An introduction to

Quantum Annealing

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Outline

1. Quantum Annealing
    a) Adiabatic Theorem
    b) Transverse Field Ising Hamiltonian
2. D-Wave's Implementation
    a) The Qubit
    b) Coupling Qubits
    c) Processor
    d) Advantage
3. Lab Tour
4. Q&A

Quantum Annealing

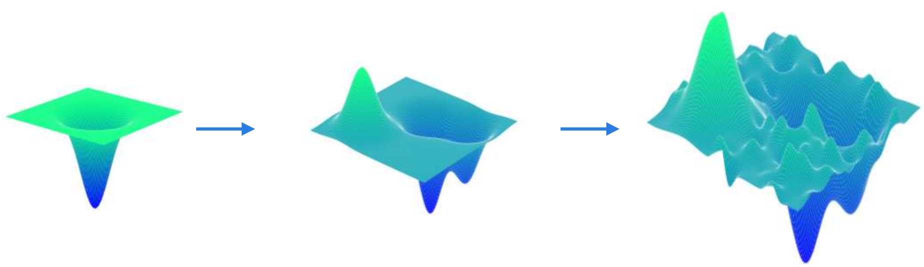
Quantum Annealing

What is quantum annealing?

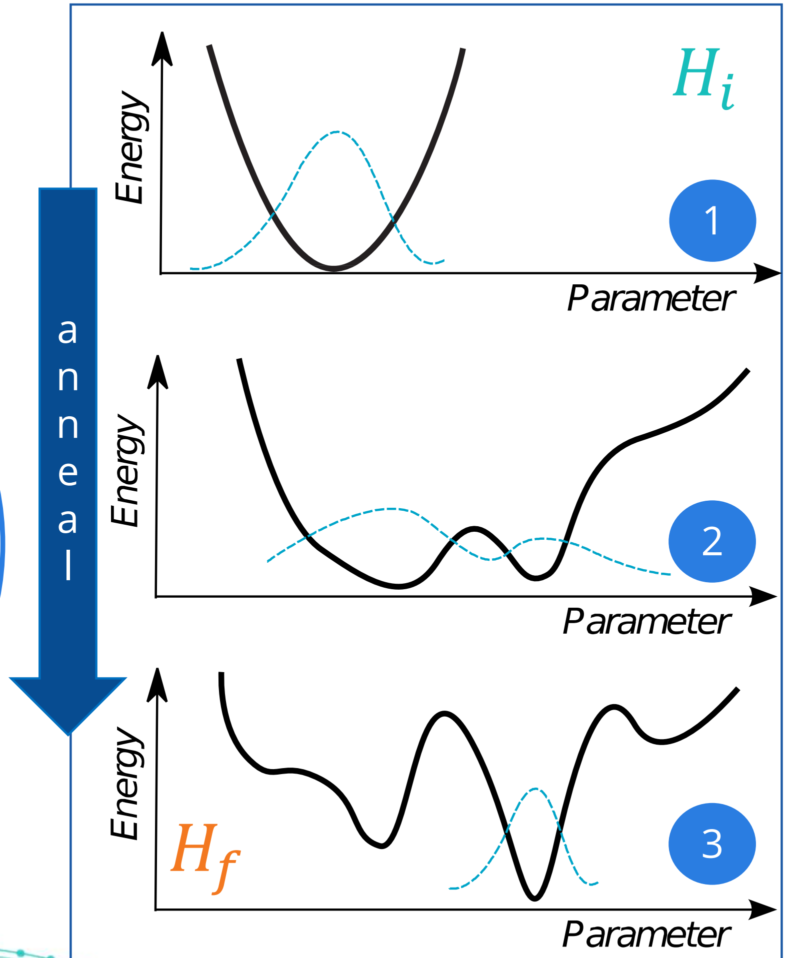
- A process that uses quantum mechanical effects to find a global minimum of an objective function
- Naturally solves optimization problems

Adiabatic Theorem

- Start a quantum system in the ground-state of a Hamiltonian (energy landscape)
- If that Hamiltonian changes slowly enough in time, the system will end up in the ground state of the final Hamiltonian



Quantum Annealing



1

- Single global minimum
- Ground state

2

- Multiple shallow minima with narrow barriers that allow tunneling
- Wavefunction delocalized across them

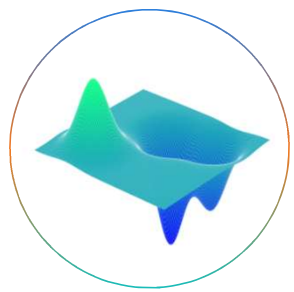
3

- Many deep minima
- Wavefunction localized around global minimum
- The ground state is a classical spin state

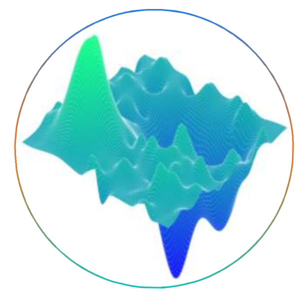
Quantum Annealing - Transverse Field Ising Hamiltonian

$$H_{ising} = \underbrace{\frac{-A(s)}{2}\left(\sum_{i} \sigma_z^{(i)}\right)}_{\text{Transverse Field Hamiltonian}} + \underbrace{\frac{B(s)}{2}\left(\sum_{i} h_i \sigma_z^{(i)} + \sum_{i>j} J_{i,j} \sigma_z^{(i)} \sigma_z^{(j)}\right)}_{\text{Problem Hamiltonian}}$$

