Date: / /

1. Find the eigen value and eigen vector from the given data point.

x	2.5	0.5	2.2	1.9	3.1	2.3	2	1	1.5	1.1
y	2.4	0.7	2.9	2.2	3.0	2.7	1.6	1.1	1.6	0.9

→ Solution,

x	y	$a = x - \bar{x}$	$b = y - \bar{y}$	a^2	b^2	ab
2.5	2.4	0.69	0.49	0.476	0.24	0.338
0.5	0.7	-1.31	-1.21	1.716	1.464	1.585
2.2	2.9	0.39	0.99	0.152	0.98	0.386
1.9	2.2	0.09	0.29	0.008	0.084	0.026
3.1	3.0	1.29	1.09	1.664	1.188	1.406
2.3	2.7	0.49	0.79	0.24	0.624	0.387
2	1.6	0.19	-0.31	0.036	0.096	-0.058
1	1.1	-0.81	-0.81	0.656	0.656	0.656
1.5	1.6	-0.31	0.31	0.096	0.096	-0.0961
1.1	0.9	-0.71	-1.01	0.504	1.02	0.717
				$\Sigma a^2 = 5.548$	$\Sigma b^2 = 6.448$	$\Sigma ab = 5.346$

From table,

$\Sigma x = 18.1 \quad \Sigma y = 19.1$

Now,

$$\bar{x} = \frac{\Sigma x}{n} = \frac{18.1}{10} = 1.81$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{19.1}{10} = 1.91$$

Now, the next step is to compute the covariance matrix of given dataset (sometimes also called as the variance covariance matrix).

So, we can compute the covariance of two variables x and y using the following formula

$$Cov(x,y) = \frac{1}{n-1} \sum_{i=1}^{n} (x_1 - \bar{x})(y_1 - \bar{y})$$

$$= \frac{1}{n-1} \Sigma ab$$

$$= \frac{5.346}{9}$$

$$= 0.594$$

$$Cov(y,x) = \frac{1}{n-1} \sum_{i=1}^{n} (y_1 - \bar{y})(x_1 - \bar{x})$$

$$= \frac{1}{n-1} \Sigma ba$$

$$= \frac{5.346}{9}$$

$$= 0.594$$

$$Cov(x,x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x})$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} a^2$$

$$= \frac{5.548}{9}$$

$$= 0.616$$

Date: / /

$$\mathrm{Cov}(y,y) = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \overline{y})(y_i - \overline{y})$$

$$= \frac{1}{n-1} \sum b^2$$

$$= \frac{6.448}{9}$$

$$= 0.716$$

Using the above, we can find the covarience matrix of our dataset. Also, the results would be a square matrix of $d \times d$ dimensions.

Let A be the convarience matrix of our dataset.

$$A = \begin{bmatrix} \mathrm{cov}(x,x) & \mathrm{cov}(x,y) \\ \mathrm{cov}(y,x) & \mathrm{cov}(y,y) \end{bmatrix} \rightarrow \text{For } 2\times2 \text{ matrix.}$$

For 3X3 convariance,

$$A = \begin{bmatrix} \mathrm{cov}(x,x) & \mathrm{cov}(x,y) & \mathrm{cov}(y,z) \\ \mathrm{cov}(y,x) & \mathrm{cov}(y,y) & \mathrm{cov}(y,z) \\ \mathrm{cov}(z,x) & \mathrm{cov}(z,y) & \mathrm{cov}(z,z) \end{bmatrix}$$

$$A = \begin{bmatrix} 0.616 & 0.594 \\ 0.594 & 0.716 \end{bmatrix}$$

Next step is to compare the eigenvalue and eigenvector.

We know,

$|A - \lambda I| = 0$

where, λ is the eigenvalue of A.

$$\begin{bmatrix} 0.616 - \lambda & 0.594 \\ 0.594 & 0.716 - \lambda \end{bmatrix} = 0$$

So,

$\lambda^2 - (\text{sum of diagonal element})\lambda + |A| = 0$

or, $\lambda^2 - 1.332\lambda + 0.088 = 0$

Solving eqn we get,

$\lambda = 1.262, 0.0697$

Now,

Eigenvector associated with eigenvalue $(\lambda) = 1.262$

$$Ax = \lambda x.$$

$$\begin{bmatrix} 0.616 & 0.594 \\ 0.594 & 0.716 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1.262 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Solving we get,

$$0.616x_1 + 0.594x_2 = 1.262x_1$$
$$0.594x_1 + 0.716x_2 = 1.262x_2$$

So, $0.646x_1 = 0.594x_2$

$0.668x_1 = 0.716x_2$

$\therefore x_1 = 0.9969x_2$

Date: / /

We know,

$$\left\| \begin{bmatrix} 0.9969 \\ 1 \end{bmatrix} \right\| = \sqrt{(0.9969)^2 + 1^2} = 1.4120$$

$$= \begin{bmatrix} 0.9969/1.4120 \\ 1/1.4120 \end{bmatrix}$$

$$= \begin{bmatrix} 0.706 \\ 0.708 \end{bmatrix}$$

Thus, eigenvalue of 1.262 have eigenvalue.
$$\begin{bmatrix} 0.706 \\ 0.708 \end{bmatrix}$$

Also,

Eigenvector associated with the eigenvalue $(\lambda) = 0.069$

$$Ax = \lambda x.$$

$$\begin{bmatrix} 0.616 & 0.594 \\ 0.594 & 0.716 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.069 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Solving we get,

$$0.616x_1 + 0.594x_2 = 0.069x_1$$
$$0.594x_1 + 0.716x_2 = 0.069x_2$$

So, $0.547x_1 = -0.594x_2$

$$0.525x_1 = -0.716x_2$$

$\therefore x_1 = -1.222x_2$

We know,

$$\left\| \begin{bmatrix} -1.222 \\ 1 \end{bmatrix} \right\| = \sqrt{(-1.222)^2 + 1^2} = 1.579.$$

$$\begin{bmatrix} -1.222/1.579 \\ 1/1.579 \end{bmatrix} = \begin{bmatrix} -0.7739 \\ 0.6333 \end{bmatrix}$$

$\therefore$ Eigenvalue of 0.069 has eigenvector $\begin{bmatrix} -0.7739 \\ 0.6333 \end{bmatrix}$