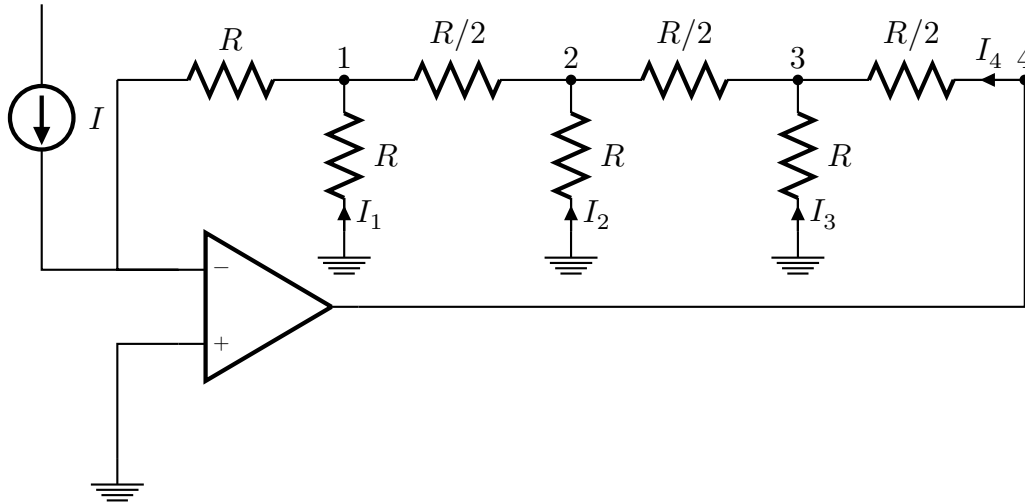


1 2.27

The circuit in Fig below can be consider to be an extension of the circuit in Fig. 2.8



- (a) Find the resistance looking into node 1, 2, 3, 4.

Ans:

$$R_1 = R$$

$$R_2 = (R_1 || R) + R/2 = R$$

$$R_3 = (R_2 || R) + R/2 = R$$

$$R_4 = (R_3 || R) + R/2 = R$$

- (b) Find the currents I_1, I_2, I_3, I_4 , in terms of I

Ans:

$$I_1 = IR/R = I$$

$$I_2 = ((I + I_1)(R/2) + IR)/R = 2I$$

$$I_3 = (4IR/2 + 2IR)/R = 4I$$

$$I_4 = -(4I + 4I) = -8I$$

- (c) Find the voltages at node 1, 2, 3, 4.

Ans:

$$V_1 = -IR$$

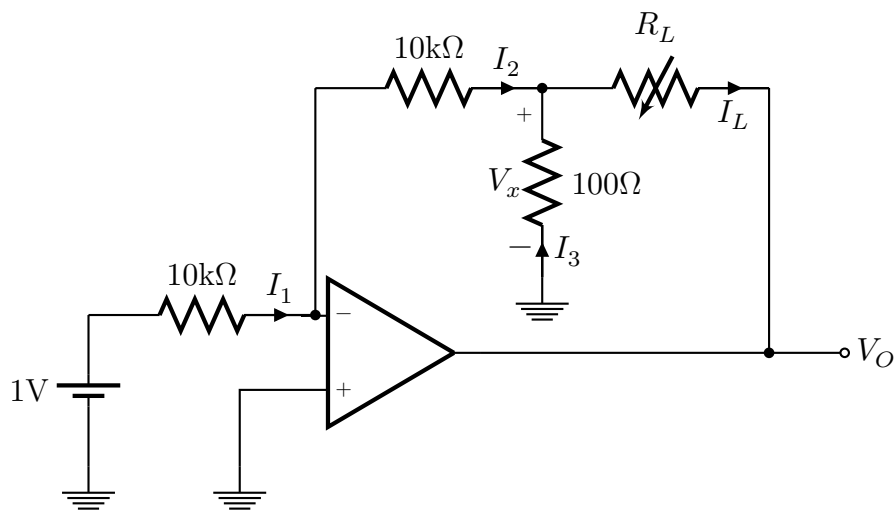
$$V_2 = V_1 + 2IR/2 = -2IR$$

$$V_3 = V_2 + 4IR/2 = -4IR$$

$$V_4 = V_3 + 8IR/2 = -8IR$$

2 2.28

The circuit below utilizes an ideal op amp.



