Feedback — Week 2 - Problem Set

Help

You submitted this homework on **Mon 14 Apr 2014 2:26 PM PDT**. You got a score of **9.00** out of **9.00**.

Question 1

Consider the following five events:

1. Correctly guessing a random 128-bit AES key on the first try.
2. Winning a lottery with 1 million contestants (the probability is $1/10^6$).
3. Winning a lottery with 1 million contestants 5 times in a row (the probability is $(1/10^6)^5$).
4. Winning a lottery with 1 million contestants 6 times in a row.
5. Winning a lottery with 1 million contestants 7 times in a row.

What is the order of these events from most likely to least likely?

Your Answer		Score	Explanation
○ 2, 4, 3, 1, 5			
◉ 2, 3, 4, 1, 5	✔	1.00	• The probability of event (1) is 1/2^128. • The probability of event (5) is 1/(10^6)^7 which is about 1/2^{139}. Therefore, event (5) is the least likely. • The probability of event (4) is 1/(10^6)^6 which is about 1/2^{119.5} which is more likely than event (1). • The remaining events are all more likely than event (4).
○ 2, 3, 1, 5, 4			
○ 2, 3, 4, 5, 1			
Total		1.00 / 1.00	

Question 2

Suppose that using commodity hardware it is possible to build a computer for about $200 that can brute force about 1 billion AES keys per second. Suppose an organization wants to run an exhaustive search for a single 128-bit AES key and was willing to spend 4 trillion dollars to buy these machines (this is more than the annual US federal budget). How long would it take the organization to brute force this single 128-bit AES key with these machines? Ignore additional costs such as power and maintenance.

Your Answer		Score	Explanation
○ More than a million years but less than a billion (10^9) years			
◉ More than a billion (10^9) years	✔	1.00	The answer is about 540 billion years. • # machines = 4*10^12/200 = 2*10^10 • # keys processed per sec = 10^9 * (2*10^10) = 2*10^19 • # seconds = 2^128 / (2*10^19) = 1.7*10^19 This many seconds is about 540 billion years.
○ More than a month but less than a year			
○ More than a day but less than a week			
○ More than a 100 years but less than a million years			
Total		1.00 / 1.00	

Question 3

Let $F : \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}^n$ be a secure PRF (i.e. a PRF where the key space, input

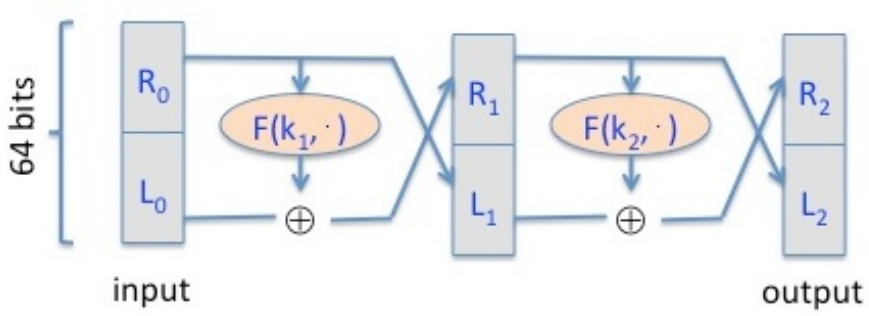
space, and output space are all $\{0,1\}^n$) and say $n = 128$. Which of the following is a secure PRF (there is more than one correct answer):

