# Introduction to Artificial Intelligence

Lecture 2: Solving problems by searching

LIÈGE université

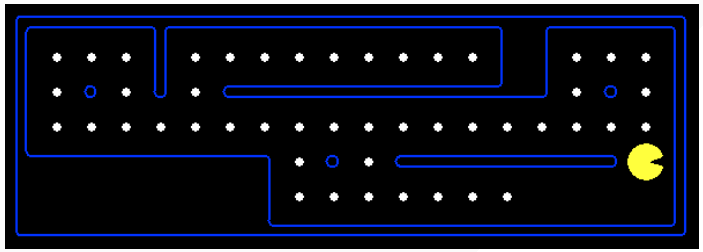
# Today

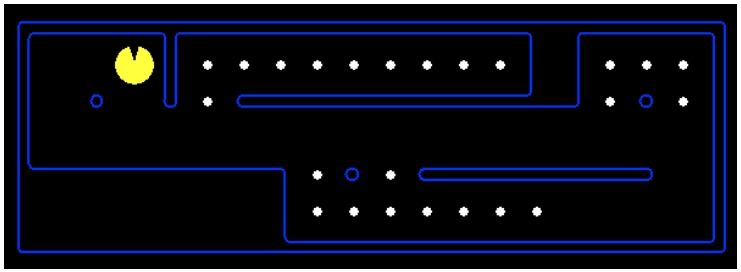
- Agents that plan ahead
- Search problems
- Uninformed search methods
  - Depth-first search
  - Breadth-first search
  - Uniform-cost search
- Informed search methods
  - A*
  - Heuristics

# Reflex agents

Reflex agents:

- Select actions on the basis of the current percept.
- May have a model of the world current state.
- Do not consider the future consequences of their actions.
- Consider only how the world is now.





[Q] Can a reflex agent be rational?

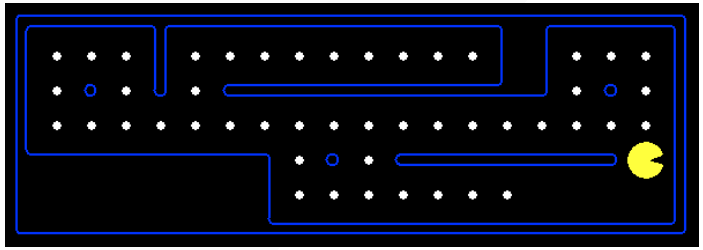
# Problem-solving agents

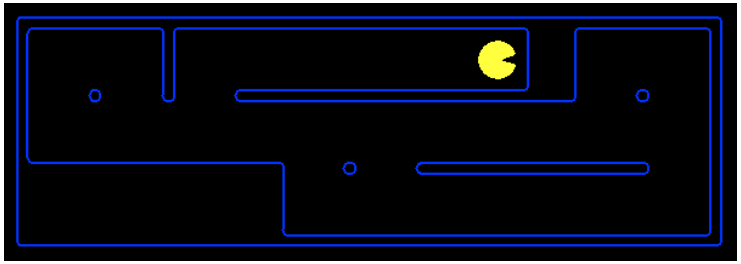
Assumptions:

- Observable, deterministic (and known) environment.

Problem-solving agents:

- Take decisions based on (hypothesized) consequences of actions.
- Must have a model of how the world evolves in response to actions.
- Formulate a goal.
- Consider how to world would be.





```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  persistent: seq, an action sequence, initially empty
              state, some description of the current world state
              goal, a goal, initially null
              problem, a problem formulation

  state ← UPDATE-STATE(state, percept)
  if seq is empty then
      goal ← FORMULATE-GOAL(state)
      problem ← FORMULATE-PROBLEM(state, goal)
      seq ← SEARCH(problem)
      if seq = failure then return a null action
  action ← FIRST(seq)
  seq ← REST(seq)
  return action
```

**Figure 3.1** A simple problem-solving agent. It first formulates a goal and a problem, searches for a sequence of actions that would solve the problem, and then executes the actions one at a time. When this is complete, it formulates another goal and starts over.

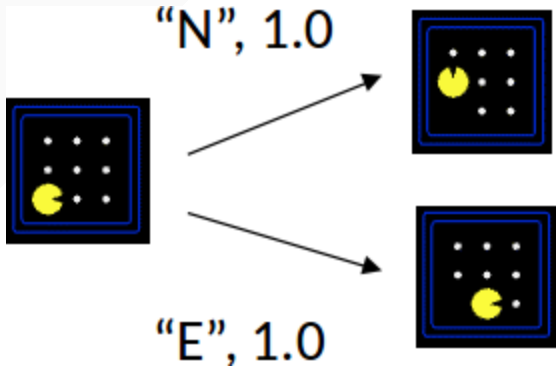
Offline vs. Online solving:

- This is offline problem solving. The solution is executed "eyes closed", ignoring the percepts.
- Online problem solving involves acting without complete knowledge.

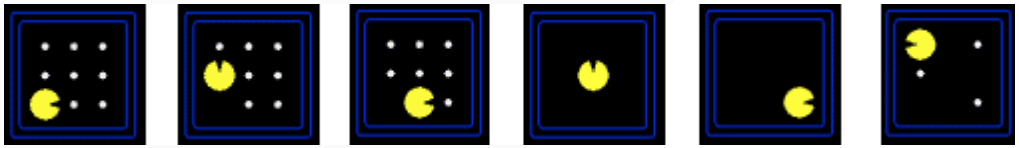
# Search problems

A search problem consists of the following components:

- The initial state of the agent.
- A description of the actions available to the agent given a state , denoted .
- A transition model that returns the state that results from doing action in state .
  - We say that is a successor of if there is an acceptable action from to .



- Together, the initial state, the actions and the transition model define the state space of the problem, i.e. the set of all states reachable from the initial state by any sequence of action.
  - The state space forms a directed graph:
    - nodes = states
    - links = actions
  - A path is a sequence of states connected by actions.



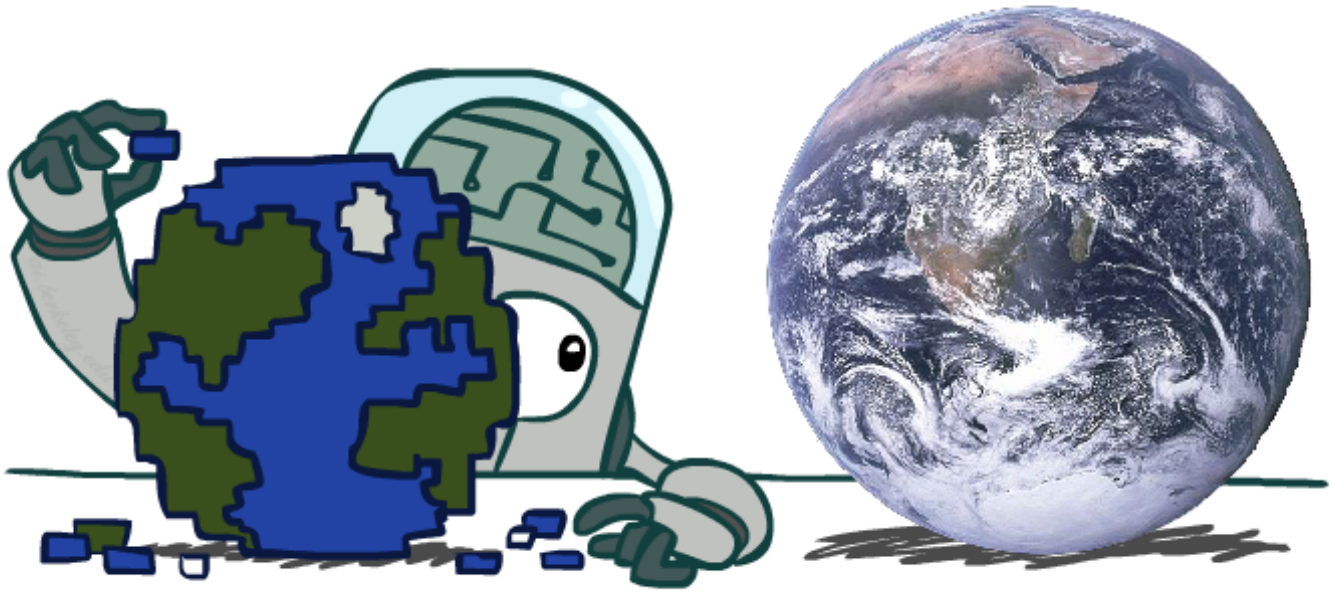
- A goal test which determines whether the solution of the problem is achieved in state .
- A path cost that assigns a numeric value to each path.
  - We may also assume that the path cost corresponds to a sum of positive step costs associated to the action in leading to .

A solution to a problem is an action sequence that leads from the initial state to a goal state.

- A solution quality is measured by the path cost function.
- An optimal solution has the lowest path cost among all solutions.