Transverse Field Hamiltonian



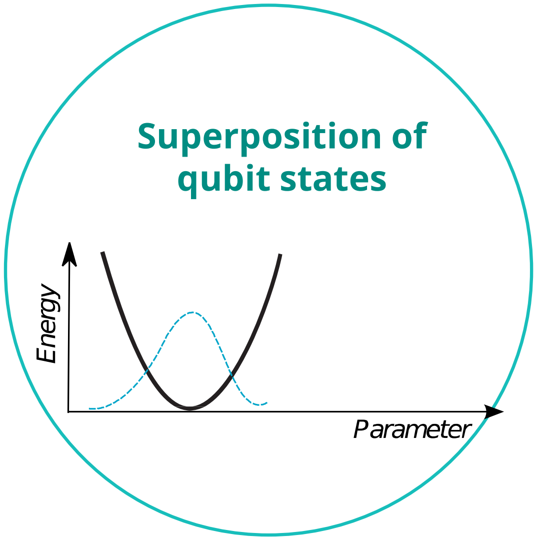
Transverse Field + Problem Hamiltonian

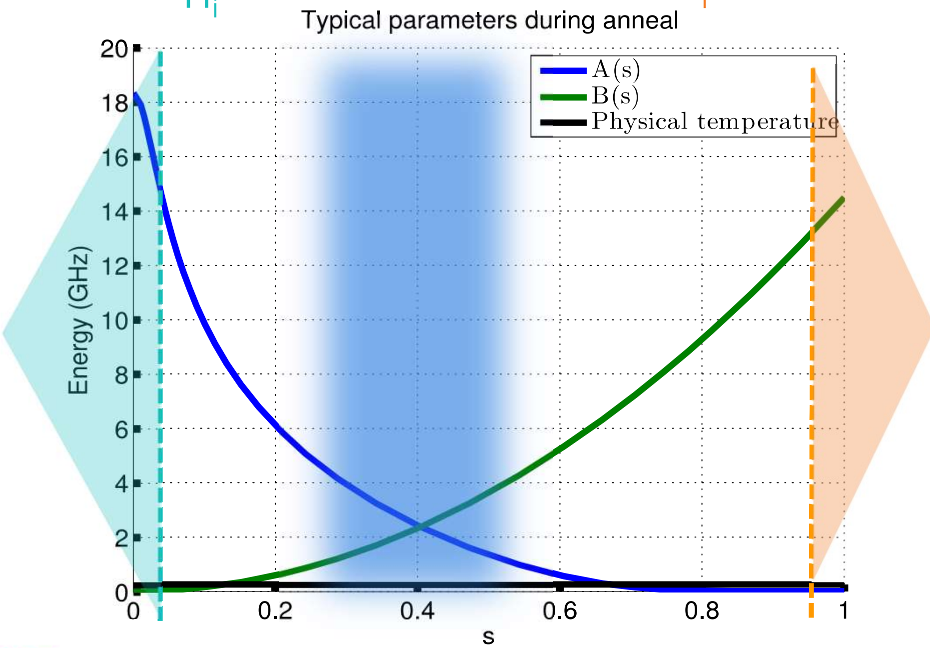


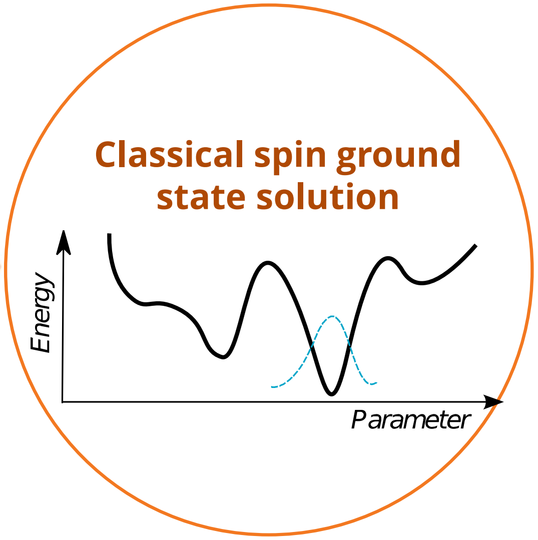
Problem Hamiltonian

Quantum Annealing - Transverse Field Ising Hamiltonian

$$H(s) = \underbrace{-A(s)\left[\sum_{i} \boldsymbol{\sigma}_{\boldsymbol{x}}^{(\boldsymbol{i})}\right]}_{H_i} + \underbrace{B(s)\left[-\sum_{i} h_i \sigma_z^{(i)} + \sum_{i,j>i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}\right]}_{H_f}$$



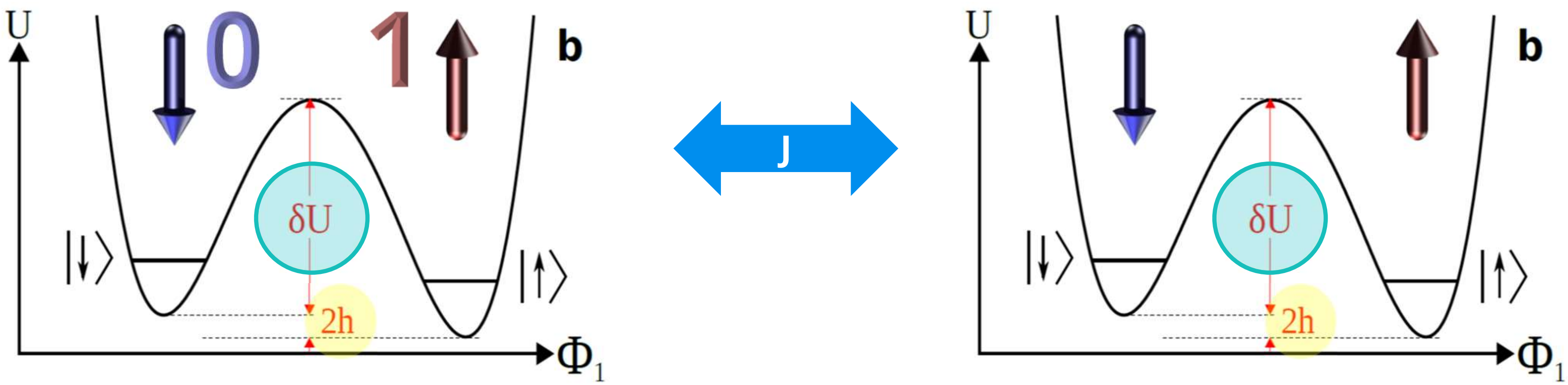




anneal

Quantum Annealing - Transverse Field Ising Hamiltonian

$$H(s) = -A(s)\left[\sum_i \boldsymbol{\sigma}_x^{(i)}\right] + B(s)\left[-\sum_i h_i \sigma_z^{(i)} + \sum_{i,j>i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$



