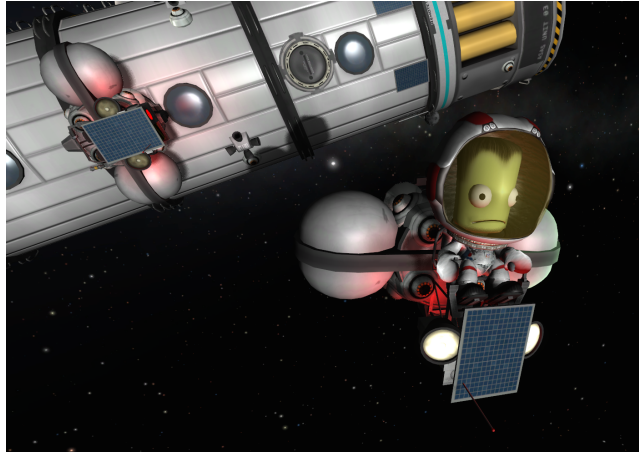## Exercise 7 - Time Optimal Direct Multiple Shooting

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The aim of this exercise is to setup and solve an optimal control problem using direct multiple shooting.

Minimum time Kerbal Retrieval

Jebediah Kerman has gone for an EVA and lost track of time. He can't remember when atmospheric re-entry is scheduled, but he believes it is very soon. He needs to get back to his spaceship as quickly as possible.

Jebediah has mass 30kg including space suit but not including fuel. He is currently carrying 10kg of fuel. He is 50m away from his ship, with zero relative velocity. He wants to return to the ship as quickly as possible (to have equal position and zero relative velocity), while still conserving 4kg of fuel for emergencies.

Model the Kerbal as having three states: position p, velocity v, and fuel mass m_F. The space suit has a rocket booster (control u) which can fire forwards or reverse, and the differential equation governing the state is:

$$\frac{d}{dt} \begin{pmatrix} p \\ v \\ m_F \end{pmatrix} = \begin{pmatrix} v \\ u/\left(30 + m_F\right) \\ -u^2 \end{pmatrix} \tag{1}$$

Tasks

1. Write down the continuous time optimal control problem.

2. Discretize this problem using direct multiple shooting, and write down the NLP.

3. Using an RK4 integrator, implement this NLP and solve it with `fmincon`. Use number of control intervals $N = 40$. Use nonlinear constraints ONLY for the dynamics constraints. For all other constraints, use lower and upper bounds on design variables (this includes equality constraints where upper bounds equal lower bounds).

4. Plot p, v, m_F, and u versus time.