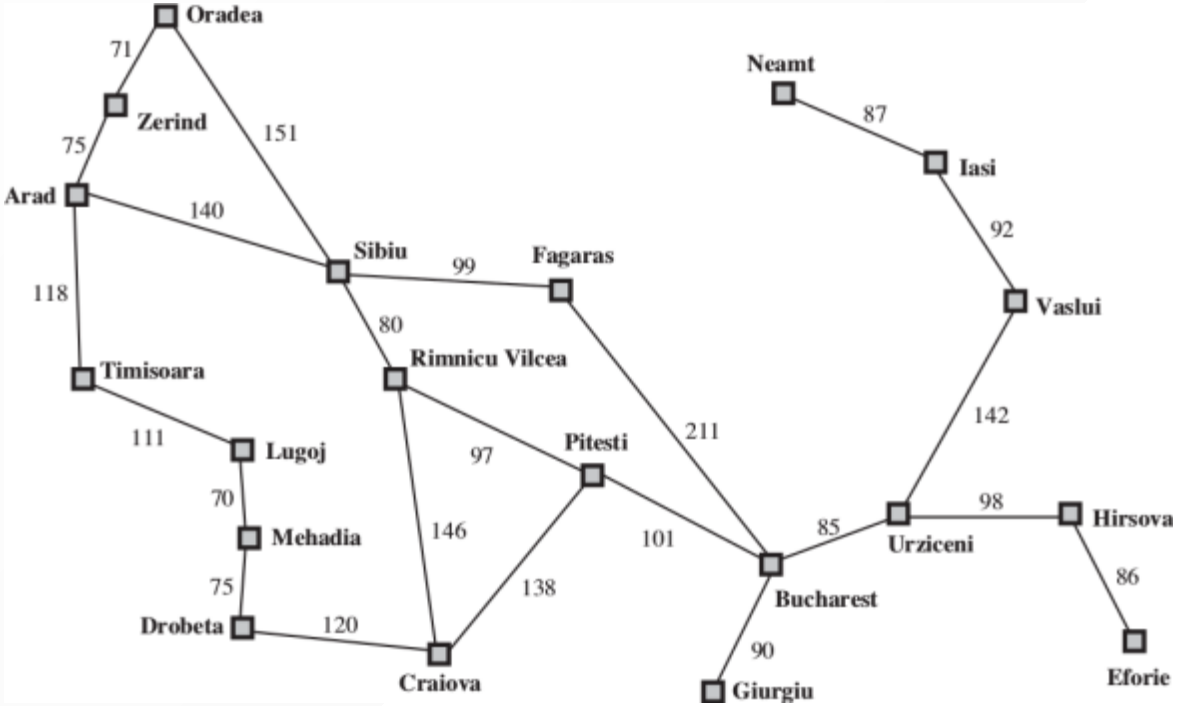
[Q] What if the environment is partially observable? non-deterministic?

# Search problems are models



Credits: UC Berkeley, CS188

# Example: Traveling in Romania

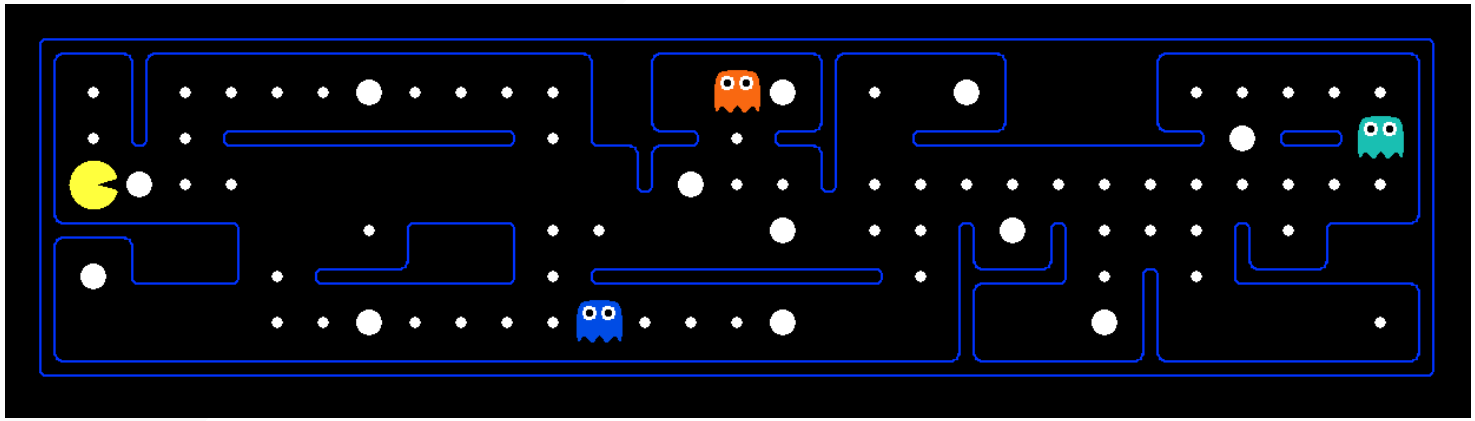



- Initial state = the city we start in.
  - $s_0 = \text{in}(\text{Arad})$
- Actions = Going from the current city to the cities that are directly connected to it.
  - $\text{actions}(s_0) = \{\text{go}(\text{Sibiu}), \text{go}(\text{Timisoara}), \text{go}(\text{Zerind})\}$

- Transition model = The city we arrive in after driving to it.
  - $\text{result}(\text{in}(Arad), \text{go}(Zerind)) = \text{in}(Zerind)$
- Goal test: whether we are in Bucharest.
  - $s \in \{\text{in}(\text{Bucharest})\}$
- Step cost: distances between cities.

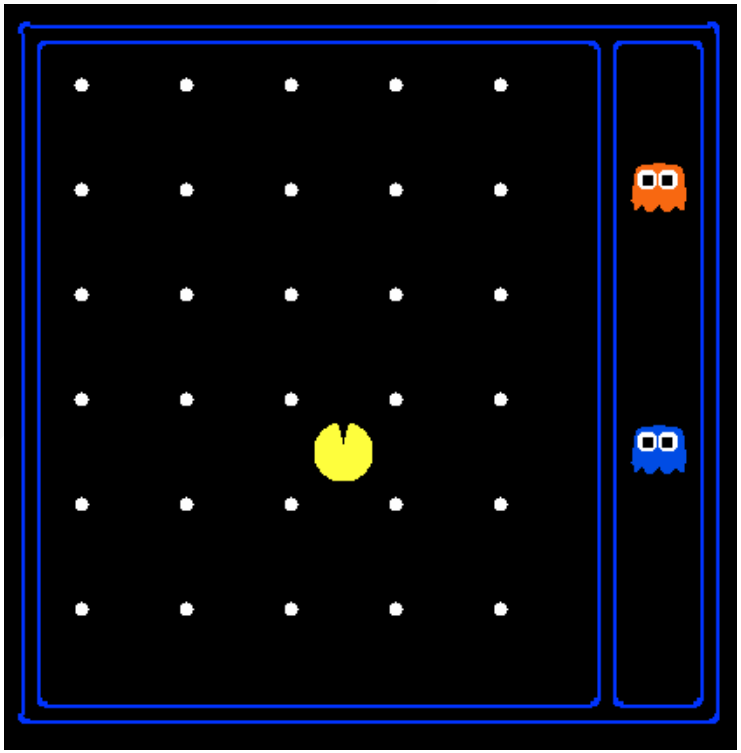
# Selecting a state space

- Real world is absurdly complex.
  - The world state includes every last detail of the environment.
  - State space must be abstracted for problem solving.
- A search state keeps only the details needed for planning.
  - Example: eat-all-dots
    - States: $\{(x, y), \text{dot booleans}\}$
    - Actions: NSEW
    - Transition: update location and possibly a dot boolean
    - Goal test: dots all false



# State space size

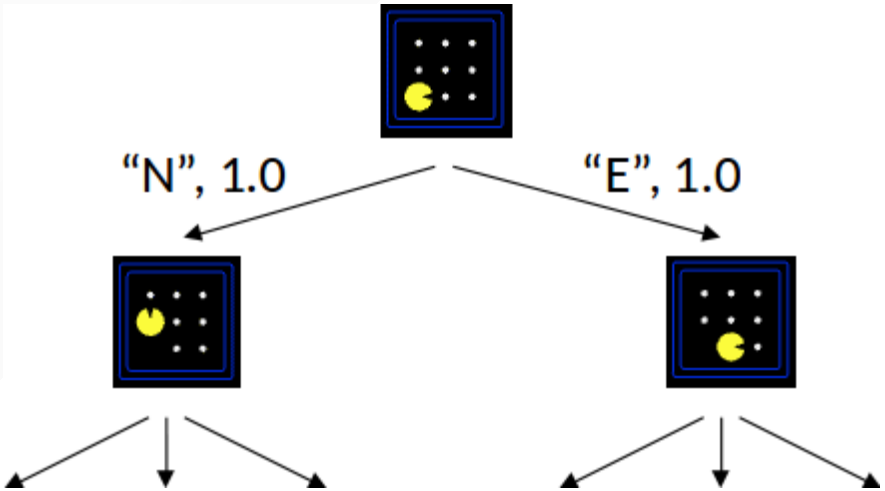
- World state:
  - Agent positions: 120
  - Found count: 30
  - Ghost positions: 12
  - Agent facing: NSEW
- How many?
  - World states?
    - $120 \times 2^{30} \times 12^2 \times 4$
  - States for eat-all-dots?
    - $120 \times 2^{30}$



# Search trees

The set of possible acceptable sequences starting at the initial state form a search tree:

- Nodes correspond to states in the state space, where the initial state is the root node.
- Branches correspond to applicable actions.
  - Child nodes correspond to successors.
- For most problems, we can never actually build the whole tree.



