# MATH 451/551 Midterm 1

Name: _____

This exam contains 12 pages (including this cover page) and 11 questions. Total of points is 42.

Grade Table (for instructor use only)

| Question | Points | Score |
|:---:|:---:|:---:|
| 1 | 3 | |
| 2 | 3 | |
| 3 | 4 | |
| 4 | 3 | |
| 5 | 5 | |
| 6 | 3 | |
| 7 | 3 | |
| 8 | 5 | |
| 9 | 5 | |
| 10 | 5 | |
| 11 | 3 | |
| Total: | 42 | |

1. (3 points) Show that $(A \cup B \cup C)^c = A^c B^c C^c$. You may use DeMorgan's Law.

2. (3 points)  Show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$$

3. (a) (2 points) Let $P(A) = .5$ and $P(B^c) = .3$ and $P(A \cup B) = .8$. What is $P(AB)$?

(b) (1 point) What is $P(A^c B)$?

(c) (1 point) What is $P(AB^c)$?

4. (3 points) Let $P$ be a probability measure. Show that if $A \subset B$ then $P(A) \leq P(B)$.

5. (5 points) You are dealt 3 cards from a deck of 52. What is the probability that they all have the same suit?

6. (3 points) A box contains eight balls. Four of the balls have an "A" written on them and four of them have a "B" written on them. You draw out the balls one by one (without replacement) and place them in a row. How many distinct strings of "A"s and "B"s are there?

7. (3 points) A group of n friends contains Larry and Rex. If the friends line up randomly, what is the probability Rex is standing next to Larry?

8. (5 points) Of students at W&M, 55% are women and 45% are men. 1 in 100 men have the flu and 2 in 100 women have the flu. If a randomly chosen student has the flu, what is the probability they are a man?

9. (5 points) A car factory has three machines labeled A, B, and C. A makes 20% of the cars, B makes 30% of the cars, and C makes 50% of the cars. 6% of the cars made by A are defective, 7% of the cars made by B are defective, and 8% of the cars made by C are defective. What is the probability that a car is defective if it is made by this factory?

10. (5 points) Let $p$ be the probability that I find a pearl when I open an oyster. How many oysters should I open to make sure I have at least a 50% chance of finding at least one pearl? We can assume that finding a pearl in one oyster is independent of finding one in another.

11. (3 points) Let $A$ and $B$ be both disjoint and independent. Let $P(A) = 1/2$. What is $P(B)$?