- (a) Find I_1, I_2, I_3, V_x

Ans:

$$I_1 = 1V/10k\Omega = 0.1mA$$

$$I_2 = I_1 = 0.1mA$$

$$V_x = 0 - (0.1mA)(10k\Omega) = -1V$$

$$I_3 = 1/100\Omega = 10mA$$

- (b) If V_O is not to be lower than $-13V$, find the maximum allowed value of R_L

Ans:

$$\begin{aligned} V_O &= V_x + R_L I_L = V_x + R_L (I_2 + I_3) \\ &= -1V - (10.1mA)R_L \end{aligned}$$

so

$$R_L \leq 12V/10.1mA \approx 1.19$$

- (c) If R_L is varied in the range 100Ω to $1k\Omega$, what is the corresponding change in I_L and in V_O ?

Ans:

I_L would not change

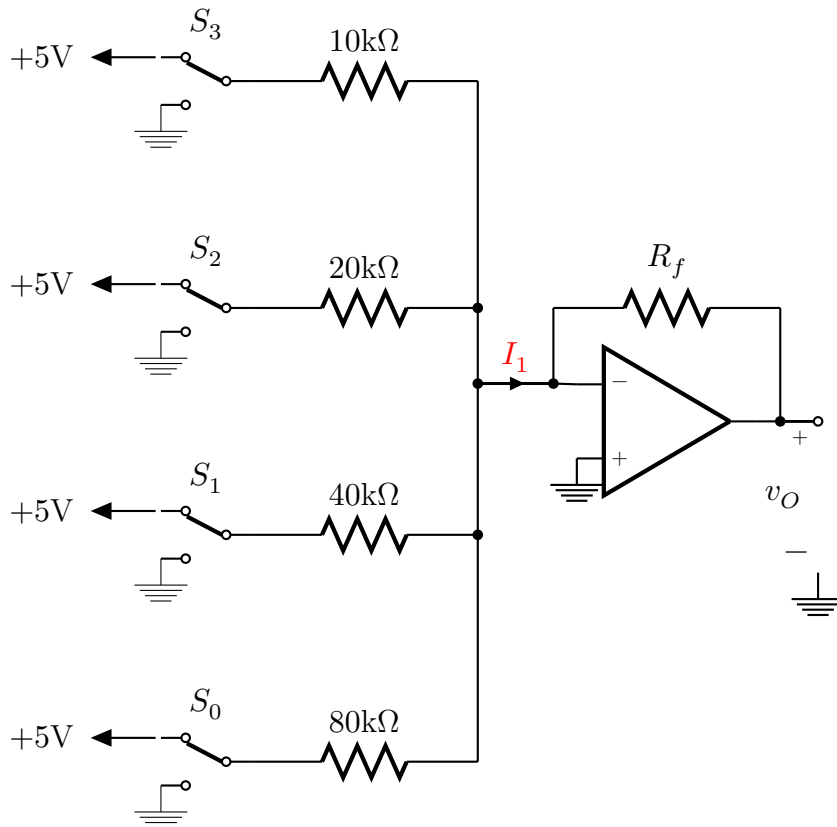
$$V_O|_{R_L=100\Omega} = 2.02V, V_O|_{R_L=1k\Omega} = 11.2V$$

3 2.37

Figure P2.37 shows a circuit for a digital-to-analog converter. The circuit accepts a 4-bit input binary word $a_3a_2a_1a_0$ where $a_i \in \{0, 1\}$. Each of the bits of the input word controls the correspondingly numbered switch. For instance, if a_2 is 0 then switch S_2 connects the $20k\Omega$ resistor to ground, while if a_2 is 1 then S_2 connects the $20k\Omega$ resistor to the $+5V$ power supply. Show that v_O is given by

$$v_O = -\frac{R_f}{16} \sum_{i=0}^3 2^i a_i$$

where R_f is in kilohms. Find the value of R_f so that v_O ranges from 0 to $-12V$.



