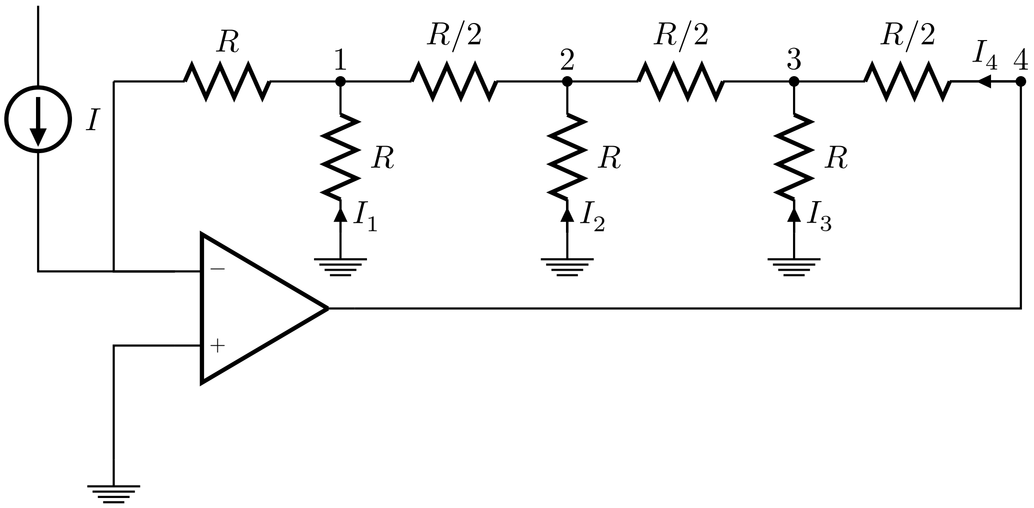
1 2.27

The circuit in Fig below can be can be consider to be an extension of the circuit in Fig. 2.8



(a) Find the resistance looking into node 1, 2, 3, 4.

Ans:

$$
\begin{aligned}
R_1 &= R \\
R_2 &= (R_1||R) + R/2 = R \\
R_3 &= (R_2||R) + R/2 = R \\
R_4 &= (R_3||R) + R/2 = R
\end{aligned}
$$

(b) Find the currents I_1, I_2, I_3, I_4, in terms of I

Ans:

$$
\begin{aligned}
I_1 &= IR/R = I \\
I_2 &= ((I + I_1)(R/2) + IR)/R = 2I \\
I_3 &= (4IR/2 + 2IR)/R = 4I \\
I_4 &= -(4I + 4I) = -8I
\end{aligned}
$$

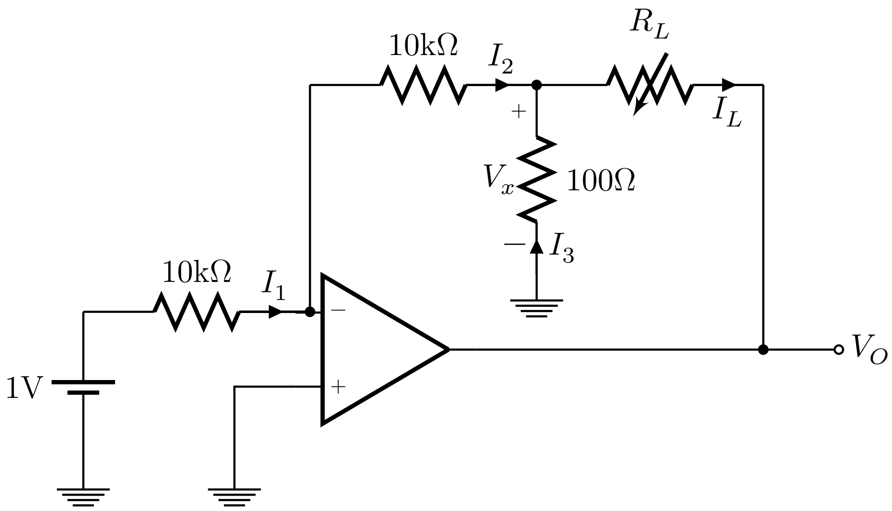
(c) Find the voltages at node 1, 2, 3, 4.

Ans:

$$
\begin{aligned}
V_1 &= -IR \\
V_2 &= V_1 + 2IR/2 = -2IR \\
V_3 &= V_2 + 4IR/2 = -4IR \\
V_4 &= V_3 + 8IR/2 = -8IR
\end{aligned}
$$

2 2.28

The circuit below utilizes an ideal op amp.



