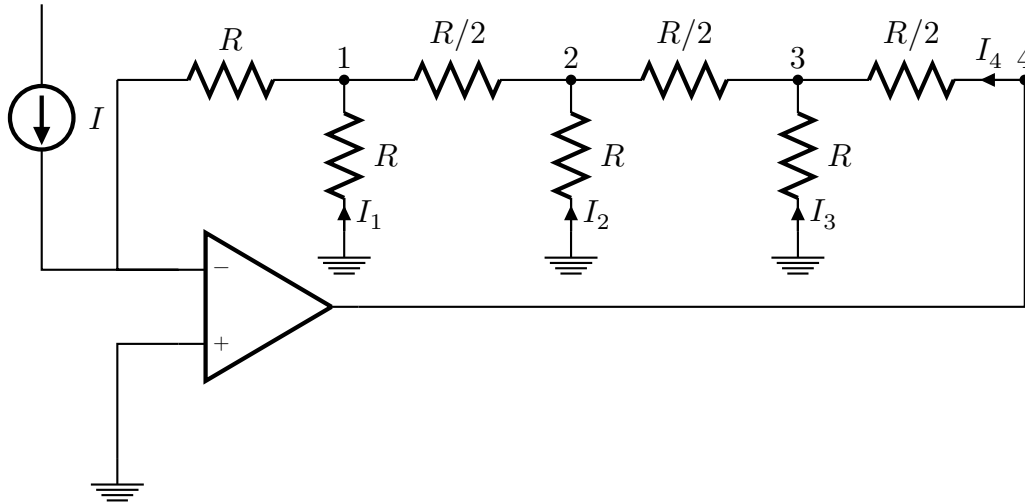


## 1 2.27

The circuit in Fig below can be consider to be an extension of the circuit in Fig. 2.8



- (a) Find the resistance looking into node 1, 2, 3, 4.

**Ans:**

$$R_1 = R$$

$$R_2 = (R_1 || R) + R/2 = R$$

$$R_3 = (R_2 || R) + R/2 = R$$

$$R_4 = (R_3 || R) + R/2 = R$$

- (b) Find the currents  $I_1, I_2, I_3, I_4$ , in terms of  $I$

**Ans:**

$$I_1 = IR/R = I$$

$$I_2 = ((I + I_1)(R/2) + IR)/R = 2I$$

$$I_3 = (4IR/2 + 2IR)/R = 4I$$

$$I_4 = -(4I + 4I) = -8I$$

- (c) Find the voltages at node 1, 2, 3, 4.

**Ans:**

$$V_1 = -IR$$

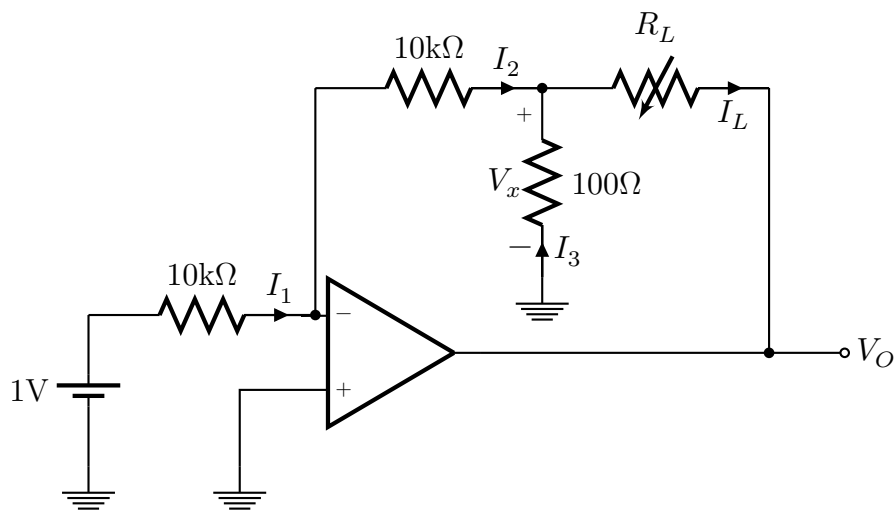
$$V_2 = V_1 + 2IR/2 = -2IR$$

$$V_3 = V_2 + 4IR/2 = -4IR$$

$$V_4 = V_3 + 8IR/2 = -8IR$$

## 2 2.28

The circuit below utilizes an ideal op amp.



