7

CHAPTER 7

Convergence Acceleration

In CFL3D, the convergence rate at which many problems are solved can be accelerated with the use of multigrid, mesh sequencing, or a combination of the two. The following sections describe the two techniques. The use of multigrid with grid overlapping and embedded grids is also discussed.

Multigrid is a *highly* recommended option available in CFL3D. The improvement in convergence acceleration afforded with multigrid makes it very worthwhile to learn about and utilize. Table 7–1 illustrates this fact for two cases, a 2-d airfoil and a 3-d forebody. A work unit is defined here to be the “equivalent” fine grid iteration. For example, on the finest grid level, 1 iteration = 1 work unit; on the next-to-finest grid level, 1 iteration = 1/8 (1/4 in 2-d) of a work unit; on the next coarser level, 1 iteration = 1/64 (1/16 in 2-d) of a work unit; and so forth. The residual and lift coefficient convergence histories for these two cases are shown in Figure 7-1 and Figure 7-2. The term “full multigrid” implies that mesh sequencing is used in conjunction with multigrid. See “Mesh Sequencing” on page 134. Note that there is an additional CPU penalty for the overhead associated with the multigrid scheme that is not reflected in the work unit measure, however, for most problems, the overhead is small.

Table 7–1. Convergence improvements with multigrid.

	Approximate Number of Work Units to Achieve the Final Lift		
Case	Without Multigrid	With Multigrid	With Full-Multigrid
2-d NACA 4412 Airfoil	10,000	1000	750
3-d F-18 Forebody	>> 1500*	500	500

*This case was stopped after 1500 iterations when it became clear that the lift coefficient was far from converged and to continue running the case would have been a waste of computer resources.

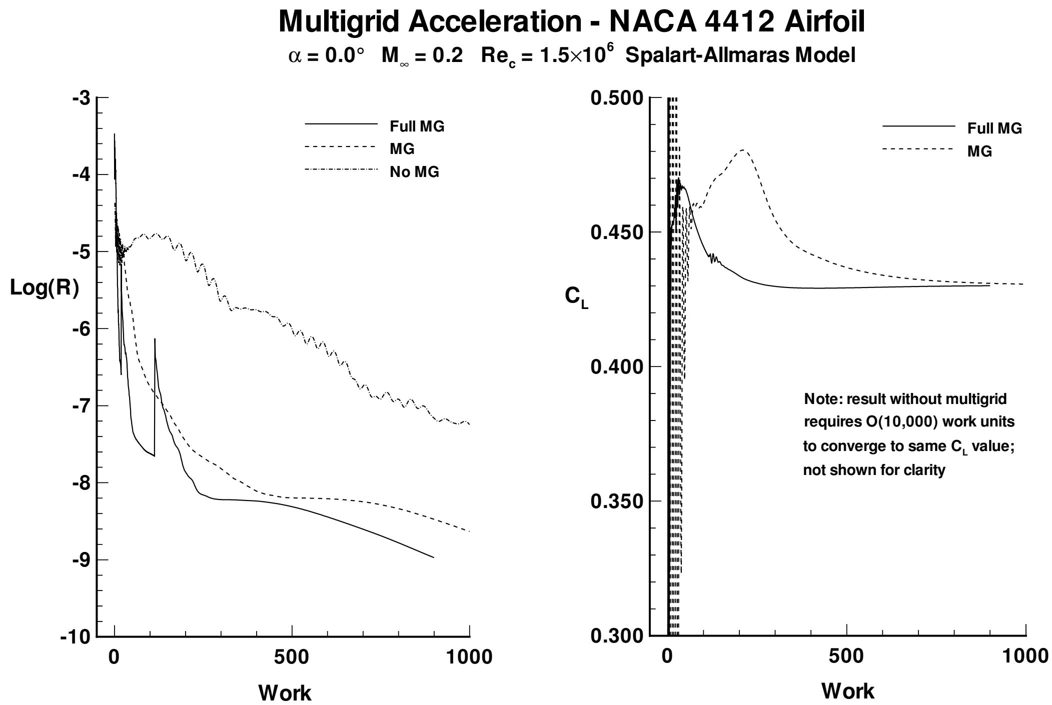


Figure 7-1. Convergence acceleration for 2-d NACA 4412 airfoil case.