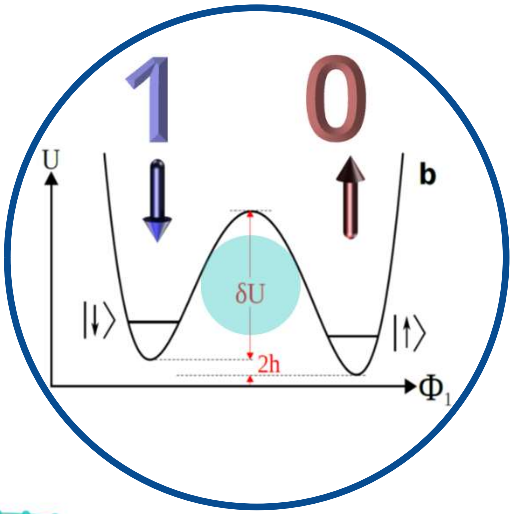
h_i and J_{ij} are specified with **programmable on-chip** control circuitry

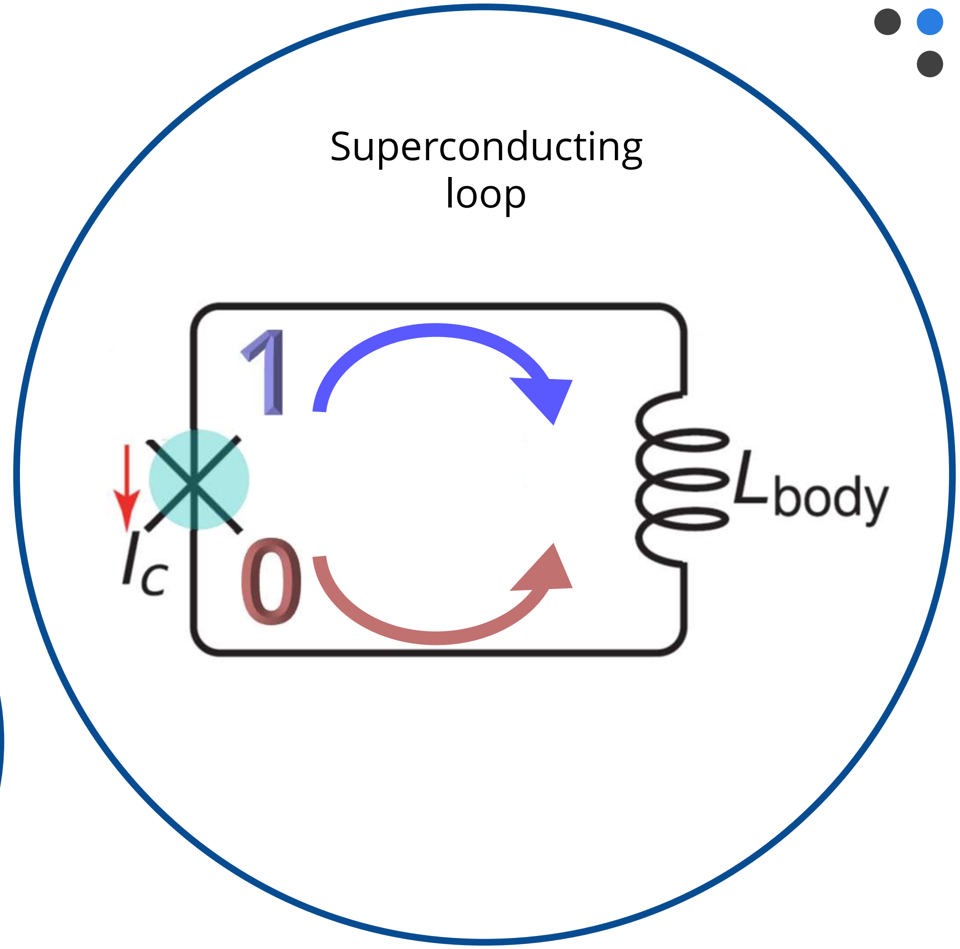
D-Wave’s Implementation

The Qubit – RF SQUID

Josephson Junction
creates barrier in potential

Critical current I_c
determines the barrier height

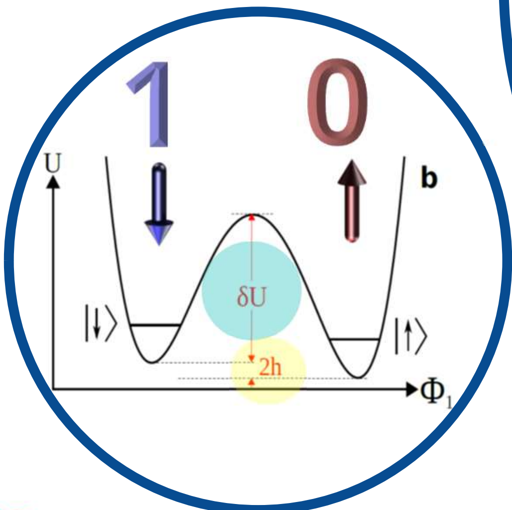


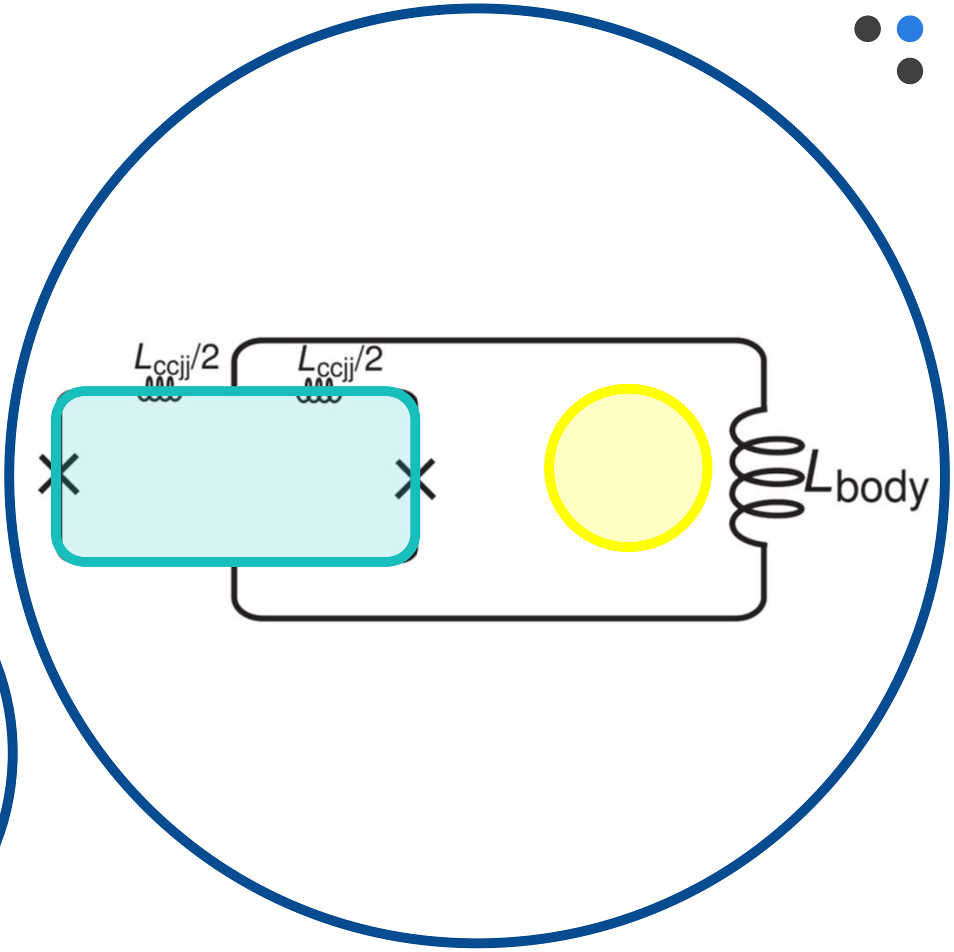


The Qubit – RF SQUID

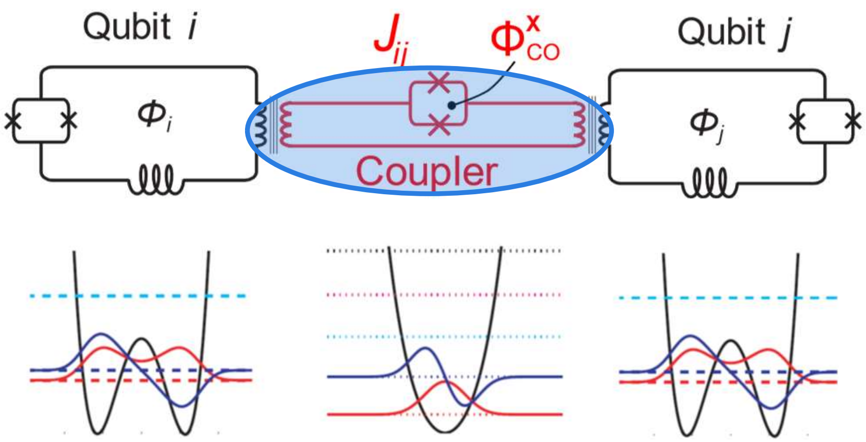
$$H(s) = -A(s)\left[\sum_i \boldsymbol{\sigma}_x^{(i)}\right] + B(s)\left[-\sum_i h_i \sigma_z^{(i)} + \sum_{i,j>i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

