APPENDIX G

Force and Moment Calculations

G.1 Forces

The forces are calculated in CFL3D as follows. Let $\tilde{\vec{F}}_l$ be the total dimensional force acting on the surface element l, with area $\tilde{s}_l$, normalized by $\tilde{q}_\infty \tilde{s}_{ref}$, where

$$\tilde{q}_\infty = \frac{1}{2}\tilde{\rho}_\infty |\tilde{\mathbf{V}}|^2_\infty \qquad \text{(G-1)}$$

and $\tilde{s}_{ref}$ is the reference area. In what follows, $\tilde{s}_{ref}$ and $\tilde{s}_l$ are taken in terms of grid dimensions. Then the dimensionless force (force coefficient) acting on element l is

$$\vec{F}_l = \frac{\tilde{\vec{F}}_l}{\tilde{q}_\infty \tilde{s}_{ref}} \qquad \text{(G-2)}$$

$\vec{F}_l$ is composed of pressure and viscous components, $\vec{F}_l^p$ and $\vec{F}_l^v$, respectively. The total force coefficient is computed by summing the contributions from all specified surface elements:

$$\vec{F} = \sum_l \vec{F}_l \qquad \text{(G-3)}$$

G.1.1 Pressure Component

$\vec{F}_l^p$ is normal to the surface.

$$\tilde{s}_{ref}\vec{F}_l^p = \frac{\tilde{p} - \tilde{p}_\infty}{\frac{1}{2}\tilde{\rho}_\infty |\tilde{\mathbf{V}}|^2_\infty}\tilde{s}_l\vec{n} = 2\frac{\tilde{p}/(\tilde{\rho}_\infty \tilde{a}^2_\infty) - \tilde{p}_\infty/(\tilde{\rho}_\infty \tilde{a}^2_\infty)}{|\tilde{\mathbf{V}}|^2_\infty / \tilde{a}^2_\infty}\tilde{s}_l\vec{n} = 2\frac{p - \frac{1}{\gamma}}{M^2_\infty}\tilde{s}_l\vec{n} \qquad \text{(G-4)}$$

Therefore

$$\tilde{s}_{ref}\vec{F}_l^p = \frac{2}{\gamma M_\infty^2}(\gamma p - 1)\tilde{s}_l\vec{n} \qquad \text{(G-5)}$$

The x, y, and z components of $\vec{F}_l^p$ are obtained by multiplying Equation (G-5) by the appropriate direction cosine. For example, if element l lies on an i = constant surface,

$$\vec{n} = \nabla\hat{\xi} \qquad \text{(G-6)}$$

$$\begin{aligned}
\tilde{s}_{ref}(\vec{F}_l^p)_x &= \frac{2}{\gamma M_\infty^2}(\gamma p - 1)\tilde{s}_l\hat{\xi}_x \\
\tilde{s}_{ref}(\vec{F}_l^p)_y &= \frac{2}{\gamma M_\infty^2}(\gamma p - 1)\tilde{s}_l\hat{\xi}_y \qquad \text{(G-7)} \\
\tilde{s}_{ref}(\vec{F}_l^p)_z &= \frac{2}{\gamma M_\infty^2}(\gamma p - 1)\tilde{s}_l\hat{\xi}_z
\end{aligned}$$

where $\hat{\xi}_x = \xi_x/|\nabla\xi|$, $\hat{\xi}_y = \xi_y/|\nabla\xi|$, and $\hat{\xi}_z = \xi_z/|\nabla\xi|$.

G.1.2 Viscous Component

$\vec{F}_l^v$ is tangential to the surface.

$$\tilde{s}_{ref}\vec{F}_l^v = \frac{\tilde{\vec{\tau}}}{\frac{1}{2}\tilde{\rho}_\infty|\tilde{\mathbf{V}}|_\infty^2}\tilde{s}_l = 2\frac{\tilde{\vec{\tau}}/(\tilde{\rho}_\infty\tilde{a}_\infty^2)}{|\tilde{\mathbf{V}}|_\infty^2/\tilde{a}_\infty^2}\tilde{s}_l = \frac{2}{M_\infty^2}\vec{\tau}\tilde{s}_l \qquad \text{(G-8)}$$

Consider the flow near a surface element l of an i = constant surface. (See Figure G-1.)

