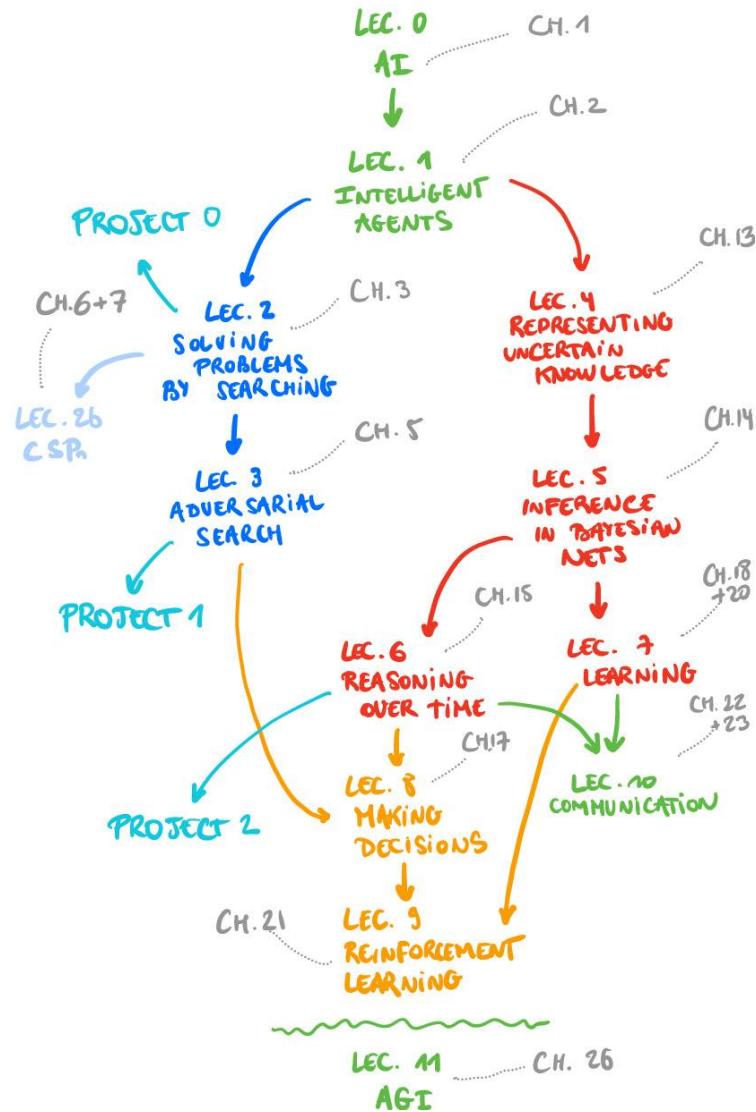


# Introduction to Artificial Intelligence

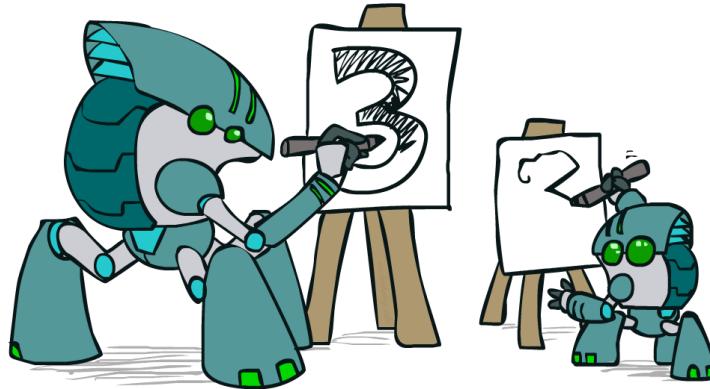
Lecture 7: Learning

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# Today



Make our agents capable of self-improvement through a **learning** mechanism.

- Bayesian learning
- Supervised learning
- Unsupervised learning (Bonus)

# Intelligence?

What we cover in this course:

- Search algorithms, using a state space specified by domain knowledge.
- (Constraint satisfaction problems, by exploiting a known structure of the states.)
- (Logical inference, using well-specified facts and inference rules.)
- Adversarial search, for known and fully observable games.
- Reasoning about uncertain knowledge, as represented using domain-motivated probabilistic models.
- Taking optimal decisions, under uncertainty and possibly under partial observation.

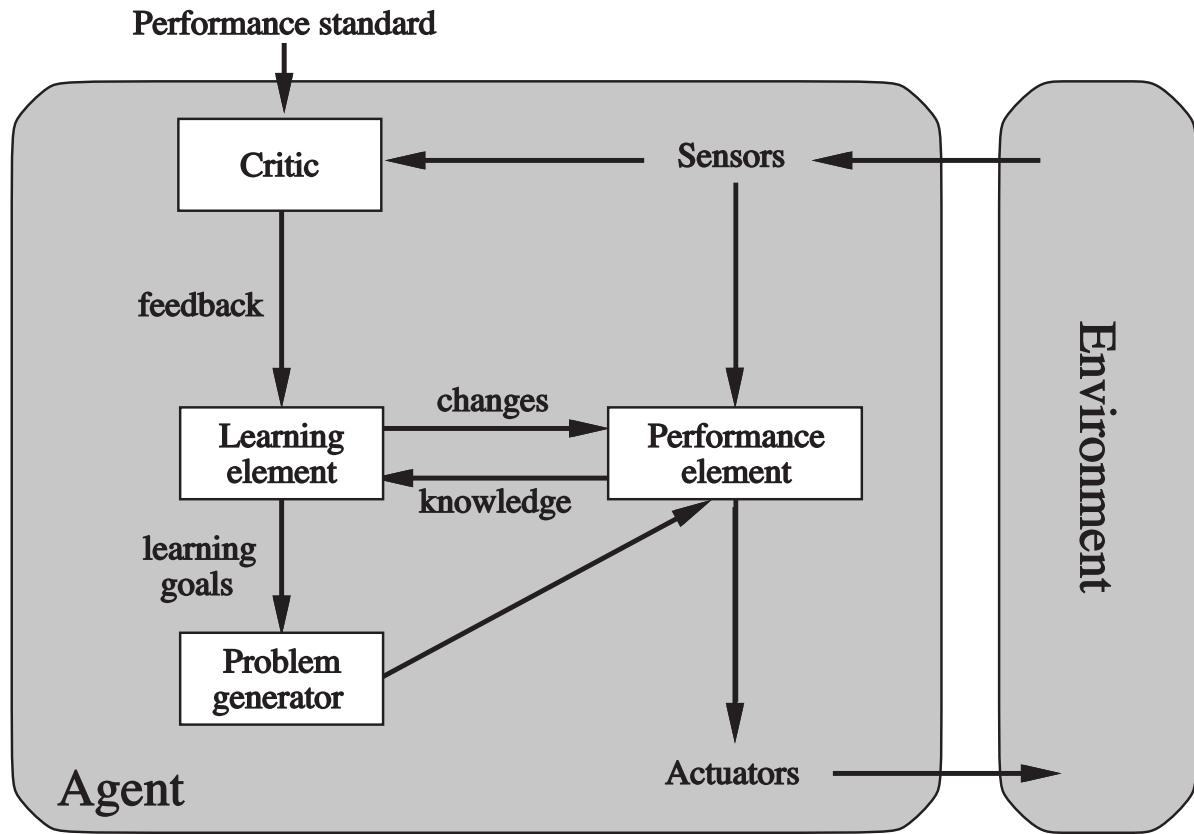
Sufficient to implement complex and rational behaviors, in some situations.

Aren't we missing something?

## Learning agents

What if the environment is **unknown**?

- Learning can be used as a system construction method.
- Expose the agent to reality rather than trying to hardcode reality into the agent's program.
- Learning provides an automated way to modify the agent's internal decision mechanisms to improve its own performance.



The design of the **learning element** is dictated by:

- What type of performance element is used.
- Which functional component is to be learned.
- How that functional component is represented.
- What kind of feedback is available.

Performance element	Component	Representation	Feedback
Alpha–beta search	Eval. fn.	Weighted linear function	Win/loss
Logical agent	Transition model	Successor–state axioms	Outcome
Utility–based agent	Transition model	Dynamic Bayes net	Outcome
Simple reflex agent	Percept–action fn	Neural net	Correct action

# Bayesian learning

# Bayesian learning

Frame **learning** as a Bayesian update of a probability distribution  $\mathbf{P}(H)$  over a hypothesis space, where

- $H$  is the hypothesis variable
- values are  $h_1, h_2, \dots$
- the prior is  $\mathbf{P}(H)$ ,
- $\mathbf{d}$  is the observed data.

