

R-square :

X	Y	$X - \bar{X}$	$Y - \bar{Y}$	$(X - \bar{X})^2$	$(Y - \bar{Y})^2$	$(X - \bar{X}) \cdot (Y - \bar{Y})$
8	3	1.6	-4	2.56	16	-6.4
2	10	-4.4	3	19.36	9	-13.2
11	3	4.6	-4	21.16	16	-18.4
6	6	-0.4	-1	0.16	1	0.4
5	8	-1.4	1	1.96	1	-1.4
4	12	-2.4	5	5.76	25	-12
12	1	5.6	-6	31.36	36	-33.6
9	4	2.6	-3	6.76	9	-7.8
6	9	-0.4	2	0.16	4	-0.8
1	14	-5.4	7	29.16	49	-37.8
64	70	0	0	118.4	166	-131

Now,

$$\bar{X} = \frac{64}{10}$$

$$= 6.4$$

$$\bar{Y} = \frac{70}{10}$$

$$= 7$$

$$m = \frac{\sum (X - \bar{X}) \cdot (Y - \bar{Y})}{\sum (X - \bar{X})^2}$$

$$= \frac{-131}{118.4}$$

$$= -1.10$$

$$\bar{Y} = m\bar{X} + c$$

$$\text{or, } 7 = (-1.10) \times 6.4 + c$$

$$\text{or, } 7 = -7.08 + c$$

$$\text{or, } 7 + 7.08 = c$$

$$\therefore c = 14.08$$

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$mx + c$

$$(-1.10) \times 8 + 14.08 = -8.8 + 14.08 = 5.28$$

$$(-1.10) \times 2 + 14.08 = -2.2 + 14.08 = 11.88$$

$$(-1.10) \times 11 + 14.08 = -12.1 + 14.08 = 1.98$$

$$(-1.10) \times 6 + 14.08 = -6.6 + 14.08 = 7.48$$

$$(-1.10) \times 5 + 14.08 = -5.5 + 14.08 = 8.58$$

$$(-1.10) \times 4 + 14.08 = -4.4 + 14.08 = 9.68$$

$$(-1.10) \times 12 + 14.08 = -13.2 + 14.08 = 0.88$$

$$(-1.10) \times 9 + 14.08 = -9.9 + 14.08 = 4.18$$

$$(-1.10) \times 6 + 14.08 = -6.6 + 14.08 = 7.48$$

$$(-1.10) \times 1 + 14.08 = -1.10 + 14.08 = 12.98$$

$y - mx + c$

$$3 - 5.28 = -2.28$$

$$10 - 11.88 = -1.88$$

$$3 - 1.98 = -1.08$$

$$6 - 7.48 = -1.48$$

$$8 - 8.58 = -0.58$$

$$12 - 9.68 = 2.32$$

$$1 - 0.88 = 0.12$$

$$4 - 4.18 = -0.18$$

$$9 - 7.48 = 1.52$$

$$14 - 12.98 = 1.02$$

$$\therefore y - mx + c = 0.2$$

Now,

$$R^2 = \frac{[\sum (y - \bar{y})^2 - \sum (y - mx + c)^2]}{\sum (y - \bar{y})^2}$$

$$= \frac{166 - 0.2}{166}$$

$$= \frac{165.8}{166}$$

$$\therefore R^2 = 0.99$$

Note :

$$R^2 = \frac{\text{Var}(y) - \text{Var}(\text{fit})}{\text{Var}(y)}$$

where,

$$\text{Var}(x) = (x - \bar{x})^2$$

$$\text{Var}(y) = (y - \bar{y})^2$$

$$\text{Var}(\text{fit}) = \{y - (mx + c)\}^2$$

1. Calculate Karl Pearson's Coefficient of correlation from the following data using moment formula.

X	12	9	8	10	11	13	7
Y	14	8	6	9	11	12	3

Solution :

COMPUTATION OF CORRELATION COEFFICIENT

X	Y	$x = X - \bar{X}$	$y = Y - \bar{Y}$	x^2	y^2	xy
12	14	2	5	4	25	10
9	8	-1	1	1	1	1
8	6	-2	-3	4	9	6
10	9	0	0	0	0	0
11	11	1	2	1	4	1
13	12	3	3	9	9	9
7	3	-3	-6	9	36	18
$\Sigma X = 70$ $\Sigma Y = 63$				$\Sigma x^2 = 28$	$\Sigma y^2 = 84$	$\Sigma xy = 46$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{70}{7} = 10$$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{63}{7} = 9$$

Now,

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}$$

$$= \frac{46}{\sqrt{28 \cdot 84}}$$

$$= \frac{46}{\sqrt{2352}} = 0.95$$

2. Calculate the coefficient of correlation from the following data of price and demand.

Price (RS)	14	16	19	22	24	30
Demand (kg)	24	22	20	24	23	26

Solution :

COMPUTATION OF CORRELATION COEFFICIENT

Price(x)	$U = x - 19$	U^2	Demand(y)	$V = y - 23$	V^2	UV
14	-5	25	24	1	1	-5
16	-3	9	22	-1	1	3
19	0	0	20	-3	9	0
22	3	9	24	1	1	3
24	5	25	23	0	0	0
30	11	121	26	3	9	33
	$\Sigma U = 11$	$\Sigma U^2 = 189$		$\Sigma V = 1$	$\Sigma V^2 = 21$	$\Sigma UV = 34$

Now,

$$\begin{aligned}
 r &= \frac{n \Sigma UV - \Sigma U \Sigma V}{\sqrt{n \Sigma U^2 - (\Sigma U)^2} \cdot \sqrt{n \Sigma V^2 - (\Sigma V)^2}} \\
 &= \frac{6 \times 34 - 11 \times 1}{\sqrt{6 \times 189 - (11)^2} \cdot \sqrt{6 \times 21 - (1)^2}} \\
 &= \frac{204 - 11}{\sqrt{1134 - 121} \cdot \sqrt{126 - 1}} \\
 &= \frac{193}{(1013 \times 125)^{1/2}} \\
 &= 0.542.
 \end{aligned}$$

→ One of the widely used mathematical methods of calculating the correlation coefficient between two variables is Karl Pearson's Correlation Coefficient. It is known as Pearsonian Correlation Coefficient.

Interpretation of correlation coefficient (r).

→ Karl Pearson's coefficient values lies between -1 and $+1$. After getting the value of r care should be taken to interpret, otherwise wrong conclusion may be obtained. However the following general rules are mentioned for interpreting the value of r .

- i) When $r=1$, there is a positively perfect correlation between the two variables.
- ii) When $r=-1$, there is a negatively perfect correlation between the two variables.
- iii) When $r=0$, the variables are uncorrelated.
- iv) Nearer the value of r to $+1$, closer will be the relationship between two variables and nearer the value of r to 0 , lesser will be the relationship.