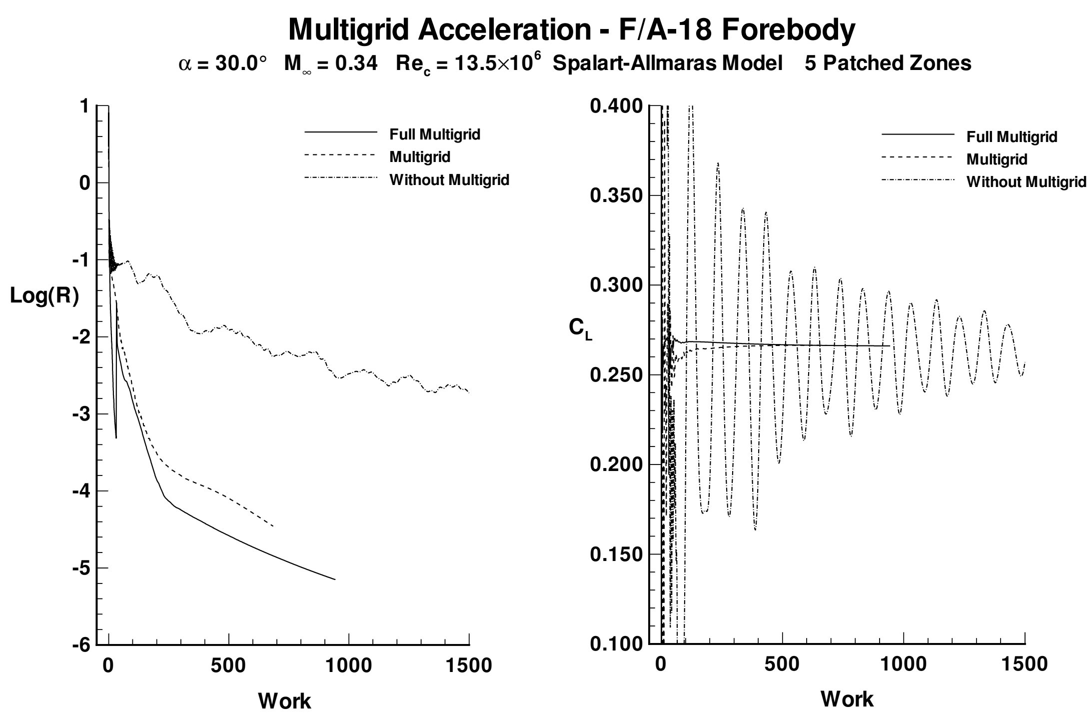


Figure 7-2. Convergence acceleration for 3-d F-18 Forebody case.

7.1 Multigrid

7.1.1 Global Grids

In a single-grid-level algorithm, the flow equations are solved only on the finest grid. The convergence rate for this algorithm is typically good during the initial stages of the computation but soon degrades significantly due to poor damping of low-frequency errors.[13] Multigrid methods, in which a sequence of coarser grids are used to eliminate the low-frequency errors, can significantly improve the convergence rate of the algorithm. The full-approximation-scheme multigrid algorithm, as developed and applied to inviscid three-dimensional flows in references 5, 6 and 7, is available in CFL3D to accelerate convergence of a solution to a steady state. A sequence of grids $G_0, G_1, \ldots, G_N$ is defined, where G_N denotes the finest grid and coarser grids are formed by successively deleting every other grid line in all three coordinate directions (except in 2-d, where the i direction is not coarsened). The fine grid serves to damp the high-frequency errors; the coarser grids damp the low-frequency errors. The coarse grids are solved with a forcing function on the right-hand side arising from restricting the residual from the finer meshes. The forcing function is the relative truncation error between the grids such that the solutions on the coarser meshes are driven by the fine grid residual. A correction is calculated on the coarser meshes and passed up to the next finest mesh using trilinear interpolation in the prolongation step.

As an example, consider a 2-d grid of dimensions 17×17 as shown in Figure 7-3. Suppose it is desired to use four levels of multigrid, i.e. the fine grid level and three coarser grid levels. The three coarser grid levels, along with their dimensions, are also shown in Figure 7-3. Figure 7-4 illustrates a typical W-cycle multigrid scheme for four grid levels. In the figure, n refers to the current iteration. The multigrid cycle begins with a Navier-Stokes calculation performed at the finest grid level specified. The fine grid residual and flow-field variables are restricted to the next grid level where they are utilized to perform a Navier-Stokes calculation at this level. The process continues until the coarsest grid level is reached. After a Navier-Stokes calculation is performed at this level, the solution is prolongated back “up” the cycle. With the W-cycle, additional calculations and the restrictions and prolongation steps are repeated at the coarser grid levels giving the “W” pattern illustrated in Figure 7-4. The other option available is a V-cycle. An example of a V-cycle for the four grid level case is illustrated in Figure 7-5. Generally, a W-cycle is more effective since additional work is done at the coarse grid level. However, this may be case dependent. Note that, if only two grid levels are specified, a V-cycle will always be performed even if a W-cycle is specified in the input file.

