_0$;
3) *Output $\mathcal{L}_{all}$, end of the algorithm.*

The link order listed in $\mathcal{L}_{all}$ is the scheduling order and $\mathcal{L}_{all}$ is also in the program to track all the links that have been scheduled. In the algorithm, the third line in step (2) is the key to guarantee that the sufficient condition specified in Lemma 5.1 is satisfied. In other words, only if all the outgoing links of a node have been scheduled (stored by $\mathcal{L}_{all}$), the incoming links of that node can be added to the set $\mathcal{L}$ to get scheduled.

Before we prove the correctness of the algorithm, we state three properties of a loop-free directed graph in the following lemma, which is proved in Appendix E.

Lemma 5.2: For a loop-free directed graph assumed for the network topology, we have the following properties:
   1) Removing links from a loop-free network creates no loops.
   2) For a connected loop-free network, there exists at least one node that has only incoming links.
   3) All the directed paths in the network terminate at the root and there exists a directed path between the root and every other node.

Lemma 5.3: The delay of the schedule constructed by the algorithm above is T.

   Proof: See Appendix F. ∎

We now give an example to illustrate how the algorithm works. Considering the topology given in Fig. 7 where the node index is labeled, the algorithm works as follows:

Algorithm:
1) *Initialization*:
   $\mathcal{Q} = \{3, 2, 4\}$, $\mathcal{L} = \{\}$;
   $\mathcal{L} = \{3 \to 5, 2 \to 5, 4 \to 5\}$;
   $\mathcal{L}_{all} = \{3 \to 5, 2 \to 5, 4 \to 5\}$;
   Schedule all links in $\mathcal{L}$;
2) *Nodes 3 and 4 in $\mathcal{Q}$ are not leaf nodes*:
   $\mathcal{Q}_0 = \{\}$, $\mathcal{L} = \{\}$;
   $\mathcal{Q}_0 = \{3\}$ and $\mathcal{L} = \{3 \to 4\}$;
   $\mathcal{L}_{all} = \{3 \to 4, 3 \to 5, 2 \to 5, 4 \to 5\}$,
   $\mathcal{Q} = \{3\}$;
   Schedule all links in $\mathcal{L}$;
3) *Node 3 in $\mathcal{Q}$ is not a leaf node*:
   $\mathcal{Q}_0 = \{\}$, $\mathcal{L} = \{\}$;
   $\mathcal{Q}_0 = \{1, 2\}$ and $\mathcal{L} = \{2 \to 3, 1 \to 3\}$;
   $\mathcal{L}_{all} = \{2 \to 3, 1 \to 3, 3 \to 4, 3 \to 5, 2 \to 5, 4 \to 5\}$,
   $\mathcal{Q} = \{1, 2\}$;
   Schedule all links in $\mathcal{L}$;
4) *All nodes in $\mathcal{Q}$ are leaf nodes $\Rightarrow$ End of the algorithm.*

Therefore, the scheduling result for the network shown in Fig. 7 is $\mathcal{L}_{all} = \{2 \to 3, 1 \to 3, 3 \to 4, 3 \to 5, 2 \to 5, 4 \to 5\}$.
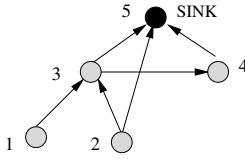
Fig. 7. Scheduling example.

B. Delay-Energy Tradeoff

In the previous sections, we have assumed that during each frame of length T, R_iT bits are sent to the SINK from node i. Note that if we transmit $\sum_{i=1}^{N-1} R_iT$ bits to the SINK in time $\hat{T} < T$, then we can reduce the delay to $\hat{T}$ (using the results in the previous subsection). Here, we consider the tradeoff between $\hat{T}$ and the average power consumption, or equivalently the energy consumption to transmit R_iT bits from each node i to the SINK. Therefore, we can calculate the Pareto-optimal [26], [27] delay-energy tradeoff which characterizes the minimum possible delay for a given energy consumption (or vice versa). The optimal tradeoff curve defines the boundary of the achievable delay-energy region. It can be computed by varying the value of the weighting factor β ($0 \le \beta < \infty$) in the following convex optimization problem,

