Exercises for Lecture Course on Optimal Control and Estimation (OCE)
Albert-Ludwigs-Universität Freiburg – Summer Term 2014

Exercise 2 - Nonlinear Programs (NLPs)

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Aim of this exercise is to get used to forming and solving NLPs. We will start by solving for the configuration of a hanging chain by minimizing energy. Then we will estimate parameters of an airplane from noisy trajectory measurements by minimizing the error between measurements and a simulated trajectory.

1 Problem of Hanging Chain

We want to model a chain hanging between two supports. Let's discretise it with N_p mass points. Each mass k has position $(x_k, y_k), k = 0 \dots N_p - 1$. Each mass is connected to the next by a link which may compress but not stretch:

$$(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2 \leq r^2, \quad k = 0 \dots N_p - 2 \tag{1}$$

where $r = 1.4 * 2/N_p$. The equilibrium point of the system minimises the gravitational potential energy, which is

$$V_{\text{chain}}(x, y) = g \sum_{i=0}^{N_p-1} m_i y_i, \tag{2}$$

Finally, there are the constraints that the ends of the chain are pinned to fixed positions:

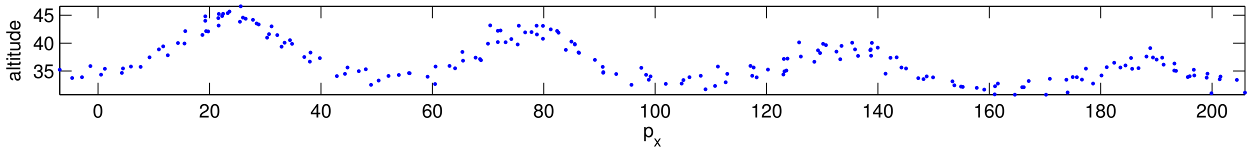
$$\begin{aligned} -1 &= x_0 && (3)\\ 1 &= x_{N_p-1} && (4)\\ 1 &= y_0 && (5)\\ 1 &= y_{N_p-1} && (6)\\ & && (7) \end{aligned}$$

Tasks:

1. Write down this problem in the standard NLP form. Is this a convex problem?
2. Solve the problem using MATLAB's `fmincon` and visualize the solution by plotting (x, y). Use $N_P = 40$. The options:
   ```
   opts = optimset('Display','iter','algorithm','interior-point','MaxFunEvals',100000);
   ```
   will probably help. Your fmincon call should look something like:
   ```
   xopt = fmincon(@fobj, x0, [], [], [], [], [], [], @nonlcon, opts);
   ```
   Don't forget to look at the terminal window and confirm that fmincon finished successfully.
3. Introduce a constraint that the chain is resting partially on a curved inclined plane:
   $$y_k \geq 0.15x_k + 0.4 + 0.3x_k^2, \quad k = 0 \dots N_p - 1 \tag{8}$$
   and re-solve. Is this a convex problem? Compare the result with the previous one.
4. Change the sign of the curvature of the plane. In other words, change the plane constraint to:
   $$y_k \geq 0.15x_k + 0.4 - 0.3x_k^2, \quad k = 0 \dots N_p - 1 \tag{9}$$
   and re-solve. Is this a convex problem? Compare the result with the previous two.

2 Paper Airplane Modeling

Using the same aircraft model from Exercise 1, I have provided a noisy data set of 20 seconds of the aircraft's flight at
`https://github.com/ghorn/OCE-2014/blob/master/exercise2/airplane_data.m`



This data set contains position measurements $\hat{p}_{x,k}$ and $\hat{p}_{z,k}$ but not velocity. Since it's possible to measure some aircraft parameters with a scale and ruler, you know that mass is 2.5, S_{sref} is 0.7, and aspect ratio AR is 14. You don't know the angle of attack α or initial state x_0 so they need to be estimated.

Task

For exercise 1, you wrote a simulation function which you can think of as taking initial state x_0 and angle of attack α as inputs, and returning the simulated states over the trajectory $\bar{x}_k = [\bar{p}_{x,k}|\bar{p}_{z,k}|\bar{v}_{x,k}|\bar{v}_{z,k}]$, $k = 0 \ldots N-1$ as outputs:

$$[\bar{x}_0, \bar{x}_1, \ldots, \bar{x}_{N-1}] = f_{\mathrm{sim}}(x_0, \alpha) \tag{10}$$

Estimate angle of attack α and initial state x_0 by solving the NLP:

$$\min_{x_0,\alpha} \sum_{k=0}^{N-1} (\bar{p}_{x,k}(x_0,\alpha) - \hat{p}_{x,k})^2 + (\bar{p}_{z,k}(x_0,\alpha) - \hat{p}_{z,k})^2$$

You may need to adjust the initial guess in order to find the correct local minimum. You may also need to play with bounds on your design variables. Plot the estimated trajectory against the noisy data.