Ans:

Note that

$$I_1 = \sum_{i=0}^3 a_i(5V)/(10 \cdot 2^{(3-i)}k\Omega) = \sum_{i=0}^3 a_i/2^{(4-i)}(mA)$$

so

$$v_O = 0 - R_f I_1 = -R_f \sum_{i=0}^3 a_i 2^i / 16.$$

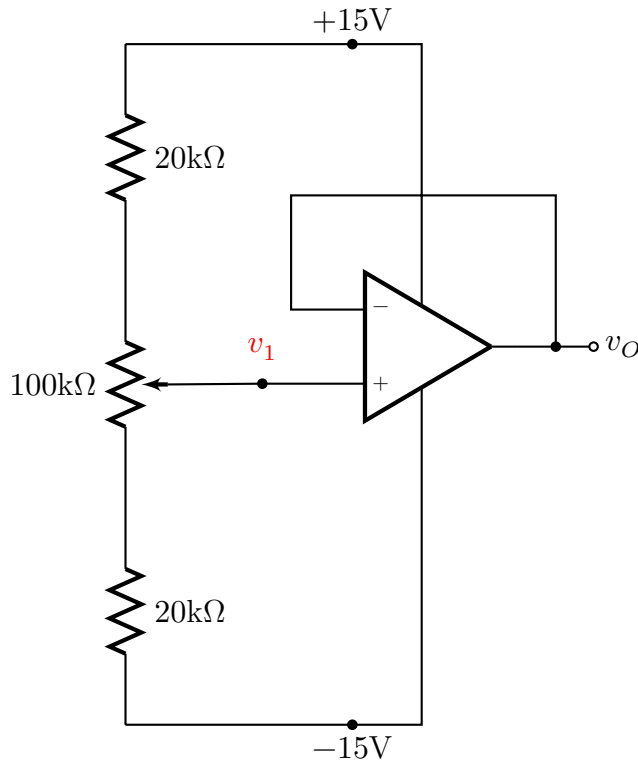
If v_O ranges from 0 V to 12 V, then

$$v_O = -\frac{R_f}{16} \sum_{i=0}^3 a_i 2^i \Rightarrow 0 \geq v_O \geq -\frac{15}{16} R_f.$$

So $R_f = 12.8\text{k}\Omega$.

4 2.51

Figure shows a circuit that provides an output voltage v_o whose value can be varied by turning the wiper of the $100\text{k}\Omega$ potentiometer. Find the range over which v_o can be varied. If the potentiometer is a "20-turn" device, find the change in v_o corresponding to each turn of the pot.



Ans:

We have

$$\begin{aligned} v_O &= v_1 \\ v_{1,\max} &= (15\text{V}) \frac{100 + 20}{100 + 20 + 20} + (-15\text{V}) \frac{20}{100 + 20 + 20} \approx 10.7\text{V} \\ v_{1,\min} &= -v_{1,\max} \end{aligned}$$

And so the voltage change each turn is $(v_{1,\max} - v_{1,\min})/20 \approx 1.07$.

5 2.63

For an instrumentation amplifier of the type shown in Fig 2.20(b), a designer proposes to make $R_2 = R_3 = R_4 = 100\text{k}\Omega$ and $2R_1 = 10\text{k}\Omega$. For ideal components, what difference-mode gain, common-mode gain, and CMRR result? Reevaluate the worst-case values for these for the situation in which all resistors are specified as $\pm 1\%$ units. Repeat the latter analysis for the case in which $2R_1$ is reduced to $1\text{k}\Omega$. What do you conclude about the importance of the relative difference gains of the first and second stages?