$$
\begin{aligned}
\min. \quad & \sum_{i=1}^{N-1}\sum_{j\in\mathcal{M}_i} t_{ij} + \beta \sum_{i=1}^{N-1}\sum_{j\in\mathcal{M}_i} \epsilon(W_{ij}, t_{ij}) \\
\text{s. t.} \quad & \sum_{j=1}^{N}(W_{ij} - W_{ji}) = R_iT, \quad i = 1, \cdots, N \\
& 2Bt_{ij} \le W_{ij} \le BC_{ij}t_{ij} \\
& i = 1, \ldots, N-1, \quad j = 1, \ldots, N
\end{aligned}
\tag{10}
$$

Specifically, when β approaches 0, all the emphasis is put on delay, which leads the optimization problem to have a solution with the smallest delay but the largest energy consumption; when β approaches ∞, all the emphasis is put on energy, which leads the optimization problem to have a solution with the smallest energy consumption but the largest delay; for values of β lying in between, we can calculate all the points on the delay-energy tradeoff curve.

To illustrate how to calculate such an delay-energy tradeoff curve, let us consider the simple string topology with four nodes as shown in Fig. 4 (just for an example), with $R_1T = 6000$ bits and $R_2T = R_3T = 0$. The delay-energy tradeoff curve is shown in Fig. 8 when only the transmission energy is considered. From the figure we see that the tradeoff curve for the rate-adaptive case is below the tradeoff curve for the case without rate adaptation. The gain is not significant in the high delay and low energy region. However, rate adaptation can be used to trade off energy consumption to decrease delay. When the circuit energy consumption is included, the tradeoff curves are shown in Fig. 9. We see that the tradeoff curve is a single point (corresponding to the single-hop transmission scheme) when rate adaptation is not allowed. This is because single-hop transmissions minimize the energy as well as the delay. The gain of rate adaptation is more significant in this case.

VI. CONCLUSIONS

We show that joint optimization across routing, MAC, and physical layers which also takes into account hardware
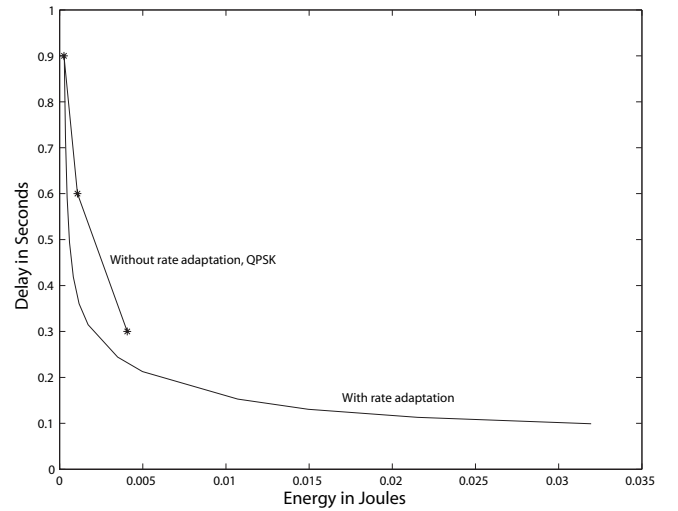

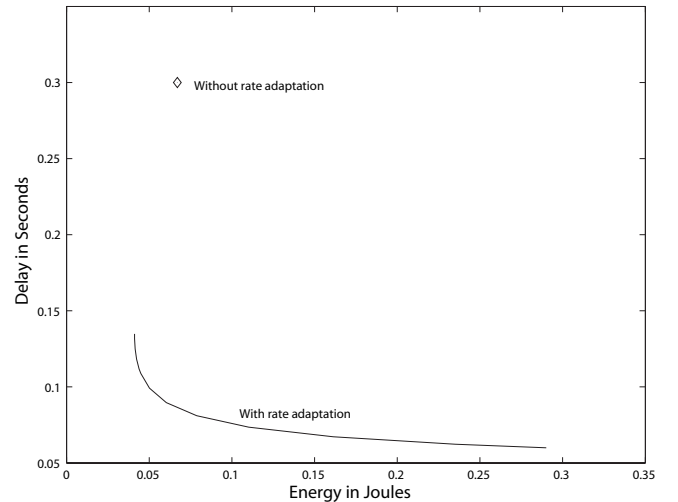Fig. 8. Delay and energy tradeoff, transmission energy only.