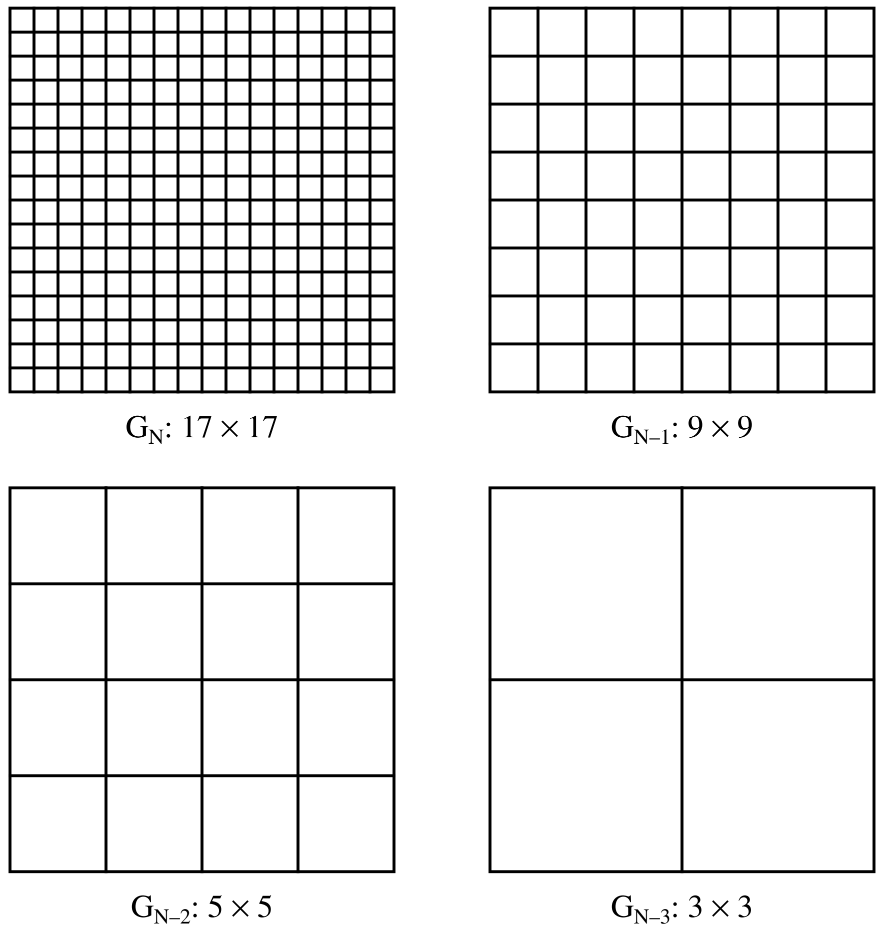


Figure 7-3. Fine grid and coarse grid levels.

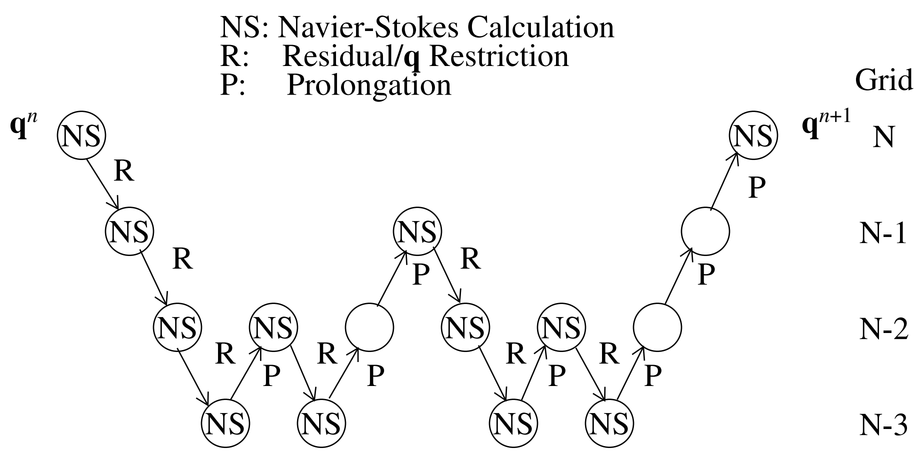


Figure 7-4. Multigrid W-cycle.

