APPENDIX E Primitive Variables

While CFL3D solves the governing equations in conservation law form, only the primitive variables are stored in permanent memory. This necessitates the use of the transformation matrix:

$$\mathbf{M} = \frac{\partial \mathbf{Q}}{\partial \mathbf{q}} \tag{E-1}$$

where

$$\mathbf{Q} = \begin{bmatrix} \rho & \rho u & \rho v & \rho w & e \end{bmatrix}^T \tag{E-2}$$

$$\mathbf{q} = \begin{bmatrix} \rho & u & v & w & p \end{bmatrix}^T \tag{E-3}$$

From Equation (A-14),

$$e = \frac{p}{\gamma - 1} + \frac{\rho}{2}(u^2 + v^2 + w^2) \tag{E-4}$$

Therefore,

$$\mathbf{M} = \begin{bmatrix} \frac{\partial\rho}{\partial\rho} & \frac{\partial\rho}{\partial u} & \frac{\partial\rho}{\partial v} & \frac{\partial\rho}{\partial w} & \frac{\partial\rho}{\partial p} \\ \frac{\partial\rho u}{\partial\rho} & \frac{\partial\rho u}{\partial u} & \frac{\partial\rho u}{\partial v} & \frac{\partial\rho u}{\partial w} & \frac{\partial\rho u}{\partial p} \\ \frac{\partial\rho v}{\partial\rho} & \frac{\partial\rho v}{\partial u} & \frac{\partial\rho v}{\partial v} & \frac{\partial\rho v}{\partial w} & \frac{\partial\rho v}{\partial p} \\ \frac{\partial\rho w}{\partial\rho} & \frac{\partial\rho w}{\partial u} & \frac{\partial\rho w}{\partial v} & \frac{\partial\rho w}{\partial w} & \frac{\partial\rho w}{\partial p} \\ \frac{\partial e}{\partial\rho} & \frac{\partial e}{\partial u} & \frac{\partial e}{\partial v} & \frac{\partial e}{\partial w} & \frac{\partial e}{\partial p} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ u & \rho & 0 & 0 & 0 \\ v & 0 & \rho & 0 & 0 \\ w & 0 & 0 & \rho & 0 \\ \frac{u^2 + v^2 + w^2}{2} & \rho u & \rho v & \rho w & \frac{1}{\gamma - 1} \end{bmatrix} \tag{E-5}$$

The inverse of $\mathbf{M}$ is

$$\mathbf{M}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{u}{\rho} & \frac{1}{\rho} & 0 & 0 & 0 \\ -\frac{v}{\rho} & 0 & \frac{1}{\rho} & 0 & 0 \\ -\frac{w}{\rho} & 0 & 0 & \frac{1}{\rho} & 0 \\ \frac{\gamma-1}{2}(u^2+v^2+w^2) & -u(\gamma-1) & -v(\gamma-1) & -w(\gamma-1) & 1 \end{bmatrix} \tag{E-6}$$