# Introduction

We present here a novel approach withing the `conviqt` algorithm, in presence of a spinning Half-Wave plate (HWP) polarization modulator accounting for the polarisation properties of the beam. In fact, it produces a rotation of the beam which is no more stationary in the sky when traditional convolution methodologies are employed . Convolution methods working in harmonic space usually create data-cubes where for each (θ, ϕ, ψ) position and orientation in the sky (with a suitable grids in the three dimensions) the convolution result is stored. The actual value of the convolved signal for a generic pointing direction and orientation is obtained via interpolation of the data-cube. The HWP adds an extra-dimension of the data-cube and the required resolution and number of detectors of modern experiments make the problem almost unfeasible.

# Notes on the TEB convolution implementation TOAST

With this approach we aim at including the polarized component of the beam that were neglected in the implementation of the TOAST operator `SimWeightedConviqt` link , where the same perfect co-polar beam from total intensity is adopted also for the polarization part. This means that for total intensity the beam is made only of its $I$ (total intensity) component and for the two linear polarisation directions only the $Q$ $(U)$ component is non-zero and it is equal to the $I$ component).

In the following, we briefly describe the previous implementation.

We firstly, convert the `GRASP` $\theta, \phi$ simulated grid files into the equivalent $IQU$ Stokes beam maps.

We then generated 3 beam $IQU$ maps so that :

- $b^I = (b^I, 0, 0)$
- $b^Q = (0, b^I, 0)$
- $b^U = (0, 0, b^I)$

with $b^I$ being the total intensity component of the initial `GRASP` beam.

Each beam map is then expanded into $TEB$ spherical harmonic coefficients, $b^X_{\ell,m}$. We remark here that

$$b^{I,E}_{\ell m} = b^{I,B}_{\ell m} = b^{Q,T}_{\ell m} = b^{U,T}_{\ell m} = 0,$$

which implies that for each beam map only the convolution for the non-zero component is performed with the relative sky harmonic coefficients, $a_{\ell m}$ .

The final convolved TOD is thus :

$$d_t \propto \sum_{\ell m s} b^{I*}_{\ell s} a_{\ell m} + (b^{Q*}_{\ell s} a_{\ell m} + b^{U*}_{\ell s} a_{\ell m}) e^{2i\psi + 4i\alpha_{HWP}} {}_{\pm 2}Y_{\ell m}$$

The new convolution algorithm( link ) assumes that the unpolarized and polarized properties of the `GRASP` $IQU$ Stokes beam maps are encoded into two beams:

- $b^T_{\ell m}$, the total intensity beam
- $b^P_{\ell m} = b^E_{\ell m} + i b^B_{\ell m}$ , the *polarized* beam obtained by expanding into $E$ and $B$ mode the $QU$ beam maps.

The convolved TOD is then:

$$d_t \propto \sum_{\ell m s} b_{\ell s}^{T*} a_{\ell m}^{T} + b_{\ell s}^{P*} (a_{\ell m}^{E} + i a_{\ell m}^{B}) e^{2i\psi + 4i\alpha_{HWP}} {}_{\pm 2}Y_{\ell m}$$

It is immediate to show that the convolution of the polarization   signals is :

- $Q \propto b_{\ell s}^{E} a_{\ell m}^{E} + b_{\ell s}^{B} a_{\ell m}^{B}$
- $U \propto b_{\ell s}^{E} a_{\ell m}^{B} - b_{\ell s}^{B} a_{\ell m}^{E}$