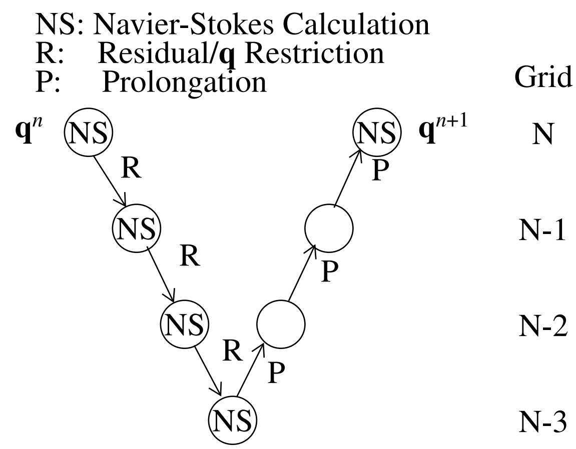


Figure 7-5. Multigrid V-cycle.

For a multigrid W-cycle, the pertinent lines of input for the example in Figure 7-3 would look something like:

Line Type								
5	DT	IREST	IFLAGTS	FMAX	IUNST	CFLTAU		
	-1.0	0	0	1.0	0	10.0		
6	NGRID	NPLOT3D	NPRINT	NWREST	ICHK	I2D	NTSTEP	ITA
	1	0	0	0	0	1	1	1
7	NCG	IEM	IADVANCE	IFORCE	IVISC(I)	IVISC(J)	IVISC(K)	
	3	0	0	1	0	0	0	
8	IDIM	JDIM	KDIM					
	2	17	17					
20	MSEQ	MGFLAG	ICONSF	MTT	NGAM			
	1	1	0	0	2			
22	NCYC	MGLEVG	NEMGL	NITFO				
	200	4	0	0				
23	MIT1	MIT2	MIT3	MIT4	MIT5			
	1	1	1	1	1			

7.1.2 A Word About Grid Dimensions

When determining how many multigrid levels are available for a particular grid, use the following formula:

$$m_c = \frac{m_f - 1}{2} + 1 = \frac{m_f + 1}{2} \tag{7-1}$$

where m_f is a fine-grid dimension (**idim**, **jdim**, or **kdim**) and m_c is the corresponding coarse-grid dimension. Then rename m_c as m_f and compute Equation (7-1) again to determine an even coarser grid dimension. When m_c is an even number, that is as coarse as that grid dimension can go.

For example, suppose a grid has **idim** = 37, **jdim** = 65, **kdim** = 65. The coarse grid below that would have **idim** = 19, **jdim** = 33, **kdim** = 33. The next coarser grid would have **idim** = 10, **jdim** = 17, **kdim** = 17. Since **idim** is an even number, that is as coarse as the grid can go; even though **jdim** and **kdim** can be reduced to coarser numbers, the entire grid is restricted by the **idim** = 10 value.

If the use of multigrid is desirable, it is important to plan ahead in the grid generation step of the computational problem. Choose grid dimensions that are "good" multigrid numbers, such as 129 (65, 33, 17, 9, 5, 3), 73 (37, 19), and 49 (25, 13, 7). Generally, two or three coarser grid levels are satisfactory.

It is best if all grid *segments* are also multigridable (see "LT14 - I0 Boundary Condition Specification" on page 32 through "LT19 - KDIM Boundary Condition Specification" on page 35). For example, a face with **jdim** = 65 might have a portion from $j = 1$ to 17 as a one-to-one interface, a portion from $j = 17$ to 41 as a viscous wall, and a portion from j = 41 to 65 as a patched interface. Each of these segment lengths (7, 25, and 25) is multigridable down three levels. Note that CFL3D will sometimes work fine even if some grid segments are not multigridable: the code can assign indices on the coarser levels that denote different physical locations than the indices on the fine grid and still converge on the finest level. However, this does *not* work all the time. For example, if a C-mesh has a wrap-around dimension of **jdim** = 257 and the wake extends from $j = 1$ to 40 and $j = 218$ to 257 (with one-to-one point matching), the code will create a coarser level with the wake from $j = 1$ to 20 and $j = 109$ to 129 (using Equation (7-1)). On this level, $j = 20$ is a *physically different point* from j = 109 (it is not even the actual trailing edge), and will yield a boundary condition error when the code is run, because these points are expected to match in a one-to-one fashion. It is currently not necessary to specify laminar regions with multigridable numbers. See *Note* (3) on page 29 in the LT9 - Laminar Region Specification description.

It is also important to consider the grid *geometry* when defining the grid dimensions. If multigrid is used, be sure to have any important geometric features (such as corners) located at multigridable points. Otherwise, on coarser levels, the geometry may change significantly, resulting in poor multigrid performance.

For a handy reference, the following table lists the grid sizes (< 1000) that are multigridable to three additional levels:

Table 7–2. Grid sizes multigridable to three additional level.