Fig. 9. Delay and energy tradeoff, circuit energy consumption included.

power consumption is feasible and beneficial. The associated problems can be relaxed to convex optimization problems for which powerful low-complexity solution techniques exist. The results show that significant energy savings are possible compared with traditional non-optimized protocols. We find that the optimal transmission scheme can be a mix of single-hop and multihop routing. Minimization of total energy favors fewer hops compared to minimization of just transmission energy. Also, numerical examples show that significant energy savings are possible when link adaptation is used. For any loop-free network with a single SINK node, we construct an algorithm to achieve the minimum possible delay. We also characterize the optimal delay-energy tradeoff using convex optimization.

APPENDIX

A. Proof of Lemma 2.1

To prove convexity of $\epsilon(W, t)$ for $W, t \ge 0$ it is sufficient to prove convexity of the function $f(W, t) = t2^{\frac{W}{t}}$ for $W, t \ge 0$. The convexity of ϵ then follows since a linear function is

convex and the sum of two convex functions is convex [16]. The Hessian of $f(W, t)$ is given by

$$\mathbf{H} = \begin{bmatrix} (\ln 2)^2 2^{W/t}/t & -(\ln 2)^2 2^{W/t} W/t^2 \\ -(\ln 2)^2 2^{W/t} W/t^2 & (\ln 2)^2 2^{W/t} W^2/t^3 \end{bmatrix}.$$

A 2×2 matrix of the form $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$ with $a > 0$ is positive semi-definite if and only if $b^2 - ac = 0$ (Schur's complement condition [16]). Since the Hessian matrix $\mathbf{H}$ has the above property, $\mathbf{H}$ is positive semi-definite or equivalently, $f(W, t)$ is convex over W and t.

B. Proof of Lemma 3.1

Note that $x t_{iN} \left(2^{\frac{W_{iN}}{B t_{iN}}} - 1 \right)$ is a monotonically decreasing function of t_{iN}, If $\frac{\sum_{i=1}^{N-1} W_{iN}}{BT} < 2$, then $t_{iN}^* = \frac{W_{iN}}{2B}$ because the minimum constellation size is 2. Hence, this is the maximum amount of time that a node can take to send W_{iN} bits to the SINK.

Now consider the case where $2 \leq \frac{\sum_{i=1}^{N-1} W_{iN}}{BT} \leq C$. We will obtain the solution to the problem in (7) without the minimum and maximum rate constraint and show that it automatically satisfies this constraint. The Karush-Kuhn-Tucker (KKT) conditions (see, for example, [16]) for the problem can be written as

$$\sum_{1=1}^{N-1} t_{iN} = T$$

$$x 2^{\frac{W_{iN}}{B t_{iN}}} \left(1 - \ln 2 \frac{W_{iN}}{B t_{iN}} \right) = x - \nu, \quad i = 1, \dots, N-1$$

for some $\nu \in \mathbb{R}$. It is easy to see that the KKT conditions and the maximum and minimum rate constraint are satisfied by $t_{iN}^* = \frac{W_{iN}}{W_\Sigma} T$ with $\nu = x - x 2^{\frac{W_\Sigma}{BT}} \left(1 - \ln 2 \frac{W_\Sigma}{BT} \right)$ and $W_\Sigma = \sum_{i=1}^{N-1} W_{iN}$. Given the convexity of the problem, these t_{iN}^*'s are optimal.