(a) Find I_1, I_2, I_3, V_x
Ans:

$$
\begin{aligned}
I_1 &= 1\text{V}/10\text{k}\Omega = 0.1\text{mA} \\
I_2 &= I_1 = 0.1\text{mA} \\
V_x &= 0 - (0.1\text{mA})(10\text{k}\Omega) = -1\text{V} \\
I_3 &= 1/100\Omega = 10\text{mA}
\end{aligned}
$$

(b) If V_O is not to be lower than -13V, find the maximum allowed value of R_L
Ans:

$$
\begin{aligned}
V_O &= V_x + R_L I_L = V_x + R_L(I_2 + I_3) \\
&= -1\text{V} - (10.1\text{mA})R_L
\end{aligned}
$$

so

$$
R_L \leq 12\text{V}/10.1\text{mA} \approx 1.19
$$

(c) If R_L is varied in the range 100Ω to $1\text{k}\Omega$, what is the corresponding change in I_L and in V_O?

Ans:

I_L would not change

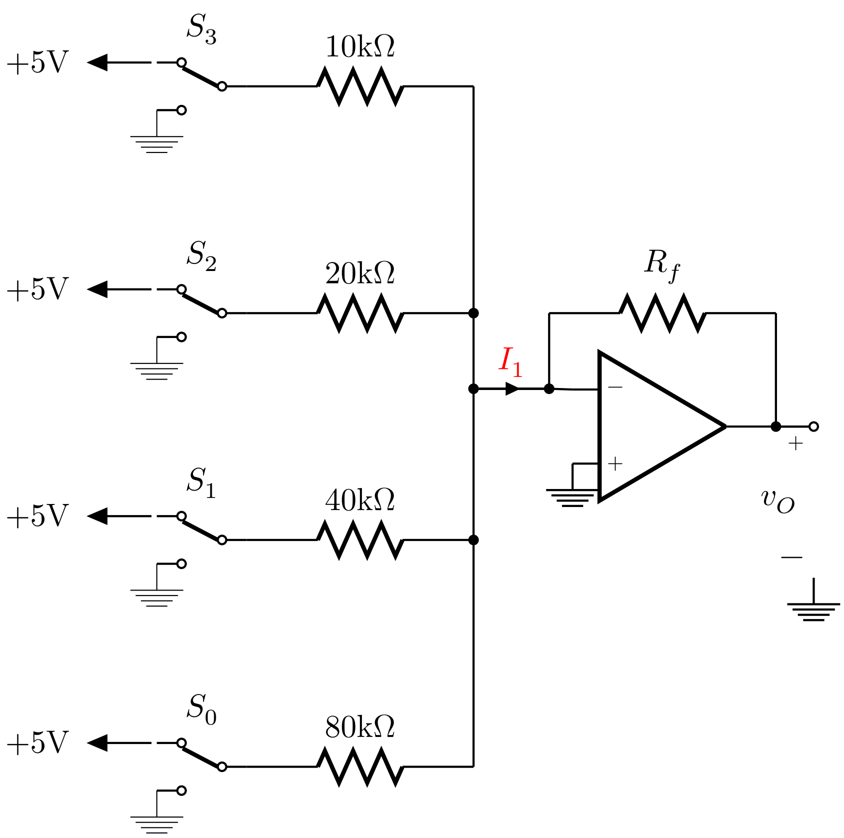
$$V_O|_{R_L=100\Omega} = -2.01\text{V}, V_O|_{R_L=1\text{k}\Omega} = -11.1\text{V}$$

3 2.37

Figure P2.37 shows a circuit for a digital-to-analog converter. The circuit accepts a 4-bit input binary word $a_3a_2a_1a_0$ where $a_i \in \{0,1\}$. Each of the bits of the input word controls the correspondingly numbered switch. For instance, if a_2 is 0 then switch S_2 connects the $20\text{k}\Omega$ resistor to ground, while if a_2 is 1 then S_2 connects the $20\text{k}\Omega$ resistor to the +5V power supply. Show that v_O is given by

$$v_O = -\frac{R_f}{16}\sum_{i=0}^{3} 2^i a_i$$

where R_f is in kiloohms. Find the value of R_f so that v_O ranges from 0 to -12V.



