Complementary Assignment 1 - INF01009
Intuition on Cross Products

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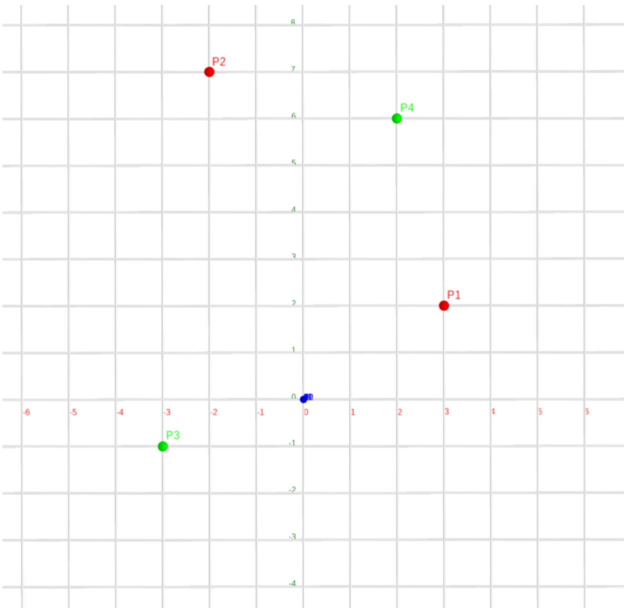

Figure 1: Points in the 2D plane

1 Line Between Points

As the specification said, we can have derive a number of things only by applying cross products to points and lines. One of them, is retrieving the line function that goes through two points. Defining (fig. 1)

$$\begin{aligned} w &= 1.0 \\ P1 &= [3, 2, w] \\ P2 &= [-2, 7, w] \\ P3 &= [-3, -1, w] \\ P4 &= [2, 6, w] \end{aligned}$$

Now, we can apply the cross product to find the line equations in a $(a, b, c) \to ax + by + c = 0$ format:

$$\begin{aligned} L1 = P1 \times P2 = [-5, -5, 25] = [-1, -1, 5] \to -x - y + 5 = 0 \\ L2 = P3 \times P4 = [-7, 5, -16] \to -7x + 5y - 16 = 0 \end{aligned}$$

This is rendered as in fig. 2.

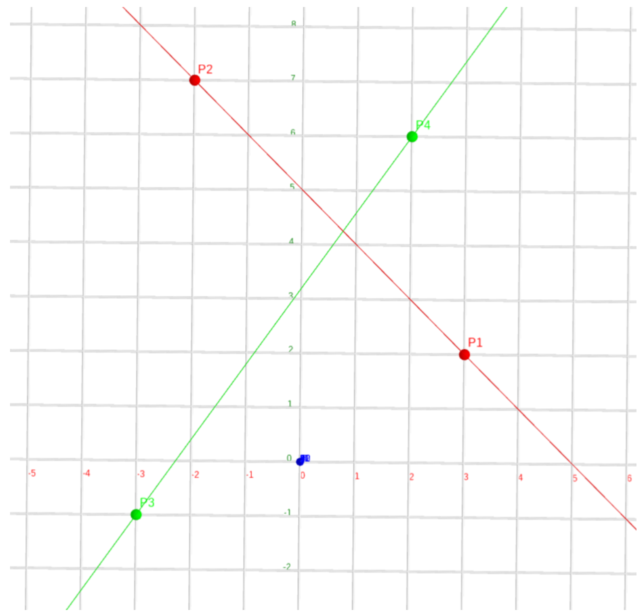

Figure 2: Lines in the 2D plane

2 Intersection of Two Lines

Now that we have the two lines, we want to find the spot where $-x - y + 2 = -7x + 5y - 16$. This can be calculated by working out the equation, or by calculating the cross product between these two equations' factors. The result is seen fig. 3.

