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- In the *axiomatic approach*, probability theory is developed based on a set of three assumptions known as the Kolmogorov axioms.

The conditional probability of event A given that event B has occurred is

$$P(A\,|\,B) = \frac{P(A \cap B)}{P(B)},$$

when $P(B) \neq 0$. The law of total probability and the rule of Bayes can be used when the sample space is partitioned by events $A_1, A_2, \ldots, A_n$. Pairwise independent and mutually independent events arise when the occurrence of an event is not influenced by the outcome of other events. We have emphasized the exact computation of probabilities in an analytic fashion in this chapter. There has been a secondary emphasis placed on Monte Carlo simulation.

2.7 Exercises

2.1 Four people play one round of the rock-paper-scissors (R-P-S) game.

(a) How many outcomes are in the sample space if players are considered distinct?

(b) If players are considered distinct, how many elements of the sample space correspond to:

- all four players getting the same symbol (for example, PPPP)?
- three players getting one symbol and the other player getting a different symbol (for example, SRSS)?
- two players getting one symbol and the two other players getting a different symbol (for example, PRPR)?
- two players getting one symbol and the two other players getting the other two symbols (for example, PPSR)?

2.2 Consider two equally-likely events A and B. If the probability that both occur is 0.2 and the probability that neither occurs is 0.3, find $P(A)$.

2.3 If A and B are disjoint events satisfying $P(A) = 0.3$ and $P(B') = 0.4$, what is $P(A' \cup B)$?

2.4 Let A_1, A_2, and A_3 be three events that partition the sample space S. Find $P\big(A_1 \cap (A_2 \cup A_3)\big)$.

2.5 Prove Bonferroni's inequality: for any two events A_1 and A_2,

$$P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1.$$

2.6 Let A_1, A_2, and A_3 form a partition of the sample space S. Find $P(A_1' \cup A_3')$.

2.7 Let E, F, and G be three events satisfying: $E \cap F = \emptyset$, $E \cap G \neq \emptyset$, and $F \cap G \neq \emptyset$, where $\emptyset$ is the null set. Draw a Venn diagram to illustrate that

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap G) - P(F \cap G).$$

2.8 For any two events A and B in S, if $P(A \cap B) = 0.4$, what is $P(A' \cup B')$?

2.9 For $P(A) = 2/3$ and $P(B) = 3/5$, what are the allowable values for $P(A \cap B)$?

2.10 If $P(A \cup B) = 3/5$ and $P(A' \cup B) = 9/10$, what is $P(B)$?

2.11 If the events A and B partition a sample space S, find $P(A' \cup B')$ and $P(A' \cap B')$.

2.12 A fair n-sided die has the numbers $1, 2, \ldots, n$ on its n faces. The die is rolled n times. A match occurs if the outcome i appears on roll i, for $i = 1, 2, \ldots, n$.

(a) Find the probability of one or more matches.

(b) Find the probability of one or more matches in the limit as $n \to \infty$.

2.13 Consider the events A and B such that $P(A \cup B) = 0.4$ and $P(A' \cup B) = 0.7$. Find $P(B)$.

2.14 Ross shuffles a deck of cards. What is the probability that the queen of hearts is the top card of the deck or the jack of spades is the bottom card of the deck?

2.15 The formula for $P(A_1 \cup A_2)$ involves three terms:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2).$$

The formula for $P(A_1 \cup A_2 \cup A_3)$ involves seven terms. How many terms are in the formula for $P(A_1 \cup A_2 \cup \ldots \cup A_n)$? Simplify your solution as much as possible.

2.16 Four tennis players split into two teams of two players at random on Monday. On Tuesday, the same four players split into two teams of two players at random, independent of how they split into teams on Monday. What is the probability that the two teams on Monday are identical to the two teams on Tuesday?

2.17 Find the probability that heads and tails alternate in the sequence of heads and tails generated by five tosses of a fair coin.

2.18 A bag contains 3 black balls and 2 gold balls. Another bag contains 2 black balls and x gold balls. One ball is selected at random from each bag. The probability that the two balls selected have a different color is 0.52. Find x.

2.19 A bridge hand consists of 13 cards dealt from a well-shuffled deck. A *yarborough* is a particularly weak bridge hand that contains no honor cards (that is, no tens, jacks, queens, kings, or aces). In other words, a yarborough is a bridge hand in which no card is higher than a nine. Charles Anderson Worsley (1809–1862), the Second Earl of Yarborough, was a British nobleman who is said to have bet 1000 to 1 that such a hand would not be dealt. Find the probability of a yarborough.

2.20 A five-card hand is dealt from a well-shuffled deck. What is the probability that the hand will contain the queen of hearts, exactly two other queens, and exactly one other face card?

2.21 Five fair dice are rolled. Find the probability that exactly two identical even numbers and exactly three identical odd numbers are rolled. The rolls 44555 and 61161 are examples of such rolls.

2.22 The integers $1, 2, \ldots, 9$ are arranged in a random order. Find the probability that the odd integers are in the odd positions.

2.23 Hailey, Anna, and Nicole are friends. They join 17 others. The 20 people divide at random into five groups of four people each. What is the probability that Hailey, Anna, and Nicole are in the same group?