Grid:	Coarser Levels:				Grid:	Coarser Levels:				Grid:	Coarser Levels:		
9	5	3	2		345	173	87	44		673	337	169	85
17	9	5	3		353	177	89	45		681	341	171	86
25	13	7	4		361	181	91	46		689	345	173	87
33	17	9	5		369	185	93	47		697	349	175	88
41	21	11	6		377	189	95	48		705	353	177	89
49	25	13	7		385	193	97	49		713	357	179	90
57	29	15	8		393	197	99	50		721	361	181	91
65	33	17	9		401	201	101	51		729	365	183	92
73	37	19	10		409	205	103	52		737	369	185	93
81	41	21	11		417	209	105	53		745	373	187	94
89	45	23	12		425	213	107	54		753	377	189	95
97	49	25	13		433	217	109	55		761	381	191	96
105	53	27	14		441	221	111	56		769	385	193	97
113	57	29	15		449	225	113	57		777	389	195	98
121	61	31	16		457	229	115	58		785	393	197	99
129	65	33	17		465	233	117	59		793	397	199	100
137	69	35	18		473	237	119	60		801	401	201	101
145	73	37	19		481	241	121	61		809	405	203	102
153	77	39	20		489	245	123	62		817	409	205	103
161	81	41	21		497	249	125	63		825	413	207	104
169	85	43	22		505	253	127	64		833	417	209	105
177	89	45	23		513	257	129	65		841	421	211	106
185	93	47	24		521	261	131	66		849	425	213	107
193	97	49	25		529	265	133	67		857	429	215	108
201	101	51	26		537	269	135	68		865	433	217	109
209	105	53	27		545	273	137	69		873	437	219	110
217	109	55	28		553	277	139	70		881	441	221	111
225	113	57	29		561	281	141	71		889	445	223	112
233	117	59	30		569	285	143	72		897	449	225	113
241	121	61	31		577	289	145	73		905	453	227	114
249	125	63	32		585	293	147	74		913	457	229	115
257	129	65	33		593	297	149	75		921	461	231	116
265	133	67	34		601	301	151	76		929	465	233	117
273	137	69	35		609	305	153	77		937	469	235	118
281	141	71	36		617	309	155	78		945	473	237	119
289	145	73	37		625	313	157	79		953	477	239	120
297	149	75	38		633	317	159	80		961	481	241	121
305	153	77	39		641	321	161	81		969	485	243	122
313	157	79	40		649	325	163	82		977	489	245	123
321	161	81	41		657	329	165	83		985	493	247	124
329	165	83	42		665	333	167	84		993	497	249	125
337	169	85	43										

7.1.3 Overlapped Grids

With the grid-overlapping scheme, interpolation stencils are determined at the finest grid level only. Therefore, information would be missing at the coarser grid levels of the multigrid scheme. To avoid the need to obtain interpolation stencils at the coarser grid levels prior to the computation and the corresponding problems that most likely will arise, the multigrid scheme has been modified to accommodate grid-overlapping cases.[24]

First, if a fine grid boundary is supplied overlap information as the boundary condition, extrapolation is used for that boundary condition on all coarser grids. Second, for the restriction step of the multigrid scheme, all points are interpolated to coarser grids regardless of whether or not they are in a hole at the fine grid level. The flow variables in the hole are simply at free-stream conditions. Finally, for the prolongation step, only corrections on field points are used to update the finest mesh solution. The hole points are not updated since they will be overwritten with free-stream values when a Navier-Stokes calculation is made at each fine grid level. Likewise, the fringe points are not corrected since they will be updated with flow information from other grids at the same fine grid level.

Note that in Chapter 3 under “LT20 - Mesh Sequencing and Multigrid” on page 35, it is stated that the W-cycle is *not* recommended with overlapped grids. Therefore, set **ngam** = 1.

7.1.4 Embedded Grids

For time advancement, embedded grids basically become an extension of the multigrid scheme. Consider the grid system shown in Figure 7-6. The finest global grid has dimensions 9×9 and there are two coarser global grids for multigrid purposes. In addition, there is a 7×13 grid embedded in the finest global grid as shown in the figure. The multigrid (W-cycle) scheme would follow the course drawn in Figure 7-7.

