CS515: Algorithms and Data Structures, Winter 2021

Homework 4*

Due: Tue, March 9, 2021

Homework Policy:

1. Students should work on group assignments in groups of preferably three people. Each group submits to CANVAS a *typeset* report in pdf format.
2. The goal of the homework assignments is for you to learn solving algorithmic problems. So, I recommend spending sufficient time thinking about problems individually before discussing them with your friends.
3. You are allowed to discuss the problems with other groups, and you are allowed to use other resources, but you *must* cite them. Also, you must write everything in your own words, copying verbatim is plagiarism.
4. *I don't know policy:* you may write "I don't know" *and nothing else* to answer a question and receive 25 percent of the total points for that problem whereas a completely wrong answer will receive zero.
5. Algorithms should be explained in plain english. Of course, you can use pseudocodes if it helps your explanation, but the grader will not try to understand a complicated pseudocode.
6. More items might be added to this list. ☺

Problem 1. Let (S, T) and (S', T') be minimum (s, t)-cuts in some flow network G. Prove that $(S \cap S', T \cup T')$, $(S \cup S', T \cap T')$ are also minimum (s, t)-cuts in G.

Problem 2. Ad-hoc networks are made up of low-powered wireless devices. In principle, these networks can be used in regions that have recently suffered from natural disasters, and in other hard-to-reach areas. The idea is that a large collection of cheap, simple devices could be distributed through the area of interest (for example, by dropping them from an airplane); the devices would then automatically configure themselves into a functioning wireless network.

These devices can communicate only within a limited range. We assume all the devices are identical; there is a distance D such that two devices can communicate if and only if the distance between them is at most D. We would like our ad-hoc network to be reliable, but because the devices are cheap and low-powered, they frequently fail. If a device detects that it is likely to fail, it should transmit its information to some other backup device within its communication range. We require each device x to have k potential backup devices, all within distance D of x; we call these k devices the backup set of x. Also, we do not want any device to be in the backup set of too many other devices; otherwise a single failure might affect a large fraction of the network.

So suppose we are given the communication radius D, parameters b and k, and an array $d[1 \ldots n, 1 \ldots n]$ of distances, where $d[i, j]$ is the distance between device i and device j. Describe

*Some of the problems are from Erickson's lecture notes. Looking into similar problems in maximum flows and minimum cu and linear programming from his notes is recommended.

an algorithm that either computes a backup set of size k for each of the n devices, such that no device appears in more than b backup sets, or reports (correctly) that no good collection of backup sets exists.

Problem 3.

(a) Give a linear-programming formulation of the minimum-cost circulation problem. You are given a flow network whose edges have both capacities and costs, and your goal is to find a feasible circulation (flow with value 0) whose total cost is as small as possible.

(b) Derive the dual of your linear program from part (a).

Problem 4. Fix a non-degenerate linear program in canonical form with d variables and $n + d$ constraints.

(a) Prove that every feasible basis has exactly d feasible neighbors.

(b) Prove that every locally optimal basis has exactly n locally optimal neighbors.