Ans:

Note that

$$I_1 = \sum_{i=0}^{3} a_i(5\text{V})/(10 \cdot 2^{(3-i)}\text{k}\Omega) = \sum_{i=0}^{3} a_i/2^{(4-i)}(\text{mA})$$

so

$$v_O = 0 - R_f I_1 = -R_f \sum_{i=0}^{3} a_i 2^i / 16.$$

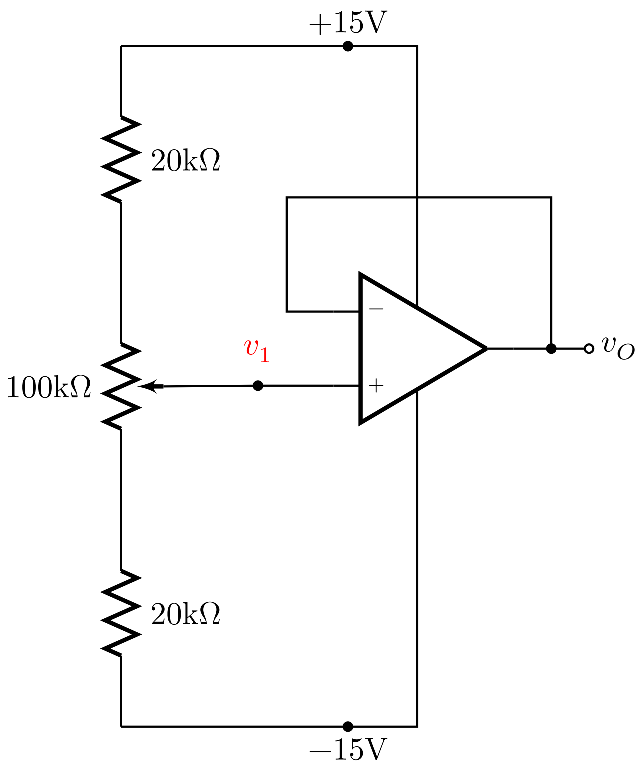
If v_O ranges from 0 V to 12 V, then

$$v_O = -\frac{R_f}{16} \sum_{i=0}^{3} a_i 2^i \Rightarrow 0 \geq v_O \geq -\frac{15}{16} R_f.$$

So $R_f = 12.8\text{k}\Omega$.

4 2.51

Figure shows a circuit that provides an output voltage v_o whose value can be varied by turning the wiper of the 100kΩpotentiometer. Find the range over which v_o can be varied. If the potentiometer is a "20-turn" device, find the change in v_o corresponding to each turn of the pot.



Ans:

We have

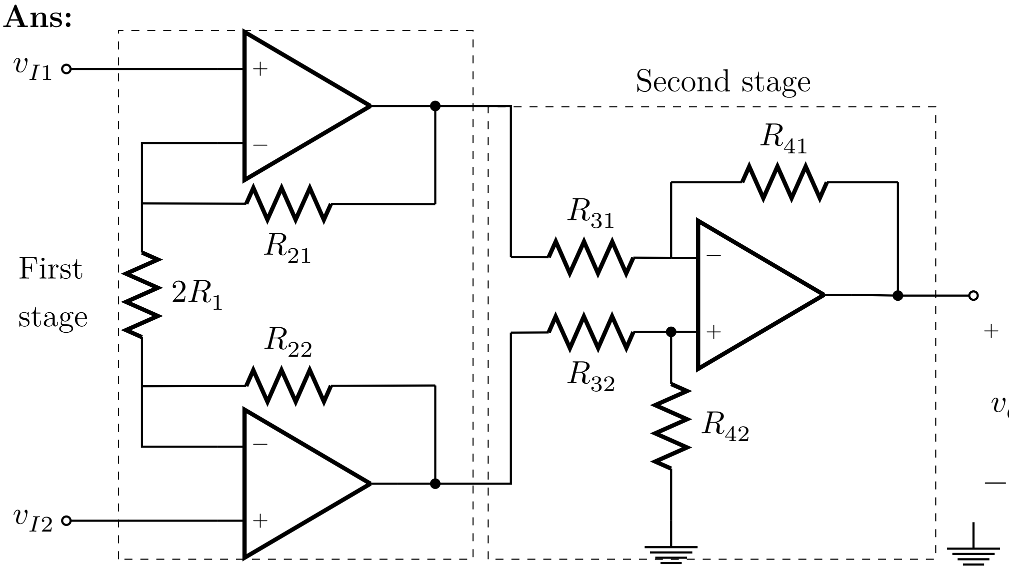
$$\begin{aligned} v_O &= v_1 \\ v_{1,\max} &= (15\text{V})\frac{100+20}{100+20+20} + (-15\text{V})\frac{20}{100+20+20} \approx 10.7\text{V} \\ v_{1,\min} &= -v_{1,\max} \end{aligned}$$

And so the voltage change each turn is $(v_{1,\max} - v_{1,\min})/20 \approx 1.07$.