# Tree search algorithms

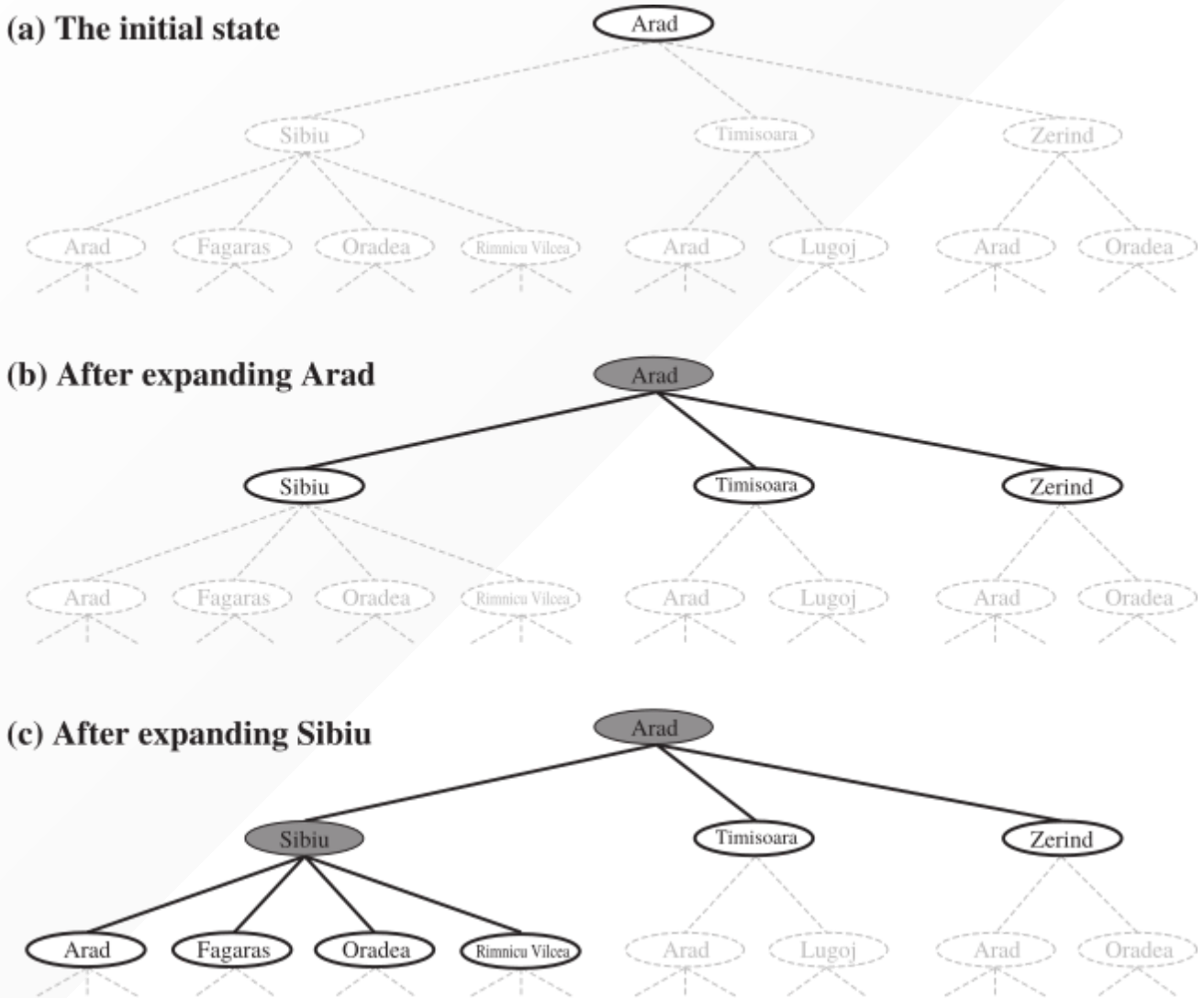
```
function TREE-SEARCH( problem) returns a solution, or failure
    initialize the frontier using the initial state of problem
    loop do
        if the frontier is empty then return failure
        choose a leaf node and remove it from the frontier
        if the node contains a goal state then return the corresponding solution
        expand the chosen node, adding the resulting nodes to the frontier
```

Important ideas:

- Fringe (or frontier) of partial plans under consideration
- Expansion
- Exploration

[Q] Which fringe nodes to explore? How to expand as few nodes as possible, while achieving the goal?

# Tree search example



# Uninformed search strategies

Uninformed search strategies use only the information available in the problem definition.

- They do not know whether a state looks more promising than some other.

Strategies:

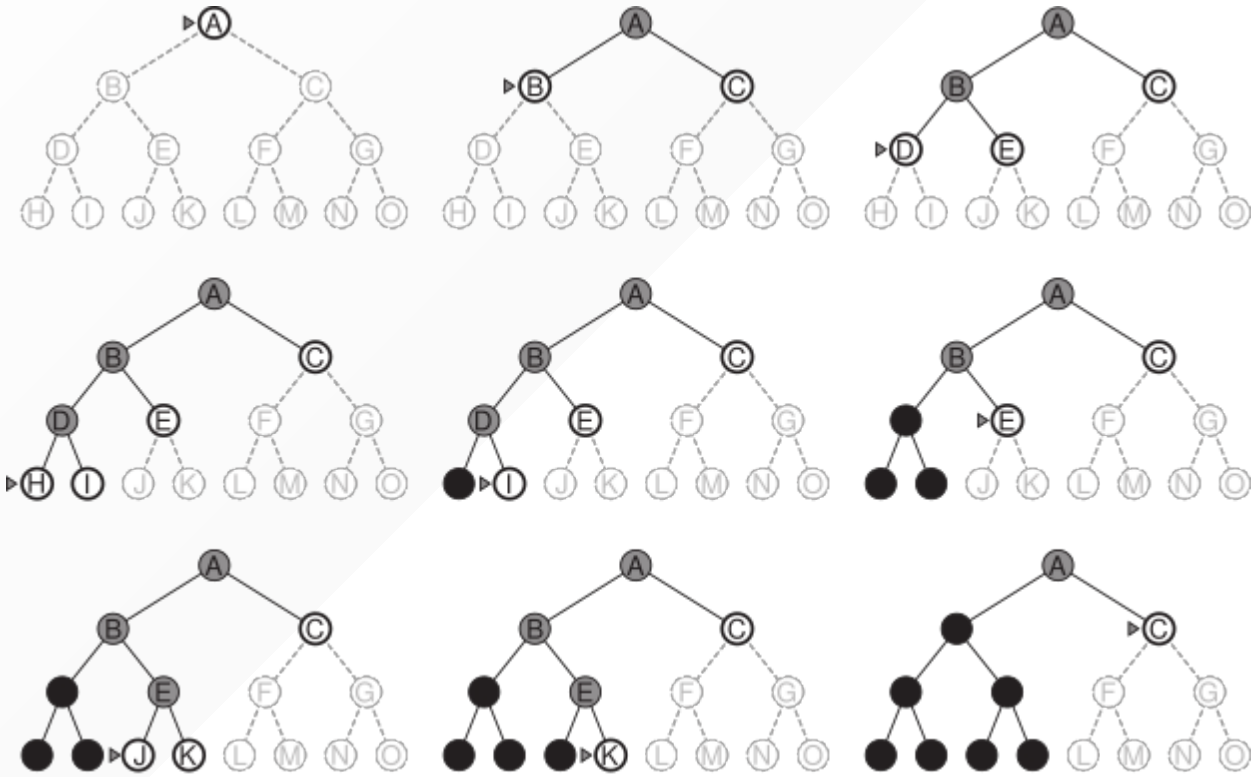
- Depth-first search
- Breadth-first search
- Uniform-cost search
- Iterative deepening

# Depth-first search

Credits: UC Berkeley, CS188

# Depth-first search

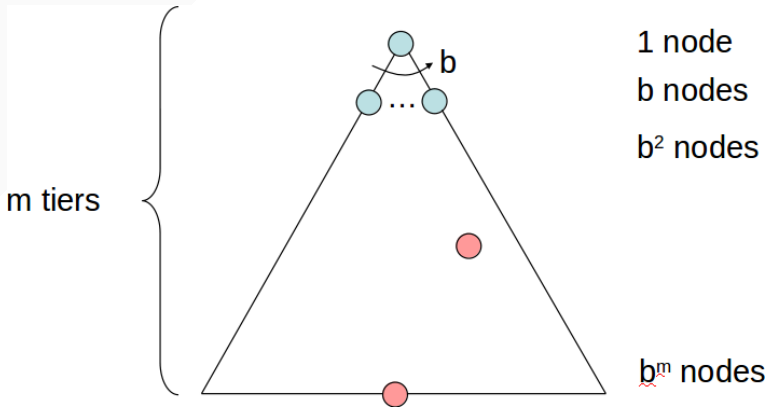
- Strategy: expand the deepest node in the fringe.
- Implementation: fringe is a LIFO stack.



