CSE 544: Principles of Database Systems

Conjunctive Queries (and beyond)

Outline

- Conjunctive queries
- Query containment and equivalence
- Query minimization
- Undecidability for relational queries

Conjunctive Queries (CQ)

- CQ = one datalog rule
- CQ = SELECT-DISTINCT-FROM-WHERE
- CQ = select/project/join (σ, Π, $\bowtie$) fragment of RA
- CQ = existential/conjunctive ($\exists$, $\wedge$) fragment of RC

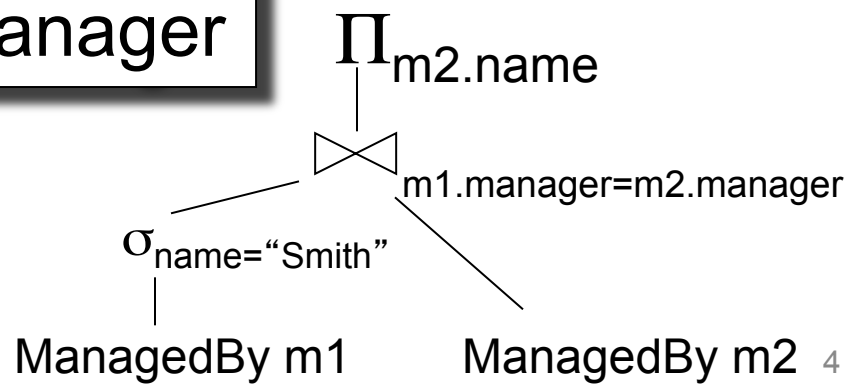
Notice: strictly speaking we are not allowed to use <, ≤, >, ≥, ≠ in CQ's. If we include these, then the language is called $CQ^{<}$, or $CQ^{\neq}$, etc.

Examples

Find all employees with same manager as "Smith":

```
q(x) :-  ManagedBy("Smith",y),   ManagedBy(x,y)
```

```
SELECT DISTINCT m2.name
FROM ManagedBy m1,
          ManagedBy m2
WHERE m1.name="Smith"
   AND m1.manager=m2.manager
```



Examples

- Example of CQ

$$q(x,y) = \exists z.(R(x,z) \wedge \exists u.(R(z,u) \wedge R(u,y)))$$

$$q(x) = \exists z.\exists u.(R(x,z) \wedge R(z,u) \wedge R(u,y))$$

- Examples of non-CQ:

$$q(x,y) = \forall z.(R(x,z) \rightarrow R(y,z))$$

$$q(x) = T(x) \vee \exists z.S(x,z)$$

Query Equivalence and Containment

Containment/equivalence are examples of static analysis

Used in:

- Query optimization
- Query rewriting using views

Query Equivalence

Definition. Queries q_1 and q_2 are **equivalent** if for every database **D**, $q_1(\mathbf{D}) = q_2(\mathbf{D})$.

Notation: $q_1 \equiv q_2$

Query Containment

Definition. Query q_1 is **contained** in q_2 if for every database **D**, $q_1(\mathbf{D}) \subseteq q_2(\mathbf{D})$.

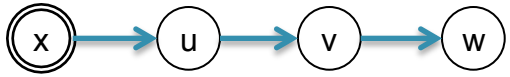
Notation: $q_1 \subseteq q_2$

Fact: $q_1 \subseteq q_2$ and $q_2 \subseteq q_1$ iff $q_1 \equiv q_2$

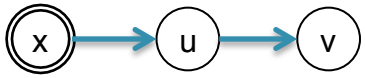
We discuss only query containment.

Example 1

Is $q_1 \subseteq q_2$?

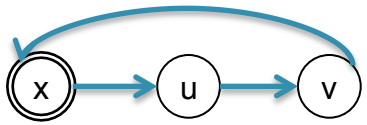


$q_1(x)$:- R(x,u), R(u,v), R(v,w)
$q_2(x)$:- R(x,u), R(u,v)



Example 2

Is $q_1 \subseteq q_2$?



$q_1(x)$:- R(x,u), R(u,v), R(v,x)
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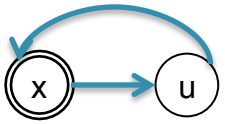
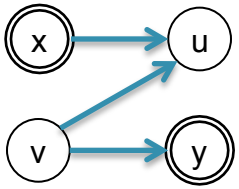


Example 3

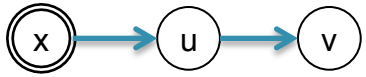
Is $q_1 \subseteq q_2$?