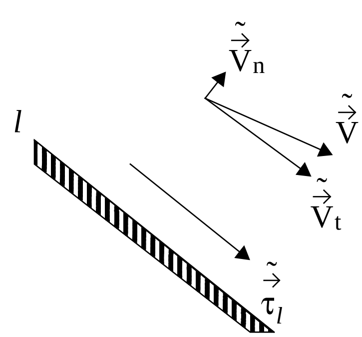

Figure G-1. Viscous force component example.

In the figure,

$$\begin{aligned}\vec{\tilde{V}} &= \text{velocity vector}\\ \vec{\tilde{V}}_n &= \text{normal velocity vector}\\ \vec{\tilde{V}}_t &= \text{tangential velocity vector (0 on the surface)}\end{aligned} \tag{G-9}$$

and

$$\vec{\tilde{V}}_t = \vec{\tilde{V}} - \vec{\tilde{V}}_n \tag{G-10}$$

$$\vec{\tilde{\tau}}_l = \tilde{\mu}\frac{\partial \vec{\tilde{V}}_t}{\partial \tilde{n}} = \tilde{\mu}\frac{\partial\left(\vec{\tilde{V}} - \vec{\tilde{V}}_n\right)}{\partial \tilde{n}} \tag{G-11}$$

where

$$\vec{\tilde{V}}_n = \left(\vec{\tilde{V}} \cdot \vec{n}\right)\vec{n} \tag{G-12}$$

($\vec{\tilde{V}} \cdot \vec{n}$ is the normalized contravariant velocity and $\vec{n}$ is the unit surface normal.) The x component of $\vec{\tilde{\tau}}_l$ is $\vec{\tilde{\tau}}_l \cdot \vec{i}$:

$$(\vec{\tilde{\tau}}_l)_x = \tilde{\mu}\frac{\partial\left(\vec{\tilde{V}} \cdot \vec{i} - \vec{\tilde{V}}_n \cdot \vec{i}\right)}{\partial \tilde{n}} = \tilde{\mu}\frac{\partial\left[\tilde{u} - \left(\vec{\tilde{V}} \cdot \vec{n}\right)\hat{\xi}_x\right]}{\partial \tilde{n}} \tag{G-13}$$

Similarly,

$$(\vec{\tilde{\tau}}_l)_y = \tilde{\mu}\frac{\partial\left[\tilde{v} - \left(\vec{\tilde{V}} \cdot \vec{n}\right)\hat{\xi}_y\right]}{\partial \tilde{n}} \tag{G-14}$$

$$(\vec{\tilde{\tau}}_l)_z = \tilde{\mu}\frac{\partial\left[\tilde{w} - \left(\vec{\tilde{V}} \cdot \vec{n}\right)\hat{\xi}_z\right]}{\partial \tilde{n}} \tag{G-15}$$

Consider the nondimensionalization of the x component of $\vec{\tilde{\tau}}_l$:

$$(\vec{\tau}_l)_x = \frac{(\tilde{\vec{\tau}}_l)_x}{\tilde{\rho}_\infty \tilde{a}_\infty^2} = \frac{1}{\tilde{\rho}_\infty \tilde{a}_\infty^2} \cdot \tilde{\mu} \frac{\partial\left[\tilde{u} - \left(\tilde{\vec{V}} \cdot \vec{n}\right)\hat{\xi}_x\right]}{\partial \tilde{n}} \tag{G-16}$$

(See Equation (G-8).) Then

$$(\vec{\tau}_l)_x = \frac{\tilde{\mu}_\infty}{\tilde{\rho}_\infty |\tilde{\mathbf{V}}|_\infty \tilde{L}_R} \frac{|\tilde{\mathbf{V}}|_\infty}{\tilde{a}_\infty} \frac{\tilde{\mu}}{\tilde{\mu}_\infty} \frac{\partial\left\{\left[\tilde{u} - \left(\tilde{\vec{V}} \cdot \vec{n}\right)\hat{\xi}_x\right] / \tilde{a}_\infty\right\}}{\partial(\tilde{n} / \tilde{L}_R)} \tag{G-17}$$

Therefore,

$$(\vec{\tau}_l)_x = \frac{M_\infty}{Re_{\tilde{L}_R}} \mu \frac{\partial[u - (\vec{V} \cdot \vec{n})\hat{\xi}_x]}{\partial n} \tag{G-18}$$

with similar expressions for $(\vec{\tau}_l)_y$ and $(\vec{\tau}_l)_z$. The derivative is evaluated using the cell-center and wall values of a cell volume like that shown in Figure G-2.