Given data, each hypothesis has a posterior probability

$$P(h_i|\mathbf{d}) = \frac{P(\mathbf{d}|h_i)P(h_i)}{P(\mathbf{d})},$$

where  $P(\mathbf{d}|h_i)$  is the likelihood of the hypothesis.

Predictions use a likelihood-weighted average over the hypotheses:

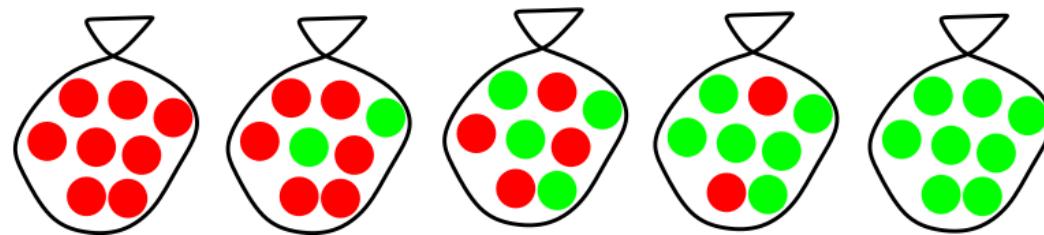
$$P(X|\mathbf{d}) = \sum_i P(X|\mathbf{d}, h_i)P(h_i|\mathbf{d}) = \sum_i P(X|h_i)P(h_i|\mathbf{d})$$

No need to pick one best-guess hypothesis!

## Example

Suppose there are five kinds of bags of candies. Assume a prior  $\mathbf{P}(H)$ :

- $P(h_1) = 0.1$ , with  $h_1$ : 100% cherry candies
- $P(h_2) = 0.2$ , with  $h_2$ : 75% cherry candies + 25% lime candies
- $P(h_3) = 0.4$ , with  $h_3$ : 50% cherry candies + 50% lime candies
- $P(h_4) = 0.2$ , with  $h_4$ : 25% cherry candies + 75% lime candies
- $P(h_5) = 0.1$ , with  $h_5$ : 100% lime candies

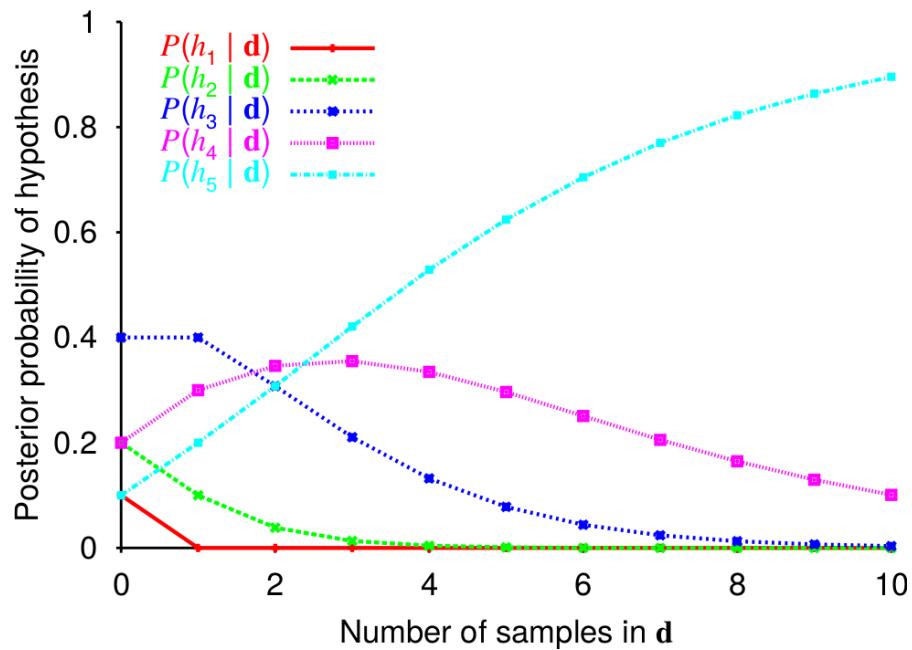


Then we observe candies drawn from some bag:

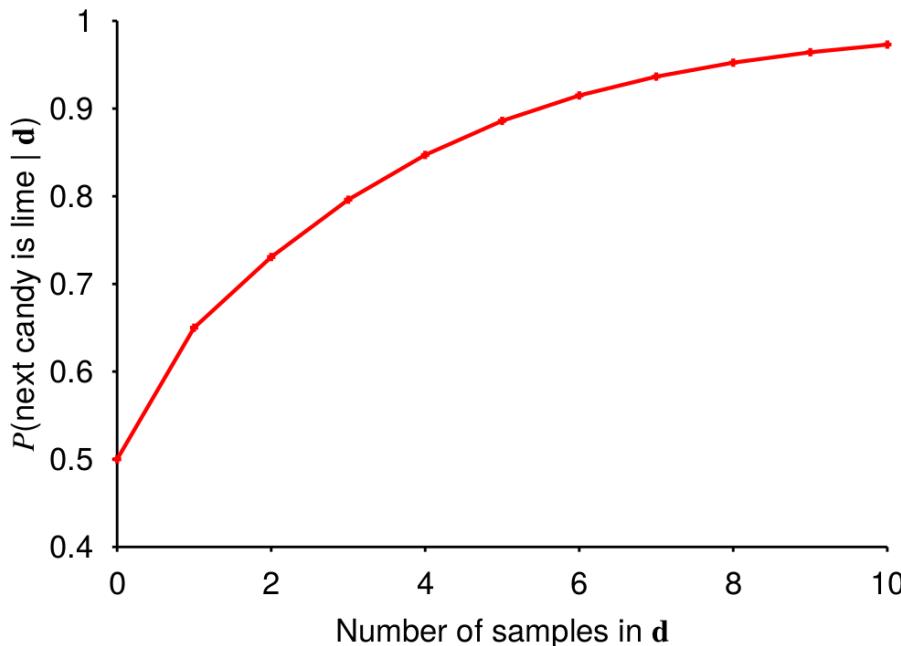


- What kind of bag is it?
- What flavor will the next candy be?

## Posterior probability of hypotheses



## Prediction probability



- This example illustrates the fact that the Bayesian prediction eventually agrees with the true hypothesis.
- The posterior probability of any false hypothesis eventually vanishes (under weak assumptions).

# Maximum a posteriori

Summing over the hypothesis space is often **intractable**.

Instead, **maximum a posteriori** (MAP) estimation consists in using the hypothesis

$$\begin{aligned} h_{\text{MAP}} &= \arg \max_{h_i} P(h_i | \mathbf{d}) \\ &= \arg \max_{h_i} P(\mathbf{d} | h_i) P(h_i) \\ &= \arg \max_{h_i} \log P(\mathbf{d} | h_i) + \log P(h_i) \end{aligned}$$

- Log terms can be viewed as (the negative number of) bits to encode data given hypothesis + bits to encode hypothesis.
  - This is the basic idea of minimum description length learning, i.e., Occam's razor.
- Finding the MAP hypothesis is often much easier than Bayesian learning.
  - It requires solving an optimization problem instead of a large summation problem.

# Maximum likelihood

For large data sets, the prior  $\mathbf{P}(H)$  becomes irrelevant.

In this case, maximum likelihood estimation (MLE) consists in using the hypothesis

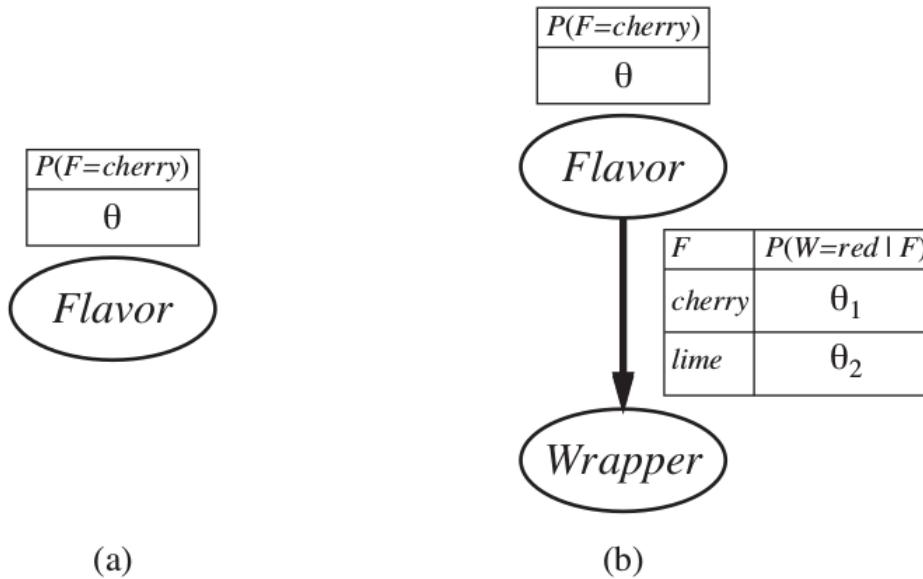
$$h_{\text{MLE}} = \arg \max_{h_i} P(\mathbf{d}|h_i).$$

- Identical to MAP for uniform prior.
- Maximum likelihood estimation is the standard (non-Bayesian) statistical learning method.