5 2.63

For an instrumentation amplifier of the type shown in Fig 2.20(b), a designer proposes to make $R_2 = R_3 = R_4 = 100\text{k}\Omega$ and $2R_1 = 10\text{k}\Omega$. For ideal components, what difference-mode gain, common-mode gain, and CMRR result? Reevaluate the worst-case values for these for the situation in which all resistors are specified as $\pm 1\%$ units. Repeat the latter analysis for the case in which $2R_1$ is reduced to $1\text{k}\Omega$. What do you conclude about the importance of the relative difference gains of the first and second stages?

Ans:



If the resistor are ideal, the circuit is same as the textbook circuit, So as in textbook, we have

$$
\begin{aligned}
A_{cm} &= 0 \\
A_d &= \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) \\
&= 1 \cdot (1 + 100/5) = 21
\end{aligned}
$$

CMRR is then $20 \log \left(\frac{A_{Id}}{A_{Icm}}\right) = \infty$.

Now if all resistors are specified as $\pm 1\%$ units. First consider the first stage gain, the common-mode gain would be maximize by letting $R_{31} = R_{42} = 101\text{k}\Omega, R_{32} = R_{41} = 99\text{k}\Omega$, and

$$
A_{cm} = \frac{R_{42}}{R_{32} + R_{42}}\left(1 - \frac{R_{41}R_{32}}{R_{31}R_{42}}\right) \approx 1.98 \times 10^{-2}\,\text{V/V}.
$$

The difference-mode gain wouldn't be affect too much by the resistor bias.
At the first stage. In the worse case, the minimize difference-mode gain by let $2R_1 = 10.1\text{k}\Omega, R_2 = 99\text{k}\Omega$.

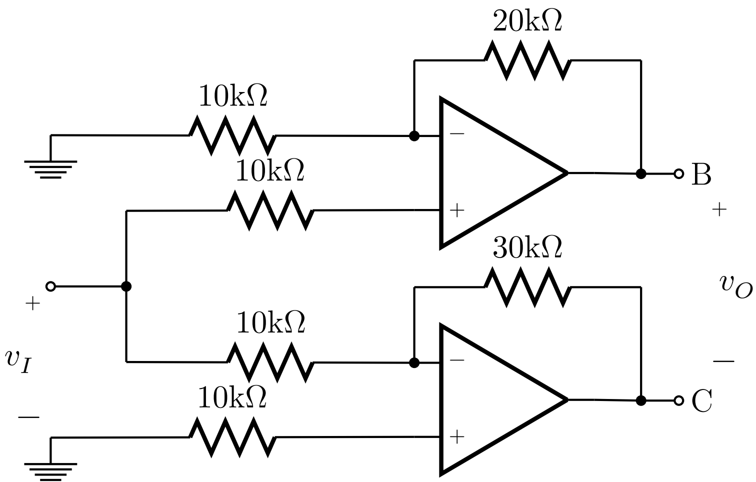
$$
A_d = \frac{2R_2 + 2R_1}{2R_1}\frac{1}{R_{31}}\frac{1}{R_{32} + R_{42}}(R_{31}R_{42} + 2R_{41}R_{42} + R_{32}R_{41}) \approx 20.4\,\text{V/V}.
$$

And the common-mode gain is 0 in every situation. So the overall CMRR is $20 \log \left(\frac{A_d}{A_{cm}}\right) \approx 60.26\text{dB}$.

If $2R_1 = 1\text{k}\Omega$, then By similar calculation, $A_d \approx 201\,\text{V/V}$. The first stage dominates the difference-mode gain. And the common-mode gain doesn't change. So CMRR is $20\log\left(\frac{A_{Id}}{A_{Icm}}\right) \approx 80.13\text{dB}$.

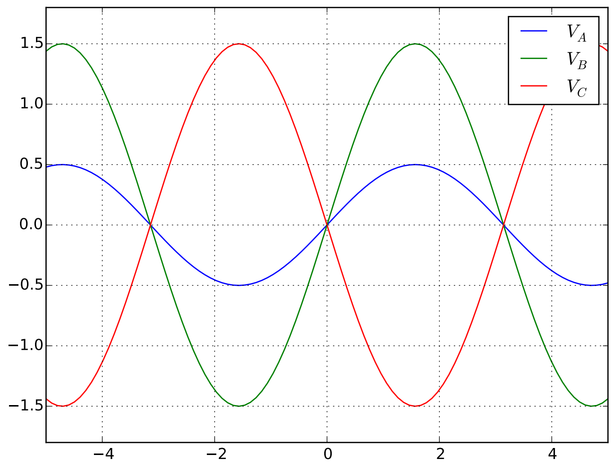
6 2.65

The circuit shown in Fig is intended to supply a voltage to floating loads (those for which both terminals are ungrounded) while making possible use of available power supply.