Ans:

For v_{Icm} , the first stage has no effect, $v_{O1} = v_{O2} = v_{Icm}$. So common-mode gain $A_{Icm} = 1\text{V/V}$. For v_{Id} , we have

$$v_{O1} - v_{O2} = \left(1 + 2\frac{100}{10}\right) v_{Id} = 21v_{Id},$$

and since $R_3 = R_4$, difference-mode gain $A_{Id} = 21\text{V/V}$. CMRR is $20 \log \left(\frac{A_{Id}}{A_{Icm}} \right) = \infty$.

If all resistors are specified as $\pm 1\%$ units, we can maximize common-mode gain by let $R_{31} = R_{42} = 101\text{k}\Omega$, $R_{32} = R_{41} = 99\text{k}\Omega$, then common-mode gain

$$A_{Icm} = \frac{R_{42}}{R_{32} + R_{42}} \left(1 - \frac{R_{41}R_{32}}{R_{31}R_{42}}\right) \approx 1.98 \times 10^{-2} \text{V/V}.$$

And we minimize difference-mode gain by let $2R_1 = 10.1\text{k}\Omega$, $R_2 = 99\text{k}\Omega$.

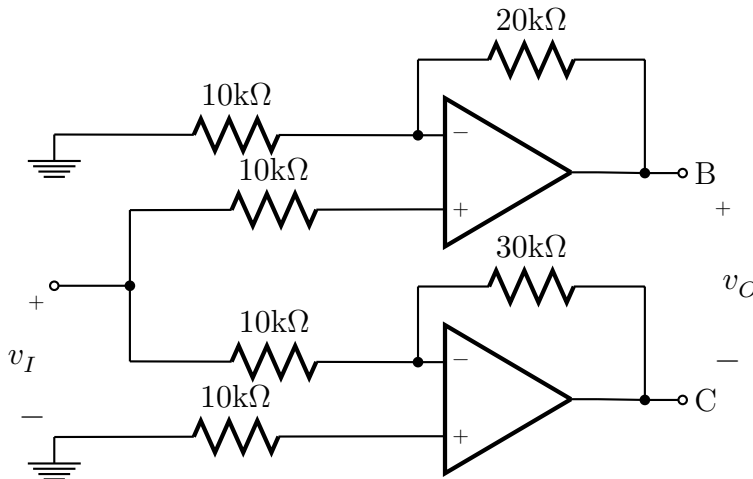
$$A_{Id} = \frac{2R_2 + 2R_1}{2R_1} \frac{R_{41} + R_{42}}{R_{31} + R_{32}} \approx 20.6 \text{V/V}.$$

CMRR is $20 \log \left(\frac{A_{Id}}{A_{Icm}} \right) \approx 60.34\text{dB}$.

If $2R_1 = 1\text{k}\Omega$, then $A_{Id} \approx 201 \text{V/V}$. The first stage dominates the difference-mode gain. And the common-mode gain doesn't change. So CMRR is $20 \log \left(\frac{A_{Id}}{A_{Icm}} \right) \approx 80.13\text{dB}$.

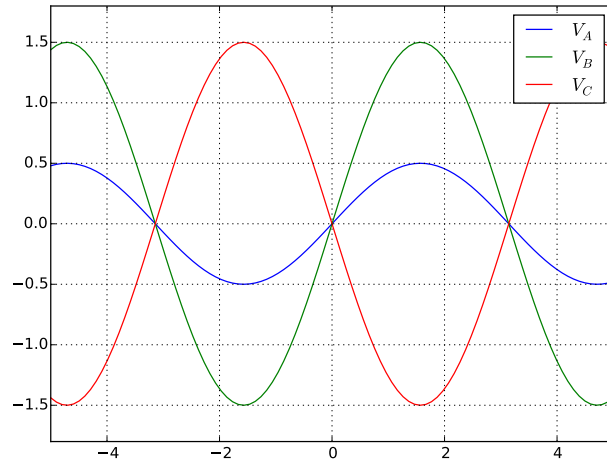
6 2.65

The circuit shown in Fig is intended to supply a voltage to floating loads (those for which both terminals are ungrounded) while making possible use of available power supply.