Your Answer		Score	Explanation
☐	$F'(k,\ x) = \begin{cases} F(k,x) & \text{when } x \neq 0^n \\ 0^n & \text{otherwise} \end{cases}$	✔ 0.17	Not a PRF. A distinguisher will query at $x = 0^n$ and output *not random* if the response is 0^n. This is unlikely to hold for a truly random function.
☑	$F'(k,x) = \text{reverse}(F(k,x))$ where reverse(y) reverses the string y so that the first bit of y is the last bit of reverse(y), the second bit of y is the second to last bit of reverse(y), and so on.	✔ 0.17	Correct. A distinguisher for F' gives a distinguisher for F.
☑	$F'(k,x) = F(k,x)[0,\ldots,n-2]$ (i.e., $F'(k,x)$ drops the last bit of $F(k,x)$)	✔ 0.17	Correct. A distinguisher for F' gives a distinguisher for F.
☐	$F'(k,x) = F(k,x) \ \|\| \ 0$ (here $\|\|$ denotes concatenation)	✔ 0.17	Not a PRF. A distinguisher will output *not random* whenever the last bit of $F(k,0^n)$ is 0.
☐	$F'(k,\ x) = \begin{cases} F(k,x) & \text{when } x \neq 0^n \\ k & \text{otherwise} \end{cases}$	✔ 0.17	Not a PRF. A distinguisher will query at $x = 0^n$ and obtain k and then query at $x=1^n$ and output *not random* if the response is $F(k,1^n)$. This is unlikely to hold for a truly random function.
☑	$F'((k_1,k_2),\ x) = F(k_1,x) \oplus F(k_2,x)$	✔ 0.17	Correct. A distinguisher for F' gives a distinguisher for F.
Total		1.00 / 1.00	

Question 4

Recall that the Luby-Rackoff theorem discussed in Lecture 3.2 states that applying a **three** round Feistel network to a secure PRF gives a secure block cipher. Let's see what goes wrong if

we only use a **two** round Feistel. Let $F : K \times \{0,1\}^{32} \rightarrow \{0,1\}^{32}$ be a secure PRF. Recall that a 2-round Feistel defines the following PRP $F_2 : K^2 \times \{0,1\}^{64} \rightarrow \{0,1\}^{64}$:



Here R_0 is the right 32 bits of the 64-bit input and L_0 is the left 32 bits.

One of the following lines is the output of this PRP F_2 using a random key, while the other three are the output of a truly random permutation $f : \{0,1\}^{64} \rightarrow \{0,1\}^{64}$. All 64-bit outputs are encoded as 16 hex characters. Can you say which is the output of the PRP? Note that since you are able to distinguish the output of F_2 from random, F_2 is not a secure block cipher, which is what we wanted to show.

Hint: First argue that there is a detectable pattern in the xor of $F_2(\cdot, 0^{64})$ and $F_2(\cdot, 1^{32}0^{32})$. Then try to detect this pattern in the given outputs.

Your Answer	Score	Explanation
On input 0^{64} the output is "4af53267 1351e2e1". On input $1^{32}0^{32}$ the output is "87a40cfa 8dd39154".		
On input 0^{64} the output is "5f67abaf 5210722b". On input $1^{32}0^{32}$ the output is "bbe033c0 0bc9330e".		
On input 0^{64} the output is "9d1a4f78 cb28d863". On input $1^{32}0^{32}$ the output is		

"75e5e3ea 773ec3e6".

◉ On input 0^{64} the output is "9f970f4e 932330e4". On input $1^{32}0^{32}$ the output is "6068f0b1 b645c008".	✔	1.00	Observe that the two round Feistel has the property that the left half of $F(\cdot, 0^{64}) \bigoplus F(\cdot, 1^{32}0^{32})$is 1^{32}. The two outputs in this answer are the only ones with this property.
Total		1.00 / 1.00	

Question 5

Nonce-based CBC. Recall that in lecture 4.4 we said that if one wants to use CBC encryption with a non-random unique nonce then the nonce must first be encrypted with an **independent** PRP key and the result then used as the CBC IV. Let's see what goes wrong if one encrypts the nonce with the **same** PRP key as the key used for CBC encryption.

Let $F : K \times \{0,1\}^{\ell} \rightarrow \{0,1\}^{\ell}$ be a secure PRP with, say, $\ell = 128$. Let n be a nonce and suppose one encrypts a message m by first computing $IV = F(k, n)$ and then using this IV in CBC encryption using $F(k, \cdot)$. Note that the same key k is used for computing the IV and for CBC encryption. We show that the resulting system is not nonce-based CPA secure.

The attacker begins by asking for the encryption of the two block message $m = (0^{\ell}, 0^{\ell})$ with nonce $n = 0^{\ell}$. It receives back a two block ciphertext (c_0, c_1). Observe that by definition of CBC we know that $c_1 = F(k, c_0)$. Next, the attacker asks for the encryption of the one block message $m_1 = c_0 \bigoplus c_1$ with nonce $n = c_0$. It receives back a one block ciphertext c_0'.

