Home assignment 1

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The strict deadline to submit your solution is October 18, 01:00 MSK. Template to compose a solution is available at
`https://www.overleaf.com/read/vknkchxdwsmk`

1. Problem 1: traveler search (1 pts)

Assume you are given the approximate distances between traveler and m objects. The coordinates of these objects are also given in some coordinate system. State the optimization problem (i.e. define the objective function, target variable, constraints if any and compose the optimization problem) to identify coordinates of the traveler location.[1]

2. Problem 2: shops and storages (2 pts)

State the optimization problem to answer the question below.

Let a_i be a number of goods items placed in the i-th storage. These goods have to be moved to m shops. The j-th shop requires b_j items of goods. The movement of the unit of goods from the i-th storage to the j-th shop costs c_{ij} and requires t_{ij} time units. How many items of goods have to be moved from every shop to supply all shops with the requested number of items?

3. Voronoi diagram (3.5 pts)

Let $\mathbf{x}_1, \ldots, \mathbf{x}_k \in \mathbb{R}^n$ be given points. Consider a set of points such that they are closer to $\mathbf{x}_0$ than to the points $\mathbf{x}_1, \ldots, \mathbf{x}_k$:

$$V = \{\mathbf{x} \in \mathbb{R}^n \mid \|\mathbf{x} - \mathbf{x}_0\|_2 \leq \|\mathbf{x} - \mathbf{x}_i\|_2, \ i = 1, \ldots, k\}$$

A set V is called Voronoi region for the point $\mathbf{x}_0$.

- (1 pts) Proof that the set V is polytope. Represent it in the form $\{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{A}\mathbf{x} \leq \mathbf{b}\}$. Notation $\mathbf{x} \leq \mathbf{y}$ means elementwise comparison.
- (1 pts) Show how to recover points $\mathbf{x}_0, \ldots, \mathbf{x}_k$ from the given polytope that is a Voronoi region for these points.
- (1 pts) Make a generalization of the Voronoi region to $p > 1$ points. Assume you are given a set of polytopes (possibly open) that composes a partition of the space. Show how to find points such that this set is a set of Voronoi regions.

[1] The same principle is a base for GPS system

- (0.5 pts) Choose any field of science that you are interested in and describe how Voronoi regions can be used in it. Wikipedia page can inspire you[2]

4. Problem 4: convexity and affinity (1.2 pts)

Check the convexity and affinity of the following sets. Explain your answer as clear as possible.

1. (0.2 pts) $\{\mathbf{x} \in \mathbb{R}^n \mid \alpha \leq \mathbf{a}^\top \mathbf{x} \leq \beta\}$
2. (0.2 pts) $\{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{a}_1^\top \mathbf{x} \leq b_1,\ \mathbf{a}_2^\top \mathbf{x} \leq b_2\}$
3. (0.4 pts) $\{\mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_0\|_2 \leq \|\mathbf{x} - \mathbf{y}\|_2,\ \forall \mathbf{y} \in S \subseteq \mathbb{R}^n\}$
4. (0.4 pts) A set of points such that the distance between them and given point $\mathbf{a}$ is less or equal than $\theta \in [0, 1]$ multiplied by distances between these points and point $\mathbf{b}$: $\{\mathbf{x} \mid \|\mathbf{x} - \mathbf{a}\|_2 \leq \theta \|\mathbf{x} - \mathbf{b}\|_2\},\ \mathbf{a} \neq \mathbf{b}$

5. Problem 5: Minkowski sum (1 pts)

1. (0.5 pts) Describe formally set $M = M_1 + M_2$, where $M_1 = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 + x_2^2 = 1\}$, and $M_2 = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 + x_2^2 = 4\}$
2. (0.5 pts) Provide two sets A and B such that at least one of them is non-convex, but the sum if these sets $A + B$ is convex.

[2] https://en.wikipedia.org/wiki/Voronoi_diagram#Applications