- (a) Find  $I_1, I_2, I_3, V_x$

**Ans:**

$$I_1 = 1V/10k\Omega = 0.1mA$$

$$I_2 = I_1 = 0.1mA$$

$$V_x = 0 - (0.1mA)(10k\Omega) = -1V$$

$$I_3 = 1/100\Omega = 10mA$$

- (b) If  $V_O$  is not to be lower than  $-13V$ , find the maximum allowed value of  $R_L$

**Ans:**

$$\begin{aligned} V_O &= V_x + R_L I_L = V_x + R_L (I_2 + I_3) \\ &= -1V - (10.1mA)R_L \end{aligned}$$

so

$$R_L \leq 12V/10.1mA \approx 1.19$$

- (c) If  $R_L$  is varied in the range  $100\Omega$  to  $1k\Omega$ , what is the corresponding change in  $I_L$  and in  $V_O$ ?

**Ans:**

$I_L$  would not change

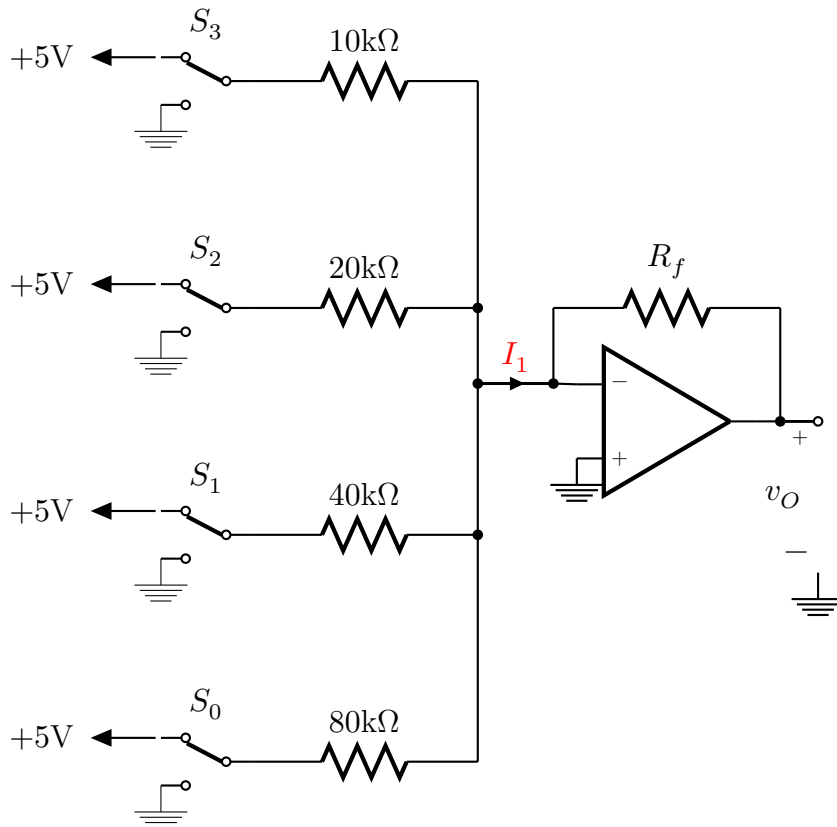
$$V_O|_{R_L=100\Omega} = 2.02V, V_O|_{R_L=1k\Omega} = 11.2V$$

### 3 2.37

Figure P2.37 shows a circuit for a digital-to-analog converter. The circuit accepts a 4-bit input binary word  $a_3a_2a_1a_0$  where  $a_i \in \{0, 1\}$ . Each of the bits of the input word controls the correspondingly numbered switch. For instance, if  $a_2$  is 0 then switch  $S_2$  connects the  $20k\Omega$  resistor to ground, while if  $a_2$  is 1 then  $S_2$  connects the  $20k\Omega$  resistor to the  $+5V$  power supply. Show that  $v_O$  is given by

$$v_O = -\frac{R_f}{16} \sum_{i=0}^3 2^i a_i$$

where  $R_f$  is in kilohms. Find the value of  $R_f$  so that  $v_O$  ranges from 0 to  $-12V$ .