## Recipe

- Choose a **parameterized** family of models to describe the data (e.g., a Bayesian network).
- Write down the log-likelihood  $L$  of the parameters  $\theta$ .
- Write down the derivative of the log likelihood of the parameters  $\theta$ .
- Find the parameter values  $\theta^*$  such that the derivatives are zero and check whether the Hessian is negative definite.

# Parameter estimation in Bayesian networks



**Figure 20.2** (a) Bayesian network model for the case of candies with an unknown proportion of cherries and limes. (b) Model for the case where the wrapper color depends (probabilistically) on the candy flavor.

## MLE, case (a)

What is the fraction  $\theta$  of cherry candies?

- Any  $\theta \in [0, 1]$  is possible: continuum of hypotheses  $h_\theta$ .
- $\theta$  is a **parameter** for this binomial family of models.

Suppose we unwrap  $N$  candies, and get  $c$  cherries and  $l = N - c$  limes. These are i.i.d. observations, therefore

$$P(\mathbf{d}|h_\theta) = \prod_{j=1}^N P(d_j|h_\theta) = \theta^c(1-\theta)^l.$$

Maximize this w.r.t.  $\theta$ , which is easier for the log-likelihood:

$$\begin{aligned} L(\mathbf{d}|h_\theta) &= \log P(\mathbf{d}|h_\theta) = c \log \theta + l \log(1-\theta) \\ \frac{dL(\mathbf{d}|h_\theta)}{d\theta} &= \frac{c}{\theta} - \frac{l}{1-\theta} = 0. \end{aligned}$$

Hence  $\theta = \frac{c}{N}$ .

## MLE, case (b)

Red and green wrappers depend probabilistically on flavor. E.g., the likelihood for a cherry candy in green wrapper:

$$\begin{aligned} P(\text{cherry, green} | h_{\theta, \theta_1, \theta_2}) \\ = P(\text{cherry} | h_{\theta, \theta_1, \theta_2}) P(\text{green} | \text{cherry}, h_{\theta, \theta_1, \theta_2}) \\ = \theta(1 - \theta_1). \end{aligned}$$

The likelihood for the data, given  $N$  candies,  $r_c$  red-wrapped cherries,  $g_c$  green-wrapped cherries, etc., is:

$$\begin{aligned} P(\mathbf{d} | h_{\theta, \theta_1, \theta_2}) &= \theta^c (1 - \theta)^l \theta_1^{r_c} (1 - \theta_1)^{g_c} \theta_2^{r_l} (1 - \theta_2)^{g_l} \\ L &= c \log \theta + l \log(1 - \theta) + \\ &\quad r_c \log \theta_1 + g_c \log(1 - \theta_1) + \\ &\quad r_l \log \theta_2 + g_l \log(1 - \theta_2) \end{aligned}$$

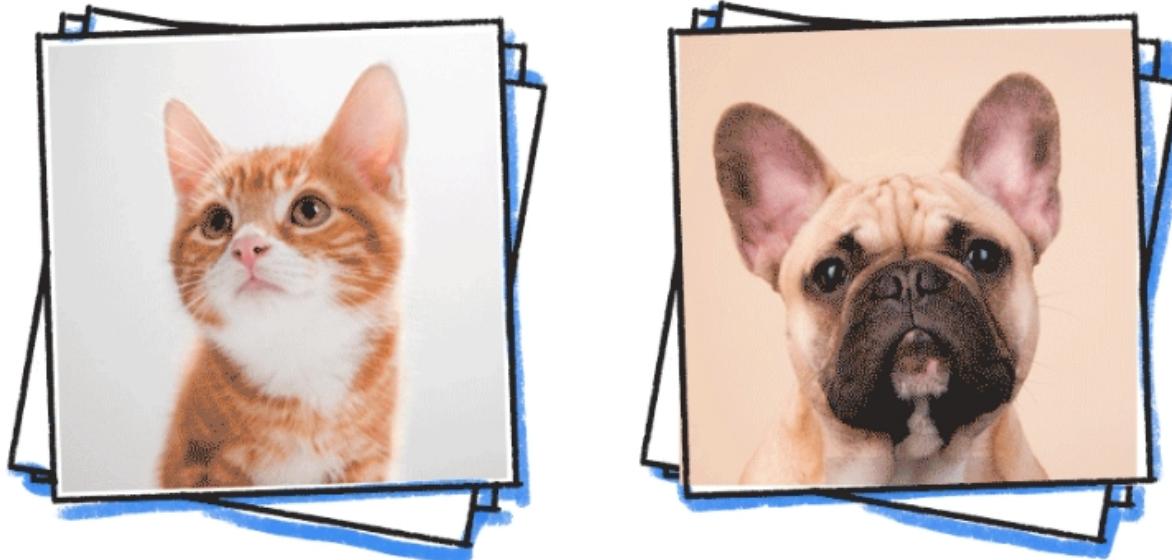
Derivatives of  $L$  contain only the relevant parameter:

$$\frac{\partial L}{\partial \theta} = \frac{c}{\theta} - \frac{l}{1-\theta} = 0 \Rightarrow \theta = \frac{c}{c+l}$$

$$\frac{\partial L}{\partial \theta_1} = \frac{r_c}{\theta_1} - \frac{g_c}{1-\theta_1} = 0 \Rightarrow \theta_1 = \frac{r_c}{r_c + g_c}$$

$$\frac{\partial L}{\partial \theta_2} = \frac{r_l}{\theta_2} - \frac{g_l}{1-\theta_2} = 0 \Rightarrow \theta_2 = \frac{r_l}{r_l + g_l}$$

