

Introduction to Artificial Intelligence

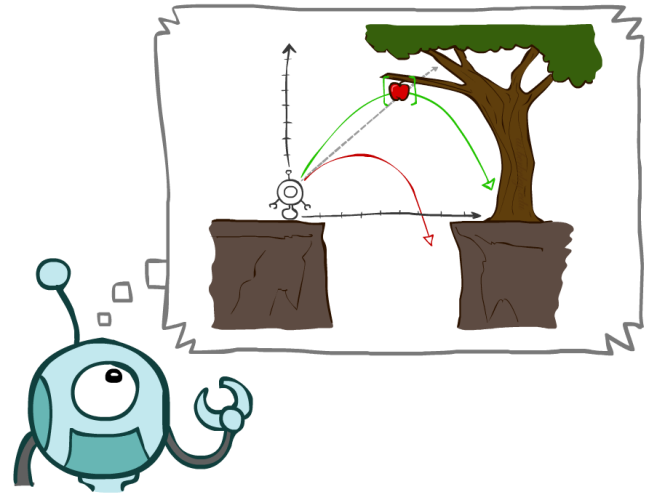
Lecture 2: Solving problems by searching

Prof. Gilles Louppe
g.louppe@uliege.be



Today

- Planning agents
- Search problems
- Uninformed search methods
 - Depth-first search
 - Breadth-first search
 - Uniform-cost search
- Informed search methods
 - A^*
 - Heuristics

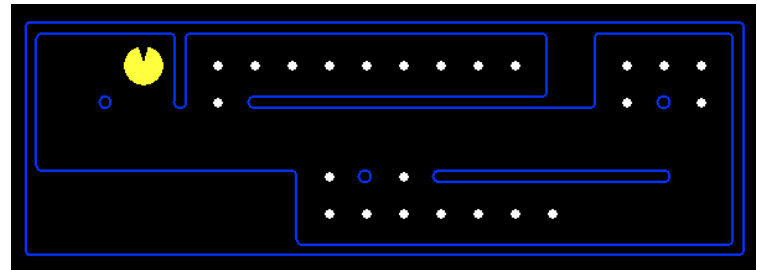
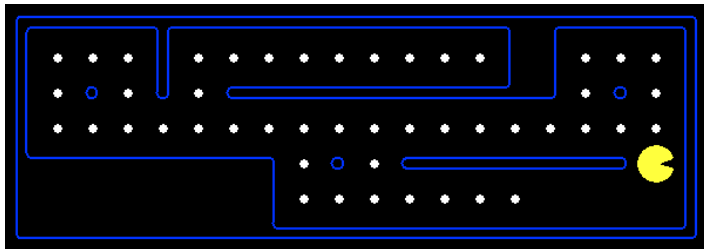


Planning agents

Reflex agents

Reflex agents

- select actions on the basis of the current percept;
- may have a model of the world current state;
- do not consider the future consequences of their actions;
- consider only **how the world is now**.



For example, a simple reflex agent based on condition-action rules could move to a dot if there is one in its neighborhood. No planning is involved to take this decision.

[Q] Can a reflex agent be rational?

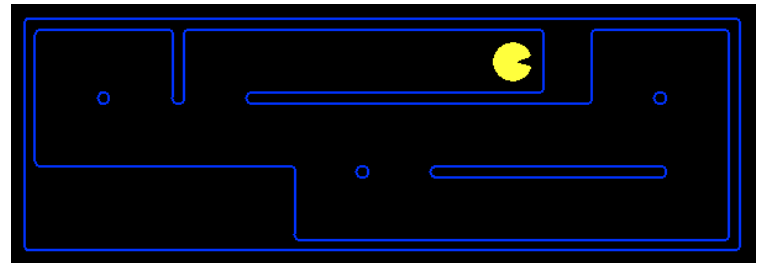
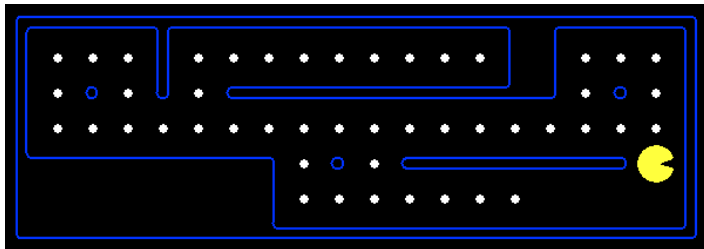
Problem-solving agents

Assumptions:

- Observable, deterministic (and known) environment.

Problem-solving agents

- take decisions based on (hypothesized) consequences of actions;
- must have a model of how the world evolves in response to actions;
- formulate a goal, explicitly;
- consider how the world would be.



A planning agent looks for sequences of actions to eat all the dots.

function SIMPLE-PROBLEM-SOLVING-AGENT(*percept*) **returns** an action

persistent: *seq*, an action sequence, initially empty

state, some description of the current world state

goal, a goal, initially null

problem, a problem formulation

state \leftarrow UPDATE-STATE(*state*, *percept*)

if *seq* is empty **then**

goal \leftarrow FORMULATE-GOAL(*state*)

problem \leftarrow FORMULATE-PROBLEM(*state*, *goal*)

seq \leftarrow SEARCH(*problem*)

if *seq* = *failure* **then return** a null action

action \leftarrow FIRST(*seq*)

seq \leftarrow REST(*seq*)

return *action*

Figure 3.1 A simple problem-solving agent. It first formulates a goal and a problem, searches for a sequence of actions that would solve the problem, and then executes the actions one at a time. When this is complete, it formulates another goal and starts over.

Offline vs. Online solving

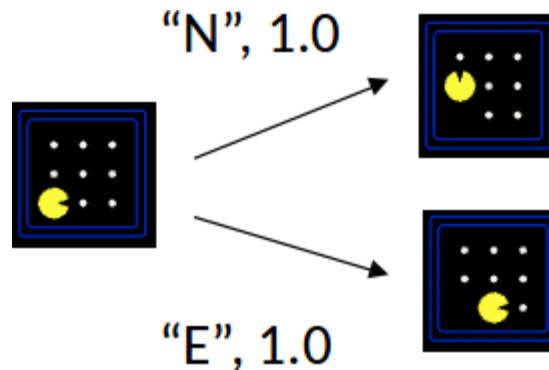
- Problem-solving agents are **offline**. The solution is executed "eyes closed", ignoring the percepts.
- **Online** problem solving involves acting without complete knowledge. In this case, the sequence of actions might be recomputed at each step.

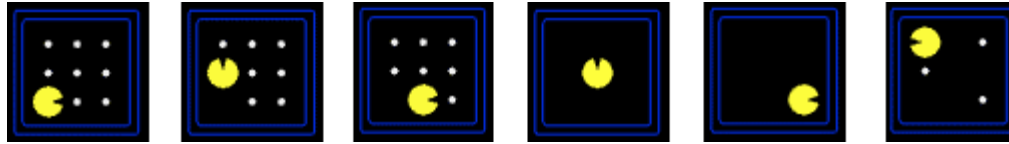
Search problems

Search problems

A **search problem** consists of the following components:

- The **initial state** of the agent.
- A description of the **actions** available to the agent given a state s , denoted $\text{actions}(s)$.
- A **transition model** that returns the state $s' = \text{result}(s, a)$ that results from doing action a in state s .
 - We say that s' is a **successor** of s if there is an acceptable action from s to s' .





- Together, the initial state, the actions and the transition model define the **state space** of the problem, i.e. the set of all states reachable from the initial state by any sequence of action.
 - The state space forms a directed graph:
 - nodes = states
 - links = actions
 - A path is a sequence of states connected by actions.
- A **goal test** which determines whether the solution of the problem is achieved in state s .
- A **path cost** that assigns a numeric value to each path.
 - In this course, we will also assume that the path cost corresponds to a sum of positive **step costs** $c(s, a, s')$ associated to the action a in s leading to s' .

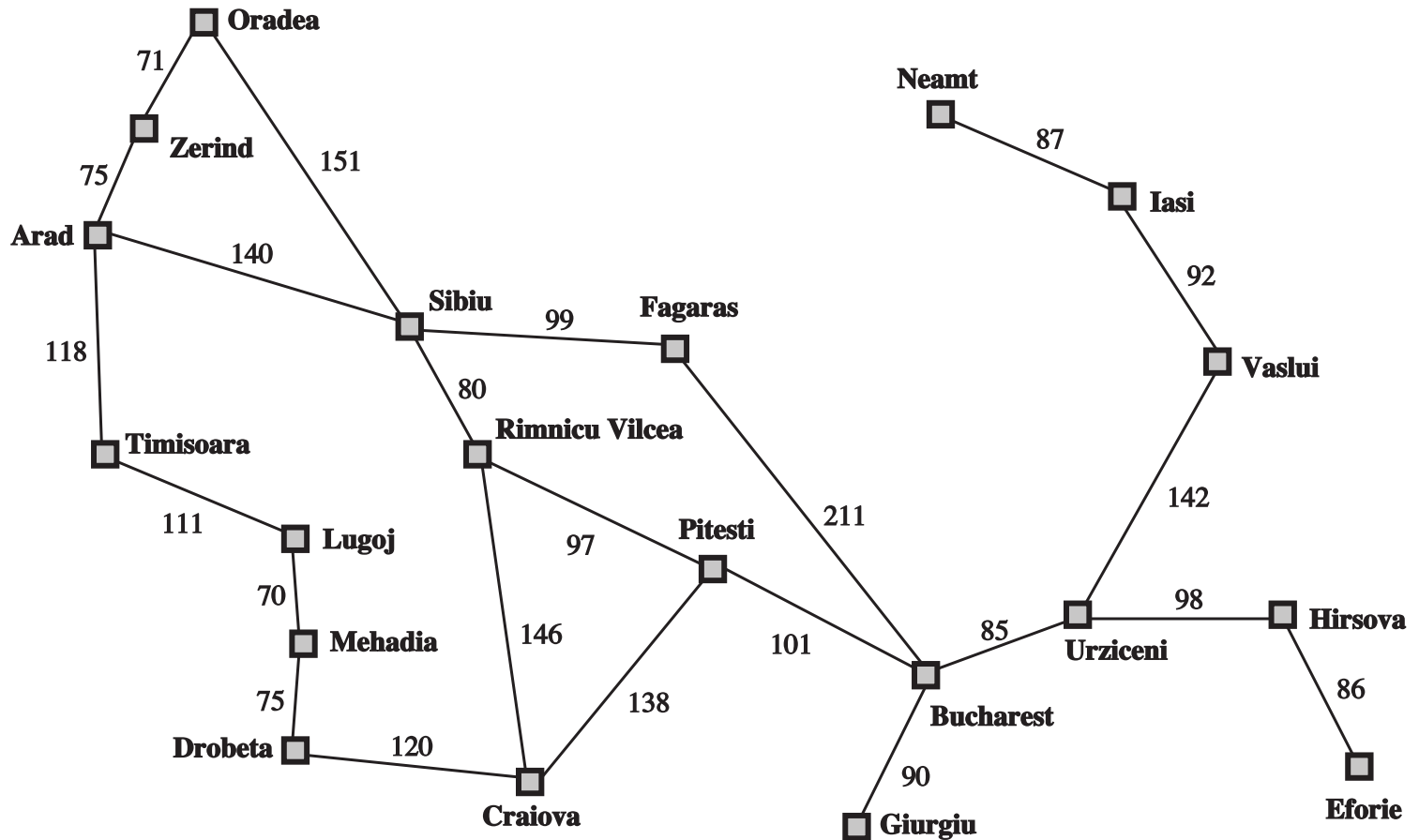
A **solution** to a problem is an action sequence that leads from the initial state to a goal state.

