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Antonio Visioli

(continued after Index)

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# Unmanned Rotorcraft Systems

 Springer

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*To  
Our UAV Research Team  
and  
Our Families*

# Series Editors' Foreword

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new philosophies, . . . , new challenges. Much of this development work resides in industrial reports, feasibility study papers and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

Autonomous control and guidance has had a long and evolving history. Long ocean voyages, large vessels and advanced propulsion systems soon led to the development of autopilots for sea-going vessels. Control developments for aerial vehicles and aircraft followed a similar pattern. Although some similar ideas prevailed for terrestrial vehicles, it took the emergence of electronic implementations for sophisticated control techniques to become more widespread in road vehicles. This field of control-technological development was designed to ease the task of the sailor, the pilot or the driver while they were still present on the bridge, in the cockpit or in the driver's seat; however, to dispatch a range of tasks in hazardous environments or to perform routine tasks over significant geographical distances, *unmanned* vehicular technology has developed and made significant strides in recent years. In space, autonomous control and guidance is a prerequisite for accomplishing many tasks involving satellites and unmanned planet "rover" vehicles. Aerial vehicles, either fixed-wing aircraft or rotorcraft are well developed for surveillance and other tasks and autonomous unmanned subsea vehicles are a critical enabler in the success of the oil industry in exploiting offshore oil resources.

In recent *Advances in Industrial Control* monographs, Guillaume J.J. Ducard considered some autonomous control aspects in *Fault-tolerant Flight Control and Guidance Systems* (ISBN 978-1-84882-560-4, 2009) for an unmanned fixed-wing aerial craft, while a little further back in time, Pedro Castillo, Rogelio Lozano, Alejandro E. Dzul reported on the *Modelling and Control of Mini-Flying Machines* (ISBN 978-1-85233-957-9, 2005). To add to this literature, the Series Editors are

now pleased to introduce this volume of *Advances in Industrial Control* entitled *Unmanned Rotorcraft Systems* authored by Guowei Cai, Ben M. Chen and Tong H. Lee that describes in comprehensive depth an exemplary development project to build an unmanned rotorcraft aerial system that was undertaken at the National University of Singapore.

Many monographs in the industrial control field, concentrate on the modeling, control design and simulation testing for a particular application but few contributions are able to emulate the authors of this one in describing the system development process, and trying to elucidate the general principles for constructing a complete system of which the control system is but one part. *Unmanned Rotorcraft Systems* has the advantage of being able to illustrate the steps of the development process through the design, construction and testing of a working prototype, the SheLion rotorcraft. Consequently, the monograph progresses through the hardware and software selection stages (Chaps. 3 and 4) and further shows how one advanced control technique (extended Kalman filtering) can be used to enhance the accuracy and performance of some low-cost technology (Chap. 5). The control system development involves modeling (Chap. 6), control design (Chap. 7) and experimental testing (Chap. 8). At each step of the way, the authors demonstrate the need to find solutions that balance technical sophistication against practical specifications, computational constraints and good operational performance. The authors also seek to identify genericity in the procedure so that the techniques can find a wider applicability in the rotorcraft and control system engineering fields. The experimental work reported involves computer simulation, hardware-in-the-loop trials and finally actual flight tests. The authors even extend an invitation to readers to view videos of the flight testing at <http://uav.ece.nus.edu.sg/>. The monograph closes with two chapters of more innovative work. One chapter (Chap. 10) describes how to achieve formation flying and the final chapter (Chap. 11) reports on the use of vision to follow a moving target. This chapter involves image processing and reports on how to integrate visual information into the rotorcraft control system.

The monograph reports a fascinating project that resulted in real-world outcomes. The comprehensive scope of the activities presented in this monograph and the demonstration of a very practical application of control engineering makes this inspirational material for a wide range of engineering students and researchers. Clearly, these rotorcraft project developments at the National University of Singapore have important industrial relevance and the monograph should be of interest to readers from the aerospace, control, signal processing and electrical engineering disciplines.

