Optimization methods. Seminar 4. Introduction to duality

Alexandr Katrutsa

Moscow Institute of Physics and Technology

November 1, 2020

Examples

- Solution of linear system with minimal norm

$$\min \|\mathbf{x}\|_2^2$$
$$\text{s.t. } \mathbf{A}\mathbf{x} = \mathbf{b}$$

- Linear programming

$$\min \mathbf{c}^\top \mathbf{x}$$
$$\text{s.t. } \mathbf{A}\mathbf{x} = \mathbf{b}$$
$$\mathbf{x} \geq 0$$

- Partition problem

$$\min \mathbf{x}^\top \mathbf{W} \mathbf{x}$$
$$\text{s.t. } x_i^2 = 1,\ i = 1, \ldots, n$$

- Semidefinite programming problem

$$\min_{\mathbf{X}} \operatorname{trace}(\mathbf{C}\mathbf{X})$$
$$\text{s.t. } \operatorname{trace}(\mathbf{A}_i \mathbf{X}) = b_i$$
$$\mathbf{X} \succeq 0,$$

More examples

- Negative entropy with linear constraints

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{R}^n} & \sum_{i=1}^{n} x_i \log x_i \\ \text{s.t. } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{1}^\top \mathbf{x} = 1 \end{aligned}$$

- Toy problem

$$\begin{aligned} \min \ & \frac{1}{2}x^2 + 2y^2 + \frac{1}{2}z^2 + x + y + 2z \\ \text{s.t. } & x + 2y + z = 4 \end{aligned}$$

- Lagrange relaxation of binary linear programming:

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{R}^n} & \ \mathbf{c}^\top \mathbf{x} \\ \text{s.t. } & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & x_i \in \{0,1\}, \quad i = 1, \ldots, n \end{aligned}$$