(a) Assuming ideal op amps, sketch the voltage waveforms at nodes B and C for a 1V peak-to-peak sine wave applied at A. Also sketch v_O

Ans:



(b) What is the voltage gain v_O/v_I?

Ans:

It is easy to see that $v_B = 3v_A, v_C = -3v_A$, so $v_O/v_I = 6$.

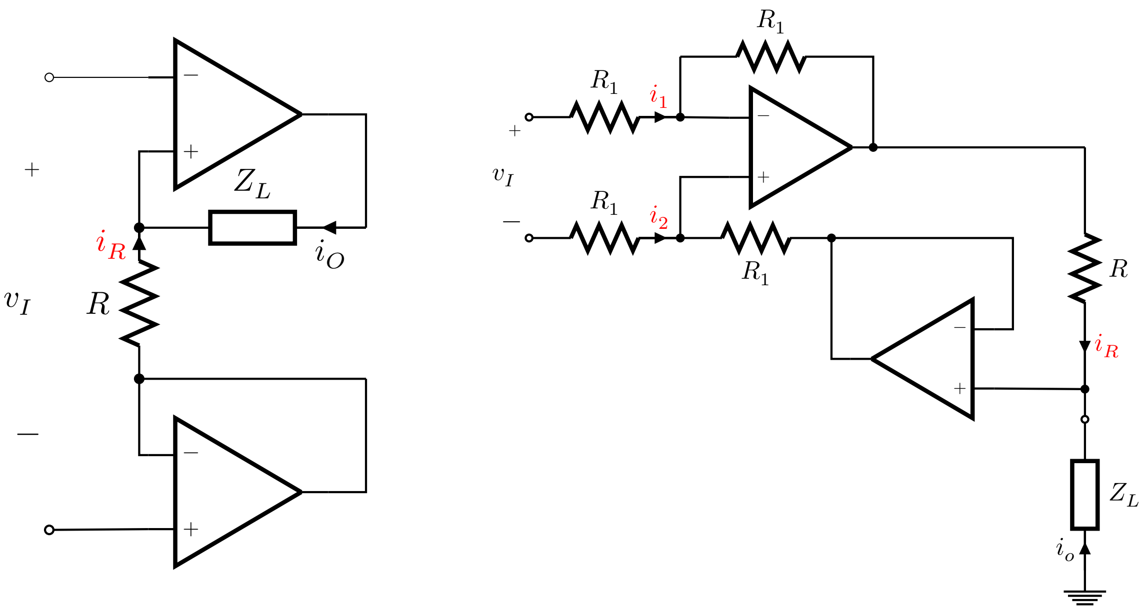
(c) Assuming that the op amps operate from ±15V power supplies and that their output saturates at ±14V (in the manner shown in Fig.1.14), what is the largest sine-wave output that can be accomodated? Specify both its peak-to-peak and rms values.

Ans:

Since $v_B = -v_C$, so if required $|v_B|, |v_C| < 14\text{V}$, The maximum output is $|v_O| = |v_B - v_C| = 28\text{V}$. and the rms value is therefore $28/\sqrt{2}\text{V} = 19.8\text{V}$.

7 2.66

The two circuits in Fig are intended to function as voltage-to-current converters; that is, they supply the load impedance Z_L with a current proportional to v_I and independent of the value of Z_L. Show that this is indeed the case, and find for each circuit i_o as a function of v_I. Comment on the differences between the two circuits.



Ans:

For fig 1, $i_O = i_R = v_I/R$ and hence proportional to v_I and independent of Z_L.

For fig 2, $i_O = i_R$, note that since the pos/neg terminal of the op amp above is virtual shorted, $v_I - i_1R_1 = -i_2R_1$,so $-i_1R_1 + i_2R_1 = -v_I$, hence we have

$$i_R = \frac{v_I - 2i_1R_1 + 2i_2r_1}{R} = -v_I/R$$

and hence proportional to v_I and independent of Z_L.

Comment: No comment.

8 2.69

An op-amp-based inverting integrator is measured at 1kHz to have a voltage gain of -100V/V. At what frequency is its gain reduced to -1V/V? What is the integrator time constant?

Ans:

We know that $|V_O|/|V_I| = 1/(\omega RC) = 1/(2\pi f RC)$, so if $\tau = RC$ is the time constant.

$$\frac{1}{2\pi \cdot 10^3 \tau} = 10^2 \;\Rightarrow\; \tau \approx 1.59 \cdot 10^{-6}\text{s}$$

and Hence at 10^5Hz, $|V_o|/|V_i| = 1/(10^5 10^{-5}) = 1$.