# Properties of search strategies

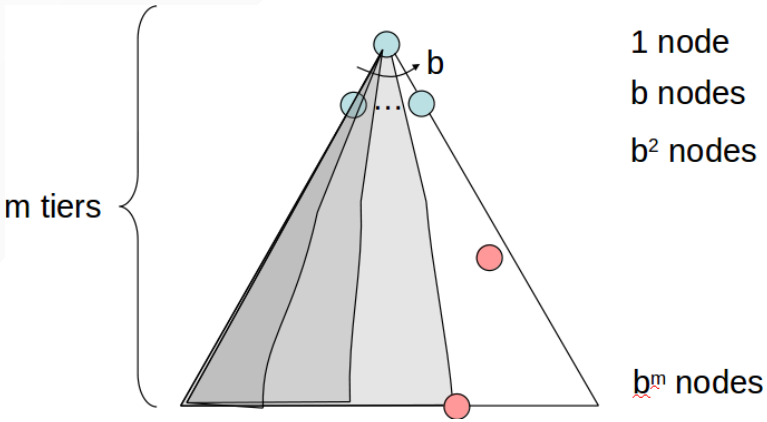
- A strategy is defined by picking the order of expansion.
- Strategies are evaluated along the following dimensions:
  - Completeness: does it always find a solution if one exists?
  - Optimality: does it always find the least-cost solution?
  - Time complexity: how long does it take to find a solution?
  - Space complexity: how much memory is needed to perform the search?
- Time and complexity are measured in terms of
  - $b$: maximum branching factor of the search tree
  - $d$: depth of the least-cost solution
  - $m$: maximum length of any path in the state space (may be $\infty$)

[Q] Number of nodes in a tree?

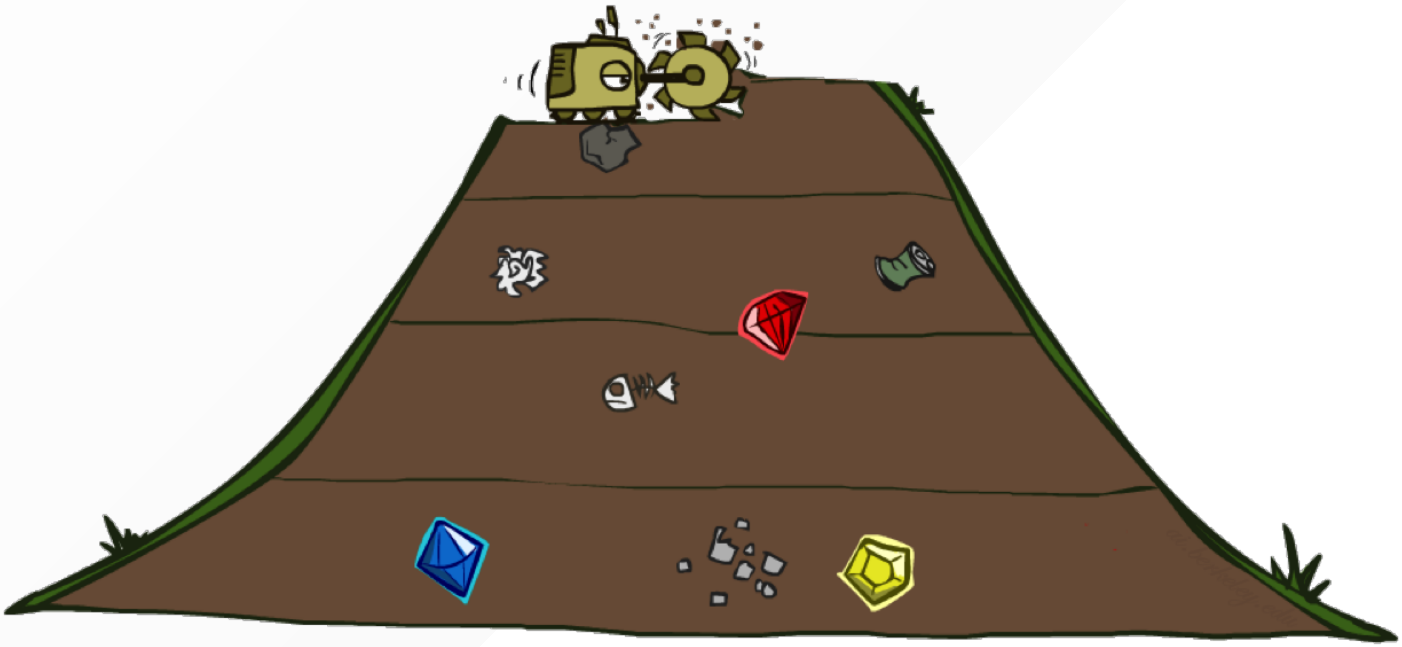


# Properties of DFS

- Completeness:
  - $m$ could be infinite, so only if we prevent cycles (more on this later).
- Optimality:
  - No, DFS finds the leftmost solution, regardless of depth or cost.
- Time complexity:
  - May generate the whole tree (or a good part of it, regardless of $d$). Therefore $O(b^m)$, which might much greater than the size of the state space!
- Space complexity:
  - Only store siblings on path to root, therefore $O(bm)$.
  - When all the descendants of a node have been visited, the node can be removed from memory.



# Breadth-first search



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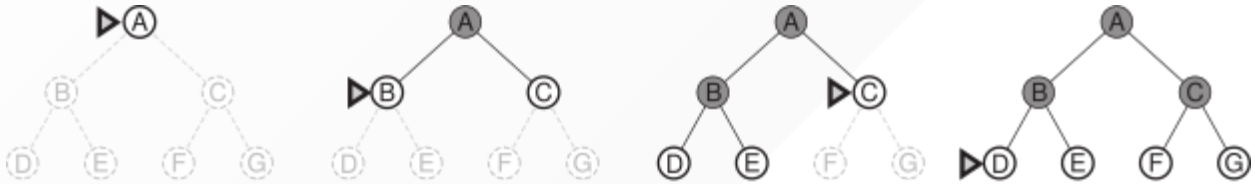
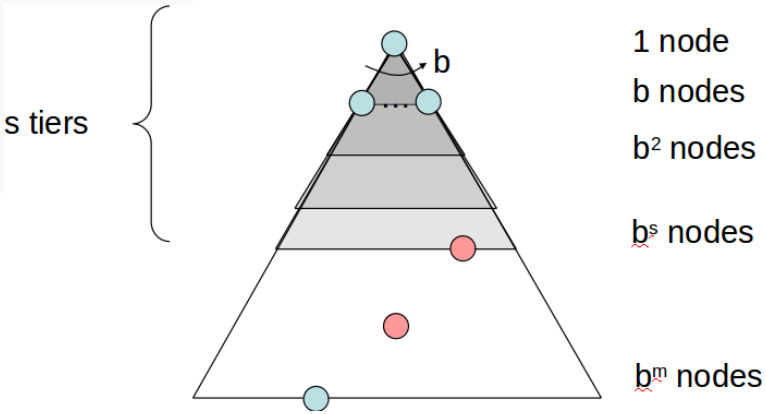
# Breadth-first search

- Strategy: expand the shallowest node in the fringe.
- Implementation: fringe is a FIFO queue.



# Properties of BFS

- Completeness:
  - If the shallowest goal node is at some finite depth $d$, BFS will eventually find it after generating all shallower nodes (provided $b$ is finite).
- Optimality:
  - The shallowest goal is not necessarily the optimal one.
  - BFS is optimal only if the path cost is a non-decreasing function of the depth of the node.
- Time complexity:
  - If the solution is a depth $d$, then the total number of nodes generated before finding this node is $b + b^2 + b^3 + ... + b^d = O(b^d)$
- Space complexity:
  - The number of nodes to maintain in memory is the size of the fringe, which will be the largest at the last tier. That is $O(b^d)$