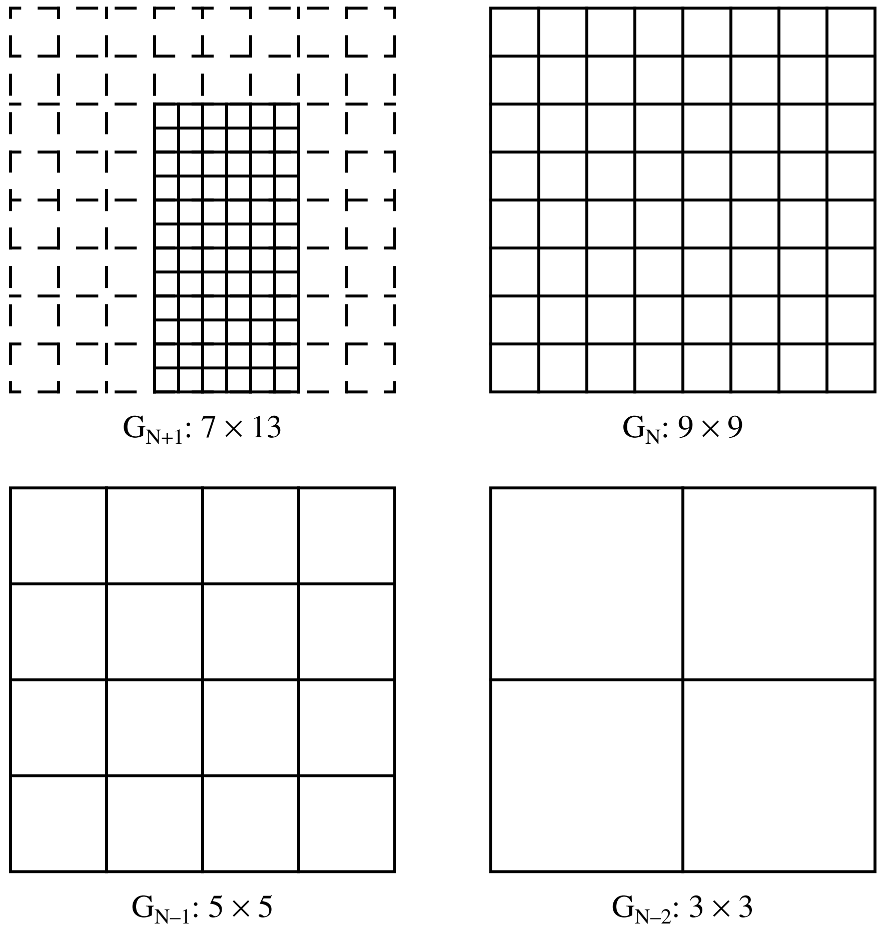


Figure 7-6. One embedded mesh in finest global grid.

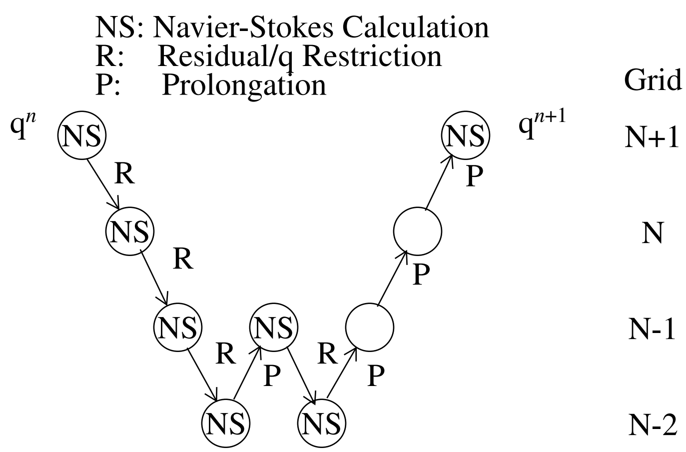


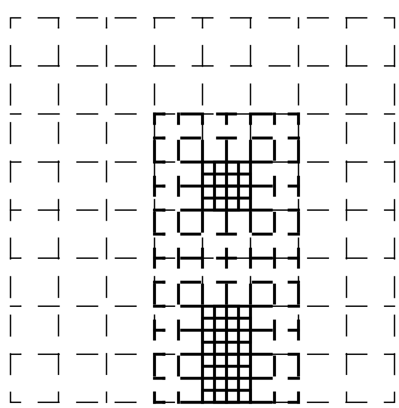
Figure 7-7. Multigrid W-cycle with one embedded grid level.