- (a) Assuming ideal op amps, sketch the voltage waveforms at nodes B and C for a 1V peak-to-peak sine wave applied at A. Also sketch v_O

Ans:



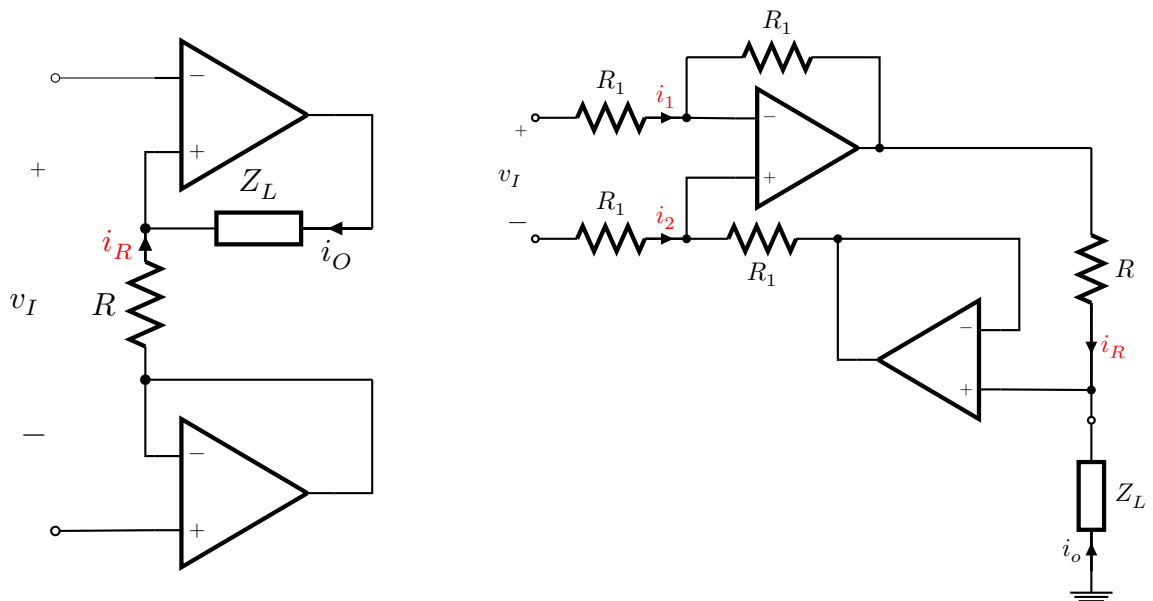
- (b) What is the voltage gain v_O/v_I ?

Ans:

It is easy to see that $v_B = 3v_A$, $v_C = -3v_A$, so $v_O/v_I = 6$.

7 2.66

The two circuits in Fig are intended to function as voltage-to-current converters; that is, they supply the load impedance Z_L with a current proportional to v_I and independent of the value of Z_L . Show that this is indeed the case, and find for each circuit i_o as a function of v_I . Comment on the differences between the two circuits.



Ans:

For fig 1, $i_O = i_R = v_I/R$ and hence proportional to v_I and independent of Z_L .

For fig 2, $i_O = i_R$, note that since the pos/neg terminal of the op amp above is virtual shorted, $v_I - i_1 R_1 = -i_2 R_1$, so $-i_1 R_1 + i_2 R_1 = -v_I$, hence we have

$$i_R = \frac{v_I - 2i_1 R_1 + 2i_2 R_1}{R} = -v_I/R$$

and hence proportional to v_I and independent of Z_L .

Comment: No comment.

8 2.69

An op-amp-based inverting integrator is measured at 1kHz to have a voltage gain of -100V/V . At what frequency is its gain reduced to -1V/V ? What is the integrator time constant?