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# Preface

In recent years, research and development of unmanned systems have gained much attention in the academic and military communities worldwide. Topics like unmanned aircraft, underwater explorers, satellites, and intelligent robotics are widely investigated as they have potential applications in the military and civilian domains. They are developed to be capable of working autonomously without the interference of a human pilot. The challenge is that they need to deal with various situations that arise in very complicated and uncertain environments, such as unexpected obstacles, enemies attacking and device failures. Besides, they are required to communicate with technical personnel in the ground station. Consideration of a wide range of factors needs to be taken. Control systems for the unmanned vehicles are required to integrate not only basic input-output control laws but also high-level functionalities for decision making and task scheduling. Software systems for unmanned vehicles are required to perform tasks from hardware driving to the management of device operations, and from traditional input-output control law implementation to task scheduling and event management.

In this monograph, the authors aim to explore the research and development of fully functional miniature unmanned-aerial-vehicle (UAV) rotorcraft, which consist of a small-scale basic rotorcraft with all necessary accessories onboard and a ground station. The unmanned system is an integration of advanced technologies developed in the communications, computing, and control areas. It is an excellent test bed for testing and implementing modern control techniques. It is, however, a highly challenging process. The flight dynamics of small-scale rotorcraft such as a hobby helicopter is similar to its full-scale counterpart but owns some unique characteristics such as the utilization of a stabilizer bar, higher rotor stiffness, and yaw rate feedback control. Besides these, the strict limitation on payload also increases the difficulty in upgrading a small-scale rotorcraft to a UAV with full capacities. Based on its various characteristics and limitations, a lightweight but effective onboard avionic system with corresponding onboard/ground software should be carefully designed to realize the system identification and automatic flight requirements. These issues will be addressed in detail in this monograph. Research on utilizing the vision-based system for accomplishing ground target tracking and following, cooperative control, and flight formation of multiple unmanned rotorcraft is also highlighted.



The intended audience of this monograph includes practicing engineers in rotorcraft industry and researchers in the areas related to the development of unmanned aerial systems. An appropriate background for this book would be some senior level and/or first-year graduate level courses in aerodynamic engineering, control engineering, electrical engineering, and/or mechanical engineering.

The authors of this monograph are thankful to the whole UAV Research Team at the National University of Singapore. We would like to thank Dr. Feng Lin, Dr. Biao Wang, Dr. Kemao Peng, Dr. Miaobo Dong, Dr. Ben Yun, Xiangxu Dong, Xiaolian Zheng, Fei Wang, Shiyu Zhao, Swee-King Phang, Kevin Ang, and Jinqiang Cui for their help and contributions. We are particularly thankful to Dr. Feng Lin for his contribution to the results presented in Chap. 11 and to Dr. Biao Wang for his help to the material given in Chap. 10 of this monograph. We would also like to extend our thanks and appreciations to Ms. Charlotte Cross, Editorial Assistant of Springer, for her kindly help and assistance, and to the Springer's copy editor and series editor for their careful reading of the entire manuscript and their invaluable comments.

We have had the benefit of the collaboration of several coworkers and discussions with international visitors, from whom we have learned a great deal. Among them are Dr. Kai-Yew Lum and Dr. Hai Lin of National University of Singapore, Dr. Chang Chen and Dr. Rodney Teo of the DSO National Laboratories of Singapore, Professor Da-Zhong Zheng of Tsinghua University, Professor Clarence de Silva of the University of British Columbia, Professor Frank Lewis of the University of Texas at Arlington, Professor Lihua Xie of Nanyang Technological University, Professor Delin Luo of Xiamen University, Professor Hai-Bin Duan of Beijing University of Aeronautics and Astronautics, Professor Wei Kang of the Naval Postgraduate School, USA, and Dr. Siva Banda of the Air Force Research Laboratory, USA. We are indebted to them for their valuable contributions and/or comments.

The second author would like to thank particularly the Defence Science and Technology Agency (DSTA), Singapore, for granting him the Temasek Young Investigator Award in 2003 to initiate his research on unmanned systems. We would also like to acknowledge Temasek Laboratories and the Temasek Defence Systems Institute, the National University of Singapore, for their financial support and research funding over the years. We are thankful to the Department of Electrical and Computer Engineering and Temasek Laboratories, the National University of Singapore, for providing us generous laboratory spaces for housing our unmanned aircraft and related research activities.