- A solution quality is measured by the path cost function.
- An **optimal solution** has the lowest path cost among all solutions.

Exercise

What if the environment is partially observable? non-deterministic?

Example: Traveling in Romania



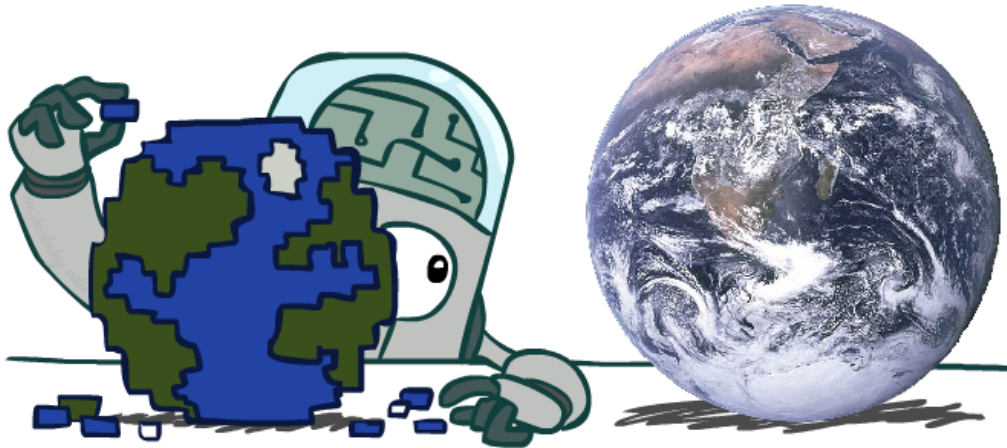
How to go from Arad to Bucharest?

- Initial state = the city we start in.
 - $s_0 = \text{in}(\text{Arad})$
- Actions = Going from the current city to the cities that are directly connected to it.
 - $\text{actions}(s_0) = \{\text{go}(\text{Sibiu}), \text{go}(\text{Timisoara}), \text{go}(\text{Zerind})\}$
- Transition model = The city we arrive in after driving to it.
 - $\text{result}(\text{in}(\text{Arad}), \text{go}(\text{Zerind})) = \text{in}(\text{Zerind})$
- Goal test: whether we are in Bucharest.
 - $s \in \{\text{in}(\text{Bucharest})\}$
- Step cost: distances between cities.

Selecting a state space

The real world is absurdly **complex**.

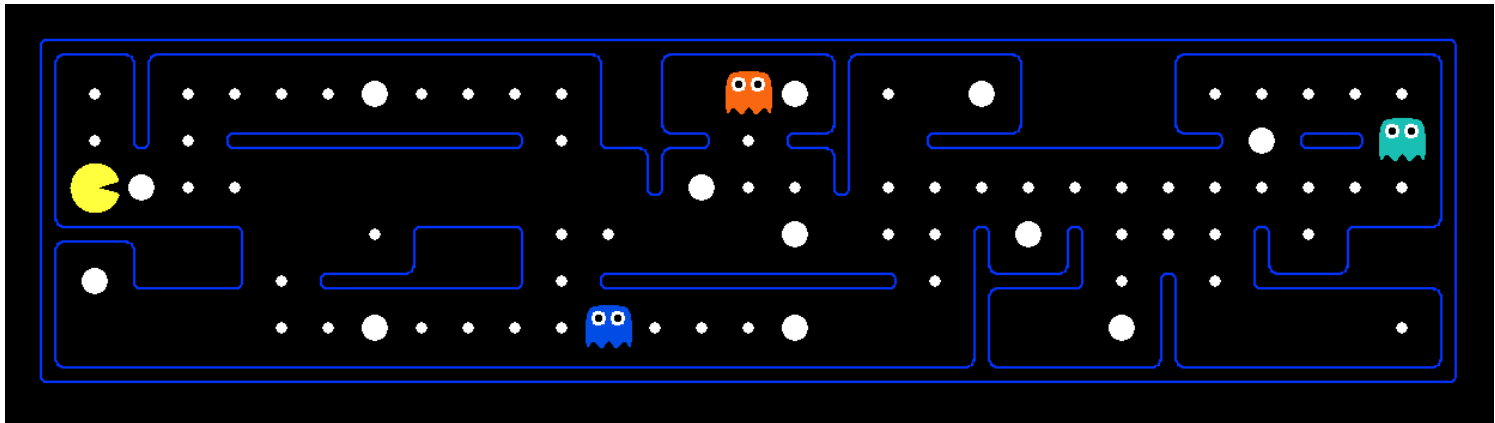
- The **world state** includes every last detail of the environment.
- A **search state** keeps only the details needed for planning.



Search problems are **models**.

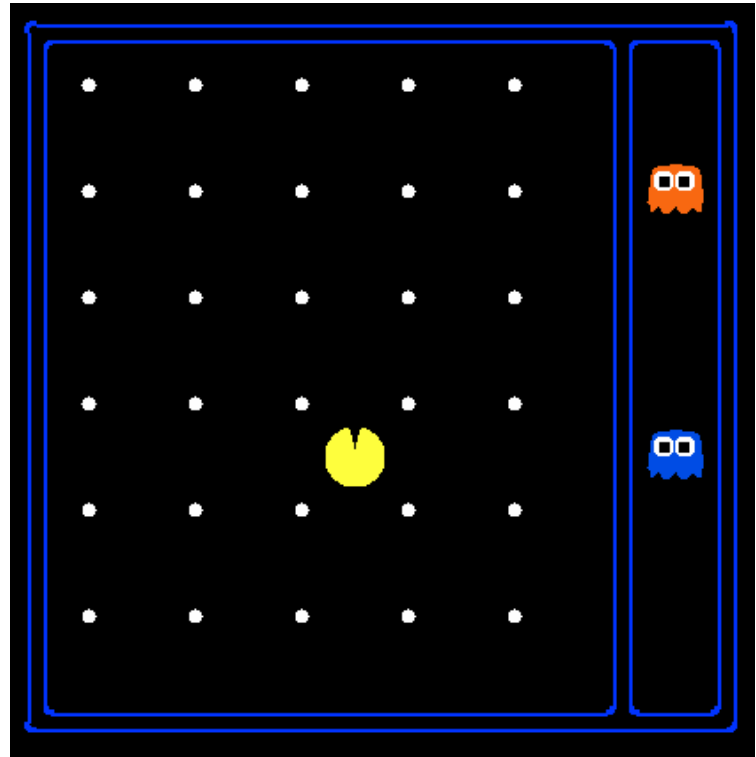
Example: eat-all-dots

- States: $\{(x, y), \text{dot booleans}\}$
- Actions: NSEW
- Transition: update location and possibly a dot boolean
- Goal test: dots all false



State space size

- World state:
 - Agent positions: 120
 - Found count: 30
 - Ghost positions: 12
 - Agent facing: NSEW
- How many?
 - World states?
 - $120 \times 2^{30} \times 12^2 \times 4$
 - States for eat-all-dots?
 - 120×2^{30}

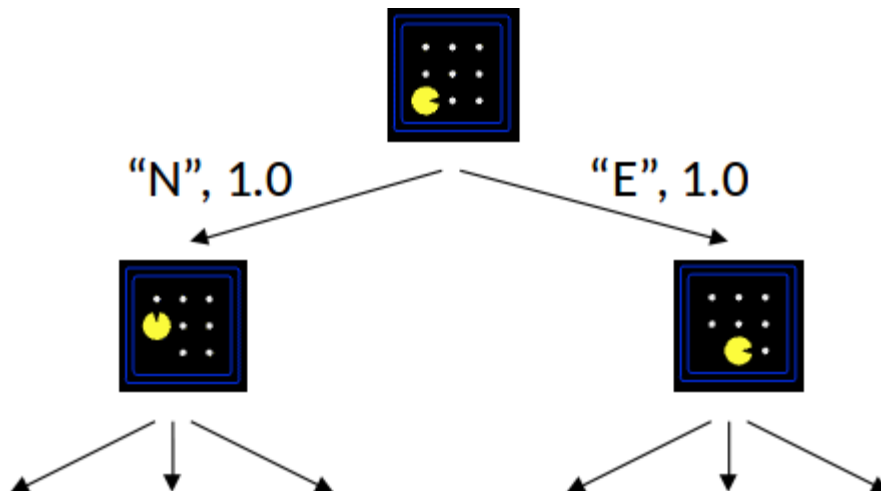


Search trees

The set of acceptable sequences starting at the initial state form a **search tree**.

- Nodes correspond to states in the state space, where the initial state is the root node.
- Branches correspond to applicable actions, with child nodes corresponding to successors.

For most problems, we can never actually build the whole tree. Yet we want to find some optimal branch!