9 2.79

Derive the transfer function of the circuit in Fig and show that it can be written in the form

$$\frac{|V_o|}{|V_i|} = \frac{-R_2/R_1}{(1+(\omega_1/(\mathrm{i}\omega)))\,(1+(\mathrm{i}\omega/(\omega_2)))}$$

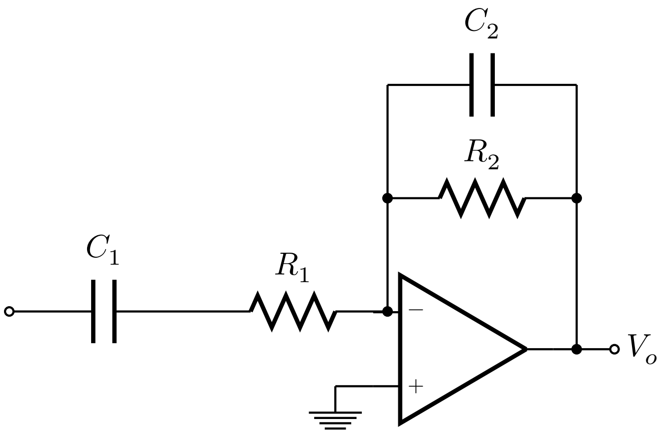
Where $\omega_1 = 1/(C_1R_1)$ and $\omega_2 = 1/(C_2R_2)$. Assuming that the circuit is designed such that $\omega_2 \gg \omega_1$, find approximate expressinos for the transfer function in the following frequency regions:

(a) $\omega \ll \omega_1$

(b) $\omega_1 \ll \omega \ll \omega_2$

(c) $\omega \gg \omega_2$

Use these approximations to sketch a Bode plot for the magnitude response. Observe that the circuit performs as an amplifier whose gain rolls off at the low-frequency end in the manner of a high-pass STC network, and at the high frequency end in the manner of a low-pass STC network. Design the circuit to provide a gain of 40dB in the "middle frequency range," a low-frequency 3dB point at 100kHz, and an input resistance (at $\omega \gg \omega_1$) of 1kΩ.



Ans:

$$
\begin{aligned}
\frac{V_o}{V_i} &= -\frac{1/(\mathrm{i}\omega C_2 + 1/R_2)}{R_1 + 1/(\mathrm{i}\omega C_1)} \\
&= \frac{-1}{\left(\mathrm{i}\omega C_2 + \frac{1}{R_2}\right)\left(R_1 + \frac{1}{\mathrm{i}\omega C_1}\right)} \\
&= \frac{-R_2/R_1}{(1 + \mathrm{i}\omega C_2 R_2)\left(\frac{1}{\mathrm{i}\omega C_1 R_1} + R_2\right)} \\
&= \frac{-R_2/R_1}{(1 + \omega_1/(\mathrm{i}\omega))(1 + \mathrm{i}(\omega/\omega_2))}
\end{aligned}
$$

Let the transfer function be $f(\omega)$ and $-R_2/R_1 = \alpha$

(a) $\omega \ll \omega_1 \ll \omega_2 \Rightarrow 1 + \mathrm{i}(\omega/\omega_2) \approx 1$ and $1 + \omega_1/(\mathrm{i}\omega) \approx \omega/(\mathrm{i}\omega_1)$, Hence $f(\omega) \approx \alpha \mathrm{i}\omega/\omega_1$

(b) $\omega_1 \ll \omega \ll \omega_2 \Rightarrow 1 + \mathrm{i}(\omega/\omega_2) \approx 1$ and $1 + \omega_1/(\mathrm{i}\omega) \approx 1$, Hence $f(\omega) \approx \alpha$

(c) $\omega_1 \ll \omega_2 \ll \omega \Rightarrow 1 + \mathrm{i}(\omega/\omega_2) \approx \mathrm{i}\omega/\omega_2$ and $1 + \omega_1/(\mathrm{i}\omega) \approx 1$, Hence $f(\omega) \approx \alpha\omega_2/(\mathrm{i}\omega)$

Plot:

To satisfied the design, the input resistance

$$
Z = R + \frac{1}{\mathrm{i}\omega_1 C} \approx R \quad (\text{if } \omega_1 \gg \omega)
$$

so $R_1 = 1\mathrm{k}\Omega$, and for the gain to be 40dB

$$
10^{40/20} = G \approx \frac{R_2}{R_1} \;\Rightarrow\; R_2 = 100R_1 = 100\mathrm{k}\Omega
$$

