Feedback — Week 3 - Problem Set

Help

You submitted this homework on **Fri 18 Apr 2014 1:08 PM PDT**. You got a score of **10.00** out of **10.00**.

Question 1

Suppose a MAC system (S, V) is used to protect files in a file system by appending a MAC tag to each file. The MAC signing algorithm S is applied to the file contents and nothing else. What tampering attacks are not prevented by this system?

Your Answer		Score	Explanation
Changing the first byte of the file contents.			
Changing the last modification time of a file.	✔	1.00	The MAC signing algorithm is only applied to the file contents and does not protect the file meta data.
Replacing the tag and contents of one file with the tag and contents of a file from another computer protected by the same MAC system, but a different key.			
Appending data to a file.			
Total		1.00 / 1.00	

Question 2

Let (S, V) be a secure MAC defined over (K, M, T) where $M = \{0,1\}^n$ and $T = \{0,1\}^{128}$ (i.e. the key space is K, message space is $\{0,1\}^n$, and tag space is $\{0,1\}^{128}$). Which of the following is a secure MAC: (as usual, we use $\|$ to denote string concatenation)

Your Answer		Score	Explanation
☐ $S'(k,m) = \begin{cases} S(k,1^n) & \text{if } m = 0^n \\ S(k,m) & \text{otherwise} \end{cases}$ and $V'(k,m) = \begin{cases} V(k,1^n,t) & \text{if } m = 0^n \\ V(k,m,t) & \text{otherwise} \end{cases}$	✔	0.17	This construction is insecure because an adversary can request the tag for the message 0^n and output the result as a valid forgery for the message 1^n.
☑ $S'((k_1,k_2),\ m) = \big(S(k_1,m), S(k_2,m)\big)$ and $V'\big((k_1,k_2),m,(t_1,t_2)\big) = \big[V(k_1,m,t_1) \text{ and } V(k_2,m,t_2)\big)$ (i.e., $V'\big((k_1,k_2),m,(t_1,t_2)\big)$outputs ``1'' if both t_1 and t_2 are valid tags)	✔	0.17	a forger for (S',V') gives a forger for (S,V).
☑ $S'(k,\ m) = \big[t \leftarrow S(k,m),\ \text{output } (t,t)\ \big)$ and $V'\big(k,m,(t_1,t_2)\big) = \begin{cases} V(k,m,t_1) & \text{if } t_1 = t_2 \\ \text{"0"} & \text{otherwise} \end{cases}$ (i.e., $V'\big(k,m,(t_1,t_2)\big)$only outputs "1" if t_1 and t_2 are equal and valid)	✔	0.17	a forger for (S',V') gives a forger for (S,V).
☐ $S'(k,m) = S(k,\ \ m[0,\ldots,n-2]\Vert 0)$ and $V'(k,m,t) = V(k,\ \ m[0,\ldots,n-2]\Vert 0,\ \ t)$	✔	0.17	This construction is insecure because the tags on $m = 0^n$ and $m = 0^{n-1}1$ are the same. Consequently, the attacker can request the tag on $m = 0^n$ and

			output an existential forgery for $m = 0^{n-1}1$.
☑ $S'(k,m) = S(k,m)[0,\ldots,126]$ and $V'(k,m,t) = \left[V(k,\ m,\ t\|0) \text{ or } V(k,\ m,\ t\|1)\ \right]$ (i.e., $V'(k,m,t)$ outputs ``1" if either $t\|0$ or $t\|1$ is a valid tag for m)	✔	0.17	a forger for (S', V') gives a forger for (S, V).
☐ $S'(k,m) = S(k,m)$ and $V'(k,m,t) = \left[V(k,\ m,\ t) \text{ or } V(k,\ m \oplus 1^n,\ t)\right]$ (i.e., $V'(k,m,t)$ outputs ``1" if t is a valid tag for either m or $m \oplus 1^n$)	✔	0.17	This construction is insecure because a valid tag on $m = 0^n$ is also a valid tag on $m = 1^n$. Consequently, the attacker can request the tag on $m = 0^n$ and output an existential forgery for $m = 1^n$.
Total		1.00 / 1.00	

Question 3

Recall that the ECBC-MAC uses a fixed IV (in the lecture we simply set the IV to 0). Suppose instead we chose a random IV for every message being signed and include the IV in the tag. In other words, $S(k,m) := \left(r,\ \ \mathrm{ECBC}_r(k,m)\right)$ where $\mathrm{ECBC}_r(k,m)$ refers to the ECBC function using r as the IV. The verification algorithm V given key k, message m, and tag (r,t)

outputs ``1'' if $t = \mathrm{ECBC}_r(k, m)$ and outputs ``0'' otherwise.