Tree search algorithms

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE(node)) then return node
    fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

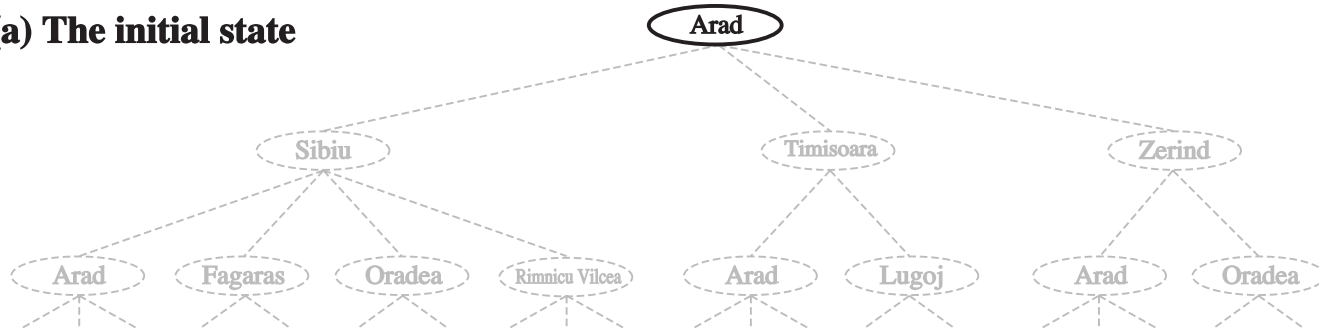
Important ideas

- Fringe (or frontier) of partial plans under consideration
- Expansion
- Exploration

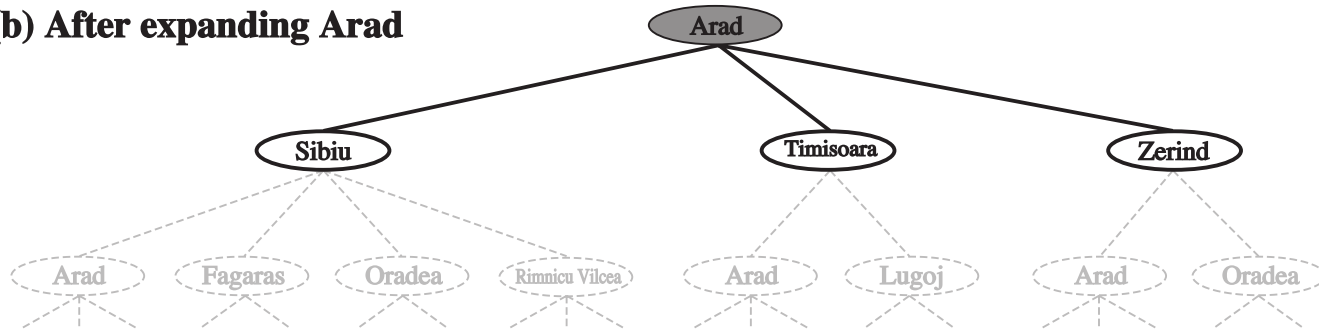
Exercise

Which fringe nodes to explore? How to expand as few nodes as possible, while achieving the goal?

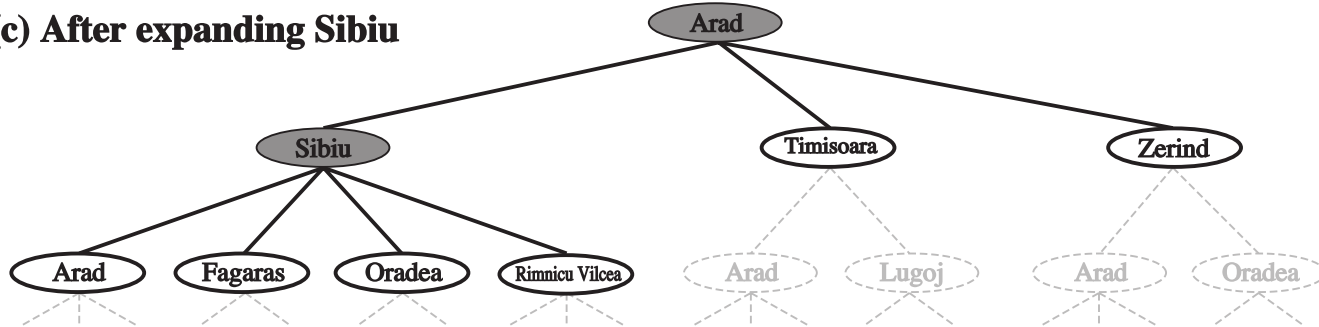
(a) The initial state



(b) After expanding Arad



(c) After expanding Sibiu



Uninformed search strategies

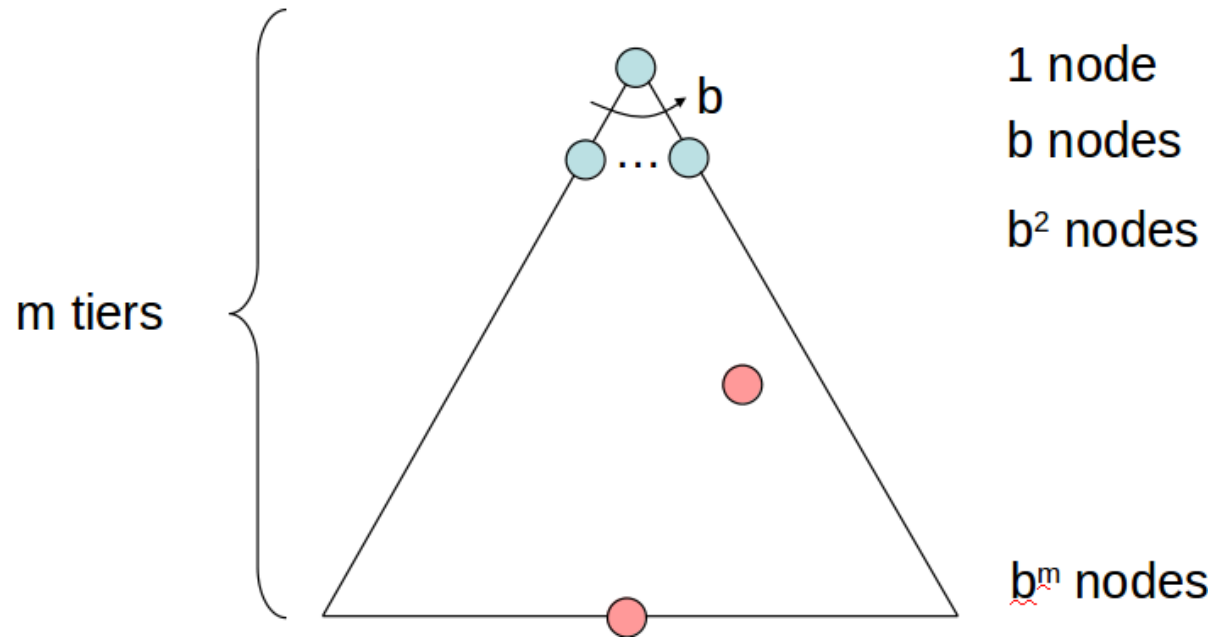
Uninformed search strategies use only the information available in the problem definition. They do not know whether a state looks more promising than some other.

Strategies

- Depth-first search
- Breadth-first search
- Uniform-cost search
- Iterative deepening

Properties of search strategies

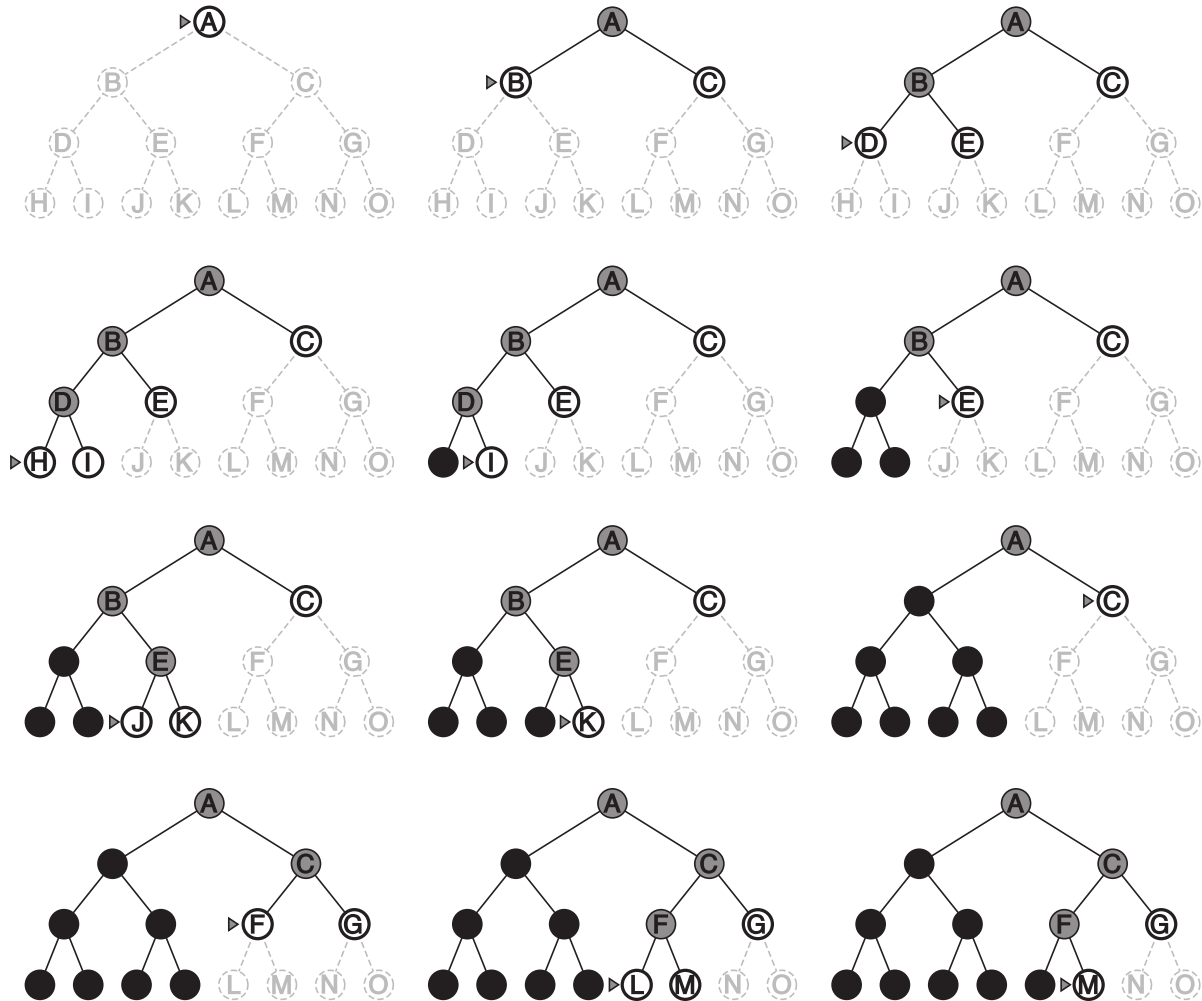
- A strategy is defined by picking the **order of expansion**.
- Strategies are evaluated along the following dimensions:
 - **Completeness**: does it always find a solution if one exists?
 - **Optimality**: does it always find the least-cost solution?
 - **Time complexity**: how long does it take to find a solution?
 - **Space complexity**: how much memory is needed to perform the search?
- Time and complexity are measured in terms of
 - **b** : maximum branching factor of the search tree
 - **d** : depth of the least-cost solution
 - the depth of **s** is defined as the number of actions from the initial state to **s** .
 - **m** : maximum length of any path in the state space (may be ∞)