What relation holds between c_0, c_1, c_0'? Note that this relation lets the adversary win the nonce-based CPA game with advantage 1.

Your Answer	Score	Explanation
○ $c_0 = c_0'$		
○ $c_0' = c_0 \bigoplus 1^{\ell}$		

◉ $c_1 = c_0'$	✔	1.00	This follows from the definition of CBC with an encrypted nonce as defined in the question.
○ $c_1 = c_0$			
Total		1.00 / 1.00	

Question 6

Let m be a message consisting of ℓ AES blocks (say $\ell = 100$). Alice encrypts m using CBC mode and transmits the resulting ciphertext to Bob. Due to a network error, ciphertext block number $\ell/2$ is corrupted during transmission. All other ciphertext blocks are transmitted and received correctly. Once Bob decrypts the received ciphertext, how many plaintext blocks will be corrupted?

Your Answer		Score	Explanation
○ $1 + \ell/2$			
◉ 2	✔	1.00	Take a look at the CBC decryption circuit. Each ciphertext blocks affects only the current plaintext block and the next.
○ ℓ			
○ $\ell/2$			
○ 0			
Total		1.00 / 1.00	

Question 7

Let m be a message consisting of ℓ AES blocks (say $\ell = 100$). Alice encrypts m using

randomized counter mode and transmits the resulting ciphertext to Bob. Due to a network error, ciphertext block number $\ell/2$ is corrupted during transmission. All other ciphertext blocks are transmitted and received correctly. Once Bob decrypts the received ciphertext, how many plaintext blocks will be corrupted?

Your Answer		Score	Explanation
0			
$1+\ell/2$			
$\ell/2$			
1	✔	1.00	Take a look at the counter mode decryption circuit. Each ciphertext block affects only the current plaintext block.
ℓ			
Total		1.00 / 1.00	

Question 8

Recall that encryption systems do not fully hide the **length** of transmitted messages. Leaking the length of web requests has been used to eavesdrop on encrypted HTTPS traffic to a number of web sites, such as tax preparation sites, Google searches, and healthcare sites. Suppose an attacker intercepts a packet where he knows that the packet payload is encrypted using AES in CBC mode with a random IV. The encrypted packet payload is 128 bytes. Which of the following messages is plausibly the decryption of the payload:

Your Answer	Score	Explanation
'If qualified opinions incline to believe in the exponential conjecture, then I think we cannot afford not to make use of it.'		

○ 'The most direct computation would be for the enemy to try all 2^r possible keys, one by one.'			
◉ 'An enciphering-deciphering machine (in general outline) of my invention has been sent to your organization.'	✔	1.00	The length of the string is 106 bytes, which after padding becomes 112 bytes, and after prepending the IV becomes 128 bytes.
○ 'The significance of this general conjecture, assuming its truth, is easy to see. It means that it may be feasible to design ciphers that are effectively unbreakable.'			
Total		1.00 / 1.00	

Question 9

Let $R := \{0,1\}^4$ and consider the following PRF $F : R^5 \times R \to R$ defined as follows:

$$F(k,x) := \begin{cases} t = k[0] \\ \text{for i=1 to 4 do} \\ \quad \text{if } (x[i-1] == 1) \quad t = t \oplus k[i] \\ \text{output } t \end{cases}$$

That is, the key is $k = (k[0], k[1], k[2], k[3], k[4]) \in R^5$ and the function at, for example, 0101 is defined as $F(k, 0101) = k[0] \oplus k[2] \oplus k[4]$

For a random key k unknown to you, you learn that

$$F(k, 0110) = 0011 \text{ and } F(k, 0101) = 1010 \text{ and } F(k, 1110) = 0110.$$

What is the value of $F(k, 1101)$? Note that since you are able to predict the function at a new point, this PRF is insecure.

You entered:

1111

Your Answer		Score	Explanation
1111	✔	1.00	
Total		1.00 / 1.00	