The resulting MAC system is insecure. An attacker can query for the tag of the 1-block message m and obtain the tag (r, t). He can then generate the following existential forgery: (we assume that the underlying block cipher operates on n-bit blocks)

Your Answer		Score	Explanation
The tag $(r,\ t \oplus r)$ is a valid tag for the 1-block message 0^n.			
The tag $(m \oplus t,\ t)$ is a valid tag for the 1-block message 0^n.			
The tag $(r \oplus 1^n,\ t)$ is a valid tag for the 1-block message $m \oplus 1^n$.	✔	1.00	The CBC chain initiated with the IV $r \oplus m$ and applied to the message 0^n will produce exactly the same output as the CBC chain initiated with the IV r and applied to the message m. Therefore, the tag $(r \oplus 1^n,\ t)$ is a valid existential forgery for the message $m \oplus 1^n$.
The tag $(r \oplus t,\ r)$ is a valid tag for the 1-block message 0^n.			
Total		1.00 / 1.00	

Question 4

Suppose Alice is broadcasting packets to 6 recipients $B_1, \ldots, B_6$. Privacy is not important but integrity is. In other words, each of $B_1, \ldots, B_6$ should be assured that the packets he is receiving were sent by Alice.

Alice decides to use a MAC. Suppose Alice and $B_1, \ldots, B_6$ all share a secret key k. Alice computes a tag for every packet she sends using key k. Each user B_i verifies the tag when receiving the packet and drops the packet if the tag is invalid. Alice notices that this scheme is insecure because user B_1 can use the key k to send packets with a valid tag to users $B_2, \ldots, B_6$ and they will all be fooled into thinking that these packets are from Alice.

Instead, Alice sets up a set of 4 secret keys $S = \{k_1, \ldots, k_4\}$ She gives each user B_i some subset $S_i \subseteq S$ of the keys. When Alice transmits a packet she appends 4 tags to it by computing the tag with each of her 4 keys. When user B_i receives a packet he accepts it as valid only if all tags corresponding to his keys in S_i are valid. For example, if user B_1 is given keys $\{k_1, k_2\}$ he will accept an incoming packet only if the first and second tags are valid. Note that B_1 cannot validate the 3rd and 4th tags because he does not have k_3 or k_4.

How should Alice assign keys to the 6 users so that no single user can forge packets on behalf of Alice and fool some other user?

Your Answer		Score	Explanation
$S_1 = \{k_2, k_4\},\ \ S_2 = \{k_2, k_3\},\ \ S_3 = \{k_3, k_4\},\ \ S_4 = \{k_1, k_3\},$	✔	1.00	Every user can only generate tags with the two keys he has. Since no set S_i is contained in another set S_j, no user i can fool a user j into accepting a message sent by i.
$S_1 = \{k_1\},\ \ S_2 = \{k_2, k_3\},\ \ S_3 = \{k_3, k_4\},\ \ S_4 = \{k_1, k_3\},\ \ S_5$			
$S_1 = \{k_1, k_2\},\ \ S_2 = \{k_1, k_3\},\ \ S_3 = \{k_1, k_4\},\ \ S_4 = \{k_2, k_3, k_4$			
$S_1 = \{k_1, k_2\},\ \ S_2 = \{k_2, k_3\},\ \ S_3 = \{k_3, k_4\},\ \ S_4 = \{k_1, k_3\},$			

Total	1.00 / 1.00

Question 5

Consider the encrypted CBC MAC built from AES. Suppose we compute the tag for a long message m comprising of n AES blocks. Let m' be the n-block message obtained from m by flipping the last bit of m (i.e. if the last bit of m is b then the last bit of m' is $b \oplus 1$). How many calls to AES would it take to compute the tag for m' from the tag for m and the MAC key? (in this question please ignore message padding and simply assume that the message length is always a multiple of the AES block size)

Your Answer		Score	Explanation
2			
5			
4	✔	1.00	You would decrypt the final CBC MAC encryption step done using k_2, the decrypt the last CBC MAC encryption step done using k_1, flip the last bit of the result, and re-apply the two encryptions.
n			
Total		1.00 / 1.00	

Question 6

Let $H: M \to T$ be a collision resistant hash function. Which of the following is collision resistant: (as usual, we use $\|$ to denote string concatenation)

Your Answer		Score	Explanation
$H'(m) = H(m) \| H(m)$	✔	0.14	a collision finder for H' gives a collision finder for H.

☐ $H'(m) = H(0)$	✔	0.14	This construction is not collision resistant because $H(0) = H(1)$
☑ $H'(m) = H(m) \| H(0)$	✔	0.14	a collision finder for H' gives a collision finder for H.
☑ $H'(m) = H(m \| 0)$	✔	0.14	a collision finder for H' gives a collision finder for H.
☐ $H'(m) = H(\lvert m \rvert)$ (i.e. hash the length of m)	✔	0.14	This construction is not collision resistant because $H(000) = H(111)$
☐ $H'(m) = H(m) \bigoplus H(m \oplus 1^{\lvert m \rvert})$ (where $m \oplus 1^{\lvert m \rvert}$ is the complement of m)	✔	0.14	This construction is not collision resistant because $H(000) = H(111)$
☐ $H'(m) = H(m[0, \ldots, \lvert m \rvert - 2])$ (i.e. hash m without its last bit)	✔	0.14	This construction is not collision resistant because $H(00) = H(01)$
Total		1.00 / 1.00	