Ans:

We know that $|V_O|/|V_I| = 1/(\omega RC)$, so if $\tau = RC$ is the time constant.

$$\frac{1}{10^3 \tau} = 10^2 \Rightarrow \tau = 10^{-5}$$

and Hence at 10^5Hz , $|V_O|/|V_i| = 1/(10^5 10^{-5}) = 1$.

9 2.79

Derive the transfer function of the circuit in Fig and show that it can be written in the form

$$\frac{|V_o|}{|V_i|} = \frac{-R_2/R_1}{(1 + (\omega_1/(i\omega))) (1 + (\omega/(i\omega_2)))}$$

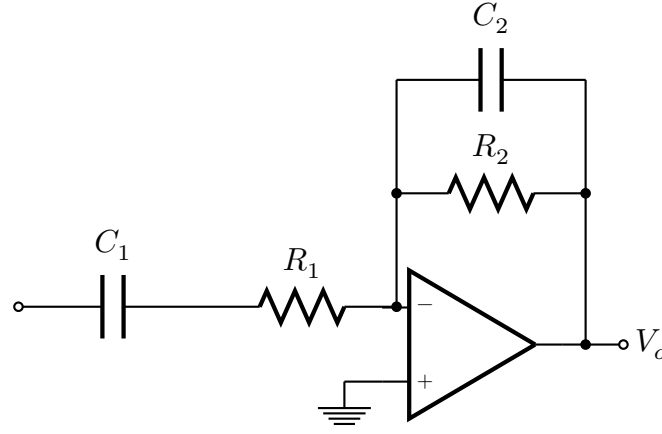
Where $\omega_1 = 1/(C_1 R_1)$ and $\omega_2 = 1/(C_2 R_2)$. Assuming that the circuit is designed such that $\omega_2 \gg \omega_1$, find approximate expressinos for the transfer function in the following frequency regions:

(a) $\omega \ll \omega_1$

(b) $\omega_1 \ll \omega \ll \omega_2$

(c) $\omega \gg \omega_2$

Use these approximations to sketch a Bode plot for the magnitude response. Observe that the circuit performs as an amplifier whose gain rolls off at the low-frequency end in the manner of a high-pass STC network, and at the high frequency end in the manner of a low-pass STC network. Design the circuit to provide a gain of 40dB in the "middle frequency range," a low-frequency 3dB point at 100kHz, and an input resistance (at



$\omega \gg \omega_1$) of $1k\Omega$.

Ans:

$$\begin{aligned}
 \frac{V_o}{V_i} &= -\frac{1/(i\omega C_2 + 1/R_2)}{R_1 + 1/(i\omega C_1)} \\
 &= \frac{-1}{\left(i\omega C_2 + \frac{1}{R_2}\right) \left(R_1 + \frac{1}{i\omega C_1}\right)} \\
 &= \frac{-R_2/R_1}{(1 + i\omega C_2 R_2) \left(\frac{1}{i\omega C_1 R_1} + R_2\right)} \\
 &= \frac{-R_2/R_1}{(1 + \omega_1/(i\omega))(1 + i(\omega/\omega_2))}
 \end{aligned}$$

Let the transfer function be $f(\omega)$ and $-R_2/R_1 = \alpha$

- (a) $\omega \ll \omega_1 \ll \omega_2 \Rightarrow 1 + i(\omega/\omega_2) \approx 1$ and $1 + \omega_1/(i\omega) \approx \omega/\omega_1$, Hence $f(\omega) \approx \alpha i\omega/\omega_1$
- (b) $\omega_1 \ll \omega \ll \omega_2 \Rightarrow 1 + i(\omega/\omega_2) \approx 1$ and $1 + \omega_1/(i\omega) \approx 1$, Hence $f(\omega) \approx \alpha$
- (c) $\omega_1 \ll \omega_2 \ll \omega \Rightarrow 1 + i(\omega/\omega_2) \approx i\omega/\omega_2$ and $1 + \omega_1/(i\omega) \approx 1$, Hence $f(\omega) \approx \alpha\omega_2/(i\omega)$

10 2.100

A designer, wanting to achieve a stable gain of $100V/V$ at $5MHz$, considers her choice of amplifier topologies. What unity-gain frequency would a single operational amplifier require to satisfy her need? Unfortunately, the best available amplifier has an f_t of $40MHz$. How many such stages would she need to achieve her goal? What is the 3dB frequency of each stage she can use? What is the overall 3dB frequency?

