Introduction to Artificial Intelligence (INFO8006)

Exercises 3 – Reasoning under uncertainty

September 16, 2022

Learning outcomes

At the end of this session you should be able to

- calculate marginal and conditional probabilities given joint distributions;
- apply and understand the Bayes rule;
- formulate a random process as a Bayesian network and conditional probability tables;
- determine whether random variables are independent given a Bayesian network;
- compute probabilities in the context of a (simple) Bayesian network.

Exercise 1 Beliefs

Let A and B be two *events* over the probability space Ω. An agent holds the beliefs $P(A) = 0.4$ and $P(B) = 0.3$. What ranges of probabilities would it be rational for the agent to hold for the events $A \cup B$ and $A \cap B$? What if the agent also believes that $P(A|B) = 0.5$?

Exercise 2 (AIMA, Ex 13.8)

	toothache		no toothache	
	catch	no catch	catch	no catch
cavity	0.108	0.012	0.072	0.008
no cavity	0.016	0.064	0.144	0.576

Given the hereabove probability table, compute the following probabilities:

1. $P(toothache)$
2. $P(cavity)$
3. $P(toothache|cavity)$
4. $P(cavity|toothache \cup catch)$

Exercise 3 (AIMA, Ex 13.15)

After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99 % accurate, *i.e.* the probability of testing positive when you do have the disease is 0.99 and the probability of testing positive when you don't is 0.01. The good news is that this is a rare disease, striking only 1 in 10 000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

Exercise 4 (AIMA, Ex 13.13)

For each of the following statements, either prove it is valid or give a counterexample.

1. If $P(a|b, c) = P(b|a, c)$, then $P(a|c) = P(b|c)$.
2. If $P(a|b, c) = P(a)$, then $P(b|c) = P(b)$.
3. If $P(a|b) = P(a)$, then $P(a|b, c) = P(a|c)$.

Exercise 5 Bag of coins

We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20 %, 60 %, and 80 %, respectively. One coin is drawn randomly from the bag (with equal probability of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2 and X_3 (heads or tail).

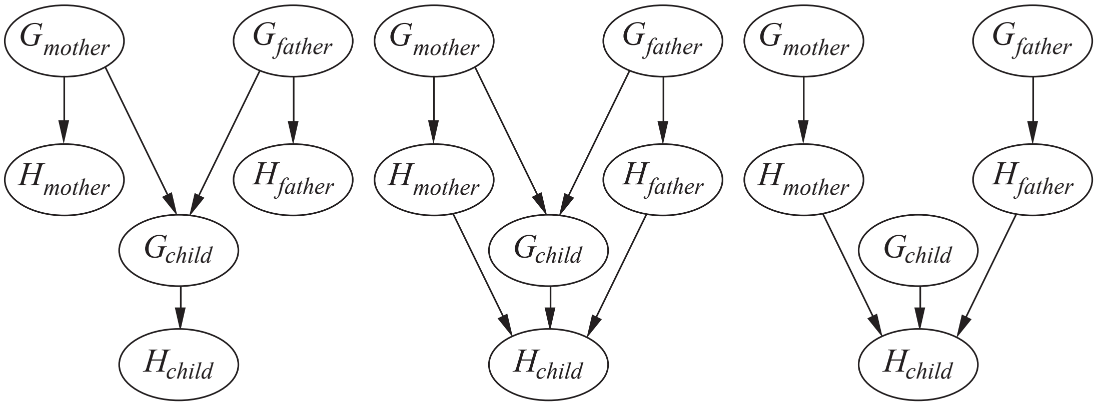
1. Draw the Bayesian network corresponding to this setup and define the corresponding conditional probability tables (CPTs).
2. Determine which coin (a, b or c) is most likely to have been drawn from the bag if the observed tosses came out heads twice and tail once.

Exercise 6 Handedness (AIMA, Ex: 14.6)

Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common *hypothesis* is that left or right-handedness is inherited by a simple mechanism; that is, there is a gene G_x, also with values l or r, and H_x turns out the same as G_x with some probability s.