**Ans:**

Note that

$$I_1 = \sum_{i=0}^3 a_i(5V)/(10 \cdot 2^{(3-i)}k\Omega) = \sum_{i=0}^3 a_i/2^{(4-i)}(mA)$$

so

$$v_O = 0 - R_f I_1 = -R_f \sum_{i=0}^3 a_i 2^i / 16.$$

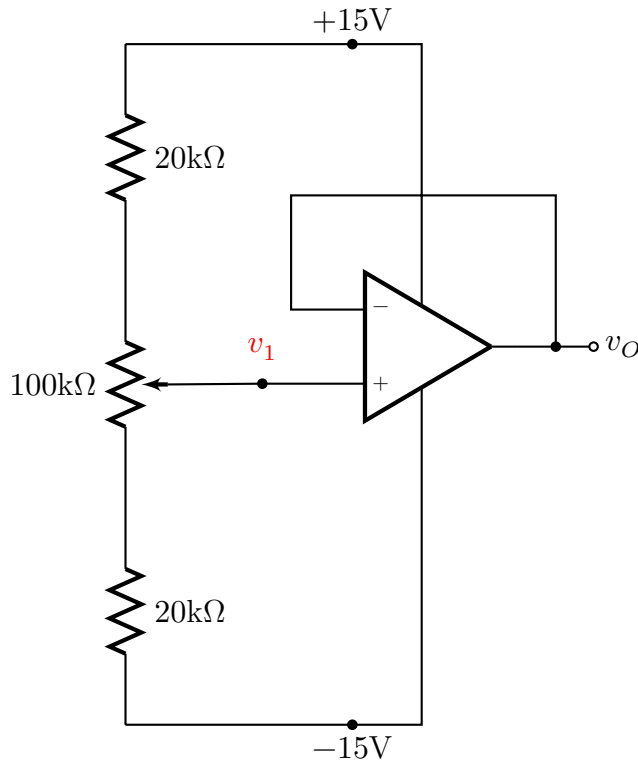
If  $v_O$  ranges from 0 V to 12 V, then

$$v_O = -\frac{R_f}{16} \sum_{i=0}^3 a_i 2^i \Rightarrow 0 \geq v_O \geq -\frac{15}{16} R_f.$$

So  $R_f = 12.8\text{k}\Omega$ .

## 4 2.51

Figure shows a circuit that provides an output voltage  $v_o$  whose value can be varied by turning the wiper of the  $100\text{k}\Omega$  potentiometer. Find the range over which  $v_o$  can be varied. If the potentiometer is a "20-turn" device, find the change in  $v_o$  corresponding to each turn of the pot.



**Ans:**

We have

$$\begin{aligned} v_O &= v_1 \\ v_{1,\max} &= (15\text{V}) \frac{100 + 20}{100 + 20 + 20} + (-15\text{V}) \frac{20}{100 + 20 + 20} \approx 10.7\text{V} \\ v_{1,\min} &= -v_{1,\max} \end{aligned}$$

And so the voltage change each turn is  $(v_{1,\max} - v_{1,\min})/20 \approx 1.07$ .

## 5 2.63

For an instrumentation amplifier of the type shown in Fig 2.20(b), a designer proposes to make  $R_2 = R_3 = R_4 = 100\text{k}\Omega$  and  $2R_1 = 10\text{k}\Omega$ . For ideal components, what difference-mode gain, common-mode gain, and CMRR result? Reevaluate the worst-case values for these for the situation in which all resistors are specified as  $\pm 1\%$  units. Repeat the latter analysis for the case in which  $2R_1$  is reduced to  $1\text{k}\Omega$ . What do you conclude about the importance of the relative difference gains of the first and second stages?

**Ans:**

For  $v_{Icm}$ , the first stage has no effect,  $v_{O1} = v_{O2} = v_{Icm}$ . So common-mode gain  $A_{Icm} = 0\text{V/V}$ . For  $v_{Id}$ , we have

$$v_{O1} - v_{O2} = \left(1 + 2\frac{100}{10}\right) v_{Id} = 21v_{Id},$$

and since  $R_3 = R_4$ , difference-mode gain  $A_{Id} = 21\text{V/V}$ . CMRR is  $20 \log \left( \frac{A_{Id}}{A_{Icm}} \right) = \infty$ .