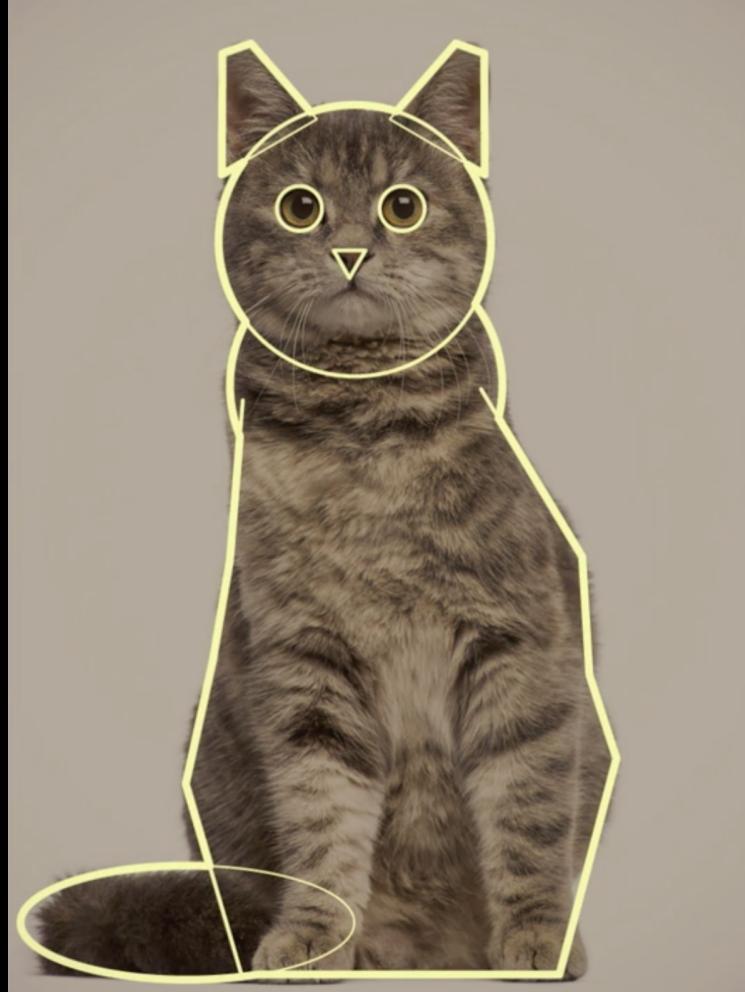
# Machine learning



### Exercise

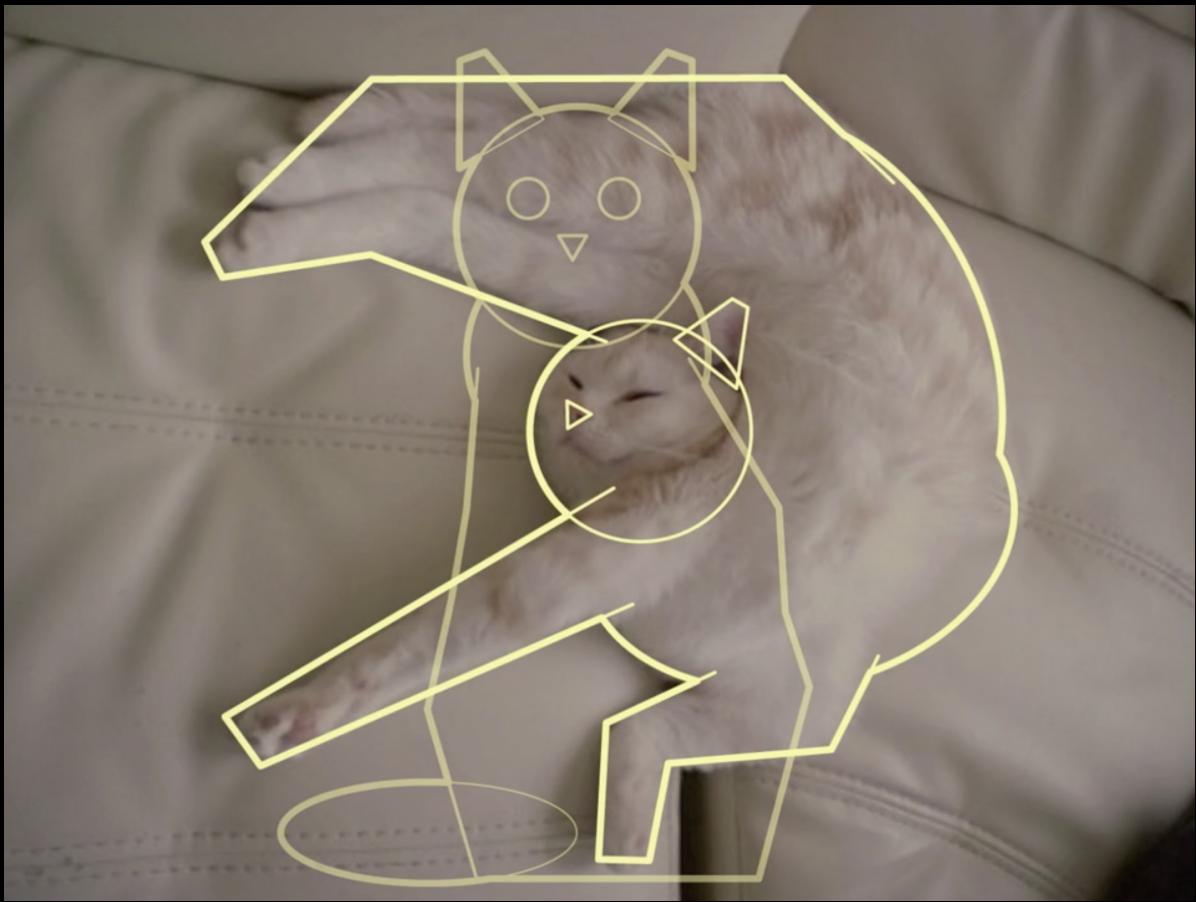
How would you write a computer program that recognizes cats from dogs?

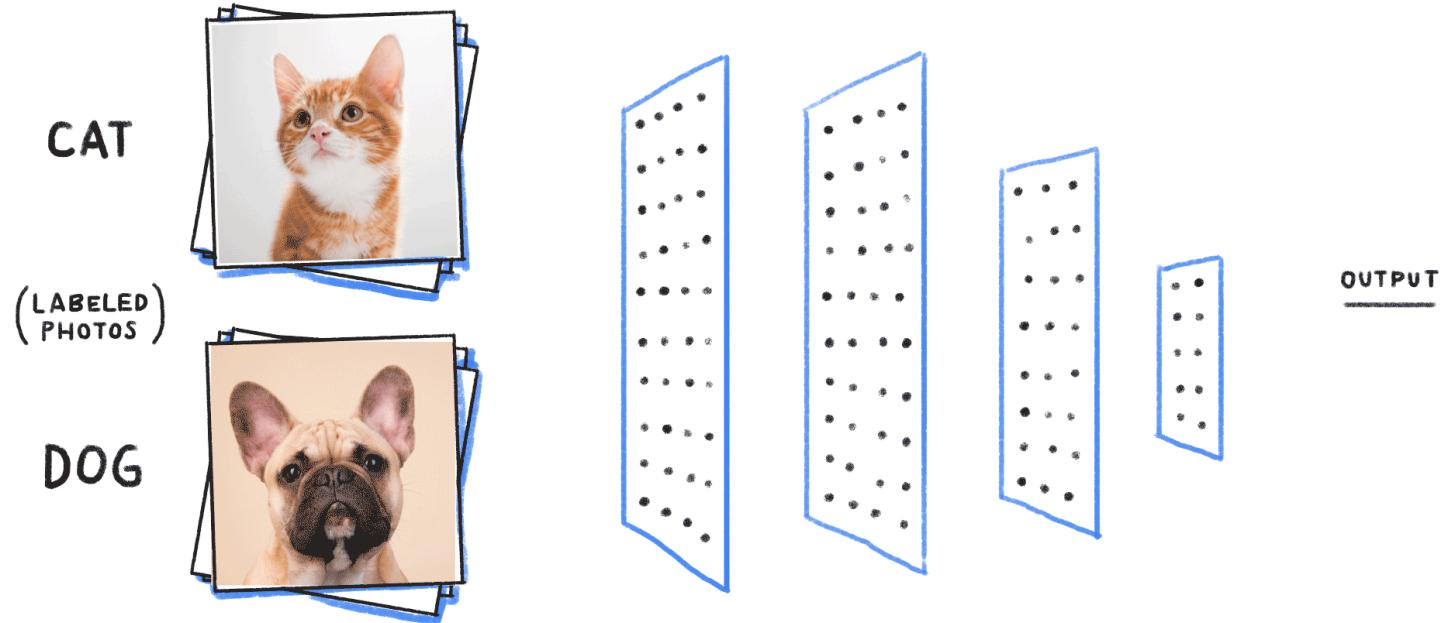




The good old-fashioned approach.







The deep learning approach.

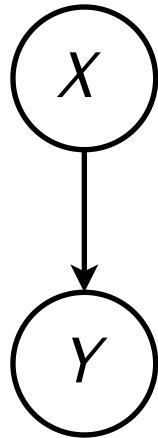
# Problem statement

Let us assume data  $\mathbf{d} \sim p(\mathbf{x}, y)$  of  $N$  example input-output pairs

$$\mathbf{d} = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \dots, (\mathbf{x}_N, y_N)\},$$

where  $\mathbf{x}_i$  are the input data and  $y_i$  was generated by an unknown function  $y_i = f(\mathbf{x}_i)$ .

From this data, we want to find a function  $h \in \mathcal{H}$  that approximates the true function  $f$ .