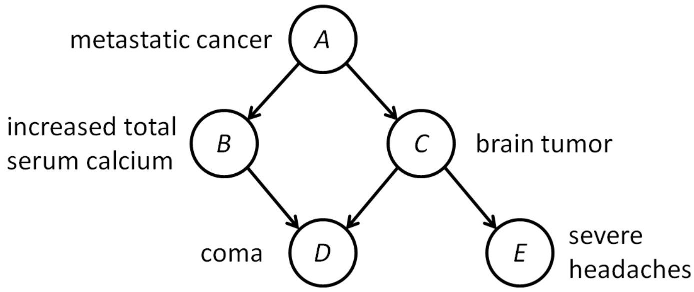
Furthermore, the gene itself is equally likely to be inherited from either of the individual's parents, with a small nonzero probability m of a random mutation flipping the gene.

1. Which of the following networks claim that $P(G_{child}|G_{mother}, G_{father}) = P(G_{child})$?



2. Which of the networks make independence claims that are consistent with the hypothesis about the inheritance of handedness ?
3. Write down the CPT for the G_{child} nodes in first network, in terms of s and m.
4. Suppose that $P(G_{mother} = l) = P(G_{father} = l) = q$. In the first network, derive an expression for $P(G_{child} = l)$ in terms of s, m and q.
5. Under conditions of genetic equilibrium, we expect the distribution of genes to be the same across generations. Use this to calculate the value of q, and, given what you know about handedness in humans, explain why the hypothesis described at the beginning of this question must be wrong.

Exercise 7 Independence



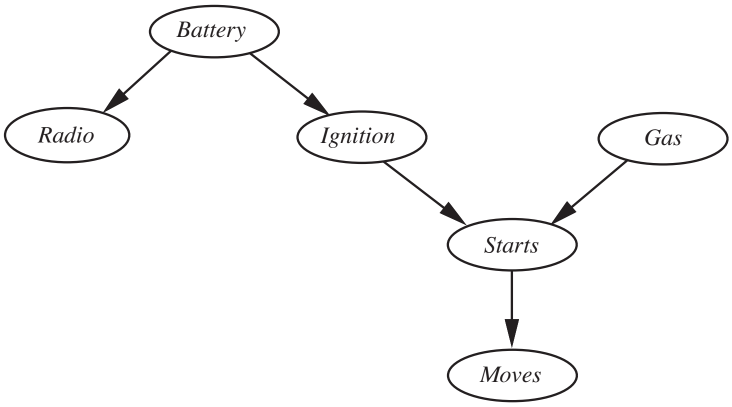
Considering the hereabove Bayesian network, which of the following statements are enforced by the network structure?

1. $P(B, C) = P(B)P(C)$
2. $P(B, C|A) = P(B|A)P(C|A)$
3. $P(B, C|A, D) = P(B|A, D)P(C|A, D)$
4. $P(B, E|A) = P(B|A)P(E|A)$
5. $P(C|A, D, E) = P(C|A, B, D, E)$
6. $P(B, E) = \sum_{a,c,d} P(E|c)P(d|B, c)P(c|a)P(B|a)P(a)$

For the same network, use inference by variable elimination to compute $P(E|A = 1, B = 1)$.

Exercise 8 Car Diagnosis (AIMA, Ex: 14.8)

Let be the following Bayesian network describing some features of a car's electrical system and engine. Each variable is Boolean, and the true value indicates that the corresponding aspect of the vehicle is in working order.



1. Extend the network with the Boolean variables IcyWeather and StarterMotor.
2. According to your knowledge of cars, give reasonable conditional probability tables for all the nodes.
3. How many independent values are contained in the joint probability distribution for eight Boolean nodes, assuming that no conditional independence relations are known to hold among them?
4. How many independent probability values do your network tables contain?

Exercise 9 Nuclear Power Plant (AIMA, Ex: 14.11)

In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables A (alarm sounds), F_A (alarm is faulty), and F_G (gauge is faulty) and the multi-valued nodes G (gauge reading) and T (actual core temperature).

1. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.
2. Suppose there are just two possible actual and measured temperatures: low (l) and high (h). The probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.
3. Suppose the alarm is always triggered by high measured temperatures, unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.
4. Suppose the gauge is not faulty and the alarm is triggered. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.

Supplementary materials

- Hygiène Mentale – La Pensée Bayésienne
- The Book of Why
- Chapters 13 and 14 of the reference textbook.