needs. A generalized micro data tabulating system, if made accessible to the research analyst, would be a powerful research tool. The Australian system can be used for research. In fact, we have used it in BLS. Earlier I cited a BLS research project which would have used a wide range of cross tabulations of labor force household reports (micro data) which traditional methods would not permit without costly programming. In point of fact, using the Australian program we derived precisely the comparisons mentioned, and many others besides, at a fraction of the cost of ad hoc programming. And, to deal with another point left dangling earlier, the system allowed us to expand sample tallies to universe statistics (estimates) by applying the blow-up factor to each case.

Screening

The last milestone, marking a return to our starting point, is the screening or editing module. I know of no generalized solution of this programming problem. And I do not know if any one has tackled it, although we are considering the problem. My hunch is that the secret of a generalized screening program lies at the heart of the general micro tabulation program, since the underlying operations are similar.

A Generalized System

Where do we stand? Some might say that the landscape is not nearly as bleak as I painted it at the outset—that is, my complaint about reinventing the wheel for each new major survey may not be valid. For, if we do have general solutions for tabulation of micro data, calculation of summary figures, storing these, retrieving and analyzing, why not use them? Programs are often transferred from one installation to another.

Here is why we can't: These general programs are very complex and represent an extension of the art of using computers. They require all the power a computer has and therefore are written, in part or entirely, in the language of the machine at the center where they were developed. Thus, if we had a "clean" micro file and wished to use present generalized programs, we could run a tabulation on a CDC 3600, store summary data and retrieve them on an IBM 7074 and analyze the results on a UNIVAC 1108—an imposing array of equipment but hardly one the typical statistician can afford.

Conclusion

My picture of the principles of processing statistical data is both incomplete and over simplified. Real life is admittedly more difficult and complex. I did not discuss, for example, sample selection, mail and control of schedules, and similar activities. Nor did I recognize that survey conditions vary and that these may impose special requirements which are not met in the general modules.

Nevertheless, I believe that my main point is valid. Having learned to process large files to gain a predefined goal, we survey statisticians now need to change directions to keep pace with the promise of the electronic computer. Rather than tailoring each special job when it arises by ad hoc means we should program the computer to do the work which most surveys have in common and then deal with the difference among surveys as they arise. There is a double profit in such a course. We would conserve resources and offer ourselves and our colleagues in other social sciences a better chance to explore the masses of data which we have collected.

Interpretation of the Kurtosis Statistic

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Abstract

A description of the kurtosis statistic has long been overlooked by authors in statistics and measurement. This study illustrates how the kurtosis statistic may be correctly interpreted when it is computed for approximately normal, rectangular, and bimodal distributions of measures. The results of the study point out that in rectangular distributions the kurtosis value is approximately -1.20 to -1.25. Other results point out that the number of cases in the tails of the distribution drastically affect the kurtosis value. Truncation in the tails of a distribution may cause the kurtosis value to change from positive to negative with only the deletion of a small number of cases. Finally, the results point out the fact that a perfectly bimodal distribution of measures has a kurtosis value of -2.00.

In the study of distributions of measures the first two moments, μ, the mean, and σ^2, the variance, are considered at length by most textbooks dealing with statistics. The third moment, α_3, the skewness, is given some emphasis, but usually only enough to enable the student to distinguish between a positive and a negative skew. The fourth moment of a distribution, α_4, the kurtosis, is typically given only limited treatment, and leaves much to be explained. The typical presentation appears to be complete if the student is informed that leptokurtic and platykurtic distributions are peaked and flat respectively.

It is the purpose of this paper to expand the somewhat limited knowledge on the subject of kurtosis. Further, this paper is designed to provide practitioners and students of educational research with enough knowledge of the fourth moment statistic, kurtosis, so that it may be interpreted correctly.

The first point to be made in this discussion is the use of an appropriate formula for kurtosis. Kurtosis is defined as: The ratio of the average of the fourth power of the deviations from the mean, to the square of the variance.

Symbolically it is:

$$\alpha_4 = \frac{\Sigma(x - \mu)^4/N}{\sigma^4}$$

Interestingly enough, when the above formula is applied the value for a normal distribution turns out to be equal to $+3$. To be in accord with the value of skewness (α_3) for a normal distribution which is zero (0), the formula for kurtosis includes a corrective factor of -3. The use of the corrective factor in computing kurtosis has the effect of making both skewness and kurtosis equal to zero for a normal distribution of measures and aids in the interpretation of both statistics.