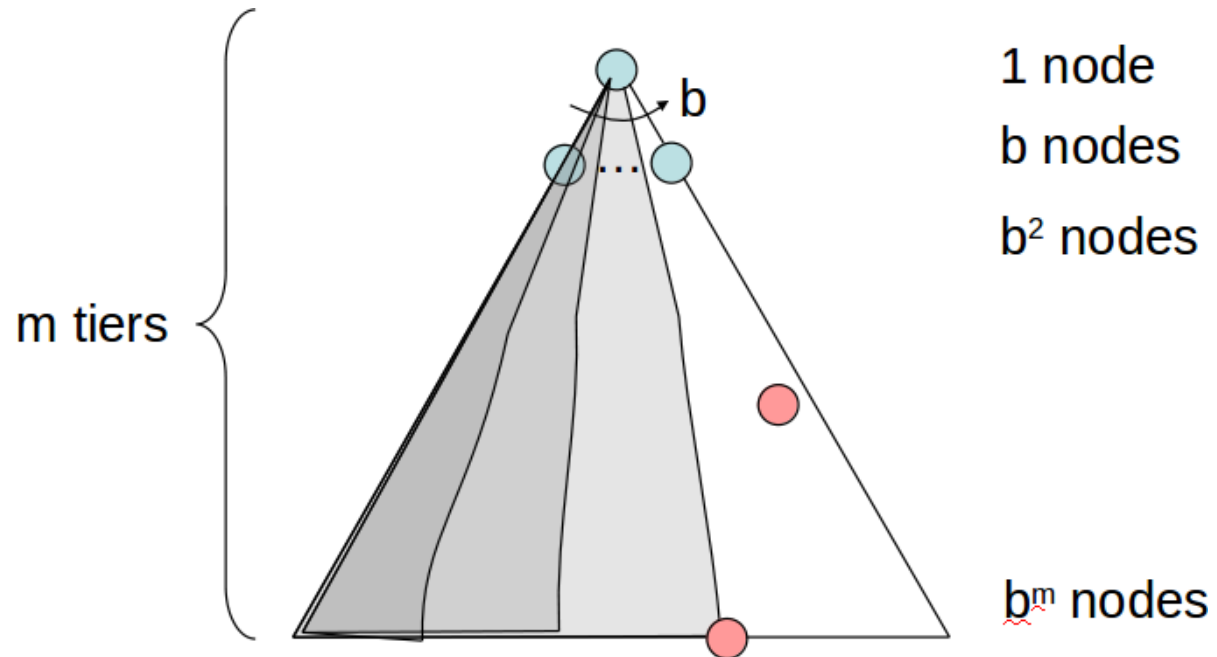


Depth-first search



- **Strategy:** expand the deepest node in the fringe.
- **Implementation:** fringe is a **LIFO stack**.

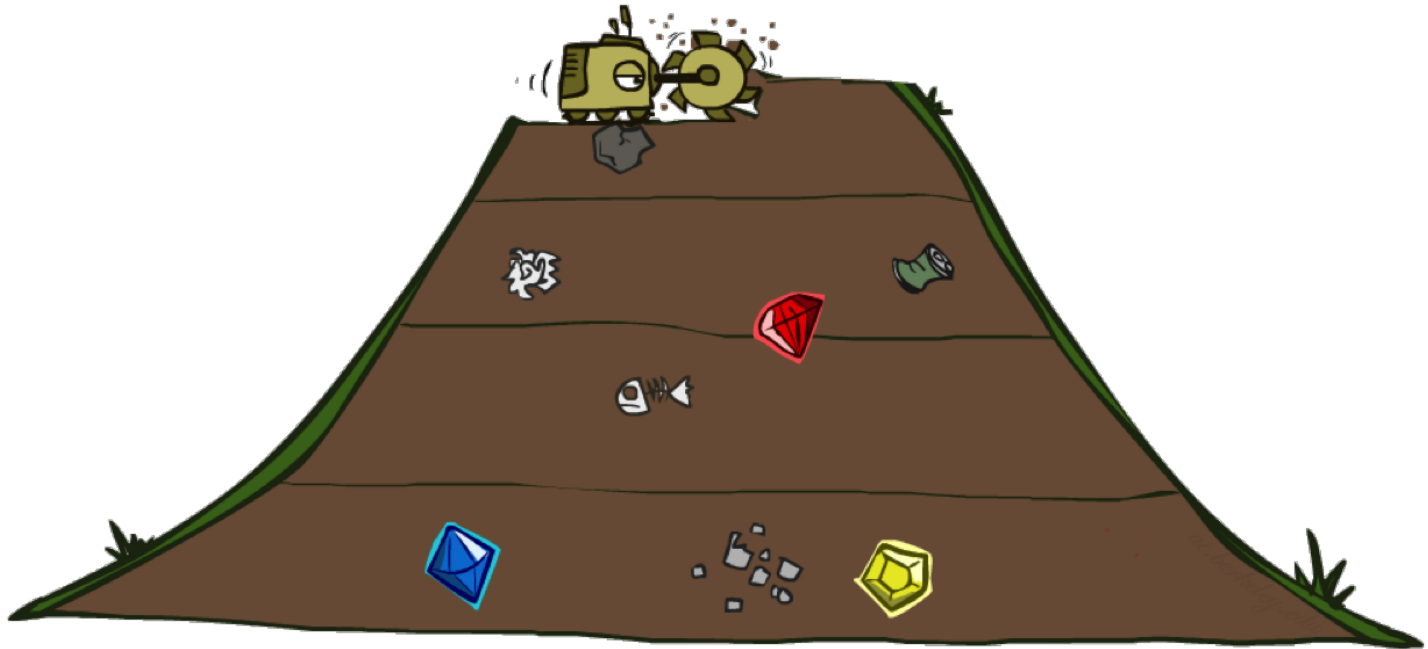




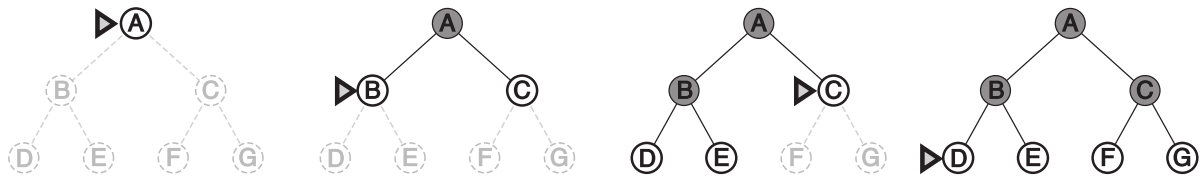
Properties of DFS

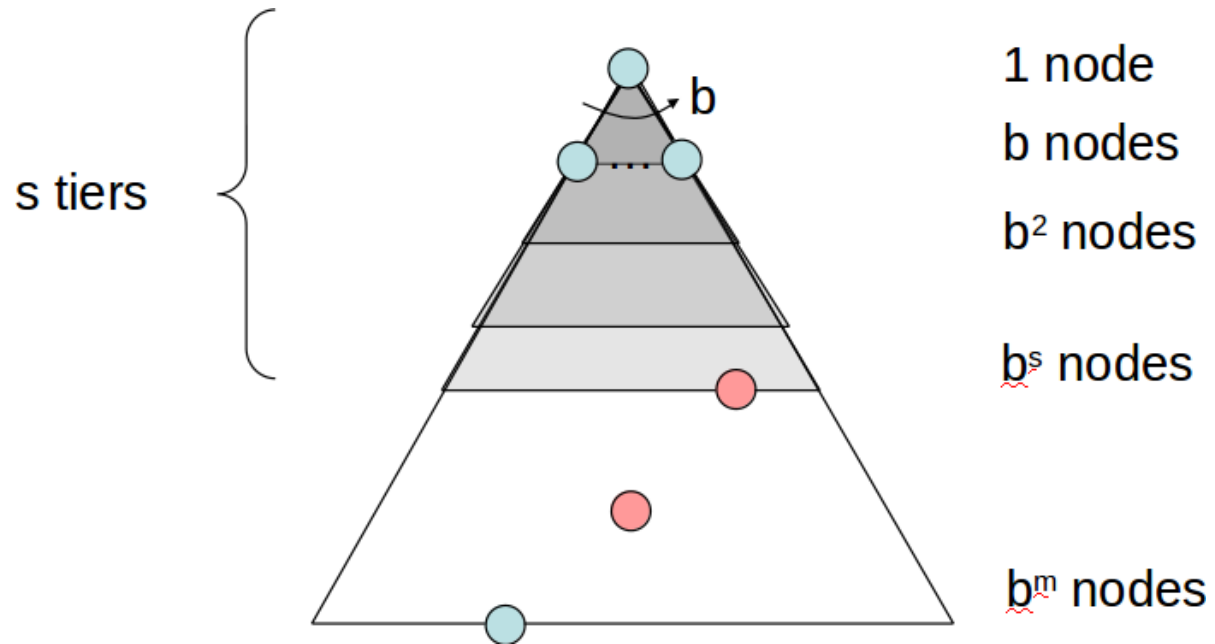
- Completeness:
 - m could be infinite, so only if we prevent cycles (more on this later).
- Optimality:
 - No, DFS finds the leftmost solution, regardless of depth or cost.
- Time complexity:
 - May generate the whole tree (or a good part of it, regardless of d). Therefore $O(b^m)$, which might be much greater than the size of the state space!
- Space complexity:
 - Only store siblings on path to root, therefore $O(bm)$.
 - When all the descendants of a node have been visited, the node can be removed from memory.

