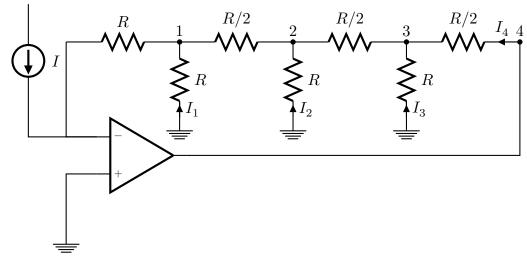
The circuit in Fig below can be can be consider to be an extension of the circuit in Fig. 2.8



(a) Find the resistance looking into node 1, 2, 3, 4. Ans:

$$\begin{split} R_1 &= R \\ R_2 &= (R||R) + R/2 = R \\ R_3 &= (R||R) + R/2 = R \\ R_4 &= (R||R) + R/2 = R \end{split}$$

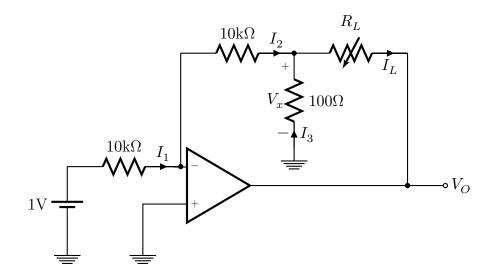
(b) Find the currents  $I_1, I_2, I_3, I_4$ , in terms of I Ans:

$$\begin{split} I_1 &= IR/R = I \\ I_2 &= ((I+I_1)(R/2) + IR)/R = 2I \\ I_3 &= (4IR/2 + 2IR)/R = 4I \\ I_4 &= (8IR/2 + 4IR)/R = 8I \end{split}$$

(c) Find the voltages at node 1, 2, 3, 4. Ans:

$$V_1 = IR$$
 
$$V_2 = V_1 + 2IR/2 = 2IR$$
 
$$V_3 = V_2 + 4IR/2 = 4IR$$
 
$$V_4 = V_3 + 8IR/2 = 8IR$$

The circuit below utilizes an ideal op amp.



(a) Find  $I_1, I_2, I_3, V_x$ Ans:

$$\begin{split} I_1 &= 1 \text{V}/10 \text{k}\Omega = 0.1 \text{mA} \\ I_2 &= I_1 = 0.1 \text{mA} \\ V_x &= 0 - (0.1 \text{mA})(10 \text{k}\Omega) = -1 \text{V} \\ I_3 &= 1/100\Omega = 10 \text{mA} \end{split}$$

(b) If  $V_O$  is not to be lower than  $-13\mathrm{V}$ , find the maximum allowed value of  $R_L$  Ans:

$$\begin{split} V_O &= V_x + R_L I_L = V_x + R_L (I_2 + I_3) \\ &= -1 \mathbf{V} - (10.1 \mathrm{mA}) R_L \end{split}$$

so

$$R_L \leq 12 \mathrm{V}/10.1 \mathrm{mA} \approx 1.19$$

(c) If  $R_L$  is varied in the range  $100\Omega$  to  $1\text{k}\Omega$ , what is the corresponding change in  $I_L$  and in  $V_O$ ?  $I_L$  would not change

$$V_O|_{R_L=100\Omega}=2.02{\rm V}, V_O|_{R_L=1{\rm k}\Omega}=11.2{\rm V}$$

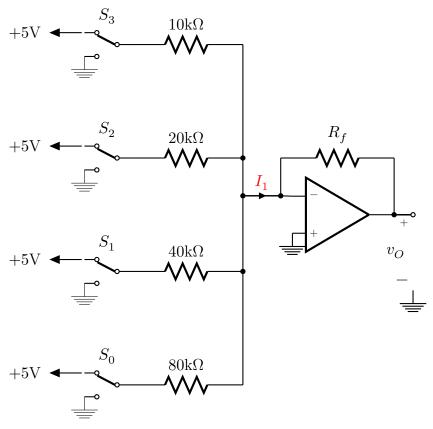
## 3 2.37

Figure P2.37 shows a circuit for a digital-to-analog converter. The circuit accepts a 4-bit input binary word  $a_3a_2a_1a_0$  where  $a_i \in \{0, 1\}$ . Each of the bits of the input word controls