Derivations of the formula for kurtosis have been set forth by Gulliksen (1950) and Horst (1966) in writings dealing with measurement applied to education. The formula used for computations is:

$$\alpha_4 = \frac{\Sigma(x - \mu)^4/N}{\sigma^4} - 3$$

With the knowledge that the kurtosis of a normal distribution as derived by the above formula is equal to zero, leptokurtic and platykurtic distributions have been defined in terms of deviations from the normal distribution. Thus, the usual definitions are:

Leptokurtic—A distribution that is "peaked,"

$$\alpha_4 > 0,$$

and

Platykurtic—A distribution that is "flat," $\alpha_4 < 0$.

Nonetheless, it is shown here that an attempt to include the kurtosis value in the definition of the distribution may lead to erroneous assumptions about the distribution. That is, defining a leptokurtic distribution as a distribution with a positive kurtosis value may not be accurate in all cases. This paper shows that leptokurtic ("peaked") distributions do not always fit the usual definition given above.

Method

In order to illustrate various characteristics of the kurtosis statistic, predetermined score distributions were used as input data for computer analyses. Histograms to illustrate the variations in the kurtosis of leptokurtic, platykurtic, and bimodal distributions and computation of all the kurtosis values were obtained from runs of a computer program.

Leptokurtic Distributions

As an orientation point, consider a perfectly symmetric distribution of measures that approximates a normal distribution. To this distribution a great many cases are added to the middle of the distribution causing a peak to occur (See Figure 1). The kurtosis value changes from zero for the normal distribution to $+.62$ indicating a leptokurtic distribution.

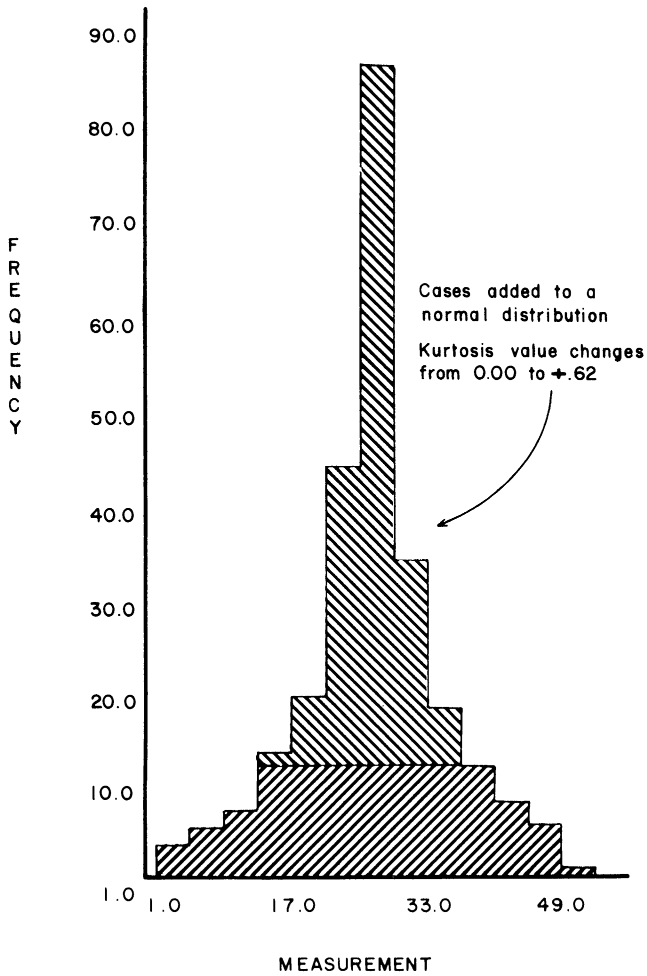

Figure 1. Illustrating the increase in kurtosis with the addition of cases to the middle of the distribution creating a peaked distribution of measurements.