If all resistors are specified as  $\pm 1\%$  units, we can maximize common-mode gain by let  $R_{31} = R_{42} = 101\text{k}\Omega$ ,  $R_{32} = R_{41} = 99\text{k}\Omega$ , then common-mode gain

$$A_{Icm} = \frac{R_{42}}{R_{32} + R_{42}} \left(1 - \frac{R_{41}R_{32}}{R_{31}R_{42}}\right) \approx 1.98 \times 10^{-2} \text{V/V}.$$

And we minimize difference-mode gain by let  $2R_1 = 10.1\text{k}\Omega$ ,  $R_2 = 99\text{k}\Omega$ .

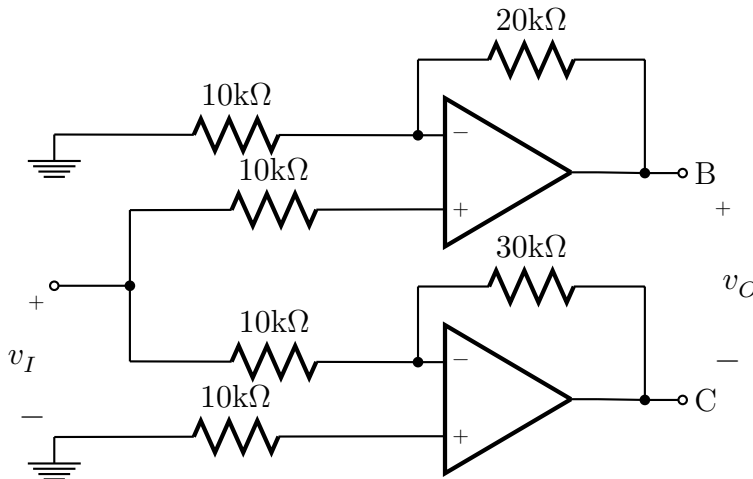
$$A_{Id} = \frac{2R_2 + 2R_1}{2R_1} \frac{R_{41} + R_{42}}{R_{31} + R_{32}} \approx 20.6 \text{V/V}.$$

CMRR is  $20 \log \left( \frac{A_{Id}}{A_{Icm}} \right) \approx 60.34\text{dB}$ .

If  $2R_1 = 1\text{k}\Omega$ , then  $A_{Id} \approx 201 \text{V/V}$ . The first stage dominates the difference-mode gain. And the common-mode gain doesn't change. So CMRR is  $20 \log \left( \frac{A_{Id}}{A_{Icm}} \right) \approx 80.13\text{dB}$ .

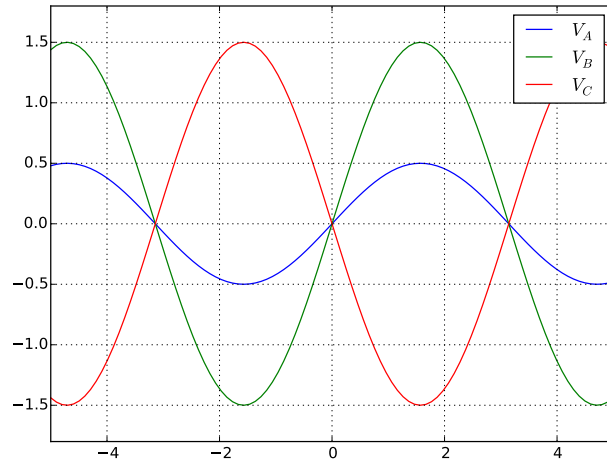
## 6 2.65

The circuit shown in Fig is intended to supply a voltage to floating loads (those for which both terminals are ungrounded) while making possible use of available power supply.



