

- 1.50 A co-ed baseball team consists of five men and four women. A man must bat first, then genders must alternate. How many batting orders are possible?
- 1.51 How many four-digit positive integers have an even hundreds digit and an odd tens digit?
- 1.52 How many two-digit numbers consist of digits which are both prime numbers?
- 1.53 Ten men decide to break into five pairs for arm wrestling matches. How many pairings are possible? Can your solution be generalized to pairing off any even number of men?
- 1.54 Amy, Blair, Cathy, Diane, and Ed are deciding on the order in which they will present their semester projects. If Blair does not want to present first and Diane does not want to present last, how many presentation orders are possible that satisfy Blair and Diane's constraints?
- 1.55 Many of the formulas in this chapter have involved the computation of $n!$, which is easy to compute for small n , but difficult for large n . Stirling's approximation is used to approximate $n!$ for large values of n . The Stirling series

$$n! = \sqrt{2\pi n} e^{-n} n^n \left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} - \frac{571}{2488320n^4} + \cdots \right)$$

indicates that $n!$ can be approximated by

$$\sqrt{2\pi n} e^{-n} n^n$$

for large values of n .

- (a) Calculate the absolute and relative error associated with using Stirling's approximation to calculate $n!$ for $n = 8, 16, 32, 64, 128$.
- (b) Use Stirling's approximation in the numerator and denominator to find

$$\lim_{n \rightarrow \infty} \frac{(n+1)!}{n!}.$$

- (c) Use a computer algebra system to calculate the exact value and Stirling's approximation to $1000!$.

- 1.56 How many ways can a mother give away 8 dogs to her 3 children if each child must receive at least 1 dog?
- 1.57 How many of the first 5000 positive integers are neither perfect squares nor perfect cubes?
- 1.58 The set A consists of all positive integers x from 1 to 15 inclusive such that $\gcd(x, 15) = 1$. List the elements of A .
- 1.59 A prime number is a positive integer that has exactly two distinct divisors: 1 and itself. Let A be the set of all positive integers. Let B be the set of all even integers. Let C be the set of all negative integers. Draw a Venn diagram that describes the relationship among the sets A , B , and C .
- 1.60 Draw a Venn diagram with events A and B and shade $A' \cap B'$.
- 1.61 Draw a Venn diagram with events A_1 , A_2 , and A_3 and shade $A_1 \cap (A_2 \cup A_3)$.

- 1.62 A 6-letter "word" is formed by selecting 6 of the 26 letters without replacement. Two examples of such words are

FRISBE and XEALRY.

Let A_1 be the set of all words beginning with X and A_2 be the set of all words ending with Y . Find the numbers of distinct words in

- (a) $A_1 \cap A_2$,
(b) $A_1 \cup A_2$.

- 1.63 Draw a Venn diagram with events A_1 , A_2 , and A_3 and shade $(A_1 \cap A_2') \cup A_3$.

- 1.64 If A and B are two events, use any of the set operations (for example, union, intersection, complement) to describe the event that neither A nor B occurs.

- 1.65 Let the set A be the perfect squares in the first 30 positive integers. Let the set B be the prime numbers in the first 30 positive integers. Find $N(A' \cap B')$, where N gives the cardinality (number of elements) in a set.

- 1.66 Shade $(B \cap (C \cap D')) \cup E$ on the Venn diagram in Figure 1.18.

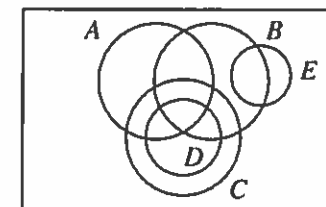


Figure 1.18: Venn diagram for events A , B , C , D , and E .

- 1.67 Two cards are drawn without replacement from a standard deck of playing cards. Draw a Venn diagram that relates the following four events.
- A : Both cards are from the same suit.
 B : Both cards have the same rank.
 C : One card is a face card and the other card is a number card.
 D : Both cards are from a red suit.
- 1.68 One hundred people are surveyed concerning their subscriptions to two newspapers. The set J consists those who subscribe to the *Wall Street Journal*. The set P consists those who subscribe to the *Washington Post*. The number of those surveyed who have one or more subscriptions is 80, that is $N(J \cup P) = 80$. Furthermore, $N(J \cup P') = 70$. How many of the 100 people surveyed subscribe to the *Wall Street Journal*?
- 1.69 Consider a universal set that consists of the first ten positive integers. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 5, 7, 9\}$, and $C = \{4, 5, 6, 7\}$. List the elements in the set $A \cap (B' \cup C)$.

1.70 A *power set* associated with a set A is a set that consists of all possible subsets of A . For example, if $A = \{a, b\}$, then the power set of A is $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. How many elements are in the power set of $A = \{a, b, c, d, e\}$?

1.71 The *exclusive or* operator \oplus for the sets A and B is defined as

$$A \oplus B = (A \cap B') \cup (A' \cap B).$$

Shade $A \oplus B \oplus C$ on a Venn diagram with the sets A , B , and C in Figure 1.19.

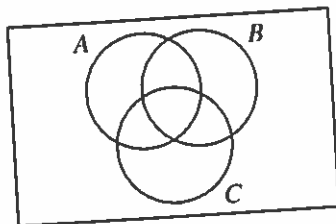


Figure 1.19: Venn diagram for events A , B , and C .

1.72 Consider the Venn diagram with three sets A , B , and C in Figure 1.20.

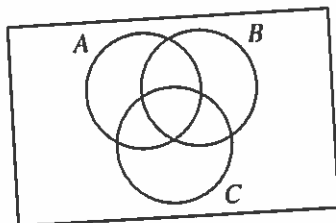


Figure 1.20: Venn diagram for events A , B , and C .

- (a) How many ways are there to shade various regions of the Venn diagram? For example, shading the entirety of A is one way to shade the Venn diagram; shading just the portion of A that does not intersect B or C is a second way to shade the Venn diagram. Assume that *not* shading any region does *not* count as shading.
- (b) Find the number of ways to shade regions of the Venn diagram associated with sets that can be written in the form

$$\{\text{set 1}\} \{\text{operator 1}\} \left(\{\text{set 2}\} \{\text{operator 2}\} \{\text{set 3}\} \right),$$

where $\{\text{set 1}\}$ is either A or A' , $\{\text{set 2}\}$ is either B or B' , $\{\text{set 3}\}$ is either C or C' , $\{\text{operator 1}\}$ is either \cap or \cup , and $\{\text{operator 2}\}$ is either \cap or \cup . An example of such a set is $A' \cap (B \cup C')$.

1.73 Of the 56 signers of the Declaration of Independence and the 39 signers of the United States Constitution, there were six men who signed both documents. How many men signed exactly one of the two documents?