Breadth-first search



- **Strategy**: expand the shallowest node in the fringe.
- **Implementation**: fringe is a **FIFO queue**.





Properties of BFS

- Completeness:
 - If the shallowest goal node is at some finite depth d , BFS will eventually find it after generating all shallower nodes (provided b is finite).
- Optimality:
 - The shallowest goal is not necessarily the optimal one.
 - BFS is optimal only if the path cost is a non-decreasing function of the depth of the node.
- Time complexity:
 - If the solution is at depth d , then the total number of nodes generated before finding this node is $b + b^2 + b^3 + \dots + b^d = O(b^d)$
- Space complexity:
 - The number of nodes to maintain in memory is the size of the fringe, which will be the largest at the last tier. That is $O(b^d)$

(demo)

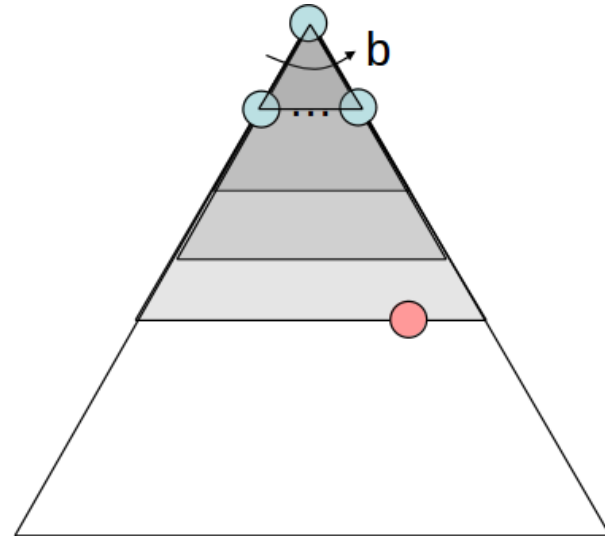
Iterative deepening

Idea: get DFS's space advantages with BFS's time/shallow solution advantages.

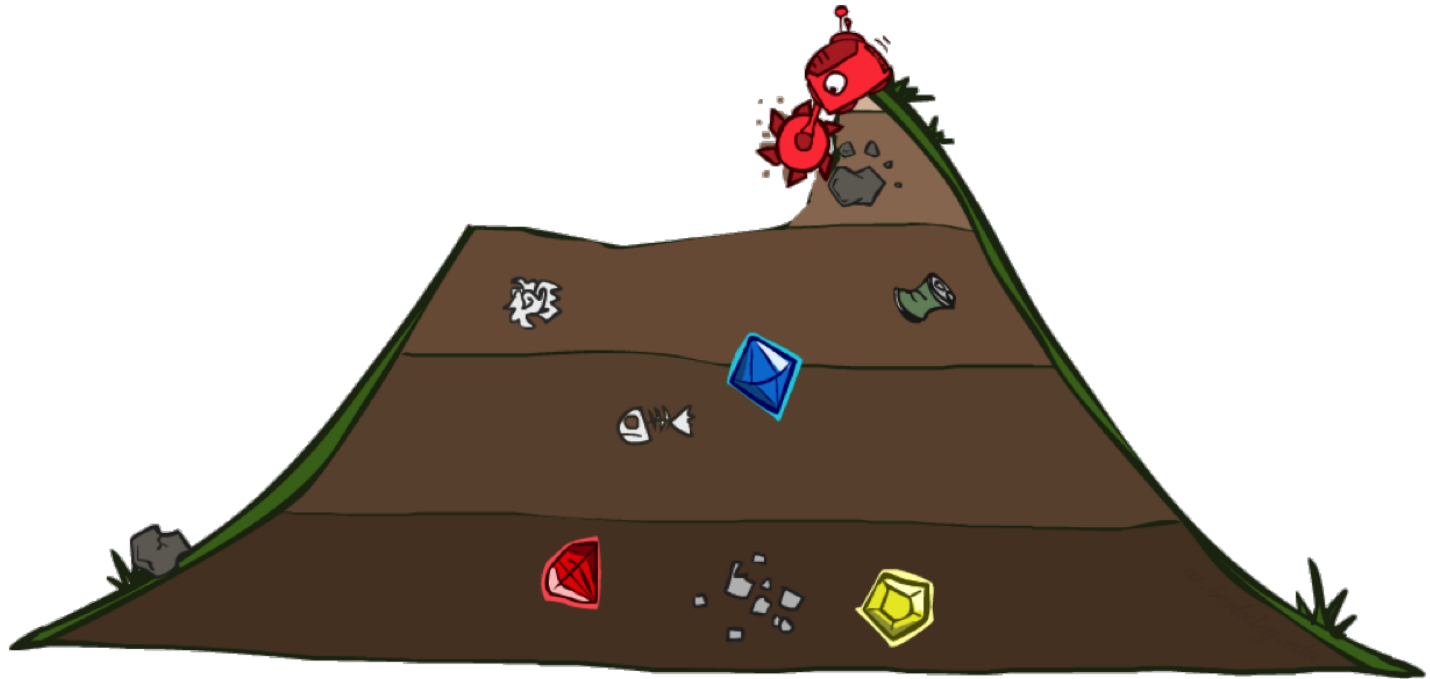
- Run DFS with depth limit 1.
- If no solution, run DFS with depth limit 2.
- If no solution, run DFS with depth limit 3.
 - ...

Exercise

- What are the properties of iterative deepening?
- Isn't this process wastefully redundant?



Uniform-cost search



- **Strategy**: expand the cheapest node in the fringe.
- **Implementation**: fringe is a **priority queue**, using the cumulative cost $g(n)$ from the initial state to node n as priority.

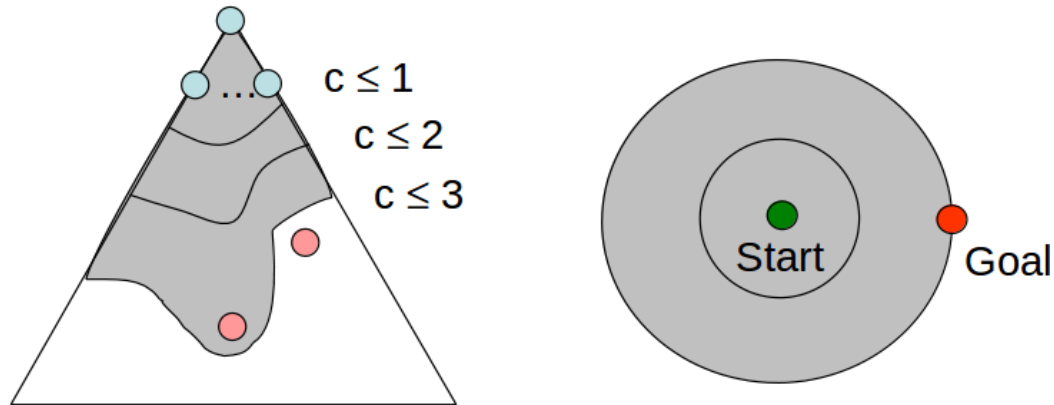


Properties of UCS

- **Completeness:**
 - Yes, if step cost are all such that $c(s, a, s') \geq \epsilon > 0$. (Why?)
- **Optimality:**
 - Yes, since UCS expands nodes in order of their optimal path cost.
- **Time complexity:**
 - Assume C^* is the cost of the optimal solution and that step costs are all $\geq \epsilon$.
 - The "effective depth" is then roughly C^* / ϵ .
 - The worst-case time complexity is $O(b^{C^* / \epsilon})$.
- **Space complexity:**
 - The number of nodes to maintain is the size of the fringe, so as many as in the last tier $O(b^{C^* / \epsilon})$.

(demo)

Informed search strategies



One of the **issues of UCS** is that it explores the state space in **every direction**, without exploiting information about the (plausible) location of the goal node.

Informed search strategies aim to solve this problem by expanding nodes in the fringe in decreasing order of **desirability**.

- Greedy search
- A*

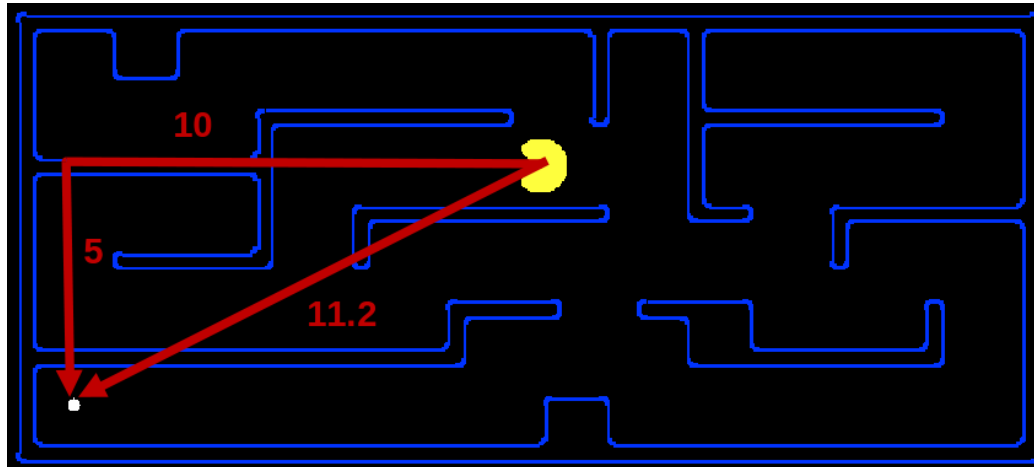
Greedy search



Heuristics

A **heuristic** (or evaluation) function $h(n)$ is:

- a function that **estimates** the cost of the cheapest path from node n to a goal state;
 - $h(n) \geq 0$ for all nodes n
 - $h(n) = 0$ for a goal state.
- is designed for a **particular** search problem.