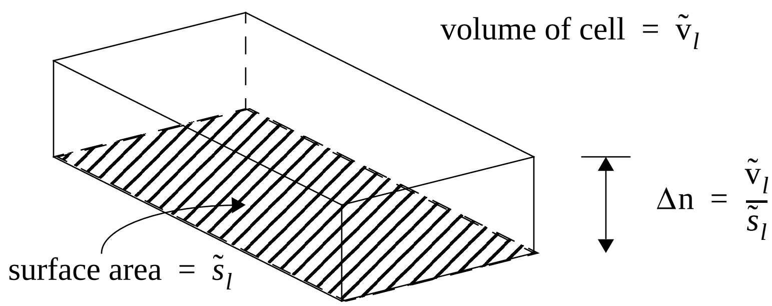

Figure G-2. Cell volume example.

That is, for the x component,

$$\frac{\partial\left[\tilde{u} - \left(\tilde{\vec{V}} \cdot \vec{n}\right)\hat{\xi}_x\right]}{\partial n} = \frac{\left[\tilde{u} - \left(\tilde{\vec{V}} \cdot \vec{n}\right)\hat{\xi}_x\right]_c - 0}{\frac{1}{2}\Delta n} \tag{G-19}$$

where the subscript c denotes the cell-center value and $\left[\tilde{u} - \left(\tilde{\vec{V}} \cdot \vec{n}\right)\hat{\xi}_x\right] \equiv 0$ on a solid wall with the no-slip assumption. So

$$(\vec{\tau}_l)_x = \frac{M_\infty}{Re_{\tilde{L}_R}}\mu\frac{\left[\tilde{u}-\left(\tilde{\vec{V}}\cdot\vec{n}\right)\hat{\xi}_x\right]_c}{\frac{1}{2}\Delta n} = \frac{2M_\infty}{Re_{\tilde{L}_R}}\mu\left(\frac{\tilde{s}_l}{\tilde{v}_l}\right)\left[\tilde{u}-\left(\tilde{\vec{V}}\cdot\vec{n}\right)\hat{\xi}_x\right]_c \quad \text{(G-20)}$$

Thus, for the x component of the viscous force,

$$\tilde{s}_{ref}(\vec{F}_l^v)_x = \frac{2}{M_\infty^2}(\vec{\tau}_l)_x\tilde{s}_l = \frac{4}{M_\infty Re_{\tilde{L}_R}}\mu[u-(\vec{V}\cdot\vec{n})\hat{\xi}_x]_c\frac{\tilde{s}_l^2}{\tilde{v}_l} \quad \text{(G-21)}$$

Similarly,

$$\tilde{s}_{ref}(\vec{F}_l^v)_y = \frac{4}{M_\infty Re_{\tilde{L}_R}}\mu[v-(\vec{V}\cdot\vec{n})\hat{\xi}_y]_c\frac{\tilde{s}_l^2}{\tilde{v}_l} \quad \text{(G-22)}$$

$$\tilde{s}_{ref}(\vec{F}_l^v)_z = \frac{4}{M_\infty Re_{\tilde{L}_R}}\mu[w-(\vec{V}\cdot\vec{n})\hat{\xi}_z]_c\frac{\tilde{s}_l^2}{\tilde{v}_l} \quad \text{(G-23)}$$

G.2 Moments

The moments due to the forces acting on an element are determined as follows. Let

$$\begin{aligned}(\vec{F}_l)_x &= (\vec{F}_l^p + \vec{F}_l^v)_x \\ (\vec{F}_l)_y &= (\vec{F}_l^p + \vec{F}_l^v)_y \\ (\vec{F}_l)_z &= (\vec{F}_l^p + \vec{F}_l^v)_z\end{aligned} \quad \text{(G-24)}$$

Figure G-2 illustrates the directions of the moments. All moments are positive if counter-clockwise when viewed from the positive axis. The conventions (assuming x points downstream and z points up) are

M_x: rolling moment; positive for counter-clockwise roll when viewed from downstream.

M_y: pitching moment; positive for pitch up.

