Jakub Kowalski, 323519; Zad 4.

$f_1(n) = n(\log n)^2$

$f_2(n) = n^2/10$

$f_3(n) = \log_2 n + 2^{10} \cdot n$

c) • $f_3(n) = O(f_1(n))$ i • $f_1(n) = O(f_2(n))$

• $f_3(n) = O(f_1(n))$

$$\lim_{n\to\infty} \frac{f_3(n)}{f_1(n)} = \lim_{n\to\infty} \frac{\log n + 2^{10} \cdot n}{n \cdot \log n \cdot \log n} = \lim_{n\to\infty} \frac{\overbrace{\frac{1}{n}}^{\to 0} + 2^{10} \cdot \overbrace{\frac{1}{\log n}}^{\to 0}}{\cancel{n \cdot \log n} \cdot \log n} =$$

$$= \lim_{n\to\infty} \frac{\overbrace{2^{10}}^{c = \text{stała}}}{\underbrace{\log n}_{\to\infty}} < \infty, \text{ zatem } f_3(n) = O(f_1(n)),$$

(od pewnego n $\log n < \sqrt{n}$
$\log^2 n < n$)
⇓

•• $f_1(n) = O(f_2(n))$

$$\lim_{n\to\infty} \frac{f_1(n)}{f_2(n)} = \lim_{n\to\infty} \frac{n \cdot \log^2 n}{n^2/10} = \lim_{n\to\infty} \frac{\log^2 n}{n/10} < \infty$$

zatem $f_1(n) = O(f_2(n))$.