- (a) Assuming ideal op amps, sketch the voltage waveforms at nodes B and C for a 1V peak-to-peak sine wave applied at A. Also sketch  $v_O$

**Ans:**



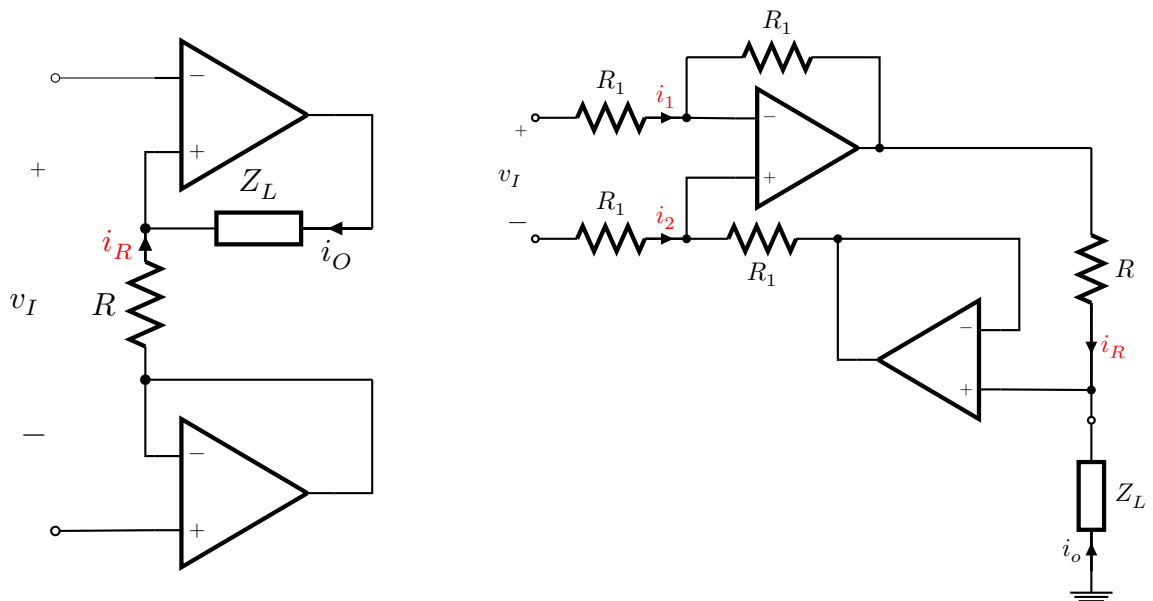
- (b) What is the voltage gain  $v_O/v_I$ ?

**Ans:**

It is easy to see that  $v_B = 3v_A$ ,  $v_C = -3v_A$ , so  $v_O/v_I = 6$ .

## 7 2.66

The two circuits in Fig are intended to function as voltage-to-current converters; that is, they supply the load impedance  $Z_L$  with a current proportional to  $v_I$  and independent of the value of  $Z_L$ . Show that this is indeed the case, and find for each circuit  $i_o$  as a function of  $v_I$ . Comment on the differences between the two circuits.



**Ans:**

For fig 1,  $i_O = i_R = v_I/R$  and hence proportional to  $v_I$  and independent of  $Z_L$ .

For fig 2,  $i_O = i_R$ , note that since the pos/neg terminal of the op amp above is virtual shorted,  $v_I - i_1 R_1 = -i_2 R_1$ , so  $-i_1 R_1 + i_2 R_1 = -v_I$ , hence we have

$$i_R = \frac{v_I - 2i_1 R_1 + 2i_2 R_1}{R} = -v_I/R$$

and hence proportional to  $v_I$  and independent of  $Z_L$ .

Comment: No comment.

## 8 2.69

An op-amp-based inverting integrator is measured at 1kHz to have a voltage gain of  $-100\text{V/V}$ . At what frequency is its gain reduced to  $-1\text{V/V}$ ? What is the integrator time constant?

**Ans:**

We know that  $|V_O|/|V_I| = 1/(\omega RC) = 1/(2\pi f RC)$ , so if  $\tau = RC$  is the time constant.

$$\frac{1}{2\pi \cdot 10^3 \tau} = 10^2 \Rightarrow \tau \approx 1.59 \cdot 10^{-6} \text{s}$$

and Hence at  $10^5 \text{Hz}$ ,  $|V_O|/|V_i| = 1/(10^5 10^{-5}) = 1$ .