the correspondingly numbered switch. For instance, if  $a_2$  is 0 then switch  $S_2$  connects the  $20 \mathrm{k}\Omega$  resistor to ground, while if  $a_2$  is 1 then  $S_2$  connects the  $20 \mathrm{k}\Omega$  resistor to the  $+5 \mathrm{V}$  power supply. Show that  $v_O$  is given by

$$v_O = -\frac{R_f}{16} \sum_{i=0}^3 2^i a_i$$

where  $R_f$  is in kiloohms. Find the value of  $R_f$  so that  $v_O$  ranges from 0 to -12V.



Ans:

Note that

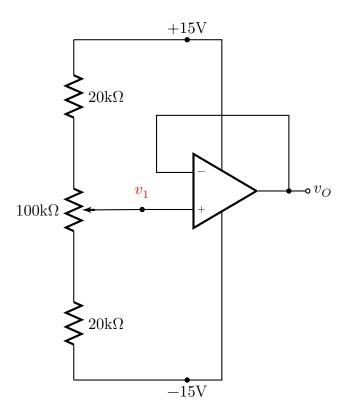
$$I_1 = \sum_{i=0}^3 a_i(5\mathrm{V})/(10 \cdot 2^{(3-i)}\mathrm{k}\Omega) = \sum_{i=0}^3 a_i/2^{(4-i)}(\mathrm{mA})$$

so

$$v_O = 0 - R_f I_1 = -R_f \sum_{i=0}^3 a_i 2^i / 16 (\mathrm{kV})$$

## 4 2.51

Figure shows a circuit that provides an output voltage  $v_o$  whose value can be varied by turning the wiper of the  $100\mathrm{k}\Omega$ potentiometer. Find the range over which  $v_o$  can be varied. If the potentiometer is a "20-turn" device, find the change in  $v_o$  corresponding to each turn of the pot.



Ans: We have

$$\begin{split} v_O &= v_1 \\ v_{1,\text{max}} &= (15\text{V}) \frac{100 + 20}{100 + 20 + 20} + (-15\text{V}) \frac{20}{100 + 20 + 20} \approx 10.7\text{V} \\ v_{1,\text{min}} &= -v_{1,\text{max}} \end{split}$$

And so the voltage change each turn is  $(v_{1,\text{max}} - v_{1,\text{min}})/20 \approx 1.07$ .

#### 5 2.63

For an instrumentation amplifier of the type shown in Fig 2.20(b), a designer proposes to make  $R_2 = R_3 = R_4 = 100 \mathrm{k}\Omega$  and  $2R_1 = 10 \mathrm{k}\Omega$ . For ideal components, what difference-mode gain, common-mode gain, and CMRR result? Reevaluate the worst-case values for these for the situation in which all resistors are specified as  $\pm 1\%$  units. Repeat the latter analysis for the case in which  $2R_1$  is reduced to  $1\mathrm{k}\Omega$ . What do you conclude about the importance of the relative difference gains of the first and second stages?