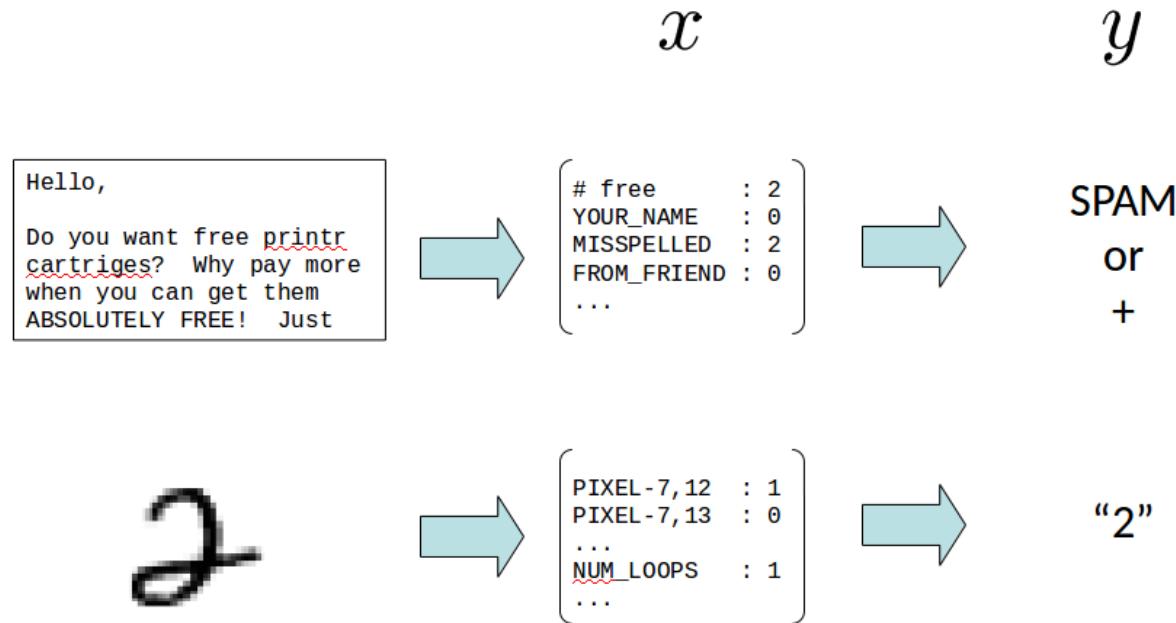


In general,  $f$  will be **stochastic**. In this case,  $y$  is not strictly a function  $x$ , and we wish to learn the conditional  $p(y|x)$ .

Most of supervised learning is actually (approximate) maximum likelihood estimation on (huge) parametric models.

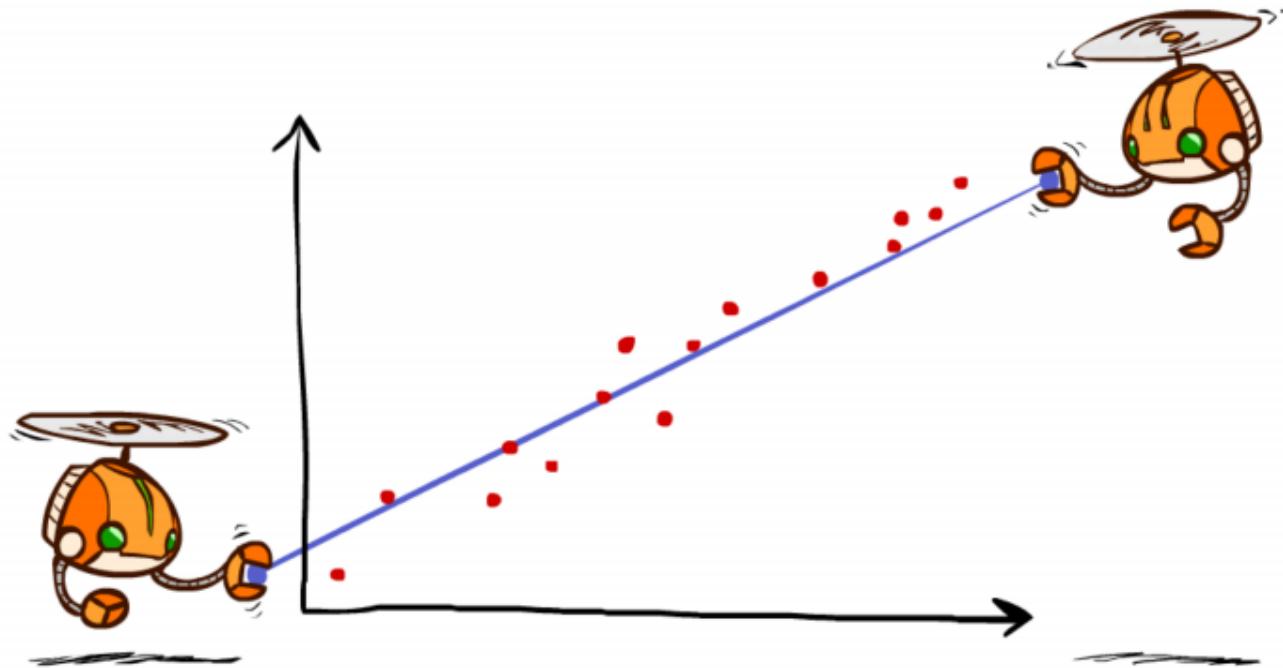
## Feature vectors

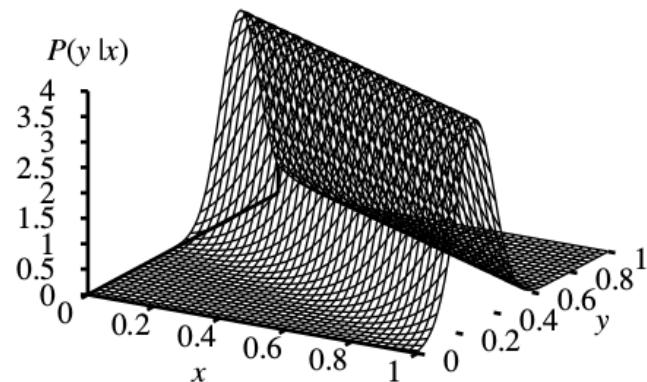
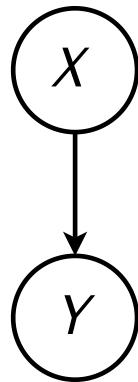
- Input samples  $\mathbf{x} \in \mathbb{R}^d$  are described as real-valued vectors of  $d$  attributes or features values.
- If the data is not originally expressed as real-valued vectors, then it needs to be prepared and transformed to this format.



# Linear regression

Let us first assume that  $y \in \mathbb{R}$ .





Linear regression considers a parameterized linear Gaussian model for its parametric model of  $p(y|\mathbf{x})$ , that is

$$p(y|\mathbf{x}) = \mathcal{N}(y|\mathbf{w}^T \mathbf{x} + b, \sigma^2),$$

where  $\mathbf{w}$  and  $b$  are parameters to determine.

To learn the conditional distribution  $p(y|\mathbf{x})$ , we maximize

$$p(y|\mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y - (\mathbf{w}^T \mathbf{x} + b))^2}{2\sigma^2}\right)$$

w.r.t.  $\mathbf{w}$  and  $b$  over the data  $\mathbf{d} = \{(\mathbf{x}_j, y_j)\}$ .

To learn the conditional distribution  $p(y|\mathbf{x})$ , we maximize

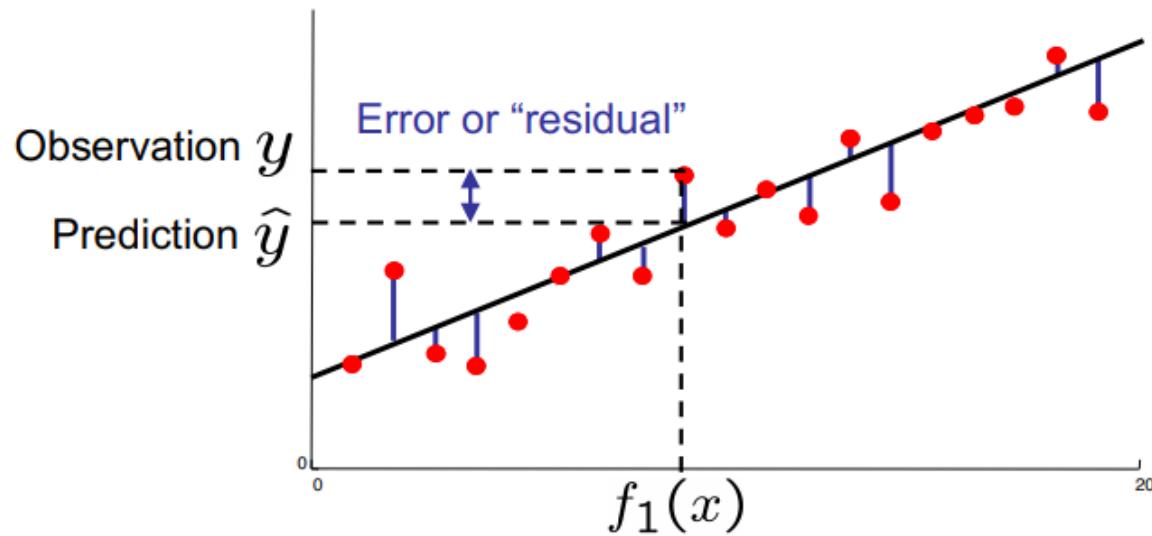
$$p(y|\mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y - (\mathbf{w}^T \mathbf{x} + b))^2}{2\sigma^2}\right)$$

w.r.t.  $\mathbf{w}$  and  $b$  over the data  $\mathbf{d} = \{(\mathbf{x}_j, y_j)\}$ .