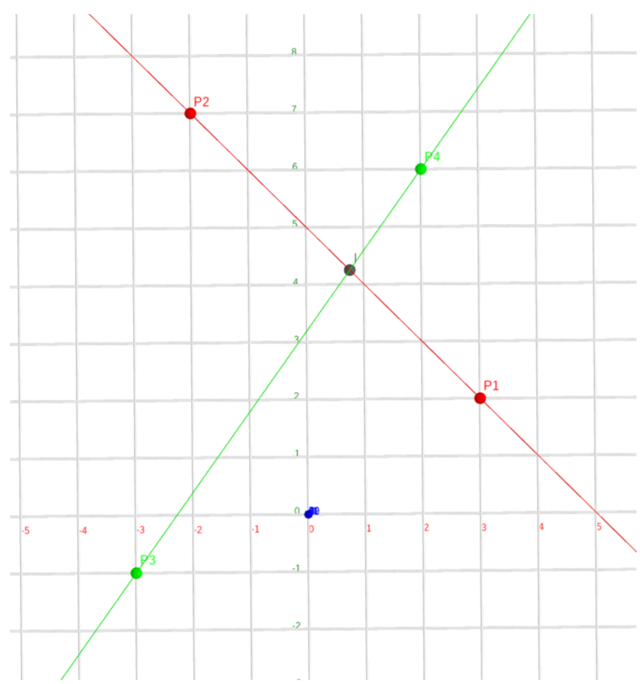

Figure 3: Intersection of two lines

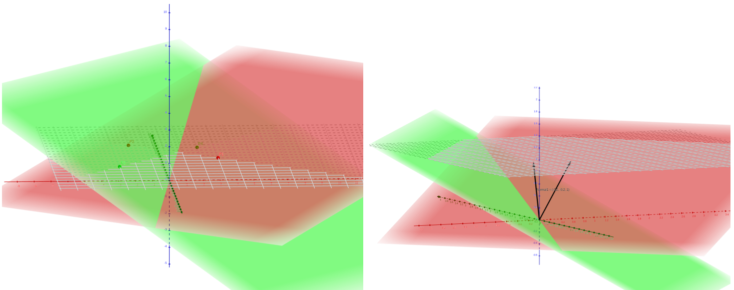

Figure 4: Planes definition based on points (left) and their normals (right)

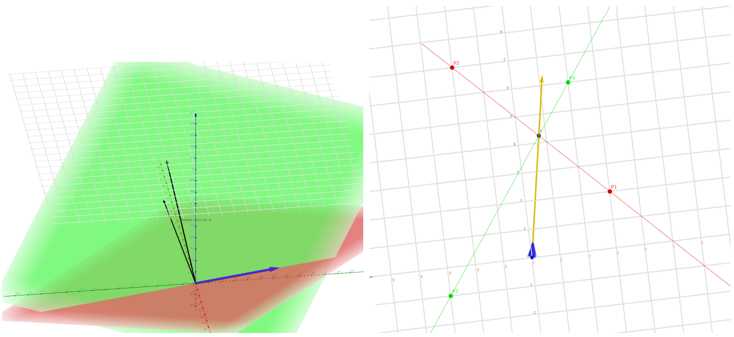

Figure 5: Cross product between the two plane normals outlines the intersection between them (left). Clearly, this outline intersects with the intersection points defined in fig. 3 (right).

3 Defining Planes

We can define planes for each of these lines. These planes must contain the two points and the origin ($[0,0,0]$). Once they are visible, we can also render their normals (fig. 4).

Once these planes are defined, obtaining a cross product of their normals should give us a perpendicular vector that is linearly independent and, thus, contained in both planes! This is seen in fig. 5.

4 Changing w

What would happen if we were to change the w value defined in the first equation set? Reading the homogeneous coordinate w as the value of the points in the *Z-axis*, we see that by decreasing it, the points would lower towards the same plane as the origin. This would make it so that the planes defined in section 3 grow perpendicular. This is shown in fig. 6.

If $w = 0$, it would be natural that the planes become the same and that the intersection would move towards infinity, as we divide the point by its homogeneous coordinate to find its position in the $w = 1$ plane.

$$
\begin{aligned}
[3,2,0] \times [-2,7,0] &= [0,0,25] \\
[-3,-1,0] \times [2,6,0] &= [0,0,-16] \\
[0,0,25] \times [0,0,16] &= [0,0,0] \\
\text{Intersection Point} &= \frac{[0,0,0]}{0}
\end{aligned}
$$

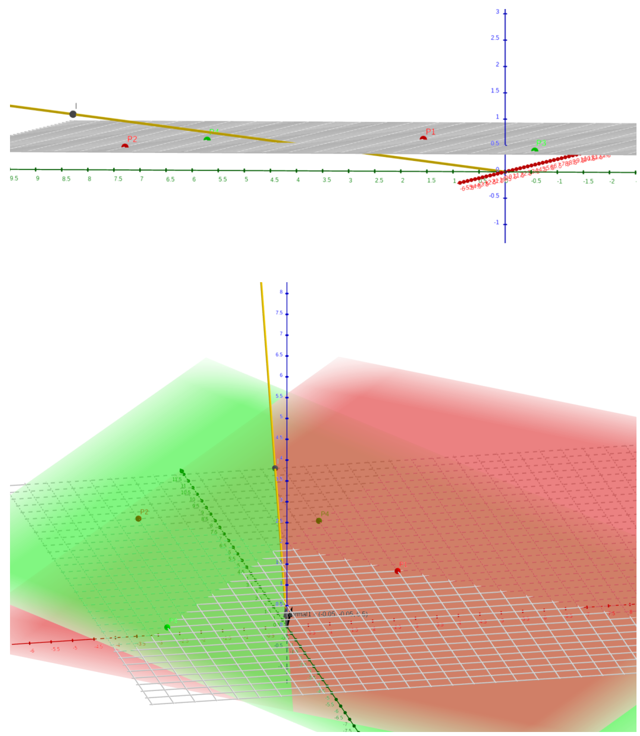

Figure 6: As w lowers its value, we see that the calculated intersection point goes further in the plane outline calculated in section 3 (top) and that the planes become more alike (bottom).