# CS515: Algorithms and Data Structures, Winter 2021

## Homework 2*

## Due: Tue, Feb 2, 2021

**Homework Policy:**

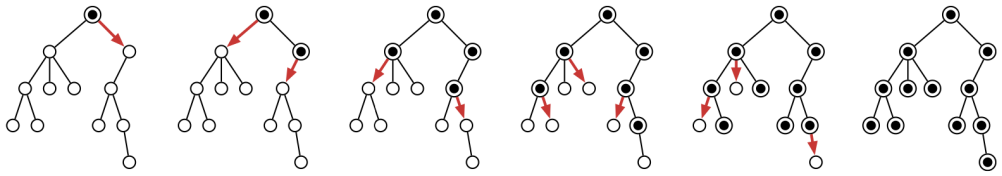
1. Students should work on group assignments in groups of preferably three people. Each group submits to CANVAS a *typeset* report in pdf format.
2. The goal of the homework assignments is for you to learn solving algorithmic problems. So, I recommend spending sufficient time thinking about problems individually before discussing them with your friends.
3. You are allowed to discuss the problems with other groups, and you are allowed to use other resources, but you *must* cite them. Also, you must write everything in your own words, copying verbatim is plagiarism.
4. *I don't know policy:* you may write "I don't know" *and nothing else* to answer a question and receive 25 percent of the total points for that problem whereas a completely wrong answer will receive zero.
5. Algorithms should be explained in plain english. Of course, you can use pseudocodes if it helps your explanation, but the grader will not try to understand a complicated pseudocode.
6. More items might be added to this list. ☺

**Problem 1.** Call a sequence $X[1..n]$ of numbers convex if $2X[i] < X[i-1] + X[i+1]$ for all $i$. Describe a polynomial time algorithm to compute the length of the longest convex subsequence of an arbitrary array $A$ of integers. Provide running time analysis and proof of correctness for your algorithm.

**Problem 2.** Suppose we are given a set $L$ of $n$ line segments in the plane, where each segment has one endpoint on the line $y = 0$ and one endpoint on the line $y = 1$, and all $2n$ endpoints are distinct. Describe a polynomial time algorithm to compute the largest subset of $L$ in which every pair of segments intersects. Provide running time analysis and proof of correctness for your algorithm.

**Problem 3.** Suppose we need to broadcast a message to all the nodes in a rooted tree. Initially, only the root node knows the message. In a single round, any node that knows the message can forward it to at most one of its children. See the figure below for an example. Design a polynomial time algorithm to compute the minimum number of rounds to broadcast a message to all nodes in a given binary tree. Analyze the running time of your algorithm, and prove that it is correct.

*Some of the problems are from the text book. Looking into similar problems from the book, chapter 3 is recommended.



**Problem 4.** Let $G = (V, E)$ be a directed acyclic graph, and let $X : V \to \mathbb{R}$ specify numbers on the vertices of $G$. Design a polynomial time algorithm to compute the longest directed path with increasing sequence of numbers on its vertices. Show that this problem is a generalization of the increasing subsequence problem that we have seen in class.