$q_1(x,y)$:- $R(x,y)$
$q_2(x,y)$:- $R(x,u)$, $R(v,u)$, $R(v,y)$

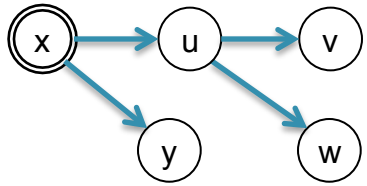


Example 4

Is $q_1 \subseteq q_2$?

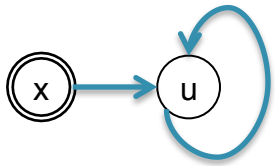


$q_1(x)$:- R(x,u), R(u,v)
$q_2(x)$:- R(x,u), R(x,y), R(u,v), R(u,w)



Example 5

Is $q_1 \subseteq q_2$?



$q_1(x)$:- R(x,u), R(u,u)
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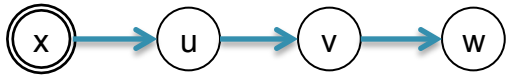

Example 6

Is $q_1 \subseteq q_2$?

$q_1(x)$:- R(x,u), R(u,”Smith”)
$q_2(x)$:- R(x,u), R(u,v)

Discussion

- We discuss only query containment for Boolean queries
  - If q_1, q_2 are Boolean queries, then containment $q_1(\mathbf{D}) \subseteq q_2(\mathbf{D})$ means implication $q_1(\mathbf{D}) \rightarrow q_2(\mathbf{D})$

- If q_1, q_2 are not Boolean, then convert them into Boolean queries by pretending that their head variables are constants
  - We must make *the same* constants in both q_1, q_2

Query Containment for CQ

Theorem [Chandra&Harel'1977]
Given two CQ, q_1, q_2, the following are equivalent:

1. For every database **D**, $q_1(\mathbf{D}) \subseteq q_2(\mathbf{D})$
2. There exists a homomorphism h : $q_2 \rightarrow q_1$
3. q_2 is true on the canonical database of q_1

Moreover, this problem is NP-complete

Query Homomorphisms

Definition A *homomorphism* h : q_2 → q_1 is a function
h: var(q_2) → var(q_1) ∪ const(q_1)
such that, for every atom R(x, y, z, …) in the query q_2,
there is an atom R(h(x), h(y), h(z), …) in the query q_1

The Homomorphism Theorem $q_1 \subseteq q_2$ iff there exists a homomorphism h : q_2 → q_1

Example 1

Is $q_1 \subseteq q_2$?

$q_1(x)$:- R(x,u), R(u,v), R(v,w)
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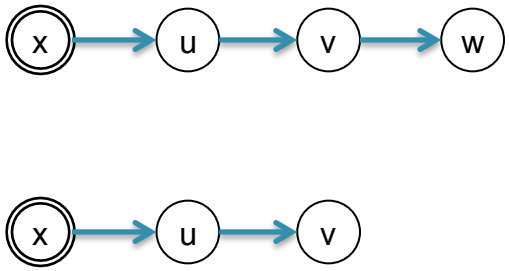

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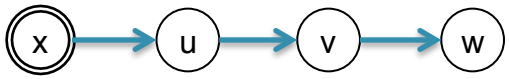

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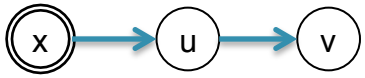
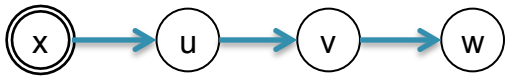


First, transform the head variables into a constant: must be the same in q_1 and q_2!

q_1 :- R("x",u), R(u,v), R(v,w)
q_2 :- R("x",u), R(u,v)

Example 1

Is $q_1 \subseteq q_2$?



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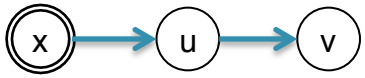

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q_2 :- R("x",u), R(u,v)

The homomorphism h : $q_2 \rightarrow q_1$ is the following:
h(u) = u, h(v) = v

In class: check that h is a homomorphism

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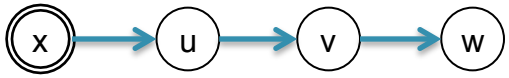
Is $q_1 \subseteq q_2$?



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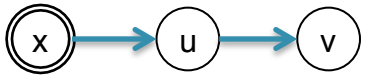

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q_1 :- R("x",u), R(u,v), R(v,w)
q_2 :- R("x",u), R(u,v)

The homomorphism h : $q_2 \rightarrow q_1$ is the following:
h(u) = u, h(v) = v

This proves $q_1 \subseteq q_2$

In class: check that h is a homomorphism

Example 2

Is $q_1 \subseteq q_2$?