Ans:

By the single pole model of the op amp. $f_t > 100 \cdot 5MHz$, provided that the low frequency gain is much greater than $100V/V$.

Let $G(\omega)$ be the gain of the best available amplifier. $G(5MHz) = 40/5 = 8$. So you will need at least 3 op amp so that $8^3 = 512 > 100$.

Now we solve for the gain of the op amp G . notice that the 3dB frequency will be

approximately at 40/GMHz, so for the total gain to be 100V/V, we have

$$\frac{G}{\sqrt{1 + \left(\frac{f}{40/G}\right)^2}} = \sqrt[3]{100}$$

Solve for G we get $G \approx 5.70$, so the 3dB frequency each stage is approximately 40MHz/5.7 \approx 7.02MHz. The over all 3dB frequency is when $1 + (f'/7.02)^2 = \sqrt[3]{2}$, and so $f' \approx 3.57$ MHz.

11 2.102

Consider an inverting summer with two inputs V_1 and V_2 and with $V_O = -(V_1 + V_2)$. Find the 3dB frequency of each of the gain functions V_O/V_1 and V_O/V_2 in terms of the op amp f_1 .

Obviously $V_O/V_1 = V_O/V_2$. Set $V_2 = 0$, We have $I_3 = I_1 - I_2 = (V_1 - V_3)/R - V_3/R$, so $-AV_3 = V_O = V_3 - (V_1 - V_3 - V_3) = 3V_3 - V_1 \Rightarrow V_O = -1/(1 + 3/A)$. Substitute $A = A_0/(1 + \omega/\omega_b)$ yields

$$G(\omega) = \frac{-1}{1 + 3(1 + \omega/\omega_b)/A_0} = \frac{-A_0}{A_0 + 3 + 3\omega/\omega_b}$$

Notice that $A_0 \gg 3$, so $\omega_{3dB} \approx A_0\omega_b/3 \approx \omega_t/3$.

12 2.104

An op amp having a slew rate of 20V/ μ s is to be used in the unity-gain follower configuration, with input pulses that rise from 0 to 3V. What is the shortest pulse that can be used while ensuring full-amplitude output? For such a pulse, describe the outputting resulting.

Ans:

Easy... $3/20 = 0.15\mu$ s.

13 1.78

A p^+n junction is one in which the doping concentration in the p region is much greater than that in the n region. In such a junction, the forward current is mostly due to hole injection across the junction. Show that

$$I \approx I_p = Aqn_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

For the specific case in which $N_D = 10^{16}/\text{cm}^3$, $N_p = 10\text{cm}^2/\text{s}$, $L_p = 10\mu\text{m}$, $A = 10^4\mu\text{m}^2$. Find I_S and the voltage V obtained when $I = 0.5\text{mA}$. Assume operation at 300K where $n_i = 1.5 \cdot 10^{10}/\text{cm}^3$.

Ans:

We have

$$I = I_p + I_n = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1)$$

but since the acceptor dominates, $N_A \gg N_D$, and hence

$$I \approx I_p = Aqn_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

A straight forward calculation yields

$$\begin{aligned} I_S = Aqn_i^2 \frac{D_p}{L_p} &\approx (10^4 \cdot 10^{-8} \text{cm}^3)(1.6 \cdot 10^{-19} \text{C})(1.5 \cdot 10^{10} / \text{cm}^3)^2 (10 \text{cm}) / (10^{-3} \text{cm}) / (10^{16} \text{cm}) \\ &\approx 3.6 \cdot 10^{-15} \text{mA} \end{aligned}$$

14 1.82

A short-base diode is one where the widths of the p and n regions are much smaller than L_n and L_p , respectively. As a result, the excess minority carrier distribution in each region is a straight line rather than the exponentials shown in Fig. 1.39.