For  $v_{Icm}$ , the first stage has no effect,  $v_{O1}=v_{O2}=v_{Icm}$ . So common-mode gain  $A_{Icm}=1{\rm V/V}.$  For  $v_{Id},$  we have

$$v_{O1} - v_{O2} = \left(1 + 2\frac{100}{10}\right)v_{Id} = 21v_{Id},$$

and since  $R_3=R_4$ , difference-mode gain  $A_{Id}=21{\rm V/V}$ . CMRR is  $20\log\left(\frac{A_{Id}}{A_{Icm}}\right)=\infty$ . If all resistors are specified as  $\pm 1\%$  units, we can maximize common-mode gain by

let  $R_{31}=R_{42}=101 \mathrm{k}\Omega, R_{32}=R_{41}=99 \mathrm{k}\Omega,$  then common-mode gain

$$A_{Icm} = \frac{R_{42}}{R_{32} + R_{42}} \left( 1 - \frac{R_{41}R_{32}}{R_{31}R_{42}} \right) \approx 1.98 \times 10^{-2} \, \mathrm{V/V}.$$

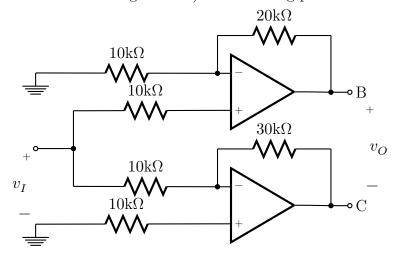
And we minimize difference-mode gain by let  $2R_1=10.1\mathrm{k}\Omega, R_2=99\mathrm{k}\Omega.$ 

$$A_{Id} = \frac{2R_2 + 2R_1}{2R_1} \frac{R_{41} + R_{42}}{R_{31} + R_{32}} \approx 20.6 \, \text{V/V}.$$

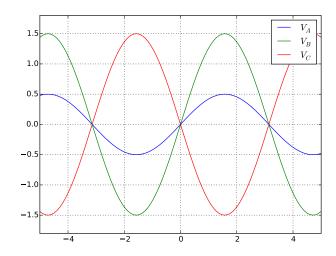
CMRR is  $20 \log \left(\frac{A_{Id}}{A_{Icm}}\right) \approx 138.95 \text{dB}$ .

## 6 2.65

The circuit shown in Fig is intended to supply a voltage to floating loads (those for which both terminals are ungrounded) while making possible use of available power supply.

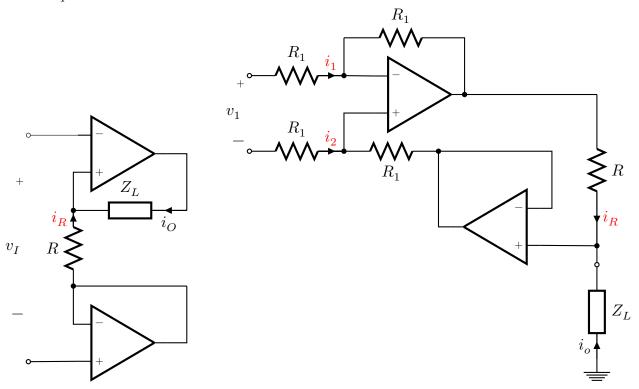


(a) Assuming ideal op amps, sketch the voltage waveforms at nodes B and C for a 1V peak-to-peak sine wave applied at A. Also sketch  $v_O$  ans:



(b) What is the voltage gain  $v_O/v_I$ ? Ans: It is easy to see that  $v_B=3v_A,v_C=-3v_A,$  so  $v_O/v_I=6.$ 

The two circuits in Fig are intended to function as voltage-to-current converters; that is, they supply the load impedance  $Z_L$  with a current proportional to  $v_I$  and independent of the value of  $Z_L$ . Show that this is indeed the case, and find for each circuit  $i_o$  as a function of  $v_I$ . Comment on the differences between the two circuits.



For fig 1,  $i_O=i_R=v_I/R$  and hence proportional to  $v_I$  and independent of  $Z_L$ . For fig 2,  $i_O=i_R$ , note that since the pos/neg terminal of the op amp above is virtual shorted,  $v_I-i_1R_1=-i_2R_1$ , so  $-i_1R_1+i_2R_1=-v_I$ , hence we have

$$i_R = \frac{v_I - 2i_1R_1 + 2i_2r_1}{R} = -v_I/R$$

and hence proportional to  $v_I$  and independent of  $Z_L$ .