## 9 2.79

Derive the transfer function of the circuit in Fig and show that it can be written in the form

$$\frac{|V_o|}{|V_i|} = \frac{-R_2/R_1}{(1 + (\omega_1/(i\omega)))(1 + (i\omega/(\omega_2)))}$$

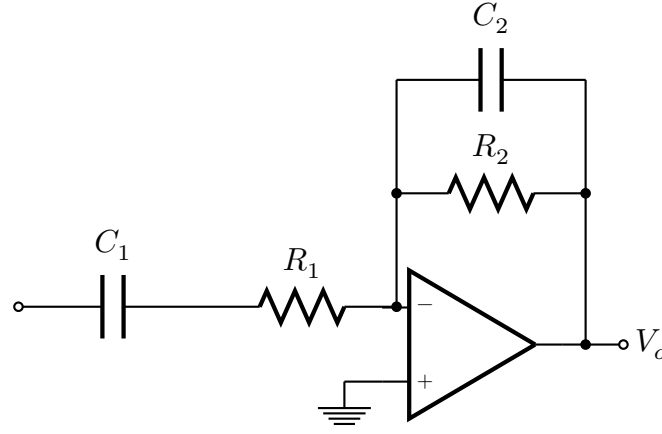
Where  $\omega_1 = 1/(C_1 R_1)$  and  $\omega_2 = 1/(C_2 R_2)$ . Assuming that the circuit is designed such that  $\omega_2 \gg \omega_1$ , find approximate expressinos for the transfer function in the following frequency regions:

(a)  $\omega \ll \omega_1$

(b)  $\omega_1 \ll \omega \ll \omega_2$

(c)  $\omega \gg \omega_2$

Use these approximations to sketch a Bode plot for the magnitude response. Observe that the circuit performs as an amplifier whose gain rolls off at the low-frequency end in the manner of a high-pass STC network, and at the high frequency end in the manner of a low-pass STC network. Design the circuit to provide a gain of 40dB in the "middle frequency range," a low-frequency 3dB point at 100kHz, and an input resistance (at



$\omega \gg \omega_1$ ) of  $1\text{k}\Omega$ .

**Ans:**

$$\begin{aligned}
 \frac{V_o}{V_i} &= -\frac{1/(i\omega C_2 + 1/R_2)}{R_1 + 1/(i\omega C_1)} \\
 &= \frac{-1}{\left(i\omega C_2 + \frac{1}{R_2}\right) \left(R_1 + \frac{1}{i\omega C_1}\right)} \\
 &= \frac{-R_2/R_1}{(1 + i\omega C_2 R_2) \left(\frac{1}{i\omega C_1 R_1} + R_2\right)} \\
 &= \frac{-R_2/R_1}{(1 + \omega_1/(i\omega))(1 + i(\omega/\omega_2))}
 \end{aligned}$$

Let the transfer function be  $f(\omega)$  and  $-R_2/R_1 = \alpha$

- (a)  $\omega \ll \omega_1 \ll \omega_2 \Rightarrow 1 + i(\omega/\omega_2) \approx 1$  and  $1 + \omega_1/(i\omega) \approx \omega/\omega_1$ , Hence  $f(\omega) \approx \alpha i\omega/\omega_1$
- (b)  $\omega_1 \ll \omega \ll \omega_2 \Rightarrow 1 + i(\omega/\omega_2) \approx 1$  and  $1 + \omega_1/(i\omega) \approx 1$ , Hence  $f(\omega) \approx \alpha$
- (c)  $\omega_1 \ll \omega_2 \ll \omega \Rightarrow 1 + i(\omega/\omega_2) \approx i\omega/\omega_2$  and  $1 + \omega_1/(i\omega) \approx 1$ , Hence  $f(\omega) \approx \alpha\omega_2/(i\omega)$

## 10 2.100

A designer, wanting to achieve a stable gain of  $100\text{V/V}$  at  $5\text{MHz}$ , considers her choice of amplifier topologies. What unity-gain frequency would a single operational amplifier require to satisfy her need? Unfortunately, the best available amplifier has an  $f_t$  of  $40\text{MHz}$ . How many such stages would she need to achieve her goal? What is the  $3\text{dB}$  frequency of each stage she can use? What is the overall  $3\text{dB}$  frequency?