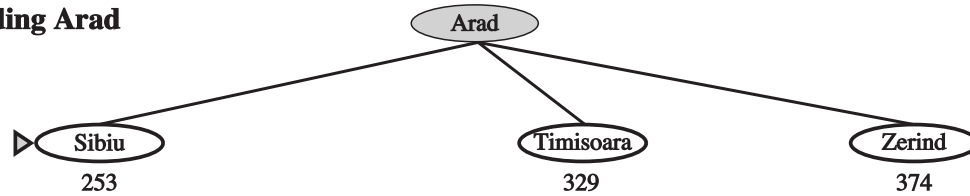
Greedy search

- **Strategy**: expand the node n in the fringe for which $h(n)$ is the lowest.
- **Implementation**: fringe is a **priority queue**, using $h(n)$ as priority.

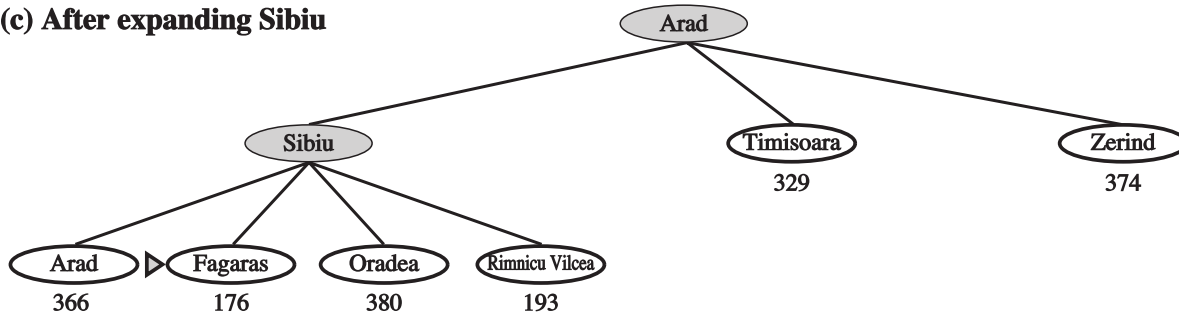
(a) The initial state



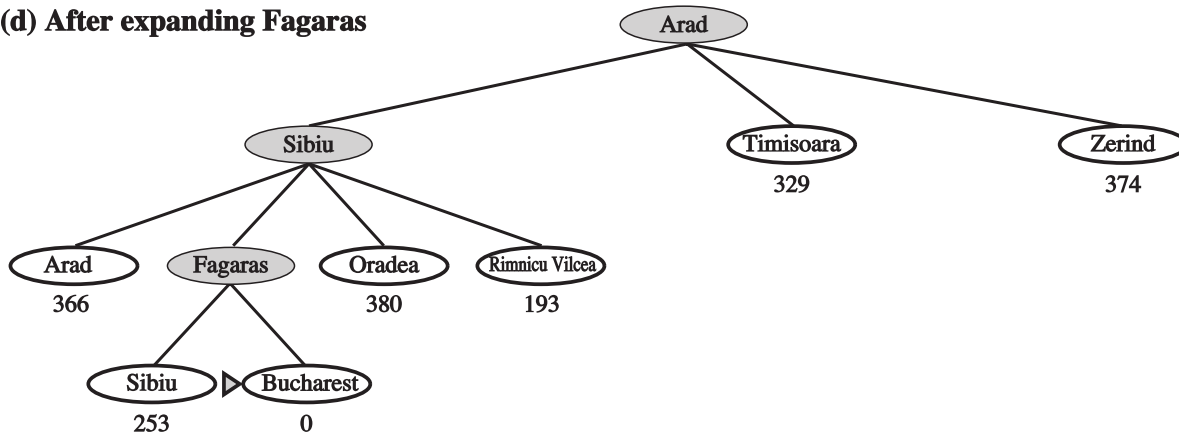
(b) After expanding Arad



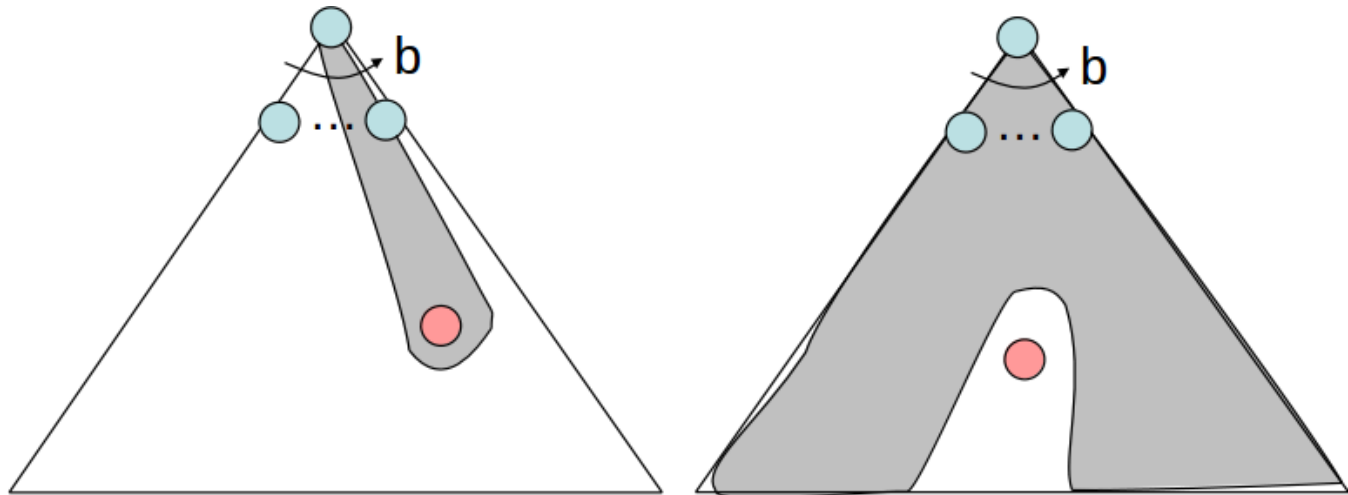
(c) After expanding Sibiu



(d) After expanding Fagaras



$h(n)$ = straight line distance to Bucharest.



At best, greedy search takes you straight to the goal.
At worst, it is like a badly-guided BFS.

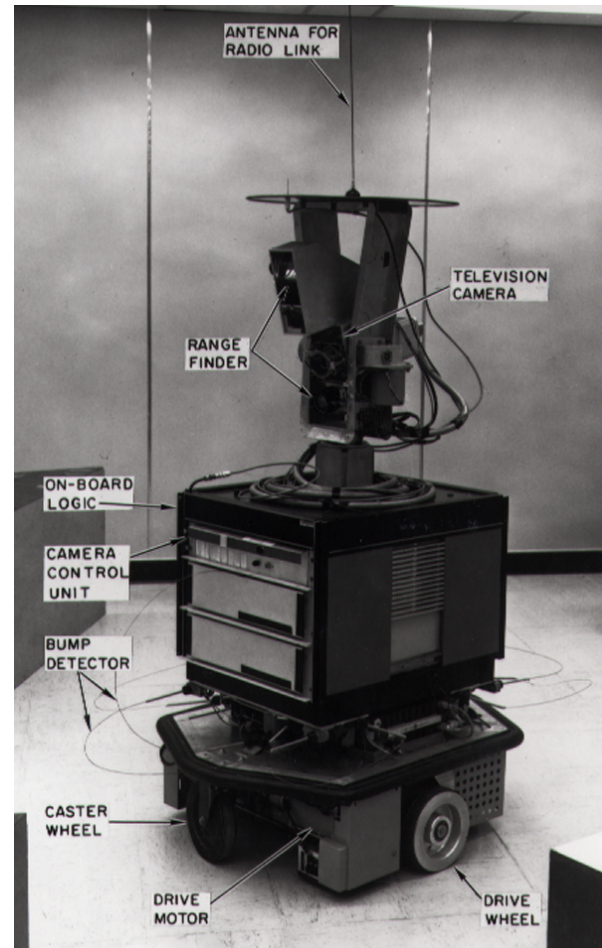
Properties of greedy search

- Completeness:
 - No, unless we prevent cycles (more on this later).
- Optimality:
 - No, e.g. the path via Sibiu and Fagaras is 32km longer than the path through Rimnicu Vilcea and Pitesti.
- Time complexity:
 - $O(b^m)$, unless we have a good heuristic function.
- Space complexity:
 - $O(b^m)$, unless we have a good heuristic function.



Shakey the Robot

- A* was first proposed in 1968 to improve robot planning.
- Goal was to navigate through a room with obstacles.



A*

- Uniform-cost orders by path cost, or backward cost $g(n)$
- Greedy orders by goal proximity, or forward cost $h(n)$
- A* combines the two algorithms and orders by the sum

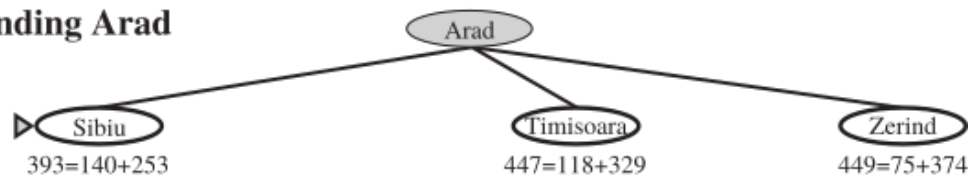
$$f(n) = g(n) + h(n)$$

- $f(n)$ is the estimated cost of cheapest solution through n .