Last, but certainly not the least, we owe a debt of gratitude to our families for their sacrifice, understanding, and encouragement during the course of preparing this monograph. It is very natural that we dedicate this work to our families and to our whole UAV Research Team.

Singapore, Singapore

Guowei Cai  
Ben M. Chen  
Tong H. Lee

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# Abbreviations

## *Acronyms*

A/D	Analog-to-Digital
AHRS	Attitude Heading Reference System
AOA	Angle Of Attack
CAM	Camera
CAMSHIFT	Continuously Adaptive Mean Shift
CCD	Charge-Coupled-Device
CEP	Circular Error Probable
CF	Compact Flash
CG	Center of Gravity
CIFER	Comprehensive Identification from FrEQUENCY Responses
CMOS	Complementary Metal-Oxide-Semiconductor
CMM	Communication
CORBA	Common Object Request Broker Architecture
CPU	Central Processing Unit
CTL	Control
D/A	Digital-to-Analog
DAQ	Data Acquisition
DARPA	Defense Advanced Research Projects Agency
DC	Direct Current
DGPS	Differential Global Positioning System
DLG	Data Logging
DOF	Degree Of Freedom
DSP	Digital Signal Processor
ECEF	Earth-Centered Earth-Fixed
EKF	Extended Kalman Filter
EMI	Electromagnetic Interference
FPS	Frames Per Second
GPS	Global Positioning System
GUI	Graphical User Interface
HITL	Hardware-In-The-Loop

HSV	Hue, Saturation, Value
IMG	Image
IMU	Inertial Measurement Unit
INS	Inertial Navigation System
I/O	Input/Output
JPEG	Joint Photographic Experts Group
Li-Po	Lithium-Polymer
LMM	Lightweight Multi-role Missile
LQR	Linear Quadratic Regulator
MAV	Micro Aerial Vehicle
MEMS	Micro-Electronic-Mechanical-System
MFC	Microsoft Foundation Class
MIMO	Multi-Input/Multi-Output
MTE	Mission Task Element
NA	Not Applicable
NAV	Navigation
NED	North-East-Down
Ni-Cd	Nickel-Cadmium
Ni-Mh	Nickel-Metal Hydride
NUS	National University of Singapore
OCF	Open Control Platform
OpenGL	Open Graphical Library
PCI	Peripheral Component Interconnect
PCM	Pulse Code Modulation
PD	Proportional-Derivative
PID	Proportional-Integral-Derivative
PPM	Pulse Position Modulation
RC	Radio-Controlled
RFI	Radio Frequency Interference
RGB	Red, Green, Blue
RPM	Revolutions Per Minute
RPT	Robust and Perfect Tracking
RTK	Real-Time Kinematic
RTOS	Real-Time Operating System
SAV	Save
SBC	Single Board Computer
SISO	Single-Input/Single-Output
SVO	Servo
TPP	Tip-Path-Plane
UAV	Unmanned Aerial Vehicle
UKF	Unscented Kalman Filter
VDE	Virtual Design Environment
WGS	World Geodetic System