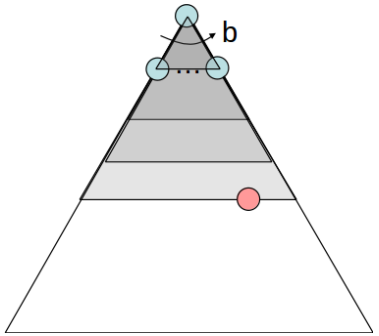


# Iterative deepening

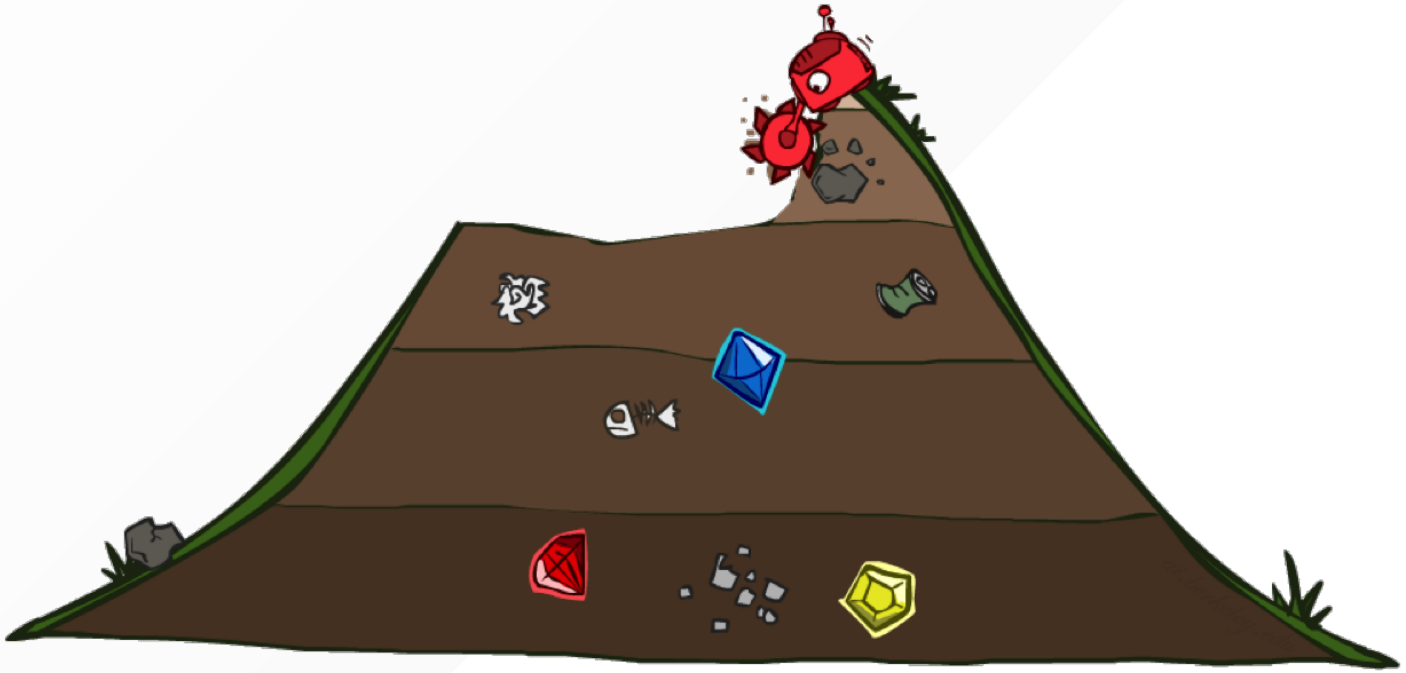
- Idea: get DFS's space advantages with BFS's time/shallow solution advantages.
  - Run DFS with depth limit 1.
  - If no solution, run DFS with depth limit 2.
  - If no solution, run DFS with depth limit 3.
  - ...

[Q] What are the properties of iterative deepening?

[Q] Isn't this process wastefully redundant?



# Uniform-cost search



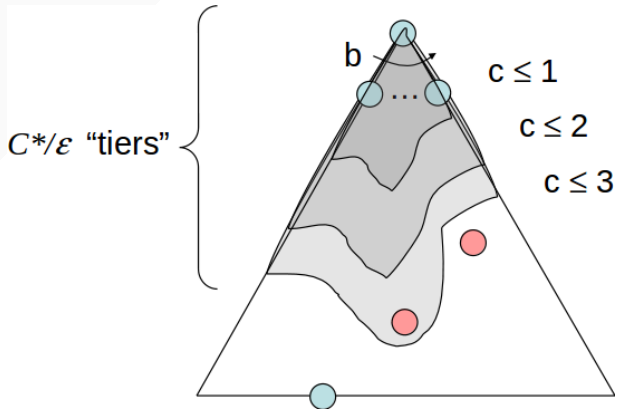
Credits: UC Berkeley, CS188

# Uniform-cost search

- Strategy: expand the cheapest node in the fringe.
- Implementation: fringe is a priority queue, using the cumulative cost $g(n)$ from the initial state to node $n$ as priority.

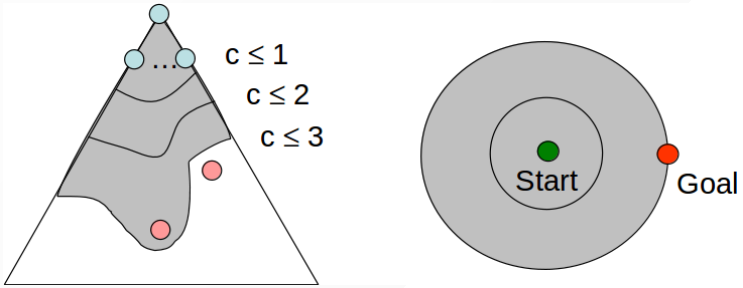
# Properties of UCS

- Completeness:
  - Yes, if step cost $\geq \epsilon > 0$.
- Optimality:
  - Yes, sinces UCS expands nodes in order of their optimal path cost.
- Time complexity:
  - Assume $C^*$ is the cost of the optimal solution and that step costs are all $\geq \epsilon$.
  - The "effective depth" is then roughly $C^*/\epsilon$.
  - The worst-case time complexity is $O(b^{C^*/\epsilon})$.
- Space complexity:
  - The number of nodes to maintain is the size of the fringe, so as many as in the last tier $O(b^{C^*/\epsilon})$.



# Informed search strategies

One of the issues of UCS is that it explores the state space in every direction, without exploiting information about the (plausible) location of the goal node.



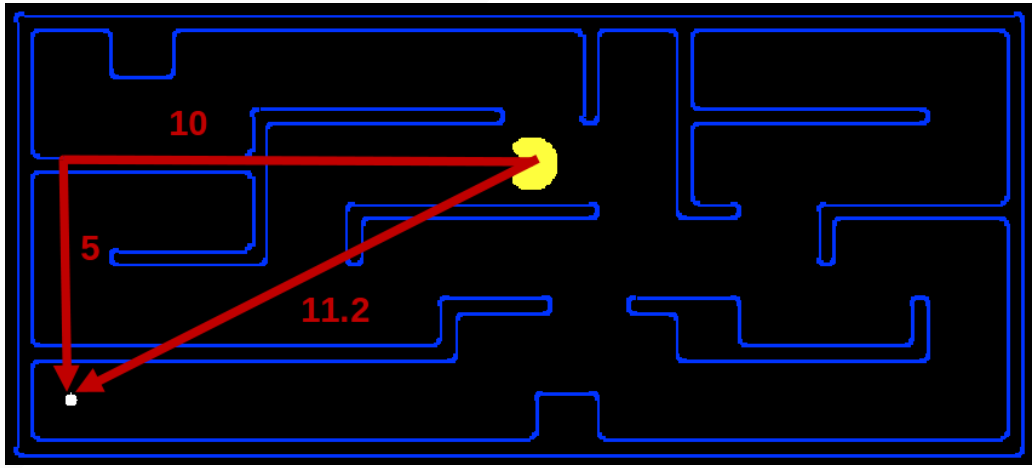

Informed search strategies aim to solve this problem by expanding nodes in the fringe in decreasing order of desirability.

- Greedy search
- A*

# Heuristics

A heuristic (or evaluation) function $h(n)$ is:

- a function that estimates the cost of the cheapest path from node $n$ to a goal state;
  - $h(n) \geq 0$ for all nodes $n$
  - $h(n) = 0$ for a goal state.
- is designed for a particular search problem.