For a multigrid W-cycle, the pertinent lines of input for the example in Figure 7-6 would look something like:

Line Type

```
5         DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
        -1.0        0        0      1.0        0     10.0
6      NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
           2        0        0        0        0        1        1        1
7        NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
           2        0        0        1        0        0        0
           0        1        0        0        0        0        0
8       IDIM     JDIM     KDIM
           2        9        9
           2        7       13

10     INEWG   IGRIDC       IS       JS       KS       IE       JE       KE
           0        0        0        0        0        0        0        0
           0        1        1        4        1        2        7        7

20      MSEQ   MGFLAG   ICONSF      MTT     NGAM
           1        1        1        0        2

22      NCYC   MGLEVG    NEMGL    NITFO
         200        3        1        0
23      MIT1     MIT2     MIT3     MIT4     MIT5
           1        1        1        1        1
```

For the boundary conditions on the embedded grid, use the appropriate physical or connectivity **bctype** on any face that lies along an edge of its parent grid and **bctype** = 0 for any face that lies wholly within its parent grid. Suppose another embedded grid level is added to the grid system as in Figure 7-8. In the figure two grids are embedded in the first embedded grid. Note that only one embedded grid *level* is added by their addition.



G_{N+2}: 5×9 and 5×5

Figure 7-8. Two embedded grids within an embedded grid.

The corresponding input would resemble:

<u>Line Type</u>

```
5        DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
       -1.0        1        0      1.0        0     10.0
6     NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
          4        0        0        0        0        1        1        1
7       NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
          2        0        0        1        0        0        0
          0        1        0        0        0        0        0
          0        2        0        0        0        0        0
          0        2        0        0        0        0        0
8      IDIM     JDIM     KDIM
          2        9        9
          2        7       13
          2        5        9
          2        5        5

10    INEWG   IGRIDC       IS       JS       KS       IE       JE       KE
          0        0        0        0        0        0        0        0
          0        1        1        4        1        2        7        7
          1        2        1        3        1        2        5        5
          1        2        1        3        9        2        5       11

20     MSEQ   MGFLAG   ICONSF      MTT     NGAM
          1        1        1        0        2

22     NCYC   MGLEVG    NEMGL    NITFO
        200        3        2        0
23     MIT1     MIT2     MIT3     MIT4     MIT5
          1        1        1        1        1
```

Note that **inewg** should be set to 0 for grids 3 and 4 if this case is restarted.

Multigrid can also be performed at the embedded grid levels themselves by setting **mgflag** = 2. The embedded grid uses the coarser grid levels in which it is embedded, so **ncg** for the embedded grid itself should remain 0. The input for this case would resemble:

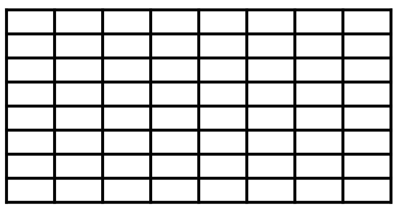
<u>Line Type</u>

```
5        DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
       -1.0        0        0      1.0        0     10.0
6     NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
          2        0        0        0        0        1        1        1
7       NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
          2        0        0        1        0        0        0
          0        1        0        0        0        0        0
8      IDIM     JDIM     KDIM
          2        9        9
          2        7       13

10    INEWG   IGRIDC       IS       JS       KS       IE       JE       KE
          0        0        0        0        0        0        0        0
          0        1        1        4        1        2        7        7

20     MSEQ   MGFLAG   ICONSF      MTT     NGAM
          1        2        1        0        2

22     NCYC   MGLEVG    NEMGL    NITFO
        200        3        1        0
23     MIT1     MIT2     MIT3     MIT4     MIT5
          1        1        1        1        1
```

7.2 Mesh Sequencing

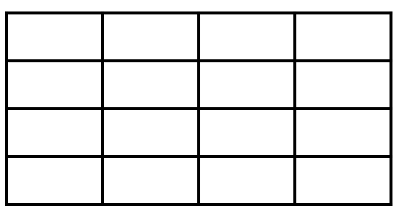
When setting up a CFD problem, initial conditions are set at every point on a grid. Usually, the closer the guess is to the final solution the quicker the case will converge. In CFL3D, the initial conditions for a problem are set at free-stream conditions for a single-grid-level case. However, if mesh sequencing is utilized, a better “guess” can be made for the initial conditions on the finer grid where the computations are most expensive.

Suppose a solution is desired on the 9×9 grid depicted in Figure 7-9. Instead of starting the solution with free-stream conditions, a solution could first be obtained on the 5×5 grid shown in Figure 7-10. While the solution on the 9×9 grid would be more accurate than that on the 5×5 grid, the coarser grid’s solution would be closer to the fine grid solution than free-stream conditions. Since the coarse grid solution can be computed more quickly than the fine grid solution, it is usually beneficial to use the coarse grid solution as the initial condition for the fine grid.



9×9

Figure 7-9. Simple grid example.



5×5

Figure 7-10. Mesh sequencing sample grid.