*Symbols*<sup>1</sup>

$A_{b_s}$	Coupling effect from $b_s$ to $a_s$ ( $s^{-1}$ )
$A_{lon}$	Linkage gain ratio of $\theta_{cyc,a_s}$ to $\delta_{lon}$ (rad)
$a_s$	Longitudinal TPP flapping angle of bare main rotor (rad)
$B_{a_s}$	Coupling effect from $a_s$ to $b_s$ ( $s^{-1}$ )
$B_{lat}$	Linkage gain ratio of $\theta_{cyc,b_s}$ to $\delta_{lat}$ (rad)
$b_{mr}$	Main rotor blade number
$b_s$	Lateral TPP flapping angle of bare main rotor (rad)
$b_{tr}$	Tail rotor blade number
$C_{D0}$	Drag coefficient of main rotor blade
$C_{lon}$	Linkage gain ratio of stabilizer bar cyclic change to $\delta_{lon}$ (rad)
$C_{l\alpha,hf}$	Lift curve slope of horizontal fin ( $rad^{-1}$ )
$C_{l\alpha,mr}$	Lift curve slope of main rotor blade ( $rad^{-1}$ )
$C_{l\alpha,sb}$	Lift curve slope of stabilizer bar paddle ( $rad^{-1}$ )
$C_{l\alpha,tr}$	Lift curve slope of tail rotor blade ( $rad^{-1}$ )
$C_{l\alpha,vf}$	Lift curve slope of vertical fin ( $rad^{-1}$ )
$c_{mr}$	Main rotor blade chord length (m)
$c_s$	Longitudinal TPP flapping angle of stabilizer bar (rad)
$c_{sb}$	Stabilizer bar paddle chord length (m)
$c_{tr}$	Tail rotor blade chord length (m)
$D_{hf}$	Horizontal fin location behind the CG (m)
$D_{lat}$	Linkage gain ratio of stabilizer bar cyclic change to $\delta_{lat}$ (rad)
$D_{tr}$	Tail rotor hub location behind the CG (m)
$D_{vf}$	Vertical fin position behind the CG (m)
$d_s$	Lateral TPP flapping angle of stabilizer bar (rad)
$e_{mr}$	Effective hinge offset of main rotor (m)
$F_b$	Aerodynamic force vector (N)
$F_{b,g}$	Gravity force vector projected onto the body frame (N)
$H_{mr}$	Main rotor hub location above the CG (m)
$H_{tr}$	Tail rotor hub location above the CG (m)
$H_{vf}$	Vertical fin location above the CG (m)
$g$	Local acceleration of gravity ( $m/s^2$ )
<b>J</b>	A diagonal matrix of moments of inertia in body frame with its main diagonal elements being $J_{xx}$ , $J_{yy}$ , $J_{zz}$ ( $kg \cdot m^2$ )
$K_I$	Integral gain of the yaw rate feedback controller
$K_p$	Proportional gain of the yaw rate feedback controller
$K_a$	Ratio of yaw rate to normalized rudder input (rad/s)
$K_{col}$	Ratio of main rotor blade collective pitch angle to collective pitch servo deflection (rad)
$K_{ped}$	Ratio of tail rotor blade collective pitch angle to rudder servo deflection (rad)
$K_{sb}$	Ratio of main rotor blade cyclic pitch to stabilizer bar flapping

<sup>1</sup>Listed in this section are all the key symbols and parameters associated with flight dynamics modeling of rotorcraft together with their physical descriptions and units (if any).