**Ans:**

By the single pole model of the op amp.  $f_t > 100 \cdot 5\text{MHz}$ , provided that the low frequency gain is much greater than  $100\text{V/V}$ .

Let  $G(\omega)$  be the gain of the best available amplifier.  $G(5\text{MHz}) = 40/5 = 8$ . So you will need at least 3 op amp so that  $8^3 = 512 > 100$ .

Now we solve for the gain of the op amp  $G$ . notice that the  $3\text{dB}$  frequency will be



approximately at  $40/G$  MHz, so for the total gain to be  $100V/V$ , we have

$$\frac{G}{\sqrt{1 + \left(\frac{f}{40/G}\right)^2}} = \sqrt[3]{100}$$

Solve for  $G$  we get  $G \approx 5.70$ , so the 3dB frequency each stage is approximately  $40\text{MHz}/5.7 \approx 7.02\text{MHz}$ . The over all 3dB frequency is when  $1 + (f'/7.02)^2 = \sqrt[3]{2}$ , and so  $f' \approx 3.57\text{MHz}$ .

## 11 2.102

Consider an inverting summer with two inputs  $V_1$  and  $V_2$  and with  $V_O = -(V_1 + V_2)$ . Find the 3dB frequency of each of the gain functions  $V_O/V_1$  and  $V_O/V_2$  in terms of the op amp  $f_1$ .

Obviously  $V_O/V_1 = V_O/V_2$ . Set  $V_2 = 0$ . We have  $I_3 = I_1 - I_2 = (V_1 - V_3)/R - V_3/R$ , so  $-AV_3 = V_O = V_3 - (V_1 - V_3 - V_3) = 3V_3 - V_1 \Rightarrow V_O = -1/(1 + 3/A)$ . Substitute  $A = A_0/(1 + \omega/\omega_b)$  yields

$$G(\omega) = \frac{-1}{1 + 3(1 + \omega/\omega_b)/A_0} = \frac{-A_0}{A_0 + 3 + 3\omega/\omega_b}$$

Notice that  $A_0 \gg 3$ , so  $\omega_{3\text{dB}} \approx A_0\omega_b/3 \approx \omega_t/3$ .

## 12 2.104

An op amp having a slew rate of  $20\text{V}/\mu\text{s}$  is to be used in the unity-gain follower configuration, with input pulses that rise from 0 to  $3\text{V}$ . What is the shortest pulse that can be used while ensuring full-amplitude output? For such a pulse, describe the outputting resulting.

**Ans:**

Easy...  $3/20 = 0.15\mu\text{s}$ .

## 13 1.78

A  $p^+n$  junction is one in which the doping concentration in the  $p$  region is much greater than that in the  $n$  region. In such a junction, the forward current is mostly due to hole injection across the junction. Show that

$$I \approx I_p = Aqn_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

For the specific case in which  $N_D = 10^{16}/\text{cm}^3$ ,  $D_p = 10\text{cm}^2/\text{s}$ ,  $L_p = 10\mu\text{m}$ ,  $A = 10^4\mu\text{m}^2$ . Find  $I_S$  and the voltage  $V$  obtained when  $I = 0.5\text{mA}$ . Assume operation at  $300\text{K}$  where  $n_i = 1.5 \cdot 10^{10}/\text{cm}^3$ .

**Ans:**

We have

$$I = I_p + I_n = Aqn_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1)$$

but since the acceptor dominates,  $N_A \gg N_D$ , and hence

$$I \approx I_p = Aqn_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

A straight forward calculation yields

$$\begin{aligned} I_S &= Aqn_i^2 \frac{D_p}{L_p} \\ &\approx (10^4 \cdot 10^{-8} \text{cm}^3)(1.6 \cdot 10^{-19} \text{C})(1.5 \cdot 10^{10} / \text{cm}^3)^2 (10 \text{cm}) / (10^{-3} \text{cm}) / (10^{16} \text{cm}) \\ &\approx 3.6 \cdot 10^{-15} \text{A} \\ I &= I_S (e^{V/V_T} - 1) \\ \Rightarrow V &= V_T \ln(I/I_S + 1) \\ &\approx 0.662 \text{mV} \end{aligned}$$

## 14 1.82

A short-base diode is one where the widths of the  $p$  and  $n$  regions are much smaller than  $L_n$  and  $L_p$ , respectively. As a result, the excess minority carrier distribution in each region is a straight line rather than the exponentials shown in Fig. 1.39.