C. Proof of Lemma 3.2

Consider an arbitrary feasible solution to the problem of minimizing the total circuit and transmission energy, where node i transmits to the SINK with constellation size b_{iN} for a fraction α_{iN} of time $\hat{T}$, where $\hat{T} \leq T$. Note that $\sum_{i=1}^{N-1} \alpha_{iN} = 1$ and $x \left(2^{b_{iN}} - 1 \right) + y$ is a convex function of b_{iN}. Hence, the total energy consumption across all the nodes to transmit $(W_{iN}, W_{2N}, \cdots, W_{(N-1)N})$ bits to the SINK satisfies

$$\sum_{i=1}^{N-1} \alpha_{iN} \hat{T} \left(x \left(2^{b_{iN}} - 1 \right) + y \right)$$

$$\geq \hat{T} \left(x \left(2^{\sum_{i=1}^{N-1} \alpha_{iN} b_{iN}} - 1 \right) + y \right)$$

$$= \sum_{i=1}^{N-1} \frac{\hat{T} \alpha_{iN} b_{iN}}{b^\dagger} \left(x \left(2^{b^\dagger} - 1 \right) + y \right)$$

$$= \sum_{i=1}^{N-1} t_{iN}^\dagger \left(x \left(2^{b^\dagger} - 1 \right) + y \right)$$

where the inequality follows Jensen's inequality. In addition, we applied $\sum_{i=1}^{N-1} \frac{\hat{T} \alpha_{iN} b_{iN}}{b^\dagger} = \hat{T}$ with $b^\dagger = \sum_{i=1}^{N-1} \alpha_{iN} b_{iN}$

in the first equality and $t_{iN}^\dagger = \frac{W_{iN}}{Bb^\dagger} = \frac{\hat{T} \alpha_{iN} b_{iN}}{b^\dagger}$ in the second equality. We see that the last line is the total energy consumption when each node sends data to the SINK with the same constellation size $b^\dagger$, where the transmission time for node i is given by $t_{iN}^\dagger$. Therefore, due to the inequality in the first line, transmitting with the same rate $b^\dagger$ for all the nodes is the optimal strategy to minimize the total energy, where $b^\dagger = \frac{\sum_{i=1}^{N-1} W_{iN}}{BT^\dagger}$ and this is equivalent to transmitting all the bits from one virtual node. The optimal transmission time $\hat{T}$ for the virtual node can be obtained by solving Eq. (8). Then the optimal transmission time for node i is given by $t_{iN}^\dagger = \frac{W_{iN}}{Bb^\dagger} = \hat{T} \frac{W_{iN}}{\sum_{i=1}^{N-1} W_{iN}}$.

D. Proof of Lemma 5.1

We assume that there are no loops formed by the directed communication routes in the network. Using loopy paths would be energy inefficient and hence can be ruled out as a solution to the minimum-energy optimization problem addressed in the first part of this paper. The proof can be constructed in a recursive way. Starting from the set $\mathcal{N}_N$, which represents the set of nodes that directly transmit data to the SINK node N, we can claim that all the data collected by nodes in $\mathcal{N}_N$ within the given period arrives at the SINK node before the end of this period under the condition given in the lemma. For $i \in \mathcal{N}_N$, let τ_i denote the time when node i begins transmission to the SINK node. Then $\forall j$, $j \in \mathcal{N}_i$, by the same argument it can be claimed that all the information collected by node j has arrived at node i before τ_i within the same period. We can extend the same argument to the nodes that send information to node j and span further until we reach the leaf nodes (*i.e.*, the source nodes without incoming links). As long as the condition described in the lemma holds for each node, it can be guaranteed that all the data generated by the source nodes arrives at the SINK node within one period of time.

E. Proof of Lemma 5.2

The first property and the third property are straightforward to prove based on our definitions and assumptions of the network topology. For the second property, we prove it by contradiction. We assume that there are K nodes in the network and each node has some outgoing links. Without loss of generality, we assume node 1 has one outgoing link that ends at node 2. Thus, node 2 cannot have its outgoing links ending at node 1. Otherwise, there will be a loop between node 1 and node 2. As a result, the outgoing links of node 2 need to end at nodes that have indices higher than 2. Again, without loss of generality, we assume node 2 has an outgoing link ending at node 3. To keep the loop-free property, the outgoing links of node 3 cannot end at either node 2 or node 1, since there already exist links from node 1 to node 2 and from node 2 to node 3. Following the same argument, for node n, its outgoing links can only end at nodes that have indices higher than n and we assume that it has an outgoing link that ends at node $n + 1$. Therefore, for node $K - 1$, it can only have one outgoing link that ends at node K. For node K, no matter where its outgoing links end, there will

be loops generated. Thus, the assumption that every node has outgoing links contradicts the loop-free property. To conclude, for a directed network to be loop-free, there exists at least one node that has only incoming links.