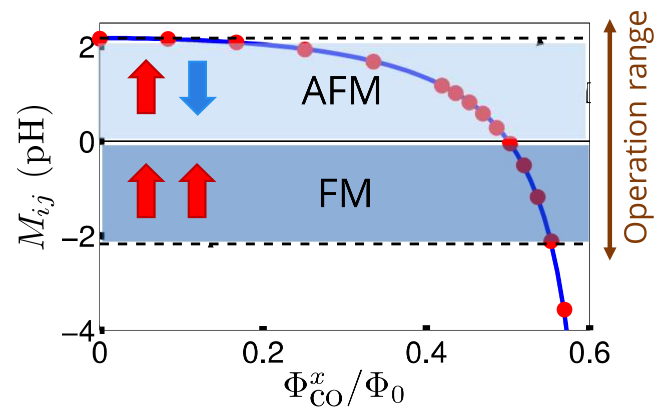
Compound Josephson Junction (CJJ) creates a **tunable** barrier





The Qubit – Coupling Qubits



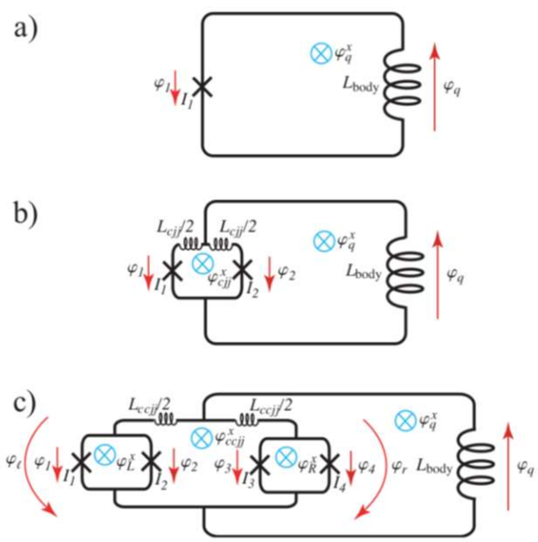


A mono-stable two-junction RF SQUID provides a tunable mutual inductance

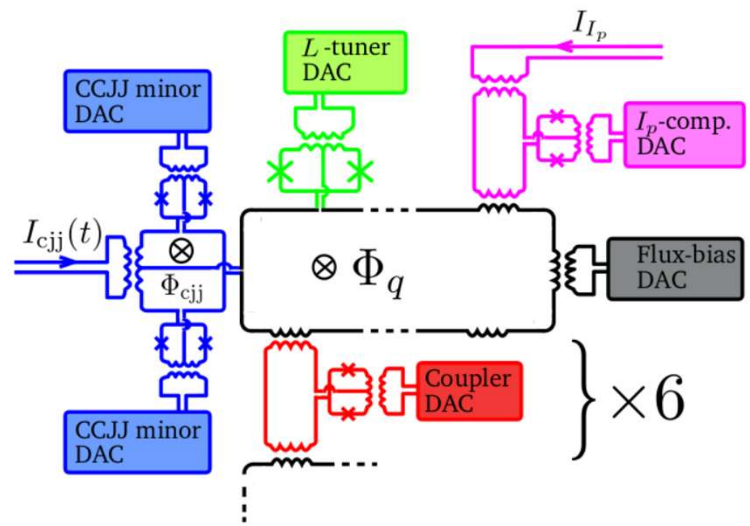
$$M_{ij} = \frac{J_{ij}}{|I_p(t)|^2}$$

$$H(s) = -A(s)\left[\sum_i \boldsymbol{\sigma}_{\boldsymbol{x}}^{(i)}\right] + B(s)\left[-\sum_i h_i \sigma_z^{(i)} + \sum_{i,j>i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}\right]$$