- (a) For the short-base diode, sketch a figure corresponding to Fig.1.39 and assume as in Fig.1.39 that $N_A \gg N_D$.

Ans:

TODO or not TODO

- (b) Following a derivation similar to that given in Section 1.11.2, show that if the widths of the p and n regions are denoted W_p and W_n then

$$I = Aqn_i^2 \left(\frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A} \right) (e^{V/V_T} - 1)$$

and

$$\begin{aligned} Q_p &= \frac{1}{2} \frac{(W_n - x_n)^2}{D_p} I_p \\ &\approx \frac{1}{2} \frac{W_n^2}{D_p} I_p, \quad \text{for } W_n \gg x_n \end{aligned}$$

Ans:

$p_n(x_n) = p_{n_0} e^{V/V_T}$ as usual, but the excess charge at W_n equals 0, so $p_n(W_n) = p_{n_0}$, hence

$$\frac{dp}{dx} = p_{n_0} (e^{V/V_T} - 1) / (W_n - x_n) = n_i^2 (e^{V/V_T} - 1) / ((W_n - x_n)N_D)$$

everywhere, hence we have

$$I_p = AJ_p = AqD_p \frac{dp}{dx} = Aqn_i^2 \frac{D_p}{N_D(W_n - x_n)} (e^{V/V_T} - 1)$$

similarly

$$I_n = AJ_n = AqD_n \frac{dp}{dx} = Aqn_i^2 \frac{D_n}{N_A(W_p - x_p)} (e^{V/V_T} - 1)$$

So

$$I = I_n + I_p = Aqn_i^2 \left(\frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A} \right) (e^{V/V_T} - 1)$$

Finally,

$$\begin{aligned} Q_p &= Aq \frac{(p_n(x_n) - p_n(W_n))(W_n - x_n)}{2} \\ &= Aq \frac{(p_n(x_n) - p_n(W_n))(W_n - x_n)^2}{2(W_n - x_n)} \\ &= Aq \frac{(W_n - x_n)^2}{D_p} I_p \\ &\approx Aq \frac{W_n^2}{2D_p} I_p \quad (\text{If } W_n \gg x_n) \end{aligned}$$

(c) Also, assuming $Q \approx Q_p, I \approx I_p$, show that

$$C_d = \frac{\tau_T}{V_T} I$$

where

$$\tau_T = \frac{1}{2} \frac{W_n^2}{D_p}$$

Ans:

$$\begin{aligned} C_d &= \frac{dQ}{dV} \approx \frac{dQ_p}{dV} \\ &= \frac{d(AqW_n^2/D_p)I_p}{2dV} \\ &= \frac{AqW_n^2}{2D_p} \frac{dI_s (e^{V/V_T} - 1)}{dV} \end{aligned}$$

If $V \gg V_T \approx 25.8\text{mV}$,

$$\frac{dI_s (e^{V/V_T} - 1)}{dV} = I_s e^{V/V_T} / V_T \approx I / V_T$$

$$\begin{aligned} C_d &\approx \frac{AqW_n^2}{2D_p} I / V_T \\ &= \frac{\tau_T}{V_T} I \end{aligned}$$

- (d) if a designer wishes to limit C_d to 8pF at $I = 1\text{mA}$, what should W_n be? Assume $D_p = 10\text{cm}^2/\text{s}$

Ans:

$$\begin{aligned} W_n &= \sqrt{\frac{2C_d D_p V_T}{I}} \\ &= \sqrt{2(8 \cdot 10^{-12}\text{F})(10^{-3}\text{m})(25.8 \cdot 10^{-3}\text{V})(10^3\text{A}^{-1})} \\ &\approx 642\text{nm} \end{aligned}$$