F. Proof. of Lemma 5.3

To prove the correctness of the scheduling algorithm, it is sufficient to prove the following.

1) Condition 1: at each iteration of step (2) in the algorithm, if any nodes in $\mathcal{Q}$ are not leaf nodes, there exists at least one node in $\mathcal{Q}$ that has only incoming links left to be scheduled (its outgoing links have been scheduled already) such that $\mathcal{Q}_0$ and $\mathcal{L}$ are non-empty at the current iteration.
2) Condition 2: all the links in the network will be traversed after the algorithm terminates.

Proof of Condition 1

To prove that the first condition is satisfied, we start from the root node N. Since there are no outgoing links for node N, all the links from the nodes in $\mathcal{N}_N$ to node N can be scheduled, which corresponds to the first step in the algorithm. If all the source nodes are connected to the root node with single-hop transmissions, the algorithm ends after the first step. Otherwise, we continue to step (2) of the algorithm. Since all the links from the nodes in $\mathcal{N}_N$ have been scheduled, we can virtually remove them from the network. According to the first property in Lemma 5.2, the remaining network is still loop-free. According to the second property in Lemma 5.2, there exists at least one node in $\mathcal{Q}$ that has only incoming links (*i.e.*, has no outgoing links). Therefore, $\mathcal{Q}_0$ and $\mathcal{L}$ are non-empty for the first iteration of step (2). After the first iteration, we again remove the scheduled links from the network. By the same argument as for the first iteration, the remaining network is still loop-free and there exists at least one node in $\mathcal{Q}$ that has only incoming links. Therefore, at each iteration, we guarantee that $\mathcal{Q}_0$ and $\mathcal{L}$ are non-empty until all the nodes in $\mathcal{Q}$ are leaf nodes. Then we stop the algorithm. Note that the network may be broken into several sub-networks after we remove the scheduled links. Since each sub-network is still loop-free, the same argument applies to each of them individually.

Proof of Condition 2

Proving that the second condition is satisfied, *i.e.*, proving that all the links in the network are traversed after the algorithm terminates, is equivalent to proving that every node that has outgoing links appears once in $\mathcal{Q}_0$ after the algorithm terminates. In other words, nodes 1 through $N-1$ must appear once in $\mathcal{Q}_0$ during the iterations of step (2) of the algorithm. We also prove this by contradiction. Let us assume that there is at least one node (denoted as node i) among nodes 1 to $N-1$ that never appears in $\mathcal{Q}_0$. Therefore, there is at least one node (denoted as node j) in the set $\mathcal{M}_i$ that never appears at $\mathcal{Q}_0$, where $\mathcal{M}_i$ is the set of nodes that node i sends data to. Otherwise, if all nodes in $\mathcal{M}_i$ appear in $\mathcal{Q}_0$, then all the outgoing links of node i must have been scheduled and node i will appear in $\mathcal{Q}_0$. By the same argument, since node j never appears in $\mathcal{Q}_0$, there must be a node in $\mathcal{M}_j$ that never appears in $\mathcal{Q}_0$. Thus, if we continue the argument and connect the related nodes with the directed links, we can construct a

special directed path such that each node along the path has at least one outgoing link that is not scheduled after the algorithm terminates. Since the root node has no outgoing links and all its incoming links were scheduled at step 1) of the algorithm, the root node can never be included into the special path. However, this contradicts the third property in Lemma 5.2, where we specify that all the directed paths must terminate at the root node. To conclude, our initial assumption that there is at least one node among nodes 1 to $N-1$ that never appears in $\mathcal{Q}_0$ is false. In other words, all the links are traversed when the algorithm terminates.

The correctness proof of the algorithm is complete.

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