Finally, to match the 3dB frequency,

$$
\begin{aligned}
2\pi \cdot 100\mathrm{Hz} = \omega_1 &= \frac{1}{C_1 R_1} \\
\Rightarrow C_1 &= 1.59\mathrm{\mu F} \\
2\pi \cdot 10^5\mathrm{Hz} = \omega_2 &= \frac{1}{C_2 R_2} \\
\Rightarrow C_2 &= 15.9\mathrm{pF}
\end{aligned}
$$

10 2.100

A designer, wanting to achieve a stable gain of 100V/V at 5MHz, considers her choice of amplifier topologies. What unity-gain frequency would a single operational amplifier require to satisfy her need? Unfortunately, the best available amplifier has an f_t of 40MHz. How many such stages would she need to achieve her goal? What is the 3dB frequency of each stage she can use? What is the overall 3dB frequency?

Ans:
By the single pole model of the op amp. $f_t > 100 \cdot 5$MHz, provided that the low frequency gain is much greater than 100V/V.
Let $G(\omega)$ be the gain of the best available amplifier. $G(5\text{MHz}) = 40/5 = 8$. So you will need at least 3 op amp so that $8^3 = 512 > 100$.
Now we solve for the gain of the op amp G. notice that the 3dB frequency will be approximately at $40/G$MHz, so for the total gain to be 100V/V at 5MHz, we have

$$\frac{G}{\sqrt{1+\left(\frac{f}{(40/G)(\text{MHz})}\right)^2}} = \sqrt[3]{100}$$

Solve for G we get $G \approx 5.70$, so the 3dB frequency each stage is approximately $40\text{MHz}/5.7 \approx 7.02$MHz. The over all 3dB frequency is when $1+(f'/7.02)^2 = \sqrt[3]{2}$, and so $f' \approx 3.57$MHz.

11 2.102

Consider an inverting summer with two inputs V_1 and V_2 and with $V_O = -(V_1 + V_2)$. Find the 3dB frequency of each of the gain functions V_O/V_1 and V_O/V_2 in terms of the op amp f_1.

Obviously $V_O/V_1 = V_O/V_2$. Set $V_2 = 0$, We have $I_3 = I_1 - I_2 = (V_1 - V_3)/R - V_3/R$, so $-AV_3 = V_O = V_3 - (V_1 - V_3 - V_3) = 3V_3 - V_1 \Rightarrow V_O = -1/(1+3/A)$. Substitute $A = A_0/(1+\omega/\omega_b)$ yields

$$G(\omega) = \frac{-1}{1+3(1+\mathrm{i}\omega/\omega_b)/A_0} = \frac{-A_0}{A_0+3+3\mathrm{i}\omega/\omega_b}$$

Notice that $A_0 \gg 3$, we approximate G to

$$G(\omega) = \frac{-1}{1+3/A_0+3\mathrm{i}\omega/(A_0\omega_b)} \approx \frac{-1}{1+3\mathrm{i}\omega/(A_0\omega_b)}$$

So $\omega_{3\text{dB}} \approx A_0\omega_b/3 \approx \omega_t/3$.

12 2.104

An op amp having a slew rate of 20V/µs is to be used in the unity-gain follower configuration, with input pulses that rise from 0 to 3V. What is the shortest pulse that can be used while ensuring full-amplitude output? For such a pulse, describe the outputing resulting.
Ans:
Easy... $3/20 = 0.15$µs.

13 1.78

A p^+n junction is one in which the doping concentration in the p region is much greater than that in the n region. In such a junction, the foward current is mostly due to hole injection across the junction. Show that

$$I \approx I_p = Aqn_i^2 \frac{D_p}{L_p N_D} \left(e^{V/V_T} - 1\right)$$

For the specific case in which $N_D = 10^{16}/\text{cm}^3, D_p = 10\text{cm}^2/\text{s}, L_p = 10\text{µm}, A = 10^4\text{µm}^2$. Find I_S and the voltage V obtained when $I = 0.5\text{mA}$. Assume operation at 300K where $n_i = 1.5 \cdot 10^{10}/\text{cm}^3$.

Ans:

We have

$$I = I_p + I_n = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right) \left(e^{(V/V_T)} - 1\right)$$

but since the acceptor dominates, $N_A \gg N_D$, and hence

$$I \approx I_p = Aqn_i^2 \frac{D_p}{L_p N_D} \left(e^{V/V_T} - 1\right)$$

A straight forward calculation yields

$$\begin{aligned}
I_S &= Aqn_i^2 \frac{D_p}{L_p} \\
&\approx (10^4 \cdot 10^{-8}\text{cm}^3)(1.6 \cdot 10^{-19}\text{C})(1.5 \cdot 10^{10}/\text{cm}^3)^2(10\text{cm})/(10^{-3}\text{cm})/(10^{16}\text{cm}) \\
&\approx 3.6 \cdot 10^{-15}\text{A} \\
I &= I_S \left(e^{V/V_T} - 1\right) \\
\Rightarrow V &= V_T \ln(I/I_S + 1) \\
&\approx 0.662\text{V}
\end{aligned}$$

14 1.82

A short-base diode is one where the widths of the p and n regions are much smaller than L_n and L_p, respectively. As a result, the excess minority carrier distribution in each region is a straight line rather than the exponentials shown in Fig. 1.39.