Now, if the bottom row of the distribution is duplicated, (one equal frequency at each existing value along the horizontal axis is added), this has the effect of truncating the tails of the distribution. The kurtosis value is reduced from $+.62$ to $+.17$ (See Figure 2). As more rows are added and truncation in the tails becomes more pronounced, the kurtosis value becomes smaller and smaller. This serves to illustrate an often overlooked point about kurtosis. In order to have a positive kurtosis value the distribution of measures must not only be peaked, but must contain a good

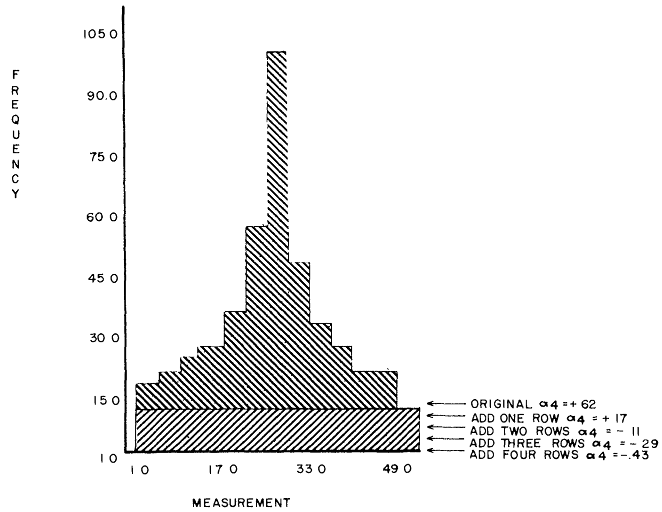

Figure 2. Illustrating the decrease in kurtosis when the bottom row of a peaked distribution is duplicated.

number of cases in the tails, i.e. a tailing off effect must be present. The mathematical explanation is that the denominator of the kurtosis formula, the variance squared, increases faster than the numerator, thus reducing the kurtosis value. These results point out that the definition of a leptokurtic distribution should not include the specification that the kurtosis value be positive.

Platykurtic Distributions

A second useful orientation point is a perfectly rectangular distribution of measures. This type of distribution is, by our definition, platykurtic.

The next distribution of scores to consider is in the shape of a horizontal rectangle for which the kurtosis is equal to -1.203. (It is interesting to note that most perfectly rectangular distributions considered in educational research have a kurtosis value of approximately -1.20 to -1.25.) If cases are added to either end of the rectangle, the kurtosis value will increase (See Figure 3). That is, the value will tend to the positive. If equal rows are added to the rectangular

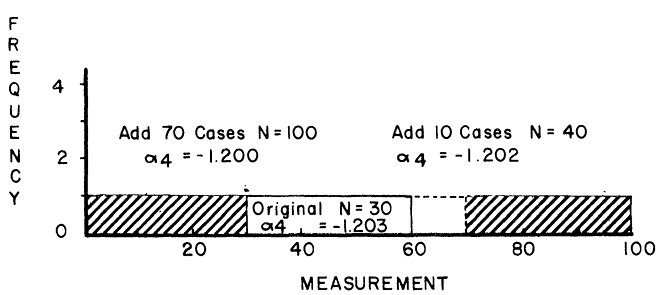

Figure 3. Illustrating the increase in kurtosis with the addition of cases horizontally to a perfectly rectangular distribution of measurements.

distribution, the kurtosis will remain constant (See Figure 4). If a rectangular distribution of measures that is oriented vertically is considered, it might be defined as a peaked distribution. This vertical orientation can result in a misinterpretation of kurtosis. It is a fact that for all perfectly rectangular distributions, no matter how oriented, the kurtosis value will fall approximately in the range -1.20 to -1.25. This value will be considerably smaller if the distribution does not have a reasonable number of intervals along the horizontal axis. Table 1 summarizes the change in kurtosis value that occurs when only a small number of intervals are used for the rectangular distribution.

These examples serve to illustrate some seldom considered points about rectangular distributions, and reemphasize the importance of the tails of the distribution in determining kurtosis.

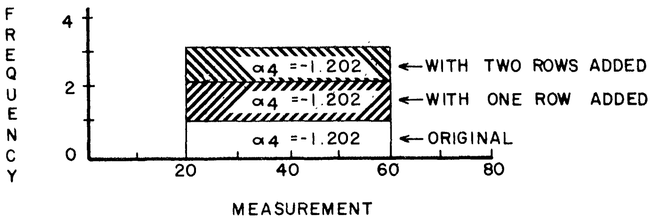

Figure 4. Illustrating the constancy of kurtosis with the addition of cases vertically to a perfectly rectangular distribution of measurements.