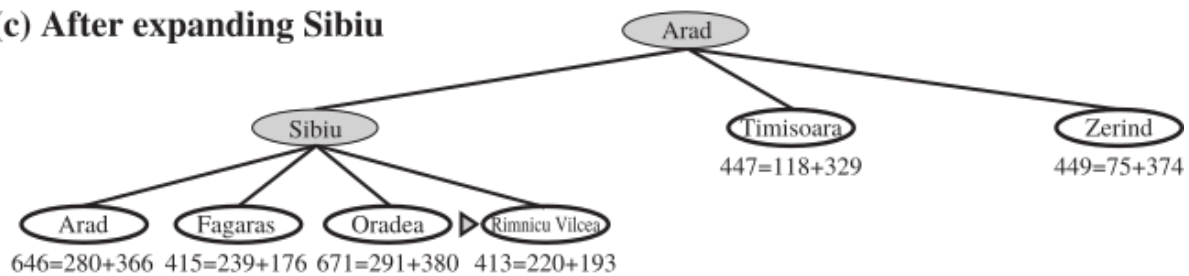
(a) The initial state



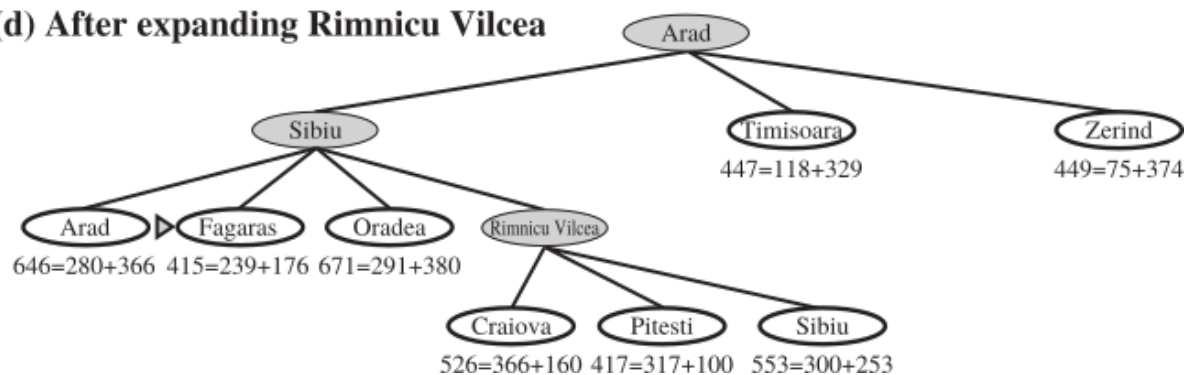
(b) After expanding Arad



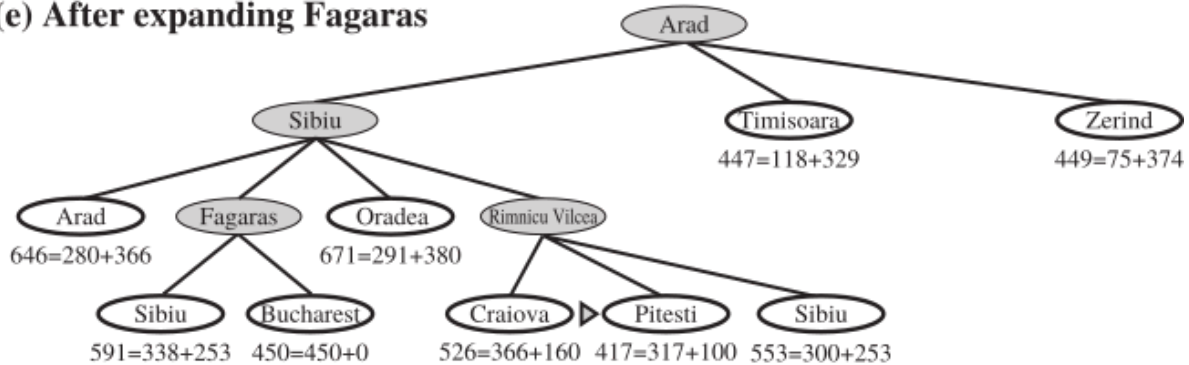
(c) After expanding Sibiu



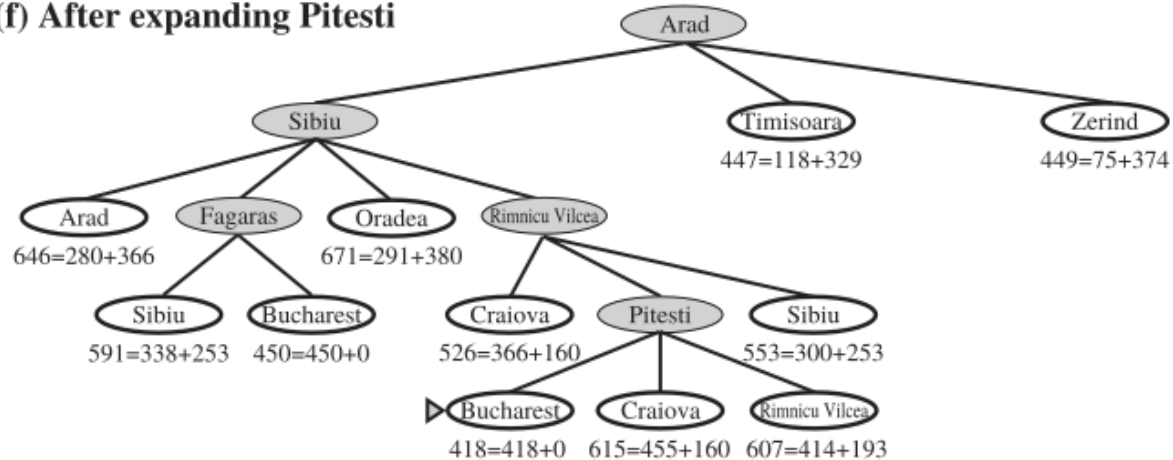
(d) After expanding Rimnicu Vilcea



(e) After expanding Fagaras



(f) After expanding Pitesti



Exercise

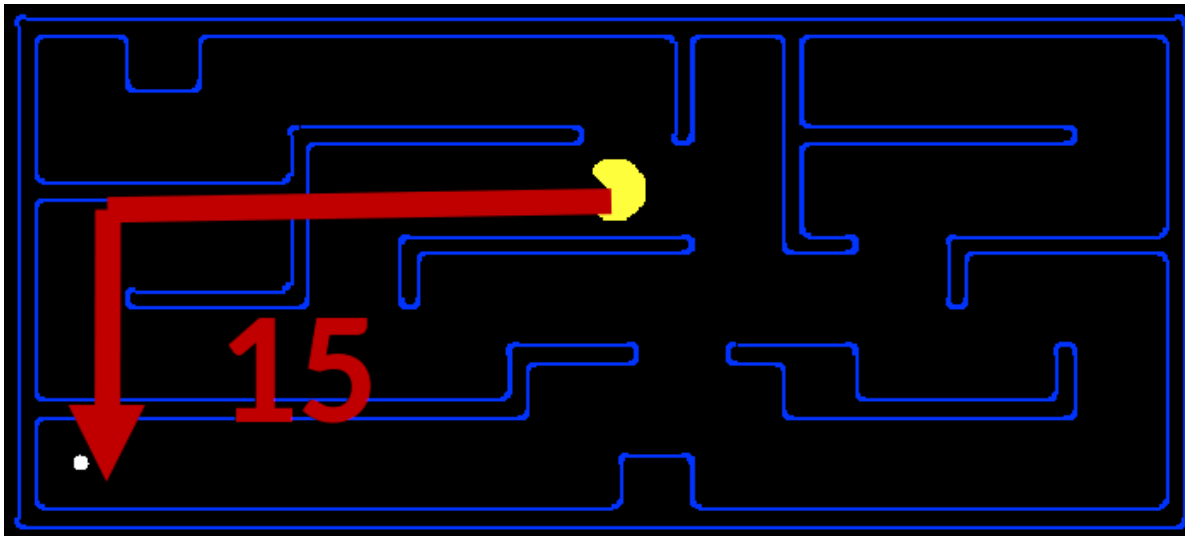
Why doesn't A* stop at step (e), since Bucharest is in the fringe?

Admissible heuristics

A heuristic h is **admissible** if

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal.



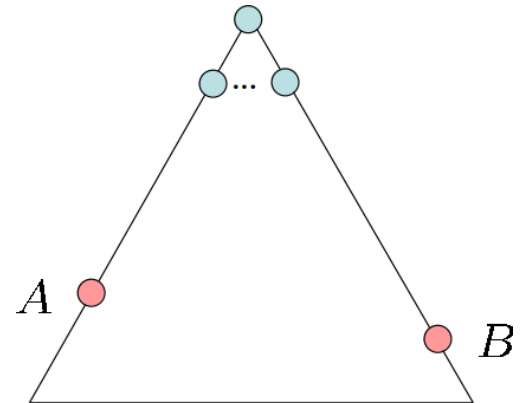
The Manhattan distance is admissible

Optimality of A*

Assumptions:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

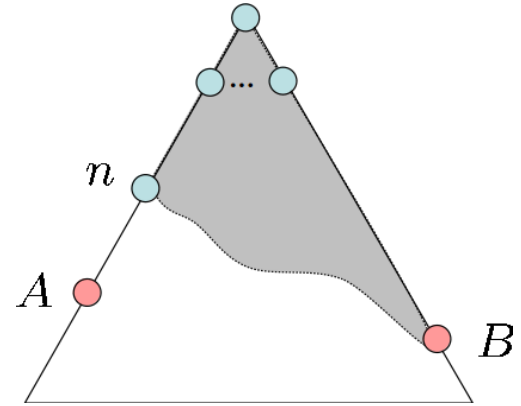
Claim: A will exit the fringe before B .



Proof

Assume B is on the fringe. Some ancestor n of A is on the fringe too.

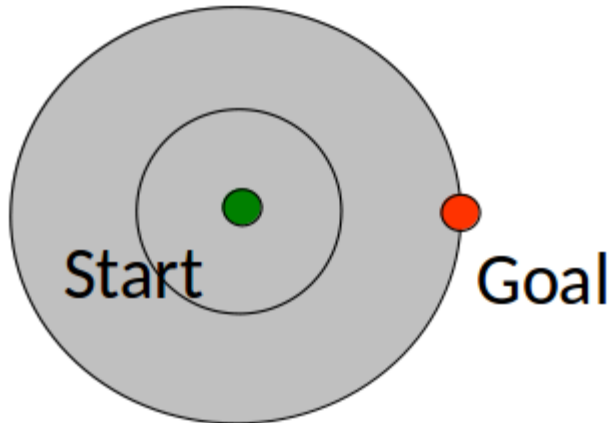
- $f(n) \leq f(A)$
 - $f(n) = g(n) + h(n)$ (by definition)
 - $f(n) \leq g(A)$ (admissibility of h)
 - $f(A) = g(A) + h(A) = g(A)$ ($h = 0$ at a goal)
- $f(A) < f(B)$
 - $g(A) < g(B)$ (B is suboptimal)
 - $f(A) < f(B)$ ($h = 0$ at a goal)
- Therefore, n expands before B .
 - since $f(n) \leq f(A) < f(B)$



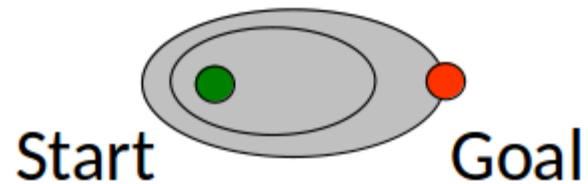
Similarly, all ancestors of A expand before B , including A . Therefore A^* is optimal.