(a) For the short-base diode, sketch a figure corresponding to Fig.1.39 and assume as in Fig.1.39 that $N_A \gg N_D$.

Ans:

TODO or not TODO

(b) Following a derivation similar to that given in Section 1.11.2, show that if the widths of the p and n regions are denoted W_p and W_n then

$$I = Aqn_i^2 \left(\frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A}\right) \left(e^{V/V_T} - 1\right)$$

and

$$\begin{aligned}
Q_p &= \frac{1}{2}\frac{(W_n - x_n)^2}{D_p} I_p \\
&\approx \frac{1}{2}\frac{W_n^2}{D_p} I_p, \quad \text{for } W_n \gg x_n
\end{aligned}$$

Ans:

$p_n(x_n) = p_{n_0} e^{V/V_T}$ as usual, but the excess charge at W_n equals 0, so $p_n(W_n) = p_{n_0}$, hence

$$\frac{\mathrm{d}p}{\mathrm{d}x} = p_{n_0}(e^{V/V_T} - 1)/(W_n - x_n) = n_i^2(e^{V/V_T} - 1)/((W_n - x_n)N_D)$$

everywhere, hence we have

$$I_p = AJ_p = AqD_p\frac{\mathrm{d}p}{\mathrm{d}x} = Aqn_i^2\frac{D_p}{N_D(W_n - x_n)}\left(e^{V/V_T} - 1\right)$$

similarly

$$I_n = AJ_n = AqD_n\frac{\mathrm{d}p}{\mathrm{d}x} = Aqn_i^2\frac{D_n}{N_A(W_p - x_p)}\left(e^{V/V_T} - 1\right)$$

So

$$I = I_n + I_p = Aqn_i^2\left(\frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A}\right)\left(e^{V/V_T} - 1\right)$$

Finally,

$$\begin{aligned}
Q_p &= Aq\frac{(p_n(x_n) - p_n(W_n))(W_n - x_n)}{2} \\
&= Aq\frac{(p_n(x_n) - p_n(W_n))(W_n - x_n)^2}{2(W_n - x_n)} \\
&= Aq\frac{(W_n - x_n)^2}{D_p} I_p \\
&\approx Aq\frac{W_n^2}{2D_p} I_p \quad (\text{If } W_n \gg x_n)
\end{aligned}$$

(c) Also, assuming $Q \approx Q_p, I \approx I_p$, show that

$$C_d = \frac{\tau_T}{V_T} I$$

where

$$\tau_T = \frac{1}{2}\frac{W_n^2}{D_p}$$

Ans:

$$\begin{aligned}
C_d &= \frac{\mathrm{d}Q}{\mathrm{d}V} \approx \frac{\mathrm{d}Q_p}{\mathrm{d}V} \\
&= \frac{\mathrm{d}(AqW_n^2/D_p)I_p}{2\mathrm{d}V} \\
&= \frac{AqW_n^2}{2D_p}\frac{\mathrm{d}I_s\left(e^{V/V_T} - 1\right)}{\mathrm{d}V}
\end{aligned}$$

If $V \gg V_T \approx 25.8\text{mV}$,

$$\frac{\mathrm{d}I_s\left(e^{V/V_T}-1\right)}{\mathrm{d}V} = I_S e^{V/V_T}/V_T \approx I/V_T$$

$$\begin{aligned} C_d &\approx \frac{AqW_n^2}{2D_p} I/V_T \\ &= \frac{\tau_T}{V_T} I \end{aligned}$$

(d) if a designer wishes to limit C_d to 8pF at $I = 1\text{mA}$, what should W_n be? Assume $D_p = 10\text{cm}^2/\text{s}$

Ans:

$$\begin{aligned} W_n &= \sqrt{\frac{2C_d D_p V_T}{I}} \\ &= \sqrt{2(8 \cdot 10^{-12}\text{F})(10^{-3}\text{m})(25.8 \cdot 10^{-3}\text{V})(10^3\text{A}^{-1})} \\ &\approx 642\text{nm} \end{aligned}$$