# Greedy search

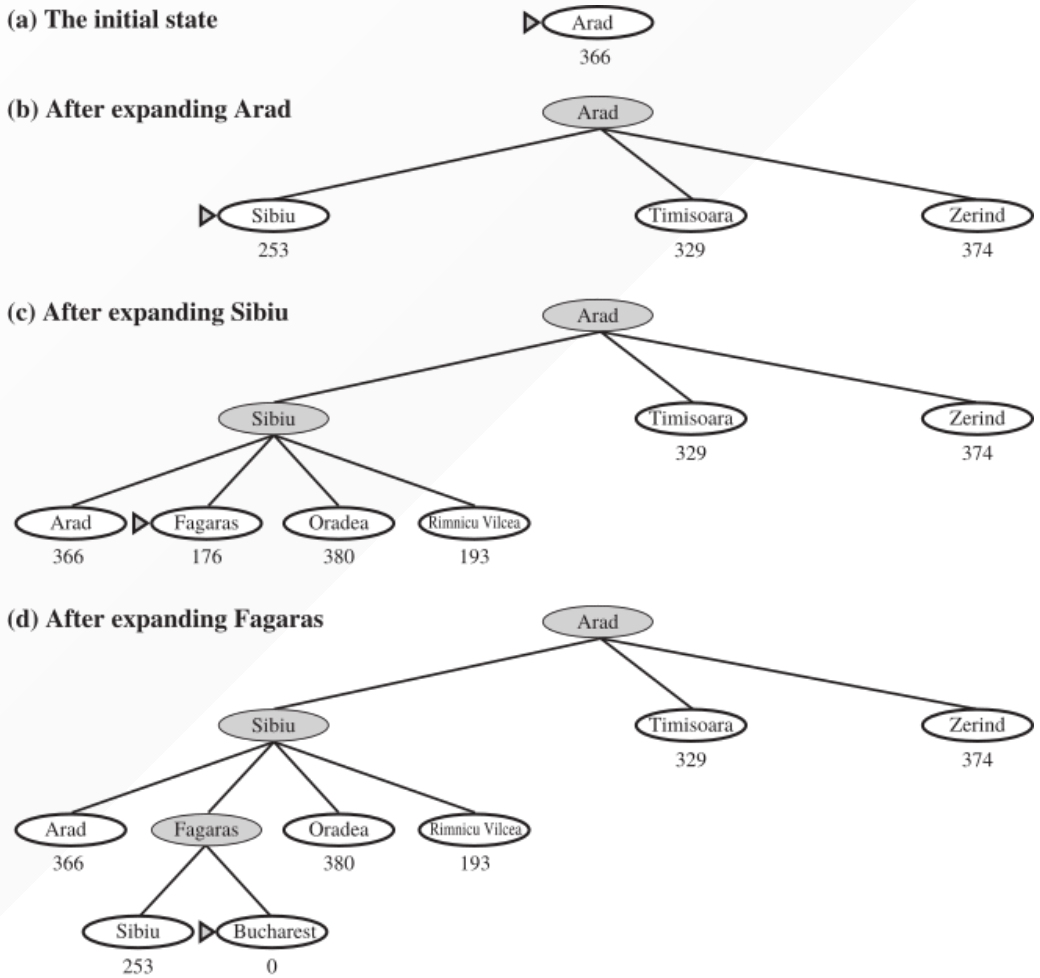
Credits: UC Berkeley, CS188

# Greedy search

- Strategy: expand the node $n$ in the fringe for which $h(n)$ is the lowest.
- Implementation: fringe is a priority queue, using $h(n)$ as priority.

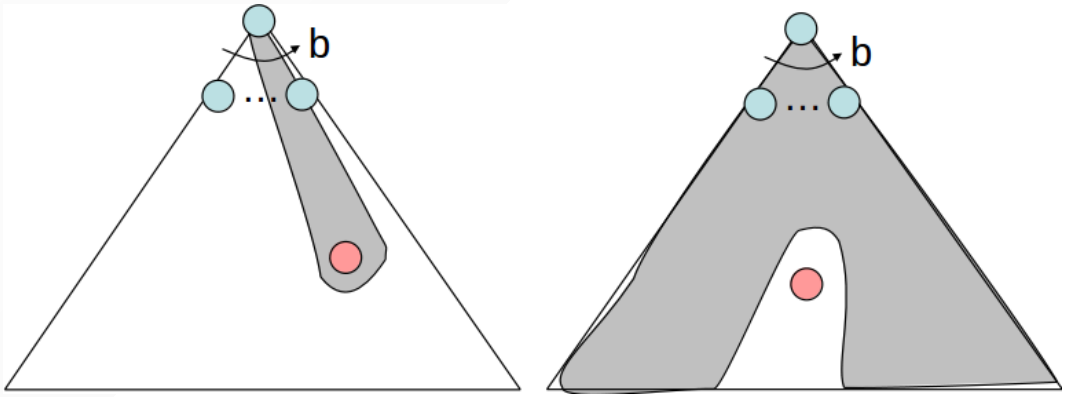
# Greedy search example

$h(n)$ = straight line distance to Bucharest.



# Properties of greedy search

- Completeness:
  - No, unless we prevent cycles (more on this later).
- Optimality:
  - No, e.g. the path via Sibiu and Fagaras is 32km longer than the path through Rimnicu Vilcea and Pitesti.
- Time complexity:
  - $O(b^m)$, unless we have a good heuristic function.
- Space complexity:
  - $O(b^m)$, unless we have a good heuristic function.



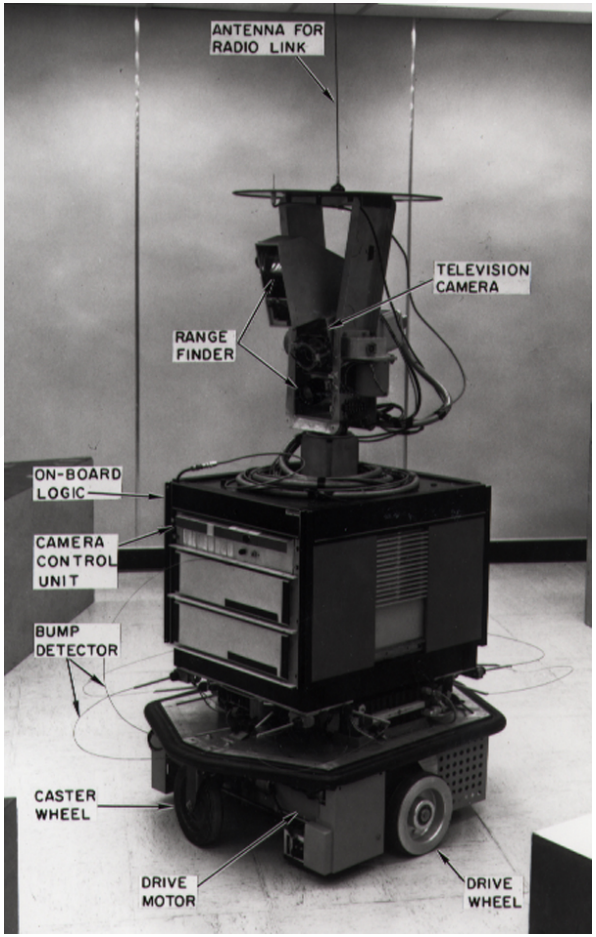

*At best, greedy search takes you straight to the goal. At worst, it is like a badly-guided BFS.*

# A*

Credits: UC Berkeley, CS188

# Shakey the Robot

- A* was first proposed in 1968 to improve robot planning.
- Goal was to navigate through a room with obstacles.



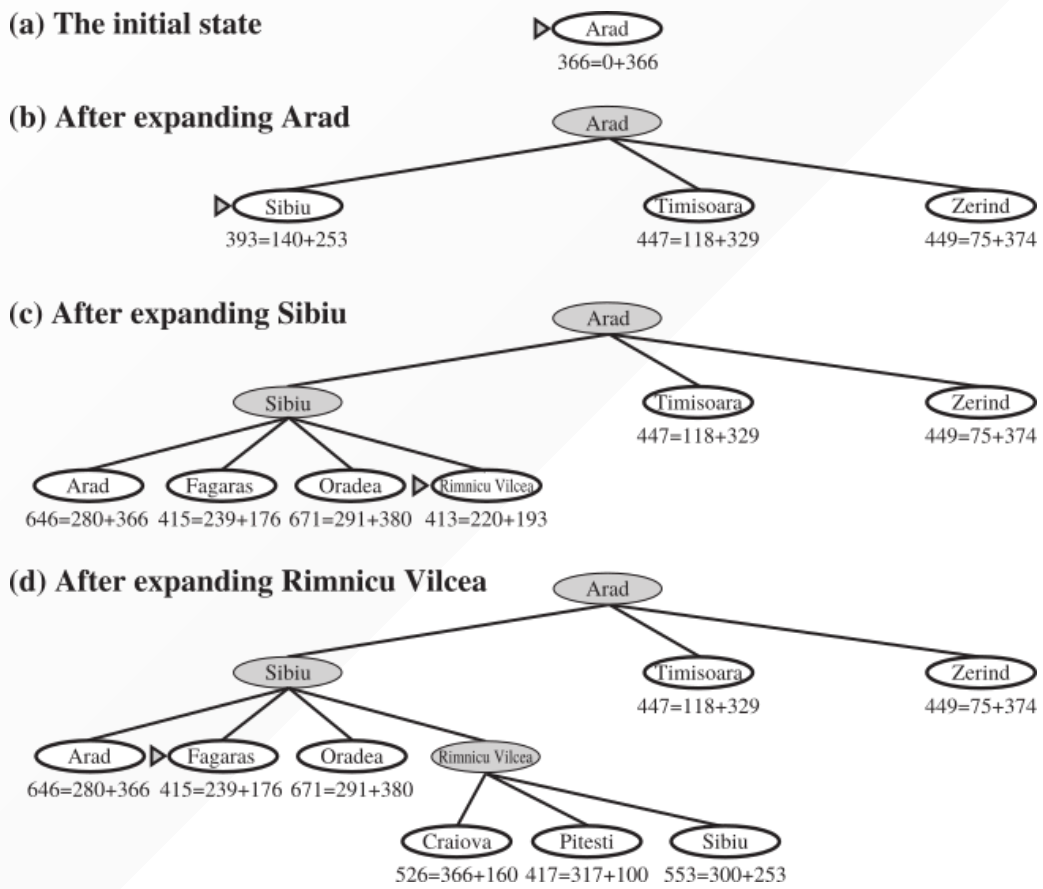
# A*

- Uniform-cost orders by path cost, or backward cost $g(n)$
- Greedy orders by goal proximity, or forward cost $h(n)$
- A* combines the two algorithms and orders by the sum

