

Exercise 4 - AD and Single Shooting

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The aim of this exercise is to implement reverse-mode AD and solve an optimal control problem with and without exact derivatives.

Dynamic System

We will use the same paper airplane system as in previous exercises. A matlab function has been provided for you which integrates the system in time, implementing:

$$x_{k+1} = f(x_k, u_k) \quad (1)$$

where $x = [p_x, p_z, v_x, v_z]$ as usual. As well as outputting x_{k+1} , this function also provides $\frac{\partial x_{k+1}}{\partial x_k}$ and $\frac{\partial x_{k+1}}{\partial u_k}$. This function is available at:

https://github.com/ghorn/OCE-2014/blob/master/exercise4/integrate_airplane_ode.m

Single Shooting NLP

We will be optimizing the following NLP:

$$\begin{aligned} & \underset{U \in \mathbb{R}^{100}}{\text{minimize}} && p_{z,N}(U) \end{aligned} \quad (2a)$$

$$\text{subject to} \quad -1^\circ \leq U_k \leq 10^\circ, \quad k = 0 \dots N-1 \quad (2b)$$

A matlab function `function [f, grad_f, X] = fobj(U)` has been provided at
<https://github.com/ghorn/OCE-2014/blob/master/exercise4/fobj.m>

This function computes $p_{z,N}$ (f) and a time history of states (X). It also returns $\frac{\partial p_{z,N}}{\partial U}$ (grad_f), but this part is incomplete - you will implement it yourself.

Tasks

1. Use `fobj.m` to solve the NLP using `fmincon` letting matlab estimate derivatives. Your `fmincon` call should look like:

```
opts = optimset('display','iter','algorithm','interior-point','MaxFunEvals',100000);  
alphasOpt = fmincon(@fobj, alphas0, [], [], [], [], lb, ub, [], opts);  
Plot  $p_x$  vs  $-p_z$  and  $\alpha$  vs time.
```

2. Using reverse mode AD, complete the missing part of `fobj.m` to compute `grad_f`.

3. Solve the NLP with `fobj.m` and `fmincon` again, this time using exact derivatives. Your `fmincon` call should look like:

```
opts = optimset('GradObj','on','display','iter','algorithm','interior-point');  
alphasOpt = fmincon(@fobj, alphas0, [], [], [], [], lb, ub, [], opts);  
Plot  $p_x$  vs  $-p_z$  and  $\alpha$  vs time.
```