M_z: yawing moment; positive for counter-clockwise yaw when viewed from above. Therefore,

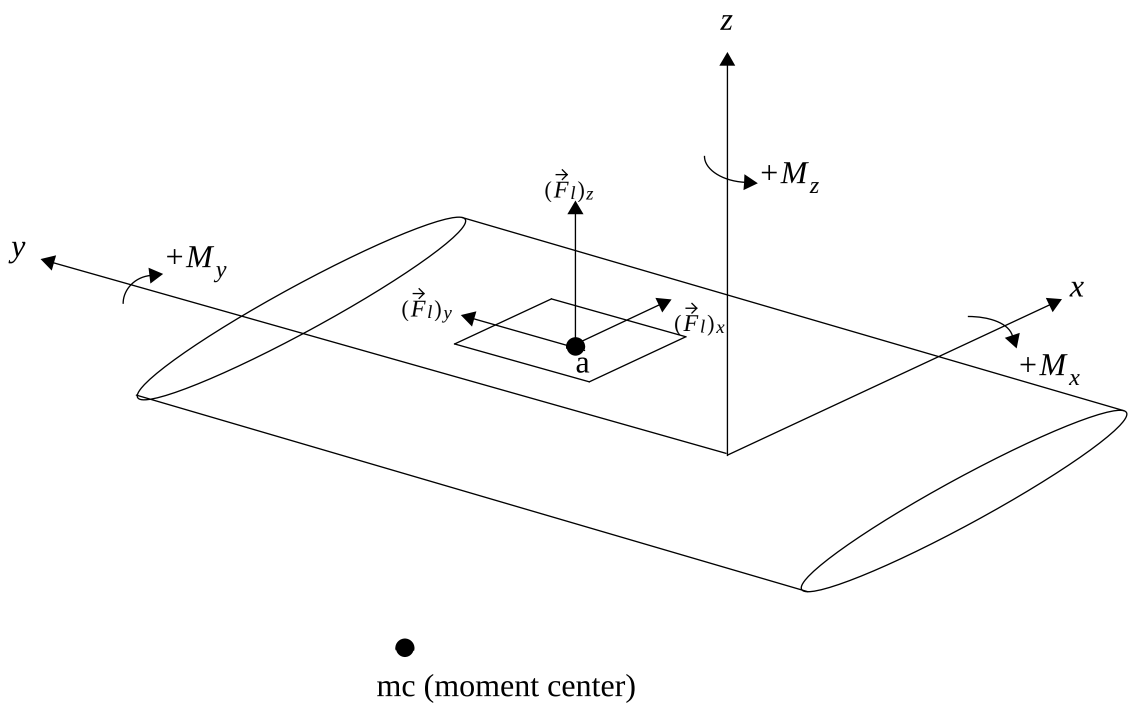

Figure G-3. Moment directions.

$$
\begin{aligned}
(\vec{M}_l)_x &= [+(\vec{F}_l)_z(\tilde{y}_a-\tilde{y}_{mc})-(\vec{F}_l)_y(\tilde{z}_a-\tilde{z}_{mc})]/\tilde{b}_{ref} \\
(\vec{M}_l)_y &= [-(\vec{F}_l)_z(\tilde{x}_a-\tilde{x}_{mc})+(\vec{F}_l)_x(\tilde{z}_a-\tilde{z}_{mc})]/\tilde{c}_{ref} \\
(\vec{M}_l)_z &= [+(\vec{F}_l)_y(\tilde{x}_a-\tilde{x}_{mc})-(\vec{F}_l)_x(\tilde{y}_a-\tilde{y}_{mc})]/\tilde{b}_{ref}
\end{aligned}
\qquad \text{(G-25)}
$$

Note that the reference length used to nondimensionalize $(\vec{M}_l)_y$ is $\tilde{c}_{ref}$, while $(\vec{M}_l)_z$ is made dimensionless with $\tilde{b}_{ref}$. This is the default for CFL3D-type grids or PLOT3D-type grids with **ialph** = 0 (see “LT3 - Flow Conditions” on page 19). However, if a PLOT3D-type grid is used with **ialph** = 1, then $(\vec{M}_l)_y$ is made nondimensional with $\tilde{b}_{ref}$ and $(\vec{M}_l)_z$ is made dimensionless with $\tilde{c}_{ref}$. $(\vec{M}_l)_x$ is always made dimensionless with $\tilde{b}_{ref}$. By switching the reference lengths based on **ialph**, the moment coefficient that is normally associated with the pitching moment is always based on $\tilde{c}_{ref}$, while the moment coefficient that is normally associated with the yawing moment is always based on $\tilde{b}_{ref}$. Because $(\vec{M}_l)_x$ always uses $\tilde{b}_{ref}$, the moment coefficient associated with the rolling moment is based on $\tilde{b}_{ref}$.