#### 8 2.69

An op-amp-based inverting integrator is measured at 1kHz to have a voltage gain of -100V/V. At what frequency is its gain reduced to -1V/V? What is the integrator time constant?

Ans: We know that  $|V_O|/|V_I|=1/(\omega RC)$  , so if  $\tau=RC$  is the time constant.

$$\frac{1}{10^3 \tau} = 10^2 \implies \tau = 10^{-5}$$

and Hence at  $10^5 \mathrm{Hz},\, |V_o|/|V_i| = 1/(10^5 10^{-5}) = 1.$ 

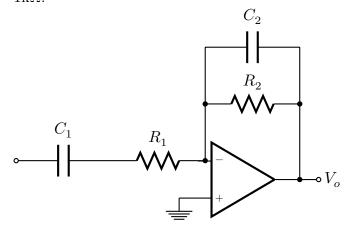
Derive the transfer function of the circuit in Fig and show that it can be written in the form

$$\frac{|V_o|}{|V_i|} = \frac{-R_2/R_1}{\left(1 + \left(\omega_1/(\mathrm{i}\omega)\right)\right)\left(1 + \left(\omega/(\mathrm{i}\omega_2)\right)\right)}$$

Where  $\omega_1 = 1/(C_1R_1)$  and  $\omega_2 = 1/(C_2R_2)$ . Assuming that the circuit is designed such that  $\omega_2 \gg \omega_1$ , find approximate expressions for the transfer function in the following frequency regions:

- (a)  $\omega \ll \omega_1$
- (b)  $\omega_1 \ll \omega \ll \omega_2$
- (c)  $\omega \gg \omega_2$

Use these approximations to sketch a Bode plot for the magnitude response. Observe that the circuit performs as an amplifier whose gain rolls off at the low-frequency end in the manner of a high-pass STC network, and at the high frequency end in the manner of a low-pass STC network. Design the circuit to provide a gain of 40dB in the "middle frequency range," a low-frequency 3dB point at 100kHz, and an input resistance (at  $\omega \gg \omega_1$ ) of 1k $\Omega$ .



Ans:

$$\begin{split} \frac{V_o}{V_i} &= -\frac{1/(\mathrm{i}\omega C_2 + 1/R_2)}{R_1 + 1/(\mathrm{i}\omega C_1)} \\ &= \frac{-1}{\left(\mathrm{i}\omega C_2 + \frac{1}{R_2}\right)\left(R_1 + \frac{1}{\mathrm{i}\omega C_1}\right)} \\ &= \frac{-R_2/R_1}{\left(1 + \mathrm{i}\omega C_2 R_2\right)\left(\frac{1}{\mathrm{i}\omega C_1 R_1} + R_2\right)} \\ &= \frac{-R_2/R_1}{\left(1 + \omega_1/(\mathrm{i}\omega)\right)(1 + \mathrm{i}(\omega/\omega_2))} \end{split}$$

Let the transfer function be  $f(\omega)$  and  $-R_2/R_1=\alpha$ 

(a) 
$$\omega \ll \omega_1 \ll \omega_2 \Rightarrow 1 + i(\omega/\omega_2) \approx 1$$
 and  $1 + \omega_1/(i\omega) \approx \omega/\omega_1$ , Hence  $f(\omega) \approx \alpha i\omega/\omega_1$ 

- (b)  $\omega_1 \ll \omega \ll \omega_2 \Rightarrow 1 + i(\omega/\omega_2) \approx 1$  and  $1 + \omega_1/(i\omega) \approx 1$ , Hence  $f(\omega) \approx \alpha$
- (c)  $\omega_1 \ll \omega_2 \ll \omega \Rightarrow 1 + \mathrm{i}(\omega/\omega_2) \approx \mathrm{i}\omega/\omega_2$  and  $1 + \omega_1/(\mathrm{i}\omega) \approx 1$ , Hence  $f(\omega) \approx \alpha\omega_2/(\mathrm{i}\omega)$

#### $10 \quad 2.100$

A designer, wanting to achieve a stable gain of  $100\mathrm{V/V}$  at 5MHz, considers her choice of amplifier topologies. What unity-gain frequency would a single operational amplifier require to satisfy her need? Unfortunately, the best available amplifier has an  $f_t$  of 40MHz. How many such stages would she need to achieve her goal? What is the 3dB frequency of each stage she can use? What is the overall 3dB frequency?