$q_1(x)$:- R(x,u), R(u,v), R(v,x)
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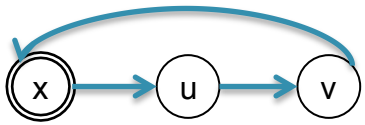

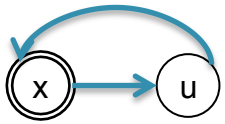

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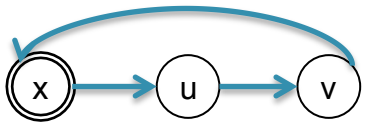
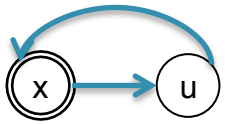




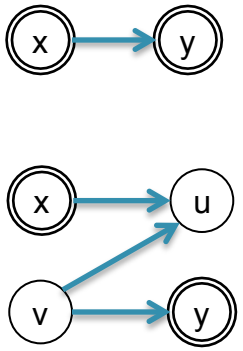
In class: check that there is no homomorphism

This proves $q_1 \not\subseteq q_2$

Example 3

Is $q_1 \subseteq q_2$?

$q_1(x,y)$:- R(x,y)
$q_2(x,y)$:- R(x,u), R(v,u), R(v,y)



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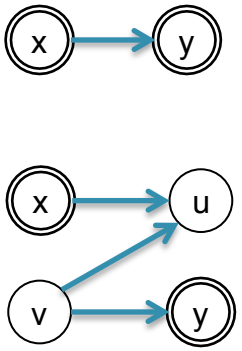

Head variables x,y are treated like constants

The homomorphism h : $q_2 \rightarrow q_1$ is the following:
h(u) = y, h(v) = x

This proves $q_1 \subseteq q_2$

Canonical Database

Definition. Given a conjunctive query q, the canonical database D_q is the following:

- The active domain = vars(q) ∪ const(q)
- Each atom in q becomes a tuple in D_q

The Canonical Database Theorem $q_1 \subseteq q_2$ iff the query q_2 is true on the canonical database D_{q1}

Example 1

Is $q_1 \subseteq q_2$?

$q_1(x)$:- R(x,u), R(u,v), R(v,w)
$q_2(x)$:- R(x,u), R(u,v)

Example 1

Is $q_1 \subseteq q_2$?

$q_1(x)$:- R(x,u), R(u,v), R(v,w)
$q_2(x)$:- R(x,u), R(u,v)

D_{q1}

R:

x	u
u	v
v	w

Compute q_2 on D_{q1}: the answer is true
Keep in mind that "x" in q_2 is a constant, it can only match x.

This proves $q_1 \subseteq q_2$

Example 2

Is $q_1 \subseteq q_2$?

$q_1(x)$:- R(x,u), R(u,v), R(v,x)
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Is $q_1 \subseteq q_2$?

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D_{q1}

R:

x	u
u	v
v	x

Compute q_2 on D_{q1}: the answer is false

This proves that $q_1 \not\subseteq q_2$

Query Containment for CQ

Theorem [Chandra&Harel'1977]
Given two CQ, q_1, q_2, the following are equivalent:

1. For every database **D**, $q_1(\mathbf{D}) \subseteq q_2(\mathbf{D})$
2. There exists a homomorphism h : $q_2 \rightarrow q_1$
3. q_2 is true on the canonical database of q_1

Moreover, this problem is NP-complete

Proof: show in class that 1 → 3 → 2 → 1

The Complexity

Theorem Checking containment of two CQ queries is NP-complete

Proof

Reduction from 3SAT

Given a 3CNF Φ

Step 1:

construct q_1 independently of Φ.

Proof

Reduction from 3SAT

Given a 3CNF Φ

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Step 2:

construct q_2 from Φ.

Proof

Reduction from 3SAT

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Claim:

there exists a homomorphism $q_2 \rightarrow q_1$ iff Φ is satisfiable

Proof

Reduction from 3SAT

Given a 3CNF Φ

Step 1:
construct q_1 independently of Φ.

Step 2:
construct q_2 from Φ.