Question 7

Suppose H_1 and H_2 are collision resistant hash functions mapping inputs in a set M to $\{0,1\}^{256}$. Our goal is to show that the function $H_2(H_1(m))$ is also collision resistant. We prove the contra-positive: suppose $H_2(H_1(\cdot))$ is not collision resistant, that is, we are given $x \neq y$ such that $H_2(H_1(x)) = H_2(H_1(y))$ We build a collision for either H_1 or for H_2. This will prove that if H_1 and H_2 are collision resistant then so is $H_2(H_1(\cdot))$. Which of the following must be true:

Your Answer	Score	Explanation
○ Either x, y are a collision for H_2 or $H_1(x), H_1(y)$ are a collision for H_1.		
○ Either $x, H_1(y)$		

are a collision for H_2 or $H_2(x), y$ are a collision for H_1.			
Either x, y are a collision for H_1 or $H_1(x), H_1(y)$ are a collision for H_2.	✔	1.00	If $H_2(H_1(x)) = H_2(H_1(y))$ then either $H_1(x) = H_1(y)$ and $x \neq y$, thereby giving us a collision on H_1. Or $H_1(x) \neq H_1(y)$ but $H_2(H_1(x)) = H_2(H_1(y))$ giving us a collision on H_2. Either way we obtain a collision on H_1 or H_2 as required.
Either x, y are a collision for H_1 or x, y are a collision for H_2.			
Total		1.00 / 1.00	

Question 8

In this question and the next, you are asked to find collisions on two compression functions:

- $f_1(x, y) = \mathrm{AES}(y, x) \bigoplus y$ and
- $f_2(x, y) = \mathrm{AES}(x, x) \bigoplus y$

where $\mathrm{AES}(x, y)$ is the AES-128 encryption of y under key x.

We provide an AES function for you to play with. The function takes as input a key k and an x value and outputs $\mathrm{AES}(k, x)$ once you press the "encrypt" button. It takes as input a key k and a y value and outputs $\mathrm{AES}^{-1}(k, y)$ once you press the "decrypt" button. All three values k, x, y are assumed to be hex values (i.e. using only characters 0-9 and a-f) and the function zero-pads them as needed.

Your goal is to find four distinct pairs $(x_1, y_1), \ (x_2, y_2), \ (x_3, y_3), \ (x_4, y_4)$ such that $f_1(x_1, y_1) = f_1(x_2, y_2)$ and $f_2(x_3, y_3) = f_2(x_4, y_4)$. In other words, the first two pairs are a collision for f_1 and the last two pairs are a collision for f_2. Once you find all four pairs, please enter them below and check your answer using the "check" button.

Note for those using the NoScript browser extension: for the buttons to function correctly please allow Javascript from class.coursera.org and cloudfront.net to run in your browser. Note also that the "save answers" button does not function for this question and the next.

You entered:

Your Answer		Score	Explanation
x1 = 00000000000000000000000000000000 y1 = 00000000000000000000000000000001 x2 = 0a5f2b5e9b9662a53b02c0de8f8a1905 y2 = 00000000000000000000000000000000	✔	1.00	You got it !
Total		1.00 / 1.00	

Question 9

You entered:

Your Answer		Score	Explanation
x3 = 00000000000000000000000000000000 y3 = 00000000000000000000000000000001 x4 = 00000000000000000000000000000001 y4 = C797D4BD0B7876B00E6C4EF6B2DAD640	✔	1.00	Awesome!
Total		1.00 / 1.00	

Question 10

Let $H: M \to T$ be a random hash function where $|M| \gg |T|$ (i.e. the size of M is much larger than the size of T). In lecture we showed that finding a collision on H can be done with $O\left(|T|^{1/2}\right)$ random samples of H. How many random samples would it take until we obtain a three way collision, namely distinct strings x, y, z in M such that $H(x) = H(y) = H(z)$?

Your Answer		Score	Explanation
$O(\lvert T\rvert^{1/4})$			
$O(\lvert T\rvert^{2/3})$	✔	1.00	An informal argument for this is as follows: suppose we collect n random samples. The number of triples among the n samples is n choose 3 which is $O(n^3)$. For a particular triple x, y, z to be a 3-way collision we need $H(x) = H(y)$ and $H(x) = H(z)$. Since each one of these two events happens with probability $1/\lvert T\rvert$ (assuming H behaves like a random function) the probability that a particular triple is a 3-way collision is $O(1/\lvert T\rvert^2)$. Using the union bound, the probability that some triple is a 3-way collision is $O(n^3/\lvert T\rvert^2)$ and since we want this probability to be close to 1, the bound on n follows.
$O(\lvert T\rvert)$			
$O(\lvert T\rvert^{1/2})$			
Total		1.00 / 1.00	