Exercises for Lecture Course on Optimal Control and Estimation (OCE)
Albert-Ludwigs-Universität Freiburg – Summer Term 2014

Exercise 6 - Dynamic Programming

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In this exercise, we use dynamic programming to solve a linear-quadratic OCP.

Dynamic System

Throughout this exercise sheet, we regard the discrete time damped-spring system

$$x_{k+1} = \begin{pmatrix} 1 & 0.02 \\ -0.1 & 0.992 \end{pmatrix} x_k + \begin{pmatrix} 0 \\ 0.02 \end{pmatrix} u_k \tag{1}$$

over the horizon of $N = 600$, with initial state $x_0 = [10, 0]$.

Tasks

1. Simulate and plot the uncontrolled system ($u = 0$) as a baseline.
2. Using dynamic programming, minimize the cost function:

$$\sum_{k=0}^{N-1} \left(x_k^T Q x_k + u_k^T R u_k \right) + x_N^T P_N x_N \tag{2}$$

   with

$$Q = \begin{pmatrix} \frac{1}{2^2} & 0 \\ 0 & \frac{1}{3^2} \end{pmatrix}$$
$$R = \left(\frac{1}{6^2} \right)$$
$$P_N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

   Plot the two states and control against the uncontrolled system.

3. Consider the infinite-horizon system ($N \to \infty$) with cost function:

$$\sum_{k=0}^{\infty} \left(x_k^T Q x_k + u_k^T R u_k \right) \tag{3}$$

   What control policy will minimize this cost function? Implement this control policy and simulate for $N = 600$. Plot this in state and control against the previous two trajectories.