By constraining the derivatives of the log-likelihood to 0, we arrive to the problem of minimizing

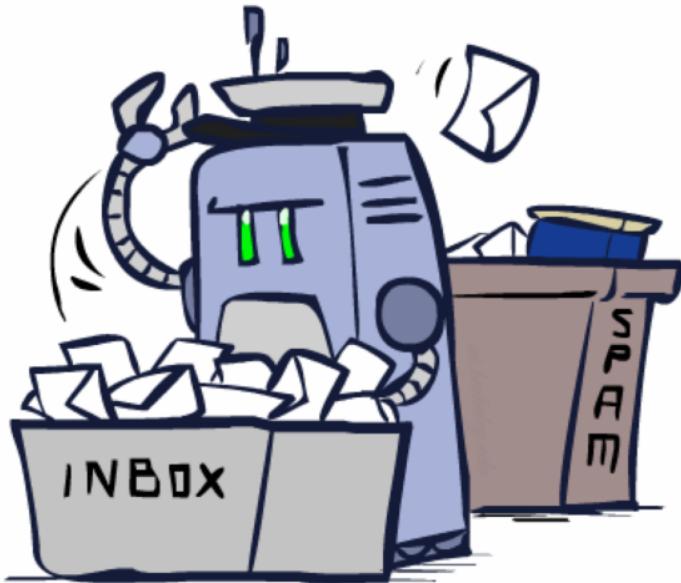
$$\sum_{j=1}^N (y_j - (\mathbf{w}^T \mathbf{x}_j + b))^2.$$

Therefore, minimizing the sum of squared errors corresponds to the MLE solution for a linear fit, assuming Gaussian noise of fixed variance.



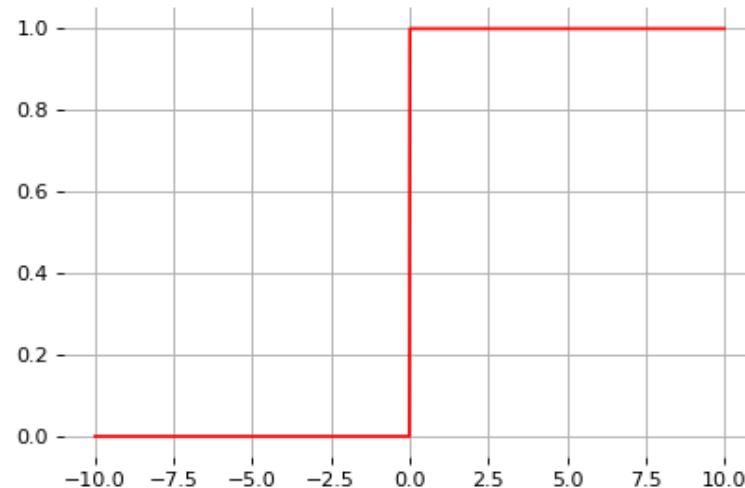
# Linear classification

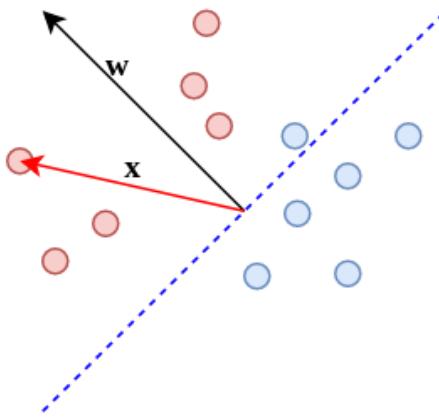
Let us now assume  $y \in \{0, 1\}$ .



The linear classifier model is a squashed linear function of its inputs:

$$h(\mathbf{x}; \mathbf{w}, b) = \text{sign}(\mathbf{w}^T \mathbf{x} + b)$$

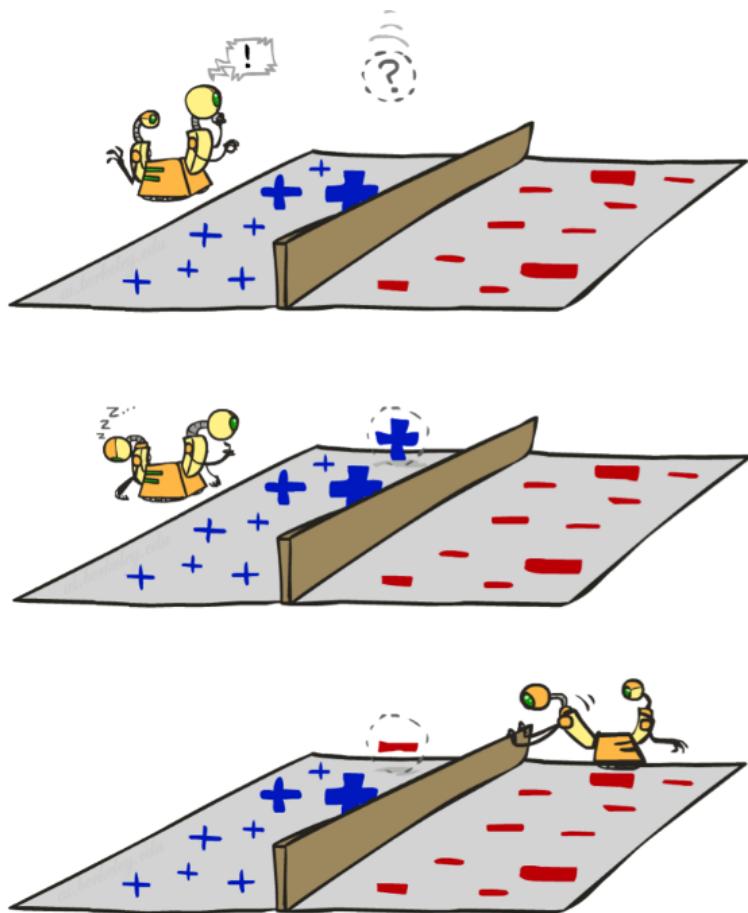
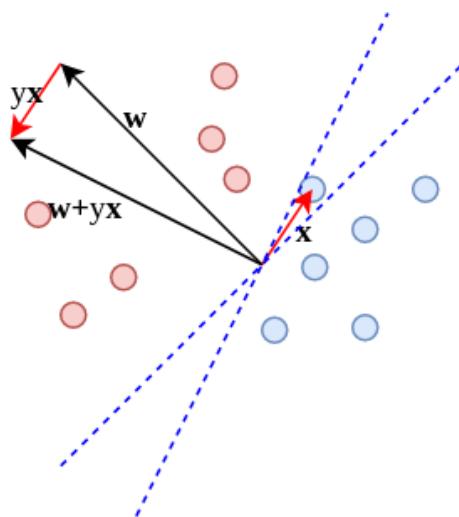




- Without loss of generality, the model can be rewritten without  $b$  as  $h(\mathbf{x}; \mathbf{w}) = \text{sign}(\mathbf{w}^T \mathbf{x})$ , where  $\mathbf{w} \in \mathbb{R}^{d+1}$  and  $\mathbf{x}$  is extended with a dummy element  $x_0 = 1$ .
- Predictions are computed by comparing the feature vector  $\mathbf{x}$  to the weight vector  $\mathbf{w}$ . Geometrically,  $\mathbf{w}^T \mathbf{x}$  corresponds to  $||\mathbf{w}|| ||\mathbf{x}|| \cos(\theta)$ .