There are two ways to implement mesh sequencing in CFL3D. The first way is to completely converge the solution on the coarse grid before mapping it up to the fine grid. For the sample grids, the pertinent input would look like:

Line Type

```
 5        DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
        -1.0        0        0      1.0        0     10.0
 6     NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
```

```
             1        0        0        0        0        1        1        1
   7       NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
             2        0        0        1        0        0        0
   8      IDIM     JDIM     KDIM
             2        9        9

  20      MSEQ   MGFLAG   ICONSF      MTT     NGAM
             2        1        0        0        1

  22      NCYC   MGLEVG    NEMGL    NITFO
           200        2        0        0
             0        3        0        0
  23      MIT1     MIT2     MIT3     MIT4     MIT5
             1        1        1        1        1
             1        1        1        1        1
```

Note that multigrid is also employed. The coarse grid solution can be examined and, if the solution on the coarse grid is not converged, the case can be restarted on that grid. After the coarse grid solution is converged, the solution is mapped to the fine grid and the calculations continue at the fine grid level. The pertinent input for this step would look something like:

Line Type

```
   5        DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
          -1.0        1        0      1.0        0     10.0
   6     NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
             1        0        0        0        0        1        1        1
   7       NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
             2        0        0        1        0        0        0
   8      IDIM     JDIM     KDIM
             2        9        9

  20      MSEQ   MGFLAG   ICONSF      MTT     NGAM
             2        1        0        0        1

  22      NCYC   MGLEVG    NEMGL    NITFO
             1        2        0        0
           200        3        0        0
  23      MIT1     MIT2     MIT3     MIT4     MIT5
             1        1        1        1        1
             1        1        1        1        1
```

Here, one iteration is performed on the coarse grid and two-hundred iterations are performed on the fine grid. At the end of this run, only the fine grid solution is available for examination. (So be sure to save the coarse grid solution *before* this step if it will be needed later for grid refinement studies, etc.) The case can be resubmitted for additional computations on the fine grid with the following input:

Line Type

```
   5        DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
          -1.0        1        0      1.0        0     10.0
   6     NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
             1        0        0        0        0        1        1        1
   7       NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
             2        0        0        1        0        0        0
   8      IDIM     JDIM     KDIM
             2        9        9

  20      MSEQ   MGFLAG   ICONSF      MTT     NGAM
```

```
                1        1        0        0        1
22           NCYC   MGLEVG    NEMGL   NITFO
              200        3        0        0
23           MIT1     MIT2     MIT3     MIT4     MIT5
                1        1        1        1        1
```

This input could be used until satisfactory convergence has been achieved.

Even a partially-converged coarse grid solution is typically a better initial guess than free-stream conditions. Therefore, the second way to set up the mesh sequencing assumes that a set number of iterations at the coarse grid level will be adequate to improve the convergence at the fine grid level. If this is the case, the coarse grid solution is mapped to the fine grid on the first submittal. For example:

Line Type

```
 5             DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
             -1.0        0        0      1.0        0     10.0
 6          NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
                1        0        0        0        0        1        1        1
 7            NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
                2        0        0        1        0        0        0
 8           IDIM     JDIM     KDIM
                2        9        9
20           MSEQ   MGFLAG   ICONSF      MTT     NGAM
                2        1        0        0        1
22           NCYC   MGLEVG    NEMGL    NITFO
              200        2        0        0
              200        3        0        0
23           MIT1     MIT2     MIT3     MIT4     MIT5
                1        1        1        1        1
                1        1        1        1        1
```

When an input like the one above is used, the coarse grid solution is not available for examination before proceeding to the fine grid. In addition, the restart file will contain the solution for the fine grid only. This is important to remember when a grid refinement study is being conducted (in which case the first approach to mesh sequencing would be beneficial). To restart from a run with the above input, use something like:

Line Type

```
 5             DT    IREST  IFLAGTS     FMAX    IUNST   CFLTAU
             -1.0        0        0      1.0        0     10.0
 6          NGRID  NPLOT3D   NPRINT   NWREST     ICHK      I2D   NTSTEP      ITA
                1        0        0        0        0        1        1        1
 7            NCG      IEM IADVANCE   IFORCE IVISC(I) IVISC(J) IVISC(K)
                2        0        0        1        0        0        0
 8           IDIM     JDIM     KDIM
                2        9        9
20           MSEQ   MGFLAG   ICONSF      MTT     NGAM
                1        1        0        0        1
22           NCYC   MGLEVG    NEMGL    NITFO
              200        3        0        0
23           MIT1     MIT2     MIT3     MIT4     MIT5
                1        1        1        1        1
```

One last note about mesh sequencing is to emphasize, as with multigrid, the advantage of "good" grid dimensions. See "A Word About Grid Dimensions" on page 127. Planning for mesh sequencing should be made at the grid generation step of the CFD problem.