Claim:
there exists a
homomorphism $q_2 \rightarrow q_1$
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Running example:

$\Phi = (\neg X_3 \vee \neg X_1 \vee X_4)$
$(X_1 \vee X_2 \vee X_3)$
$(\neg X_2 \vee \neg X_3 \vee X_1)$

Step 1: Constructing q_1

There are four types of clauses possible in a 3SAT:

Type 1 = ¬X ∨ ¬Y ∨ ¬Z

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Type 1 = ¬X ∨ ¬Y ∨ ¬Z

Type 2 = ¬X ∨ ¬Y ∨ Z

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There are four types of clauses possible in a 3SAT:

Type 1 = ¬X ∨ ¬Y ∨ ¬Z

Type 2 = ¬X ∨ ¬Y ∨ Z

Type 3 = ¬X ∨ Y ∨ Z

Step 1: Constructing q_1

There are four types of clauses possible in a 3SAT:

Type 1 = ¬X ∨ ¬Y ∨ ¬Z

Type 2 = ¬X ∨ ¬Y ∨ Z

Type 3 = ¬X ∨ Y ∨ Z

Type 4 = X ∨ Y ∨ Z

For each type, we include in q_1 a relation with all 7 satisfying assignments

R
(misses 1,1,1)

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0

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(misses 1,1,0)

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0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	1

T
(misses 1,0,0)

0	0	0
0	0	1
0	1	0
0	1	1
1	0	1
1	1	0
1	1	1

K
(misses 0,0,0)

0	0	1
0	1	0
0	1	1
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There are four types of clauses possible in a 3SAT:

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K
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0	0	1
0	1	0
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1	0	0
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1	1	0
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So which are the atoms in q_1?

Step 2: Constructing q_2

Given Φ, we construct q_2 as follows.

For each clause in Φ we create an atom in q_2:

- If the clause is of Type 1, then the atom is R;
  if the clause is of Type 2, then the atom is S; etc.
- The variables in this atom are the same as the variables in the clause

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Example:

$$\Phi = (\neg X_3 \vee \neg X_1 \vee X_4) \wedge (X_1 \vee X_2 \vee X_3) \wedge (\neg X_2 \vee \neg X_3 \vee X_1)$$

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$$q_2 = S(x_3,x_1,x_4),\ K(x_1,x_2,x_3),\ S(x_2,x_3,x_1)$$

Proof

Claim:
There exists a homomorphism $q_2 \rightarrow q_1$ iff Φ is satisfiable

1. Suppose there is a satisfying assignment θ for Φ:
   - Define the function h: $Vars(q_2) \rightarrow Const(q_1)$:
     - If $\theta(X_i) = 0$ then $h(x_i) = 0$
     - If $\theta(X_i) = 1$ then $h(x_i) = 1$
   - h is a homomorphism $h : q_2 \rightarrow q_1$ (why??)
2. Suppose there exists a homomorphism $h : q_2 \rightarrow q_1$.
   - Define the following assignment θ:
     - If $h(x_i) = 0$ then $\theta(X_i) = 0$
     - If $h(x_i) = 1$ then $\theta(X_i) = 1$
   - θ is a satisfying assignment for Φ (why??)

Proof

Claim:
There exists a homomorphism $q_2 \rightarrow q_1$ iff Φ is satisfiable

1. Suppose there is a satisfying assignment θ for Φ:
   - Define the function h: $Vars(q_2) \rightarrow Const(q_1)$:
     - If $\theta(X_i) = 0$ then $h(x_i) = 0$
     - If $\theta(X_i) = 1$ then $h(x_i) = 1$
   - h is a homomorphism $h : q_2 \rightarrow q_1$ (why??)
2. Suppose there exists a homomorphism $h : q_2 \rightarrow q_1$.
   - Define the following assignment θ:
     - If $h(x_i) = 0$ then $\theta(X_i) = 0$
     - If $h(x_i) = 1$ then $\theta(X_i) = 1$
   - θ is a satisfying assignment for Φ (why??)

This completes the proof: query containment is NP-hard.

Discussion

- We have a procedure to check if two queries are equivalent
- Will use to do query optimization (discussed next)
- NP-hard is not that scary today: SAT solvers can scale to tens of thousands of clauses

Query Minimization

Definition A conjunctive query q is minimal
if for every other conjunctive query q' ,
if $q \equiv q'$ then q' has at least as many atoms as q

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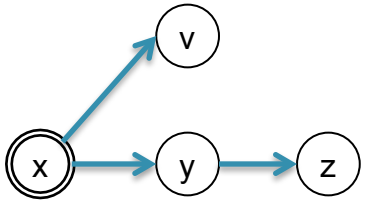
Are these queries minimal ?

q(x) :- R(x,y), R(y,z), R(x,v)



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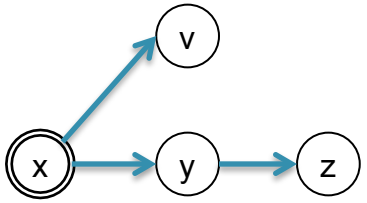
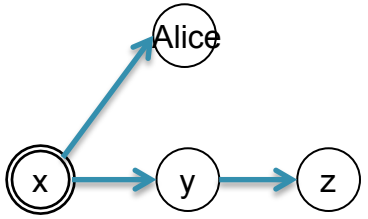


q(x) :- R(x,y), R(y,z), R(x,'Alice')



Query Minimization

- Query minimization algorithm

  Choose an atom of q, and remove it: let q' be the new query

  We already know q ⊆ q' (why ?)