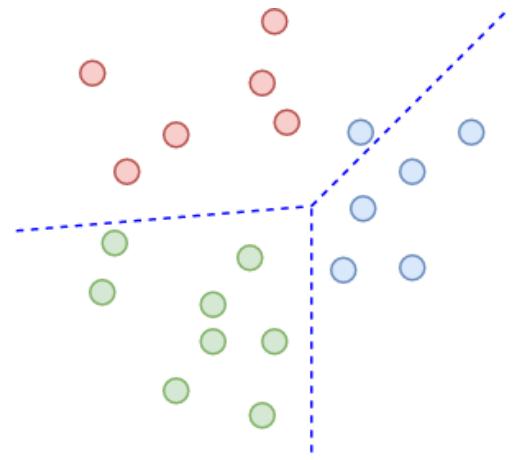
# Perceptron

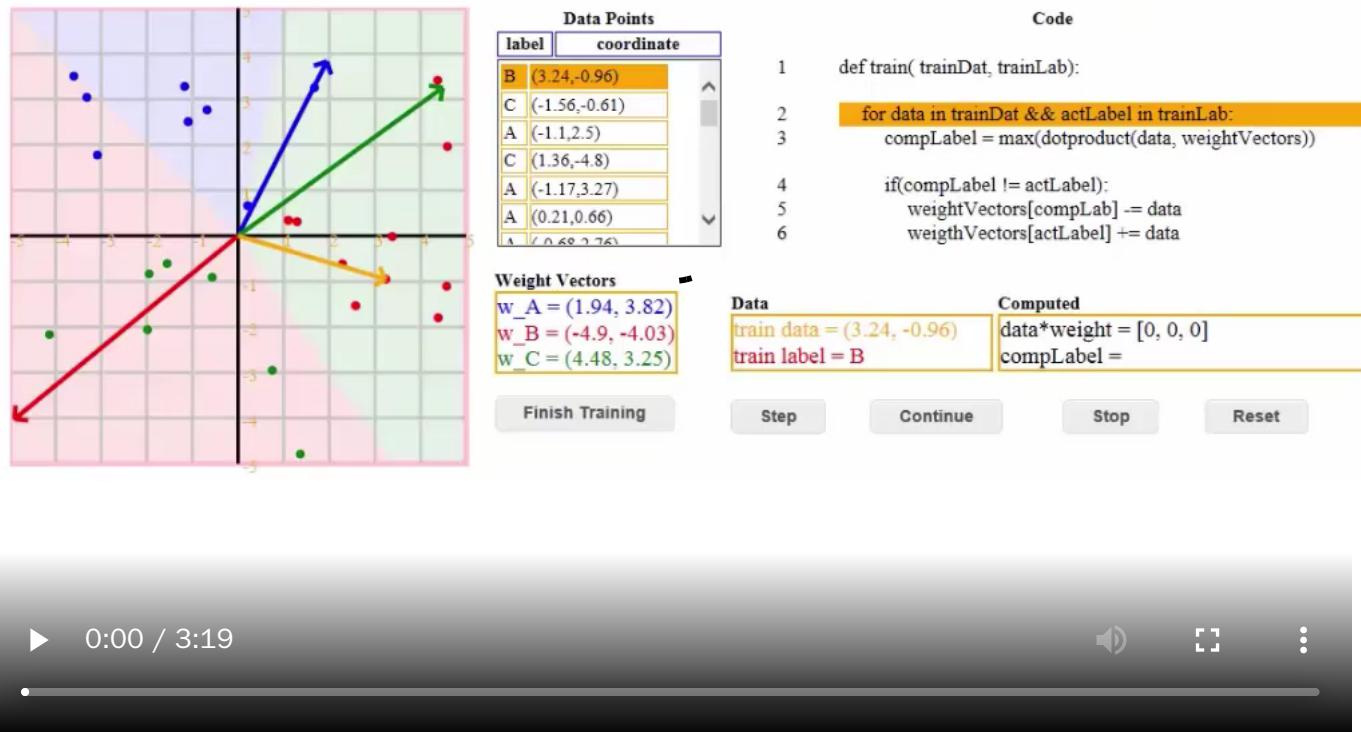
- Start with  $\mathbf{w} = 0$ .
- For each training example  $(\mathbf{x}, y)$ :
  - Classify with current weights:  
 $\hat{y} = \text{sign}(\mathbf{w}^T \mathbf{x})$
  - If  $y = \hat{y}$ , do nothing.
  - Otherwise, update parameters:  
 $\mathbf{w} = \mathbf{w} + y\mathbf{x} - (1 - y)\mathbf{x}$



## Multiclass perceptron

- If we have more than 2 classes, then
  - Define a weight vector  $\mathbf{w}_c$  for each class  $c$ .
  - The activation for class  $c$  is  $\mathbf{w}_c^T \mathbf{x}$ .
- Learning:
  - Start with  $\mathbf{w}_c = \mathbf{0}$  for all  $c$ .
  - For each training example  $(\mathbf{x}, y)$ :
    - Classify with current weights:  $\hat{y} = \arg \max_c \mathbf{w}_c^T \mathbf{x}$
    - If  $y = \hat{y}$ , do nothing.
    - Otherwise, update parameters:
      - $\mathbf{w}_y = \mathbf{w}_y + \mathbf{x}$  (raise score of right answer)
      - $\mathbf{w}_{\hat{y}} = \mathbf{w}_{\hat{y}} - \mathbf{x}$  (lower score of wrong answer).

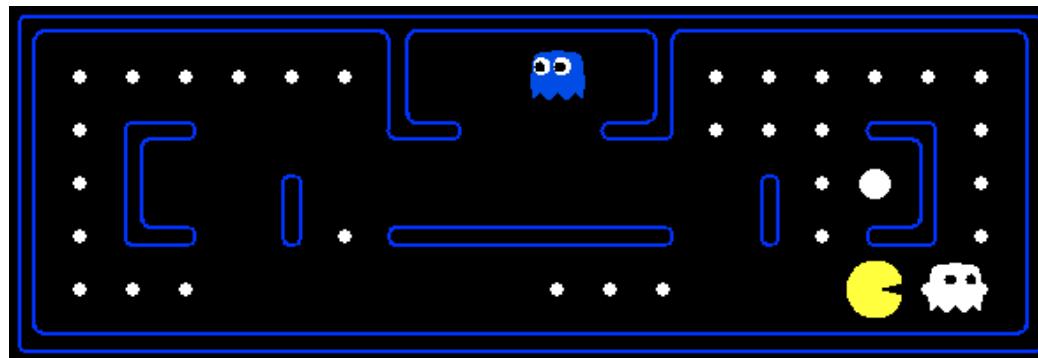


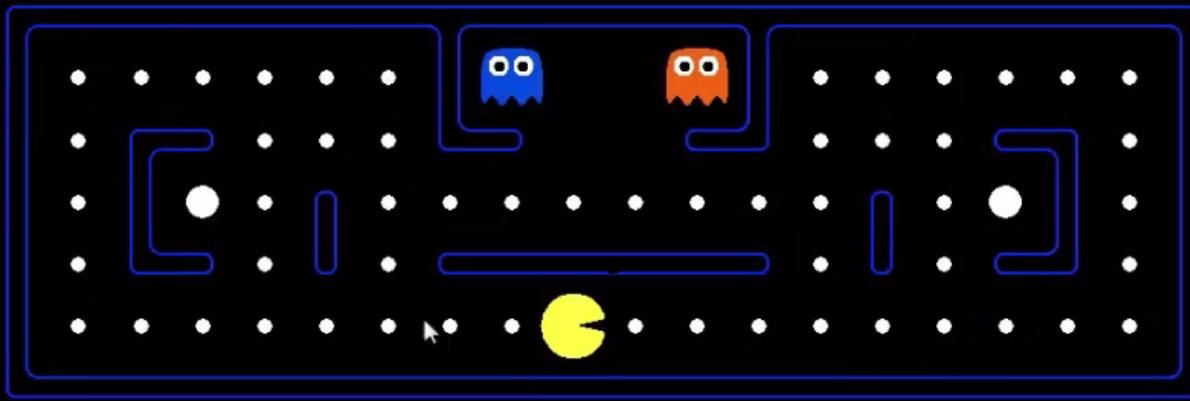


# Apprenticeship

Can we learn to play Pacman only from observations?