By the single pole model of the op amp.  $f_t > 100 \cdot 5 \text{MHz}$ . (事實上要寫出這個答案需要不少恥力,因爲這時候只剛好是 3dB frequency.)

Let  $G(\omega)$  be the gain of the best available amplifier. G(5MHz) = 40/5 = 8. So you will need at least 3 op amp so that  $8^3 = 512$ .

But actually if we consider the 3dB frequency,  $G(5\text{MHz}) = \frac{40}{2 \cdot 5} = 4$ , so you will need 4 op amp so that  $4^4 = 256$ . 剩下的需要 op amp 的 low frequency gain. 題目沒給是要算小小小。

### $11 \quad 2.102$

Consider an inverting summer with two inputs  $V_1$  and  $V_2$  and with  $V_O = -(V_1 + V_2)$ . Find the 3dB frequency of each of the gain functions  $V_O/V_1$  and  $V_O/V_2$  in terms of the op amp  $f_1$ .

#### $12 \quad 2.104$

An op amp having a slew rate of  $20V/\mu s$  is to be used in the unity-gain follower configuration, with input pulses that rise from 0 to 3V. What is the shortest pulse that can be used while ensuring full-amplitude output? For such a pulse, describe the outputing resulting.

#### 13 1.78

A  $p^n$  junction is one in which the doping concentration in the p region is much greater than that in the n region. In such a junction, the foward current is mostly due to hole injection across the junction. Show that

$$I \approx I_p = Aqn_i^2 \frac{D_p}{L_p N_D} \left( e^{V/V_T} - 1 \right)$$

For the specific case in which  $N_D=10^{16}/{\rm cm}^3, N_p=10{\rm cm}^2/{\rm s}, L_p=10{\rm \mu m}, A=10^4{\rm \mu m}^2.$  Find  $I_S$  and the voltage V obtained when  $I=0.5{\rm mA}.$  Assume operation at 300K where  $n_i=1.5\cdot 10^{10}/{\rm cm}^3$ 

## 14 1.82

A short-base diode is one where the widths of the p and n regions are much smaller than  $L_n$  and  $L_p$ , respectively. As a result, the excess minority carrier distribution in each region is a straight line rather than the exponentials shown in Fig. 1.39.

- (a) For the short-base diode, sketch a figure corresponding to Fig.1.39 and assume as in Fig.1.39 that  $N_A\gg N_D$ .
- (b) Following a derivation similar to that given in Section 1.11.2, show that if the widths of the p and n regions are denoted  $W_p$  and  $W_n$  then

$$I = Aqn_i^2 \left( \frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A} \right) \left( e^{V/V_T} - 1 \right)$$

and

$$\begin{split} Q_p &= \frac{1}{2} \frac{(W_n - x_n)^2}{D_p} I_p \\ &\approx \frac{1}{2} \frac{W_n^2}{D_p} I_p, \quad \text{for } W_n \gg x_n \end{split}$$

(c) Also, assuming  $Q \approx Q_p, I \approx I_p$ , show that

$$C_d = \frac{\tau_T}{V_T} I$$

where

$$\tau_T = \frac{1}{2} \frac{W_n^2}{D_p}$$

(d) if a designer wishes to limit  $C_d$  to 8pF at  $I=1{\rm mA},$  what should  $W_n$  be? Assume  $D_p=10{\rm cm}^2/{\rm s}$