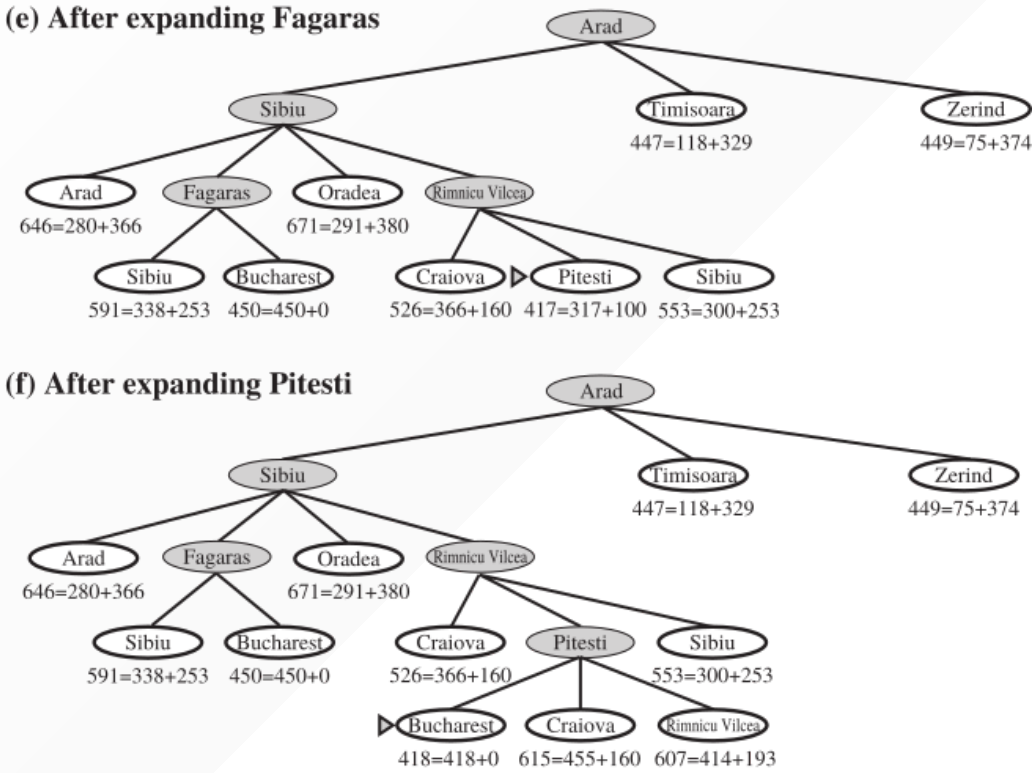
$$f(n) = g(n) + h(n)$$

- $f(n)$ is the estimated cost of cheapest solution through $n$.

# A* example


(a) The initial state
Arad
366=0+366
(b) After expanding Arad
Arad
Sibiu
Timisoara
Zerind
393=140+253
447=118+329
449=75+374
(c) After expanding Sibiu
Arad
Sibiu
Timisoara
Zerind
447=118+329
449=75+374
Arad
Fagaras
Oradea
Rimnicu Vilcea
646=280+366
415=239+176
671=291+380
413=220+193
(d) After expanding Rimnicu Vilcea
Arad
Sibiu
Timisoara
Zerind
447=118+329
449=75+374
Arad
Fagaras
Oradea
Rimnicu Vilcea
646=280+366
415=239+176
671=291+380
Craiova
Pitesti
Sibiu
526=366+160
417=317+100
553=300+253

# A* example



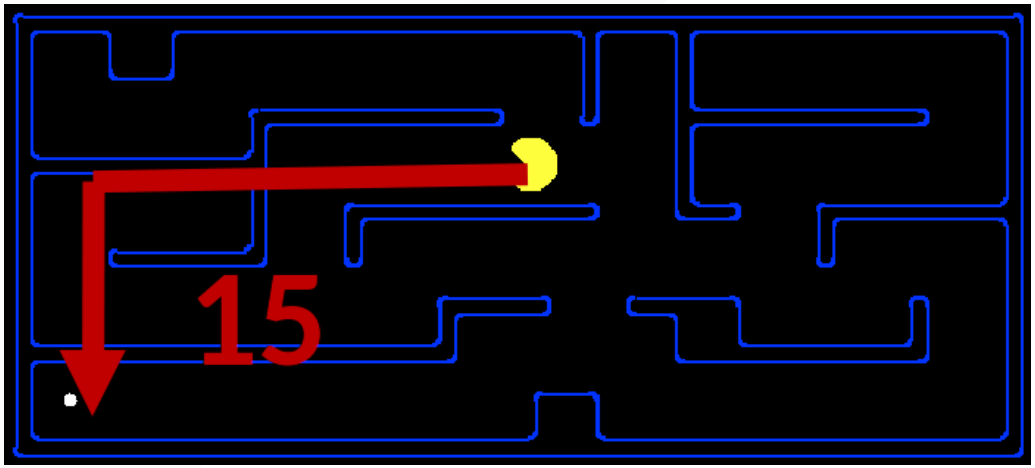

[Q] Why doesn't A* stop at step (e), since Bucharest is in the fringe?

# Admissible heuristics

A heuristic $h$ is admissible if

$$0 \leq h(n) \leq h^*(n)$$

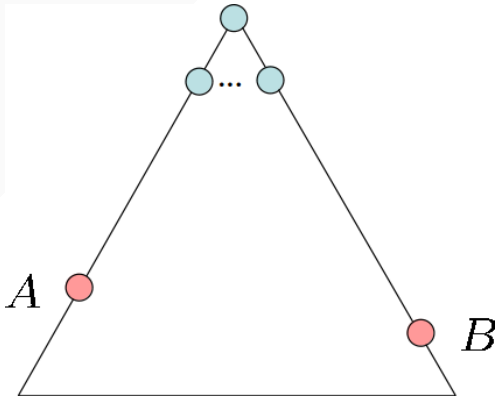
where $h^*(n)$ is the true cost to a nearest goal.




*The Manhattan distance is admissible*

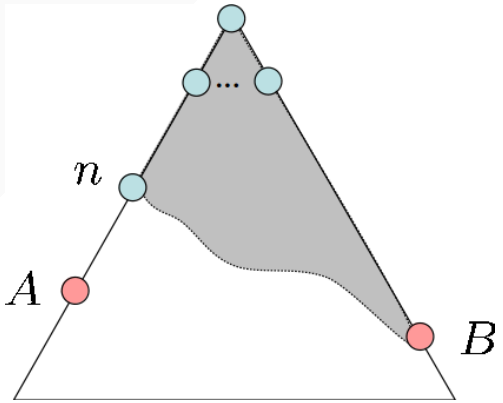
# Optimality of A* (tree search)

- Assumptions:
  - $A$ is an optimal goal node
  - $B$ is a suboptimal goal node
  - $h$ is admissible
- Claim:
  - $A$ will exit the fringe before $B$



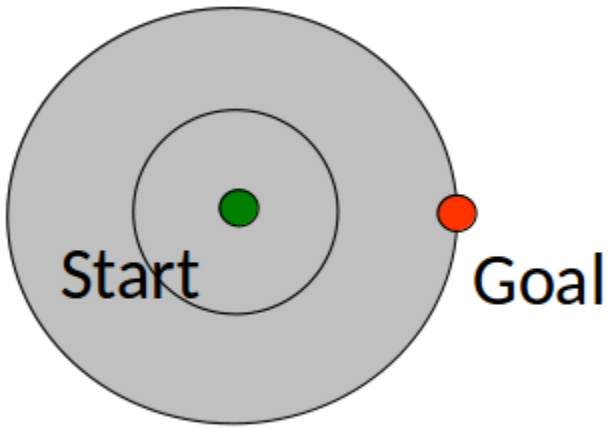
# Optimality of A* (tree search)

- Assume $B$ is on the fringe.
- Some ancestor $n$ of $A$ is on the fringe too.
- Claim: $n$ will be expanded before $B$.
  - $f(n) \leq f(A)$
    - $f(n) = g(n) + h(n)$ (by definition)
    - $f(n) \leq g(A)$ (admissibility of $h$)
    - $f(A) = g(A) + h(A) = g(A)$ ($h = 0$ at a goal)
  - $f(A) < f(B)$
    - $g(A) < g(B)$ ($B$ is suboptimal)
    - $f(A) < f(B)$ ($h = 0$ at a goal)
  - $n$ expands before $B$
    - since $f(n) \leq f(A) < f(B)$
- All ancestors of $A$ expand before $B$, including $A$. Thefore A* is optimal.

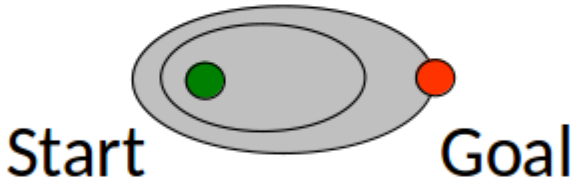


# A* contours

- $f$-costs are non-decreasing along any path.
- We can define contour levels $t$ in the state space, that include all nodes $n$ for which $f(n) \leq t$.

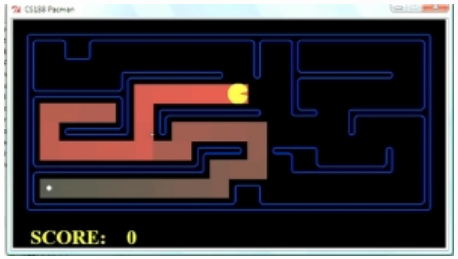



For UCS ($h(n) = 0$ for all $n$), bands are circular around the start.