- (a) For the short-base diode, sketch a figure corresponding to Fig.1.39 and assume as in Fig.1.39 that  $N_A \gg N_D$ .

**Ans:**

TODO or not TODO

- (b) Following a derivation similar to that given in Section 1.11.2, show that if the widths of the  $p$  and  $n$  regions are denoted  $W_p$  and  $W_n$  then

$$I = Aqn_i^2 \left( \frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A} \right) (e^{V/V_T} - 1)$$

and

$$\begin{aligned} Q_p &= \frac{1}{2} \frac{(W_n - x_n)^2}{D_p} I_p \\ &\approx \frac{1}{2} \frac{W_n^2}{D_p} I_p, \quad \text{for } W_n \gg x_n \end{aligned}$$

**Ans:**

$p_n(x_n) = p_{n_0} e^{V/V_T}$  as usual, but the excess charge at  $W_n$  equals 0, so  $p_n(W_n) = p_{n_0}$ , hence

$$\frac{dp}{dx} = p_{n_0} (e^{V/V_T} - 1) / (W_n - x_n) = n_i^2 (e^{V/V_T} - 1) / ((W_n - x_n) N_D)$$

everywhere, hence we have

$$I_p = AJ_p = AqD_p \frac{dp}{dx} = Aqn_i^2 \frac{D_p}{N_D(W_n - x_n)} (e^{V/V_T} - 1)$$

similarly

$$I_n = AJ_n = AqD_n \frac{dp}{dx} = Aqn_i^2 \frac{D_n}{N_A(W_p - x_p)} (e^{V/V_T} - 1)$$

So

$$I = I_n + I_p = Aqn_i^2 \left( \frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A} \right) (e^{V/V_T} - 1)$$

Finally,

$$\begin{aligned} Q_p &= Aq \frac{(p_n(x_n) - p_n(W_n))(W_n - x_n)}{2} \\ &= Aq \frac{(p_n(x_n) - p_n(W_n))(W_n - x_n)^2}{2(W_n - x_n)} \\ &= Aq \frac{(W_n - x_n)^2}{D_p} I_p \\ &\approx Aq \frac{W_n^2}{2D_p} I_p \quad (\text{If } W_n \gg x_n) \end{aligned}$$

(c) Also, assuming  $Q \approx Q_p, I \approx I_p$ , show that

$$C_d = \frac{\tau_T}{V_T} I$$

where

$$\tau_T = \frac{1}{2} \frac{W_n^2}{D_p}$$

**Ans:**

$$\begin{aligned} C_d &= \frac{dQ}{dV} \approx \frac{dQ_p}{dV} \\ &= \frac{d(AqW_n^2/D_p)I_p}{2dV} \\ &= \frac{AqW_n^2}{2D_p} \frac{dI_s (e^{V/V_T} - 1)}{dV} \end{aligned}$$

If  $V \gg V_T \approx 25.8\text{mV}$ ,

$$\frac{dI_s (e^{V/V_T} - 1)}{dV} = I_s e^{V/V_T} / V_T \approx I / V_T$$

$$C_d \approx \frac{AqW_n^2}{2D_p}I/V_T$$

$$= \frac{\tau_T}{V_T}I$$

- (d) if a designer wishes to limit  $C_d$  to 8pF at  $I = 1\text{mA}$ , what should  $W_n$  be? Assume  $D_p = 10\text{cm}^2/\text{s}$

**Ans:**

$$W_n = \sqrt{\frac{2C_d D_p V_T}{I}}$$

$$= \sqrt{2(8 \cdot 10^{-12}\text{F})(10^{-3}\text{m})(25.8 \cdot 10^{-3}\text{V})(10^3\text{A}^{-1})}$$

$$\approx 642\text{nm}$$