Optimization methods.
Seminar 5. Optimality conditions, vol. 1

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Unconstrained minimization problem

Problem: $f(x) \to \min\limits_{x \in \mathbb{R}^n}$.

Theorem

If $f(x)$ is convex and differentiable, then x^* is a solution of the problem iff $\nabla f(x^*) = 0$.

Sufficient condition for non-convex functions

Let $f : \mathbb{R}^n \to \mathbb{R}$ be twice differentiable and x^* such that $\nabla f(x^*) = 0$. If $\nabla^2 f(x^*) \succ 0$, then x^* is a strict local minimizer.

Examples

- $x_1 e^{x_1} - (1 + e^{x_1}) \cos x_2 \to \min$
- Rosenbrock function:
  $(1 - x_1)^2 + \alpha \sum_{i=2}^{n} (x_i - x_{i-1}^2)^2 \to \min, \ \alpha > 0$
- $x_1^2 + x_2^2 - x_1 x_2 + e^{x_1 + x_2} \to \min$
- $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2$
- $\min_{\mathbf{x}} \|\mathbf{x}\|_1 + \lambda \|\mathbf{x} - \mathbf{y}\|_2^2$

Recap

- Optimality conditions for unconstrained minimization problem
- Transformation of non-smooth problem to smooth one