The Qubit In Detail

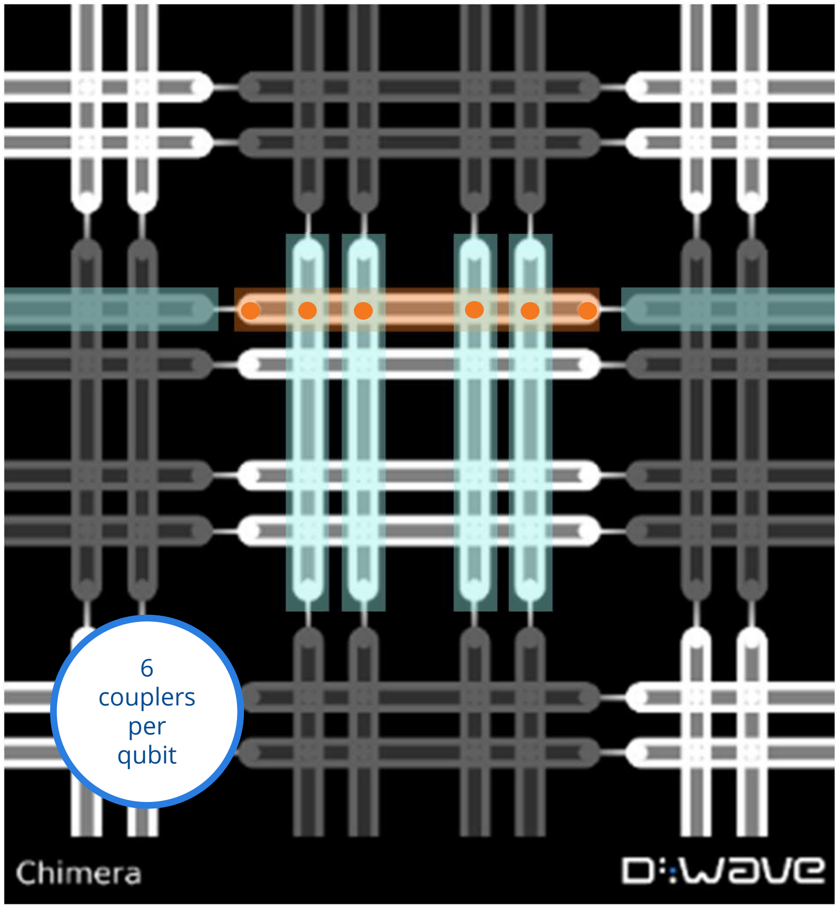


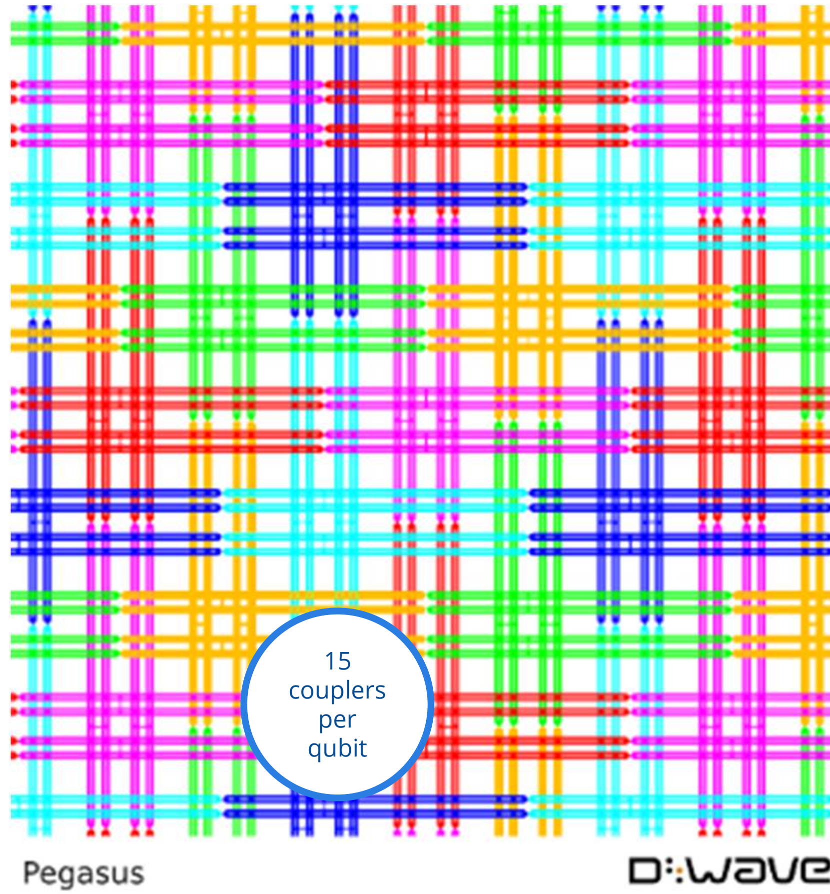
Harris et al., 2009



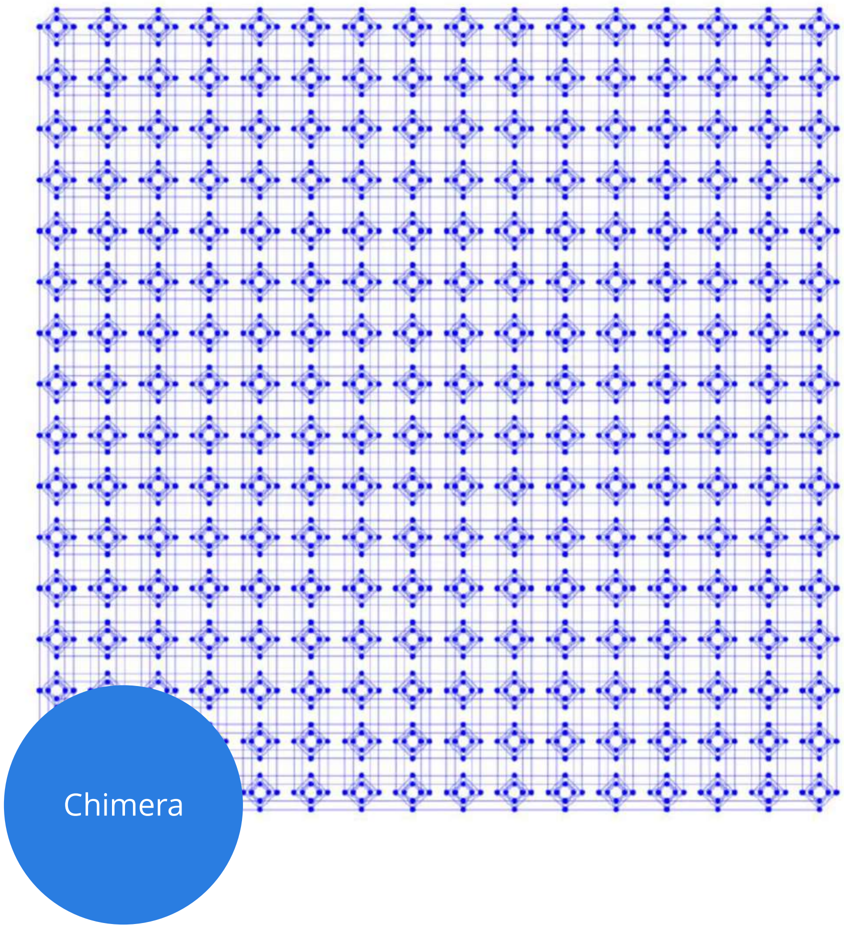
Johnson et al., 2010

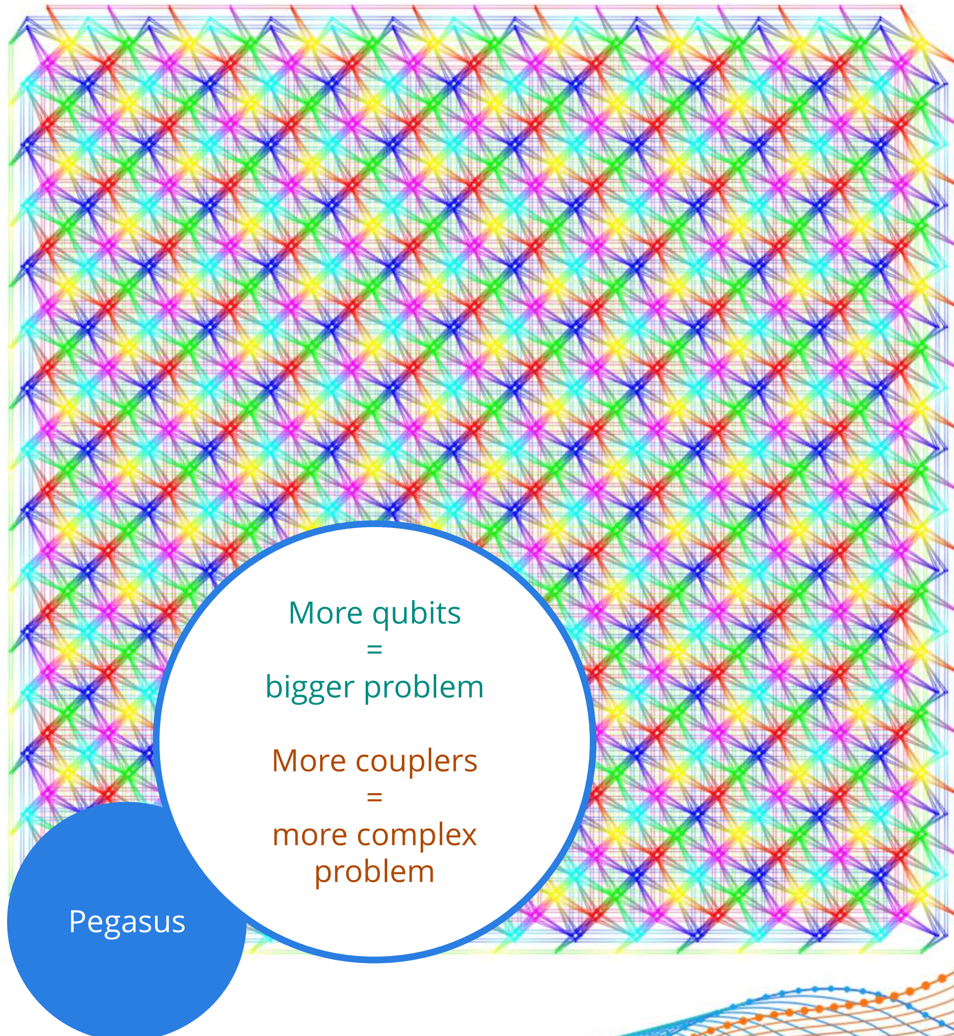
The Chip - Processor Layout





The Chip – Processor Layout





Lab Tour!

Lab Tour:
Video 1: https://youtu.be/zDotDiK2UuY?t=1
Video 2: https://youtu.be/VfxNdBTH8wY
Video 3: https://youtu.be/AGByZoYUlU0

Q&A

Additional Resources

Harris, R. et. al. Experimental Demonstration of a Robust and Scalable Flux Qubit, arXiv: 0909.4321

Harris, R. et. al. A Compound Josephson Junction Coupler for Flux Qubits With Minimal Crosstalk, arXiv: 0904.3784

Quantum Programming & Applications

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Outline

1. Binary Quadratic Model
2. Example: Antenna Selection
3. Demo
4. Applications
5. Q&A

D:Wave

Solving Real World Problems with Quantum Annealing

Quantum Annealing – How Do We Solve Real-World Problems?