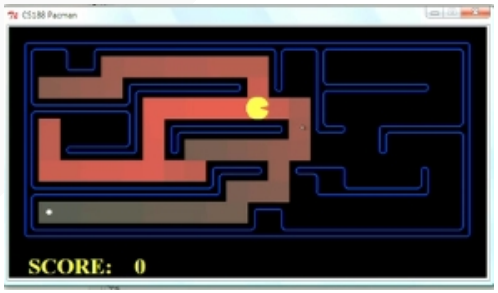

For A* with accurate heuristics, bands stretch towards the goal.

# Comparison




Greedy search


UCS

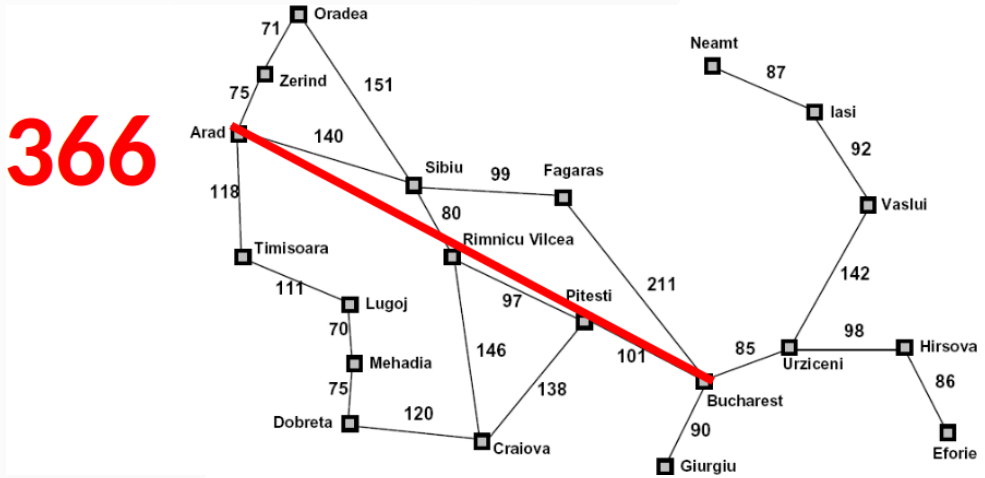



A*

Credits: UC Berkeley, CS188

# Creating admissible heuristics

- Most of the work in solving hard search problems optimally is in finding admissible heuristics.
- Admissible heuristics can be derived from the exact solutions to relaxed problems, where new actions are available.



# Dominance

- If $h_1$ and $h_2$ are both admissible and if $h_2(n) \geq h_1(n)$ for all $n$, then $h_2$ dominates $h_1$ and is better for search.
- Given any admissible heuristics $h_a$ and $h_b$,

$$h(n) = \max(h_a(n), h_b(n))$$

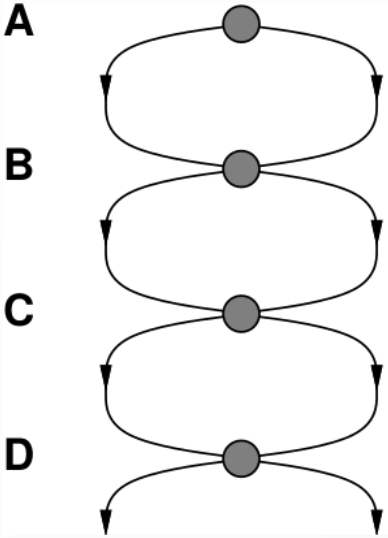
is also admissible and dominates $h_a$ and $h_b$.

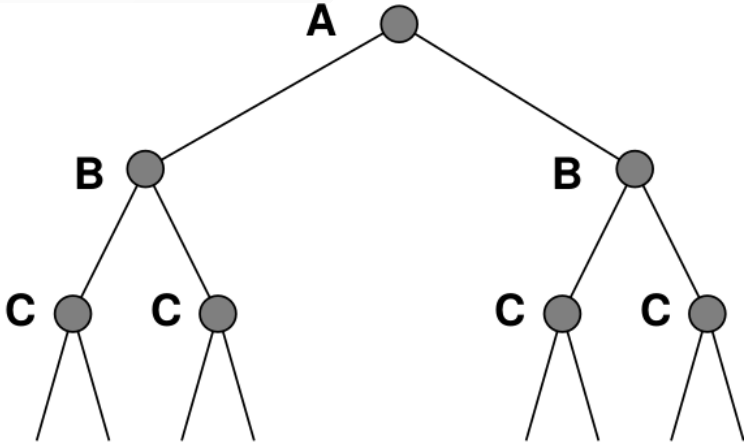
# Learning heuristics from experience

- Assuming an episodic environment, an agent can learn good heuristics by playing the game many times.
- Each optimal solution $s^*$ provides training examples from which $h(n)$ can be learned.
- Each example consists of a state $n$ from the solution path and the actual cost $g(s^*)$ of the solution from that point.
- The mapping $n \rightarrow g(s^*)$ can be learned with supervised learning algorithms.
    - Linear models, Neural networks, etc.

# Redundant paths

The failure to detect repeated states can turn a linear problem into an exponential one!



# Graph search

Redundant paths and cycles can be avoided by keeping track of the states that have been explored. This amounts to grow a tree directly on the state-space graph.

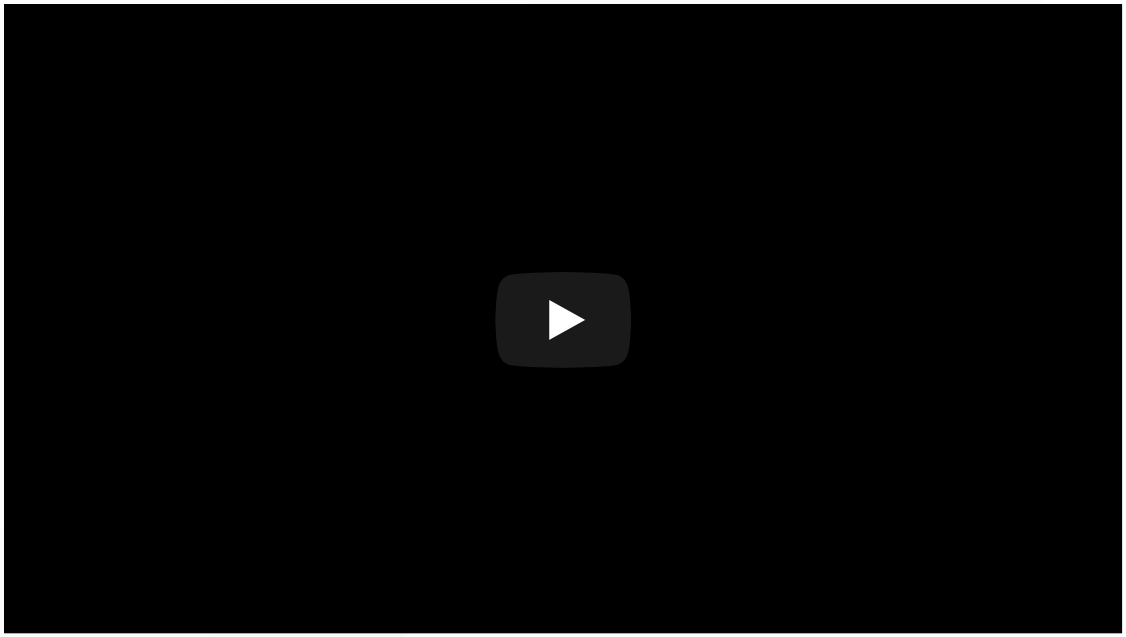
```
function GRAPH-SEARCH(problem) returns a solution, or failure
    initialize the frontier using the initial state of problem
    initialize the explored set to be empty
    loop do
        if the frontier is empty then return failure
        choose a leaf node and remove it from the frontier
        if the node contains a goal state then return the corresponding solution
        add the node to the explored set
        expand the chosen node, adding the resulting nodes to the frontier
            only if not in the frontier or explored set
```

[Q] What are the properties of DFS/GFS/UCS/GS/A* based on graph search?

# Recap example: Super Mario


- Task environment?
  - performance measure, environment, actuators, sensors?
- Type of environment?
- Search problem?
  - initial state, actions, transition model, goal test, path cost?
- Good heuristic?

# Super Mario: A* in action



# Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored.
- Variety of uninformed search strategies (DFS, BFS, UCS, Iterative deepening)
- Heuristic functions estimate costs of shortest paths.
- Good heuristic can dramatically reduce search cost.
- Greedy best-first search expands lowest $h$.
  - incomplete and not always optimal.
- A* search expands lower $f = g + h$
  - complete and optimal
- Admissible heuristics can be derived from exact solutions of relaxed problems or learned from training examples.
- Graph search can be exponentially more efficient than tree search.

# References


- Hart, Peter E., Nils J. Nilsson, and Bertram Raphael. "A formal basis for the heuristic determination of minimum cost paths." IEEE transactions on Systems Science and Cybernetics 4.2 (1968): 100-107.