Home assignment 4

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The strict deadline to submit your solution is November, 12 20:00 MSK.
Template to compose a solution is available at
`https://www.overleaf.com/read/vknkchxdwsmk`
The submission form is `https://forms.gle/55cHaUr6FJ78jbpy6`

1. Negative entropy with linear constraints

Derive the dual for the following problem. Does the strong duality hold and why? Compare the dimensions and feasible sets of the primal and dual problems and discuss what problem (dual or primal) is easier to solve.

$$\begin{aligned} \min_{\mathbf{x} \in \mathbb{R}^n} & \sum_{i=1}^n x_i \log x_i \\ \text{s.t. } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{1}^\top \mathbf{x} = 1 \end{aligned}$$

2. Toy problem

Derive the dual problem, solve it and reconstruct the solution of the primal problem from the solution of the dual one.

$$\begin{aligned} \min_{(x,y,z)} & \frac{1}{2}x^2 + 2y^2 + \frac{1}{2}z^2 + x + y + 2z \\ \text{s.t. } & x + 2y + z = 4 \end{aligned}$$

3. Lagrange relaxation of binary linear programming

Derive the dual problem of the following problem:

$$\begin{aligned} \min_{\mathbf{x} \in \mathbb{R}^n} & \mathbf{c}^\top \mathbf{x} \\ \text{s.t. } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & x_i \in \{0, 1\}, \quad i = 1, \dots, n \end{aligned}$$

Prove that the lower bound to p^* derived from the dual problem is equal to the lower bound obtained by replacing constraints $x_i \in \{0, 1\}$ with convexified constraints $0 \leq x_i \leq 1$.