Map real-world problems to binary quadratic models (BQMs)

$$BQM = \sum_{i} a_i x_i + \sum_{i>j} b_{i,j} x_i x_j$$

where x_i = binary variables, x ∈ {0,1}

a_i, $b_{i,j}$ = linear and quadratic biases

Model the problem as a set of objective function/s and constraints

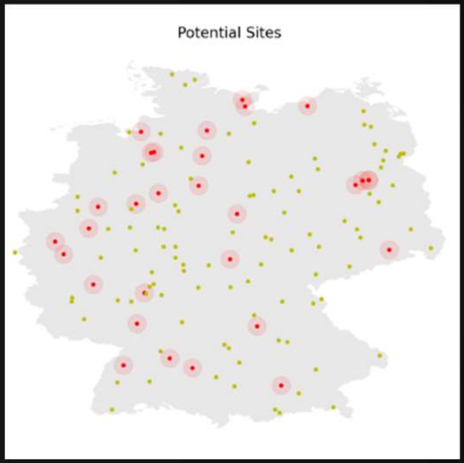
- Don’t need to know the physics to use the system
- Use binary variables as decision variables in the problem domain
- An objective function is what you’re trying to minimize or maximize
- Constraints are rules that determine solution feasibility

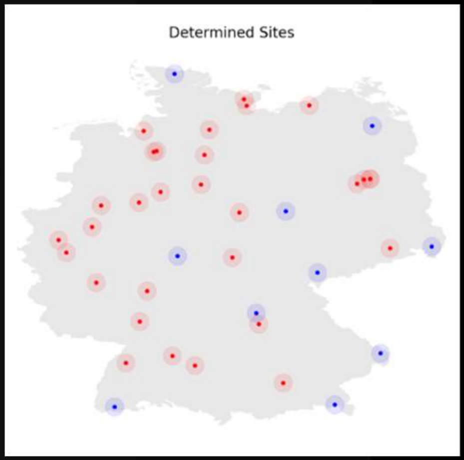
Example: Antenna Selection

Antenna Selection

Suppose a cell phone provider wants to extend their coverage over an area.

Given a set of viable sites how should they choose the optimal placements such that they *maximize coverage* without introducing interference between towers?





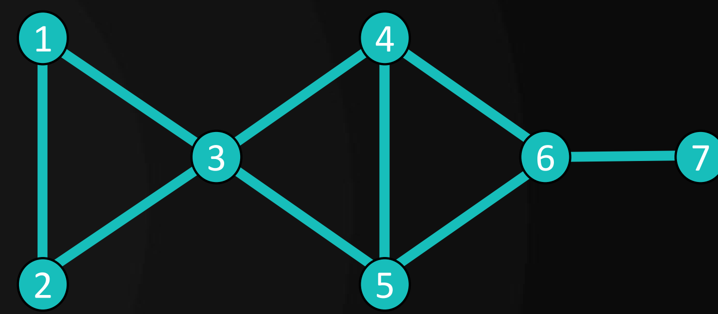
Problem Formulation: Antenna Selection

Modelling the problem

- Nodes = potential locations
- Edges = overlapping coverage

We want to *maximize the coverage* without introducing interference.

This is a *maximum independent set* problem.



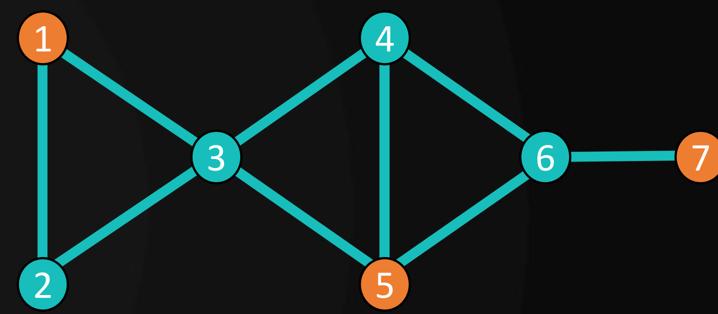
Problem Formulation: Antenna Selection

Objective

- Maximize the number of nodes chosen

Constraints

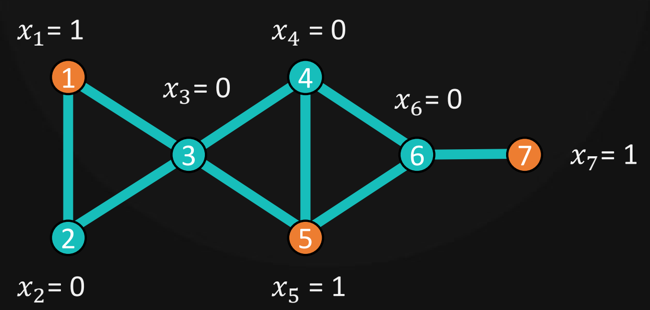
- The set must not contain any edges



Problem Formulation: Variables

We need to select antenna locations, so we can use binary variables to choose whether or not to build an antenna at a viable site.

$$x_i = \begin{cases} 1 & \text{if node i is selected as an antenna site} \\ 0 & \text{if node i is not selected} \end{cases}$$



Problem Formulation: Objective Function

We want to maximize the number of nodes selected:

$$max\left(\sum_i x_i\right)$$

Since the quantum annealer naturally finds the minima we recast this to a minimization problem:

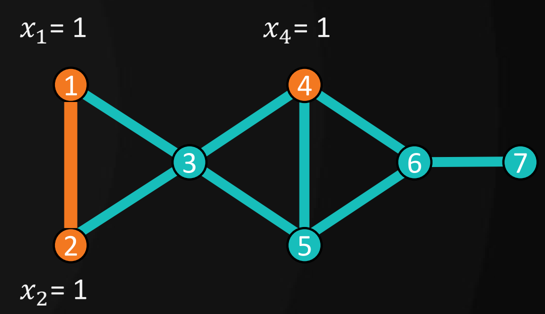
$$min\left(-\sum_i x_i\right)$$

Problem Formulation: Constraints

To ensure we don't have any interference between antennas, we need to make sure we don't have any edges in the set.