A* contours

- Assume f -costs are non-decreasing along any path.
- We can define **contour levels** t in the state space, that include all nodes n for which $f(n) \leq t$.



For UCS ($h(n) = 0$ for all n), bands are circular around the start.



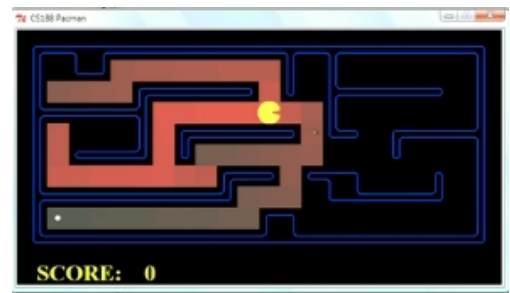
For A* with accurate heuristics, bands stretch towards the goal.



Greedy search



UCS



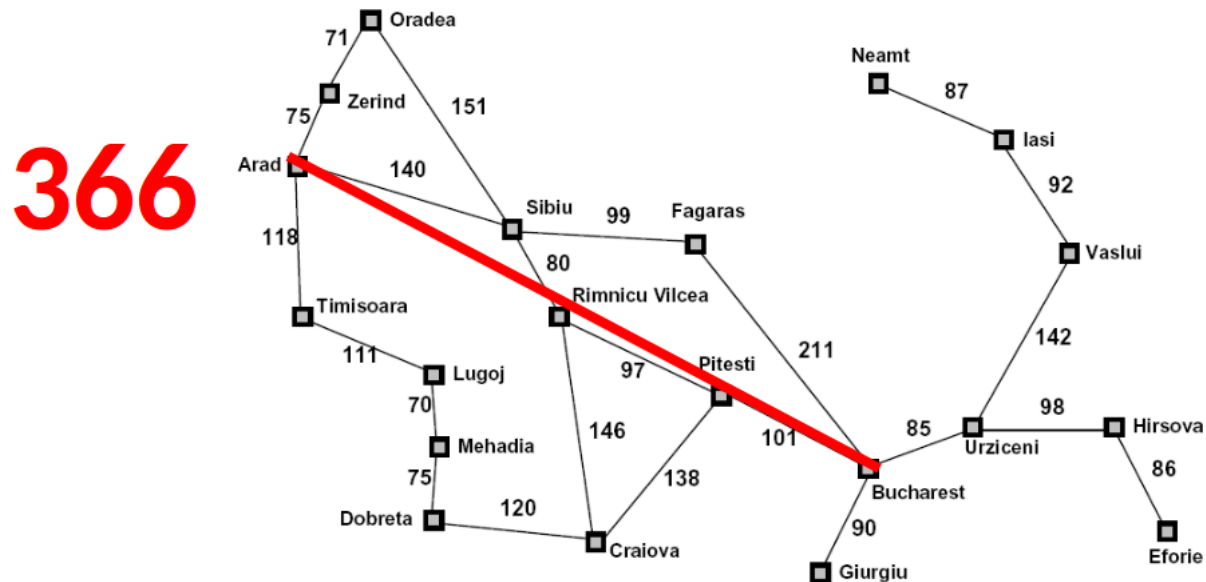
A*

(demo)

Creating admissible heuristics

Most of the work in solving hard search problems optimally is in finding admissible heuristics.

Admissible heuristics can be derived from the exact solutions to [relaxed problems](#), where new actions are available.



Dominance

- If h_1 and h_2 are both admissible and if $h_2(n) \geq h_1(n)$ for all n , then h_2 dominates h_1 and is better for search.
- Given any admissible heuristics h_a and h_b ,

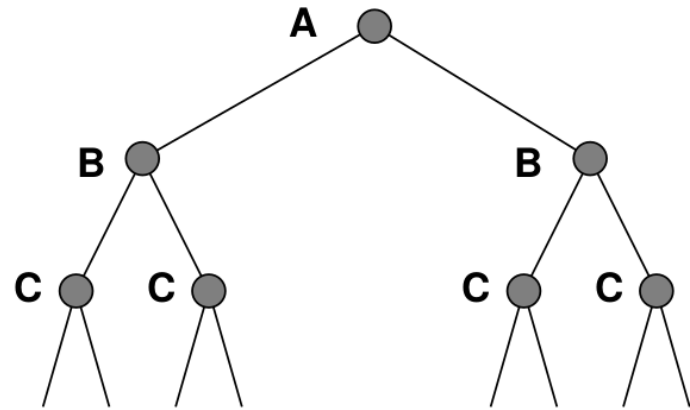
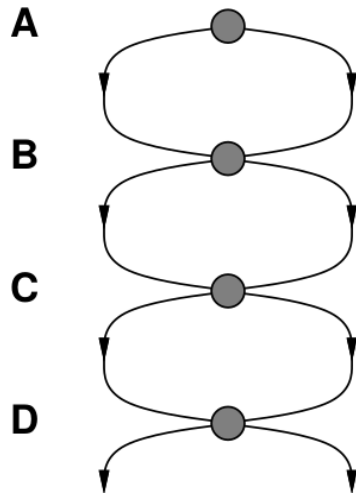
$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a and h_b .

Learning heuristics from experience

- Assuming an **episodic** environment, an agent can **learn** good heuristics by playing the game many times.
- Each optimal solution s^* provides **training examples** from which $h(n)$ can be learned.
- Each example consists of a state n from the solution path and the actual cost $g(s^*)$ of the solution from that point.
- The mapping $n \rightarrow g(s^*)$ can be learned with **supervised learning** algorithms.
 - Linear models, Neural networks, etc.

Graph search



The failure to detect **repeated states** can turn a linear problem into an exponential one. It can also lead to non-terminating searches.

Redundant paths and cycles can be avoided by **keeping track** of the states that have been **explored**. This amounts to grow a tree directly on the state-space graph.

function GRAPH-SEARCH(*problem*, *fringe*) **returns** a solution, or failure

closed \leftarrow an empty set

fringe \leftarrow INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)

loop do

if *fringe* is empty **then return** failure

node \leftarrow REMOVE-FRONT(*fringe*)

if GOAL-TEST(*problem*, STATE[*node*]) **then return** *node*

if STATE[*node*] is not in *closed* **then**

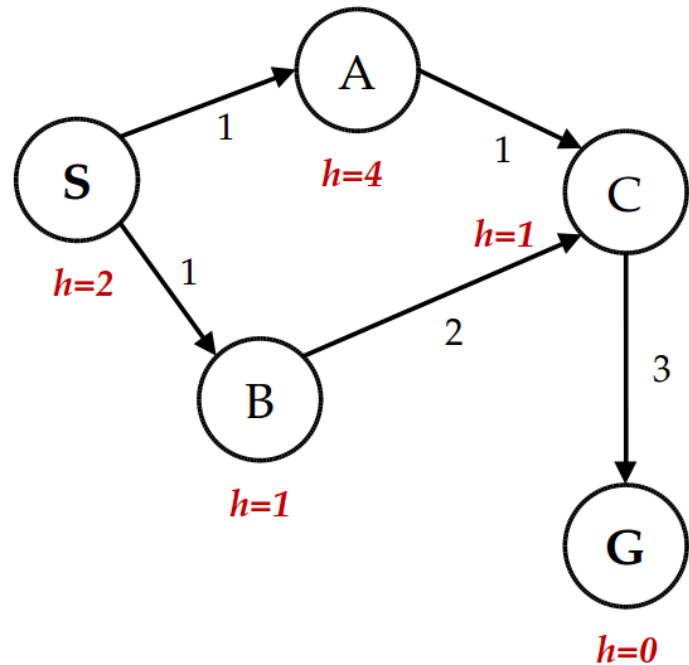
 add STATE[*node*] to *closed*

fringe \leftarrow INSERTALL(EXPAND(*node*, *problem*), *fringe*)

end

A* graph-search gone wrong?

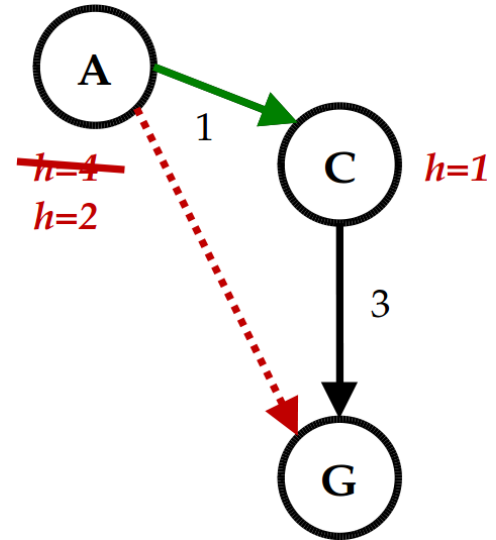
- We start at *S* and *G* is a goal state.
- Which path does graph search find?



Consistent heuristics

A heuristic h is consistent if for every n and every successor n' generated by any action a ,

$$h(n) \leq c(n, a, n') + h(n').$$



Consequences of consistent heuristics:

- $f(n)$ is non-decreasing along any path.
- $h(n)$ is admissible.
- With a consistent heuristic, graph-search A* is optimal.

Recap example: Super Mario



- Task environment?
 - performance measure, environment, actuators, sensors?
- Type of environment?
- Search problem?
 - initial state, actions, transition model, goal test, path cost?
- Good heuristic?



A* in action

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored.
- Variety of uninformed search strategies (DFS, BFS, UCS, Iterative deepening).
- Heuristic functions estimate costs of shortest paths. Good heuristic can dramatically reduce search cost.
- Greedy best-first search expands lowest h , which shows to be incomplete and not always optimal.
- A^* search expands lowest $f = g + h$. This strategy is complete and optimal.
- Graph search can be exponentially more efficient than tree search.

References

- Hart, Peter E., Nils J. Nilsson, and Bertram Raphael. "A formal basis for the heuristic determination of minimum cost paths." IEEE transactions on Systems Science and Cybernetics 4.2 (1968): 100-107.