$K_\beta$	Rotor spring constant (N·m)
$L_{mr}, M_{mr}, N_{mr}$	Aerodynamic moments generated by main rotor (kg·m <sup>2</sup> )
$L_{vf}, N_{vf}$	Aerodynamic moments generated by vertical fin (kg·m <sup>2</sup> )
$L_{tr}, N_{tr}$	Aerodynamic moments generated by tail rotor (kg·m <sup>2</sup> )
$M_b$	Aerodynamic moment vector (kg·m <sup>2</sup> )
$M_{hf}$	Aerodynamic moment generated by horizontal fin (kg·m <sup>2</sup> )
$m$	Helicopter mass (kg)
$n_{tr}$	Gear ratio of tail rotor to main rotor
$p, q, r$	Angular velocities (rad/s)
$P_c$	Climbing power of main rotor (W)
$P_i$	Induced power of main rotor (W)
$P_n$	Local NED position vector with its elements being $x_n, y_n, z_n$ (m)
$P_{pa}$	Parasitic power of main rotor (W)
$P_{pr}$	Profile power of main rotor (W)
$\mathbf{R}_{n/b}$	Rotation matrix from the body frame to the local NED frame
$Re$	Reynolds number
$R_{mr}$	Main rotor blade radius (m)
$R_{sb,in}$	Inner radius of the stabilizer bar disc (m)
$R_{sb,out}$	Outer radius of the stabilizer bar disc (m)
$R_{tr}$	Tail rotor blade radius (m)
$\mathbf{S}$	Transformation matrix from Euler angles derivatives to $\omega_{b/n}^b$
$S_{fx}$	Effective longitudinal fuselage drag area (m <sup>2</sup> )
$S_{fy}$	Effective lateral fuselage drag area (m <sup>2</sup> )
$S_{fz}$	Effective vertical fuselage drag area (m <sup>2</sup> )
$S_{hf}$	Effective horizontal fin area (m <sup>2</sup> )
$S_{vf}$	Effective vertical fin area (m <sup>2</sup> )
$T_{mr}$	Main rotor thrust (N)
$T_{tr}$	Tail rotor thrust (N)
$\mathbf{V}_a$	Velocity vector relative to the air projected onto body frame with its elements being $u_a, v_a, w_a$ (m/s)
$\mathbf{V}_b$	Velocity vector projected onto body frame with its elements being $u, v, w$ (m/s)
$\mathbf{V}_n$	Velocity vector projected onto local NED frame with its elements being $u_n, v_n, w_n$ (m/s)
$\mathbf{V}_{wind}$	Wind gust velocity vector projected onto body frame with its elements being $u_{wind}, v_{wind}, w_{wind}$ (m/s)
$v_{i,mr}$	Main rotor induced velocity (m/s)
$v_{i,tr}$	Tail rotor induced velocity (m/s)
$v_{vf}$	Local lateral airspeed at the vertical fin (m/s)
$\hat{v}_{mr}^2$	Intermediate variable in main rotor thrust calculation (m <sup>2</sup> /s <sup>2</sup> )
$\hat{v}_{tr}^2$	Intermediate variable in tail rotor thrust calculation (m <sup>2</sup> /s <sup>2</sup> )
$w_{hf}$	Local vertical airspeed at the horizontal fin (m/s)
$x, y, z$	Position coordinates in local NED frame (m)
$x_n, y_n, z_n$	Position coordinates in local NED frame (m)
$X_{mr}, Y_{mr}, Z_{mr}$	Aerodynamic forces generated by main rotor (N)

$X_{\text{fus}}, Y_{\text{fus}}, Z_{\text{fus}}$	Aerodynamic forces generated by fuselage (N)
$Y_{\text{tr}}$	Aerodynamic force generated by tail rotor (N)
$Y_{\text{vf}}$	Aerodynamic force generated by vertical fin (N)
$Z_{\text{hf}}$	Aerodynamic force generated by horizontal fin (N)
$\alpha_{\text{st}}$	Critical angle of attack in stall (rad)
$\gamma_{\text{mr}}$	Lock number of main rotor blade
$\gamma_{\text{sb}}$	Lock number of the stabilizer bar
$\delta_{\text{col}}$	Normalized collective pitch servo input $[-1, 1]$
$\delta_{\text{lat}}$	Normalized aileron servo input $[-1, 1]$
$\delta_{\text{lon}}$	Normalized elevator servo input $[-1, 1]$
$\delta_{\text{ped}}$	Normalized rudder servo input $[-1, 1]$
$\delta_{\text{ped,int}}$	Intermediate state in yaw rate feedback controller dynamics (rad)
$\bar{\delta}_{\text{ped}}$	Rudder servo actuator deflection (rad)
$\theta_{\text{col}}$	Collective pitch angle of main rotor blade (rad)
$\theta_{\text{cyc},\text{as}}$	Longitudinal cyclic pitch angle of main rotor blade (rad)
$\theta_{\text{cyc},\text{bs}}$	Lateral cyclic pitch angle of main rotor blade (rad)
$\theta_{\text{ped}}$	Collective pitch angle of tail rotor blade (rad)
$\lambda_{\text{vf}}$	Indicator of the vertical fin exposed to tail rotor wake
$\rho$	Air density ( $\text{kg}\cdot\text{m}^3$ )
$\tau_{\text{mr}}$	Time constant of bare main rotor (s)
$\tau_{\text{sb}}$	Time constant of stabilizer bar (s)
$\phi, \theta, \psi$	Euler angles (rad)
$\Omega_{\text{mr}}$	Main rotor rotating speed (rad/s)
$\Omega_{\text{tr}}$	Tail rotor rotating speed (rad/s)
$\omega_{\text{b/n}}^{\text{b}}$	Angular velocity vector with its elements being $p, q, r$ (rad/s)