TABLE 1

Increase in Kurtosis Resulting from an Increase in the Number of Intervals in a Distribution

Number of Whole Number Values Using Consecutive Integers	Kurtosis Value
2	-2.00
3	-1.50
4	-1.36
5	-1.30
6	-1.27
7	-1.25

Bimodal Distributions

For a perfectly symmetrical bimodal distribution (equal frequencies for two different values of the measure), the kurtosis value is a constant -2.00. This means that in the formula for kurtosis $\Sigma(x - \mu)^4/N$ is equal to σ^4, and the result is -2.00. That is, the numerator and the denominator are equal, yielding a value of $+1$. When 3 is subtracted from it the result is -2.00. This result is illustrated in both Figure 5

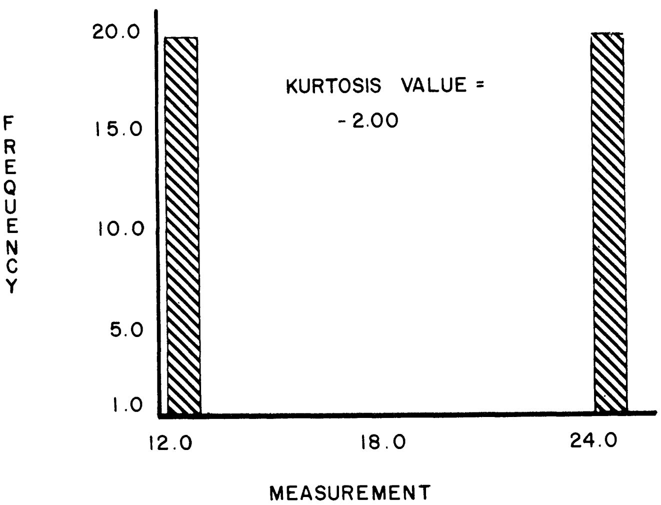

Figure 5. Illustrating the constant kurtosis value of −2.00 for a perfectly symmetrical bimodal distribution of measurements.

and Table 1. A more representative example of a perfectly symmetrical bimodal distribution is depicted in Figure 6. As cases are added to either side of the two modes, the kurtosis value increases, again illustrating the importance of the distribution tails in the computation of the kurtosis statistic.

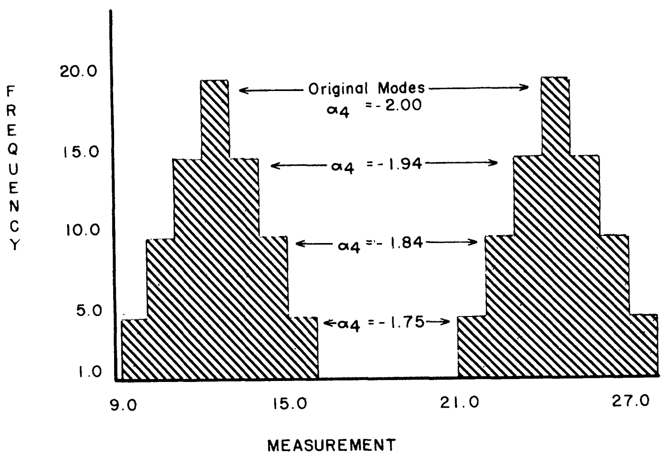

Figure 6. Illustrating the increase in kurtosis with the addition of cases to the modes of a perfectly symmetrical bimodal distribution of measurements.

Summary

It is important to remember that kurtosis is dependent upon the peak in the distribution and the distribution tails, and a major emphasis must be placed on the tails of the distribution in the determination of the fourth moment. Without a tailing off of the distribution tails, an adequate representation and subsequent interpretation of the kurtosis statistic is difficult to obtain. The effect of truncated distribution tails is a point well worth remembering when considering kurtosis. The peakedness of a distribution may also be falsely interpreted if only the kurtosis statistic is considered due to the effects of truncation.

Along with the effect of truncation, the effect of a bimodal distribution on the kurtosis value is of importance. It is difficult to determine the shape of a distribution from the kurtosis value alone, since almost any distribution may have a negative kurtosis value. Therefore, when interpreting the kurtosis statistic the interpreter should be careful to consider more than the kurtosis value of a distribution before labeling a distribution as leptokurtic or platykurtic.

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