Penalty Model

i	j	Adjacent nodes selected
0	0	No
0	1	No
1	0	No
1	1	Yes

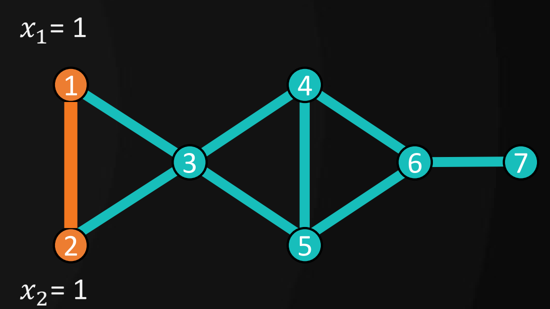


Problem Formulation: Constraints

To ensure we don't have any interference between antennas, we need to make sure we don't have any edges in the set.

Penalty Model

i	j	Adjacent nodes selected	Cost
0	0	No	0
0	1	No	0
1	0	No	0
1	1	Yes	1



Penalize this selection by 1

Problem Formulation: Constraints

To ensure we don't have any interference between antennas, we need to make sure we don't have any edges in the set.

Penalty Model

i	j	Adjacent nodes selected	Cost
0	0	No	0
0	1	No	0
1	0	No	0
1	1	Yes	1

Solve a system of equations for
$ax_i + bx_i + cx_ix_j + d = cost$

$$\sum_{i,j} x_i x_j$$

Problem Formulation: Antenna Selection

Remember we need our problem in the form

$$BQM = \sum_i a_i x_i + \sum_{i>j} b_{i,j} x_i x_j$$

We combine the objective and constraint:

$$BQM = -\sum_i x_i + \gamma \sum_{i,j} x_i x_j$$

Demo

Applications

POWERFUL HYBRID SOLVERS

CONSTRAINED QUADRATIC MODEL SOLVER

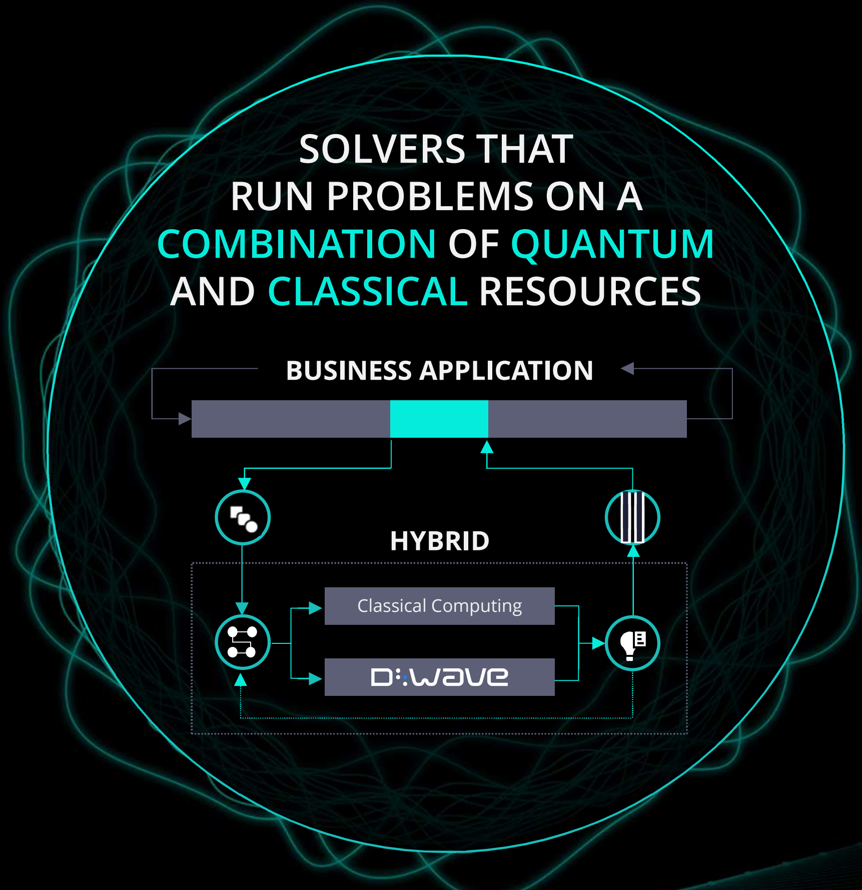
- More native representation of problem
- Unlocks larger application problems
- Inequality & equality constraints; Up to 100,000
- Binary, integer, and real/continuous variables

BINARY QUADRATIC MODEL SOLVER

- Up to 1,000,000 variables
- Enables enterprise-scale problem solving
- Accepts problems with binary variable

DISCRETE QUADRATIC MODEL SOLVER

- Expands into new problem types
- Enables optimization with option selection: e.g., Choose one of 11, 19, 29
- Accepts discrete multi-level variables



REAL-WORLD, COMMERCIAL APPLICATIONS TODAY ACROSS KEY VERTICALS

LOGISTICS

Shipping container logistics
Employee scheduling
Farm to market food delivery
Last mile vehicle routing

PHARMA

Protein folding
Clinical trials
Drug discovery

FINANCE

Portfolio risk reduction and return optimization
Marketing campaign optimization
Fraud detection

REDUCE WASTE IN THE AUTOMOTIVE SUPPLY CHAIN

80%

REDUCTION IN WASTE AND PAINT TIME

"By continuing to research and develop these types of algorithms, we hope to have a significant impact on Volkswagen's core business throughout multiple units. This application has immediate, real-world implications for production and logistics."

—VOLKSWAGEN QUANTUM COMPUTING RESEARCHER SHEIR YARKONI

LOGISTICS OPTIMIZATION AT PORT OF LA WITH QUANTUM COMPUTING

60%

MORE CARGO HANDLED EACH DAY PER CRANE

SAVANTX

D-Wave's quantum system is used as part of the SavantX HONE optimization engine at the Port of Los Angeles. The goal is to expedite delivery of containers out of the terminal while increasing the amount of cargo that can be handled.

"With HONE and D-Wave, each huge crane handled 60% more cargo per day, while the turnaround time for trucks was reduced by 12%."

— SAVANTX TEAM

Q&A

Additional Resources

Sign up for Leap: https://cloud.dwavesys.com/signup

D-Wave Applications: https://www.dwavesys.com/learn/featured-applications/

Webinars: https://www.youtube.com/user/dwavesystems