  If q' ⊆ q then permanently remove it; otherwise try another atom

- Notice: the order in which we inspect atoms doesn't matter

Query Minimization In Practice

- No database system today performs minimization
- Reason:
  - It applies only to SELECT-DISTINCT queries (doesn't work for bag semantics)
  - Users rarely write non-minimal queries

- However, non-minimal queries arise when using views intensively

Query Minimization for Views

An Employee is a “happy boater” if she is a boater and her manager is also a boater:

```
CREATE VIEW HappyBoaters

   SELECT DISTINCT E1.name, E1.manager
   FROM Employee E1, Employee E2
   WHERE E1.manager = E2.name
         and E1.boater= ‘YES’
         and E2.boater= ‘YES’
```

This query is minimal

Query Minimization for Views

Now compute the Very-Happy-Boaters

```
SELECT DISTINCT  H1.name
FROM HappyBoaters H1, HappyBoaters H2
WHERE H1.manager = H2.name
```

Query Minimization for Views

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SELECT DISTINCT  H1.name
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After expanding HappyBoaters, the query is not minimal

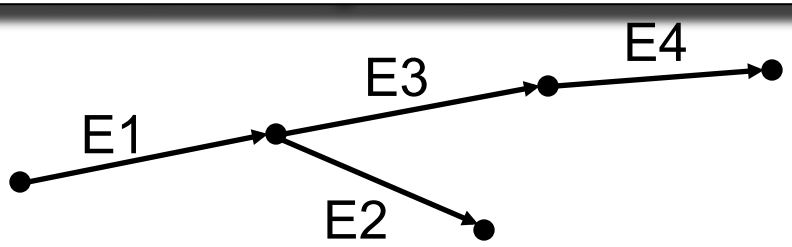
Query Minimization for Views

Now compute the Very-Happy-Boaters

```
SELECT DISTINCT  H1.name
FROM HappyBoaters H1, HappyBoaters H2
WHERE H1.manager = H2.name
```

After expanding HappyBoaters, the query is not minimal

```
SELECT DISTINCT  E1.name
FROM Employee E1, Employee E2, Employee E3, Empolyee E4
WHERE E1.manager = E2.name and E1.boater = 'YES' and E2.boater = 'YES'
    and E3.manager = E4.name and E3.boater = 'YES' and E4.boater = 'YES'
    and E1.manager = E3.name
```



E2 is redundant

Beyond CQ

- Any static analysis on relational queries (including containment, equivalence) is undecidable
- This follows from Trakthenbrot's theorem, discussed next
- Similar to Rice's undecidability theorem for Turning machines

Trakhtenbrot's Theorem

Definition A sentence φ, is called *finitely satisfiable* if there exists a finite database instance D s.t. D |= φ

Theorem [Trakhtenbrot] The following is undecidable: Given FO sentence φ, check if φ is finitely satisfiable

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Is this satisfiable?
$\exists x\, \exists y\, R(x,y) \wedge$
$\forall y\, (\exists x R(x,y) \rightarrow \exists z R(y,z)) \wedge$
$\forall x\, \forall y\, \forall z\, (R(x,y) \wedge R(x,z) \rightarrow y=z)$

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$\exists x\, (\exists y\, R(x,y) \wedge \neg (\exists u\, R(u,x)))$

Query Containment

Theorem Query containment for Relational Calculus is undecidable

Proof: By reduction from the finite satisfiability problem:

Given a sentence φ, define two queries:

$q_1 = \exists x(R(x) \wedge \varphi)$, and $q_2 = \exists x(R(x) \wedge x \neq x)$

Then $q_1 \subseteq q_2$ iff φ is not finitely satisfiable

Summary of DB Theory

- We have discussed query complexity and static analysis for conjunctive queries

- Why we care:
  - Complexity helps us understand the tradeoffs between what we can express in a language and how difficult it is to implement
  - Static analysis is critical in optimizing complex queries

- Next time: transactions