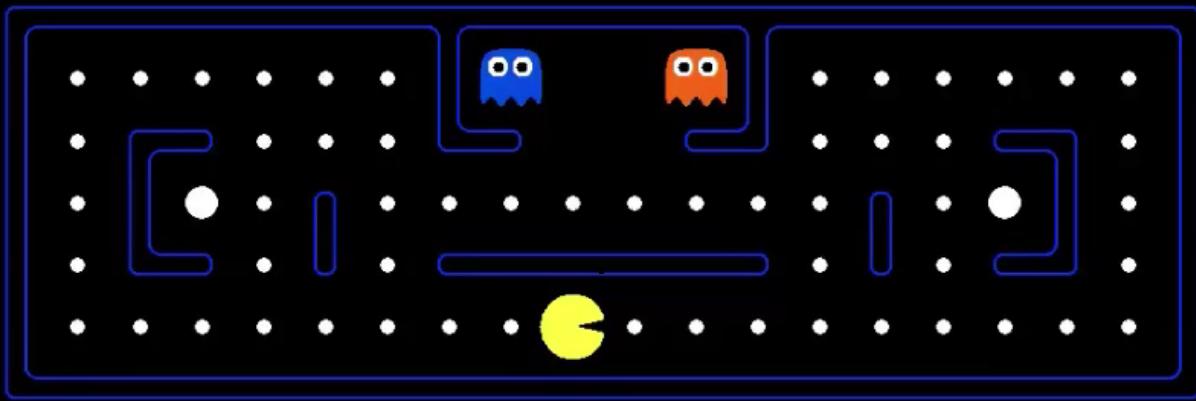
- Feature vectors  $\mathbf{x} = g(\mathbf{s})$  are extracted from the game states  $\mathbf{s}$ . Output values  $y$  corresponds to actions  $a$ .
- State-action pairs  $(\mathbf{x}, y)$  are collected by observing an expert playing.
- We want to learn the actions that the expert would take in a given situation. That is, learn the mapping  $f : \mathbb{R}^d \rightarrow \mathcal{A}$ .
- This is a multiclass classification problem.

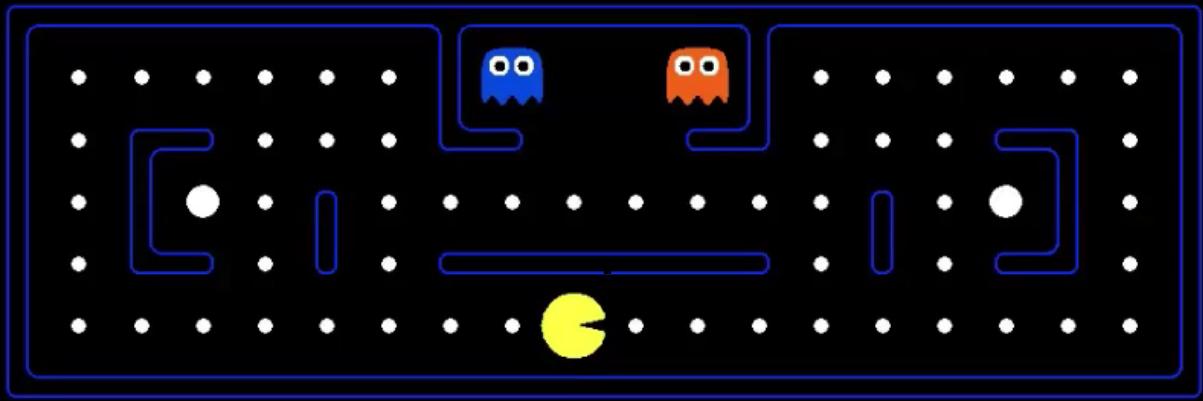




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The Perceptron agent observes a very good Minimax-based agent for two games and updates its weight vectors as data are collected.





**SCORE: 0**

▶ 0:00 / 0:21



After two training episodes, the Perceptron agents plays.  
No more Minimax!

# Deep Learning

(a short introduction)

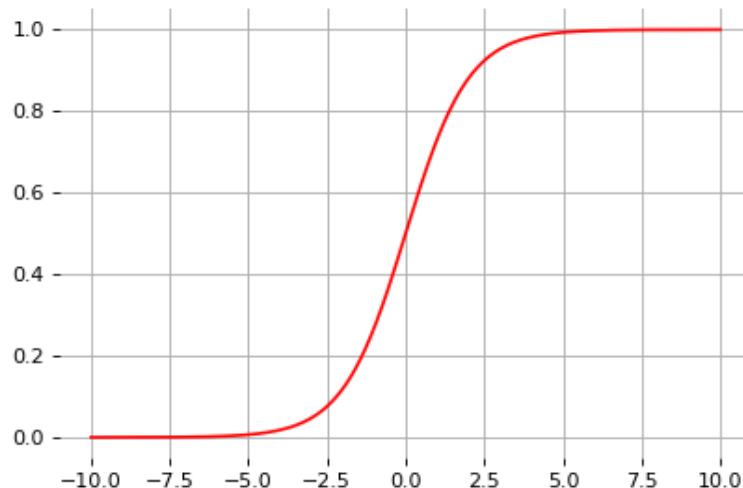
# Logistic regression

An alternative model to the classification model based on the **sign** function is to consider that  $P(Y = 1|\mathbf{x})$  varies smoothly with  $\mathbf{x}$ .

Logistic regression models the conditional as

$$P(Y = 1|\mathbf{x}) = \sigma(\mathbf{w}^T \mathbf{x} + b),$$

where the sigmoid activation function  $\sigma(x) = \frac{1}{1+\exp(-x)}$  looks like a soft heavyside:



Following the principle of maximum likelihood estimation, we have

$$\begin{aligned}
 & \arg \max_{\mathbf{w}, b} P(\mathbf{d} | \mathbf{w}, b) \\
 &= \arg \max_{\mathbf{w}, b} \prod_{\mathbf{x}_i, y_i \in \mathbf{d}} P(Y = y_i | \mathbf{x}_i, \mathbf{w}, b) \\
 &= \arg \max_{\mathbf{w}, b} \prod_{\mathbf{x}_i, y_i \in \mathbf{d}} \sigma(\mathbf{w}^T \mathbf{x}_i + b)^{y_i} (1 - \sigma(\mathbf{w}^T \mathbf{x}_i + b))^{1-y_i} \\
 &= \arg \min_{\mathbf{w}, b} \underbrace{\sum_{\mathbf{x}_i, y_i \in \mathbf{d}} -y_i \log \sigma(\mathbf{w}^T \mathbf{x}_i + b) - (1 - y_i) \log(1 - \sigma(\mathbf{w}^T \mathbf{x}_i + b))}_{\mathcal{L}(\mathbf{w}, b) = \sum_i \ell(y_i, \hat{y}(\mathbf{x}_i; \mathbf{w}, b))}
 \end{aligned}$$

This loss is an instance of the **cross-entropy**

$$H(p, q) = \mathbb{E}_p[-\log q]$$

for  $p = Y | \mathbf{x}_i$  and  $q = \hat{Y} | \mathbf{x}_i$ .

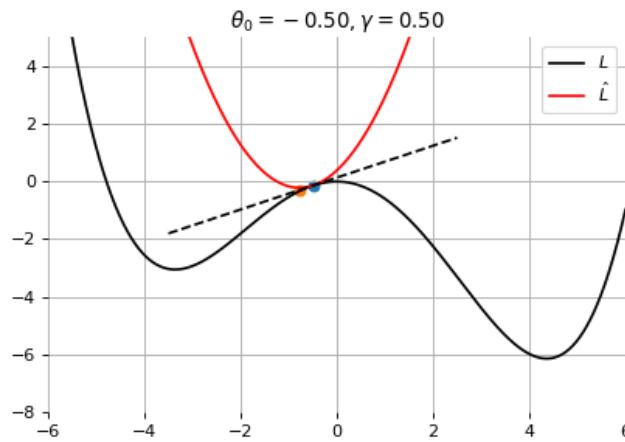
# Gradient descent

Let  $\mathcal{L}(\theta)$  denote a loss function defined over model parameters  $\theta$  (e.g.,  $\mathbf{w}$  and  $b$ ).

To minimize  $\mathcal{L}(\theta)$ , **gradient descent** uses local linear information to iteratively move towards a (local) minimum.

For  $\theta_0 \in \mathbb{R}^d$ , a first-order approximation around  $\theta_0$  can be defined as

$$\hat{\mathcal{L}}(\epsilon; \theta_0) = \mathcal{L}(\theta_0) + \epsilon^T \nabla_{\theta} \mathcal{L}(\theta_0) + \frac{1}{2\gamma} \|\epsilon\|^2.$$



A minimizer of the approximation  $\hat{\mathcal{L}}(\epsilon; \theta_0)$  is given for

$$\begin{aligned}\nabla_{\epsilon} \hat{\mathcal{L}}(\epsilon; \theta_0) &= 0 \\ &= \nabla_{\theta} \mathcal{L}(\theta_0) + \frac{1}{\gamma} \epsilon,\end{aligned}$$

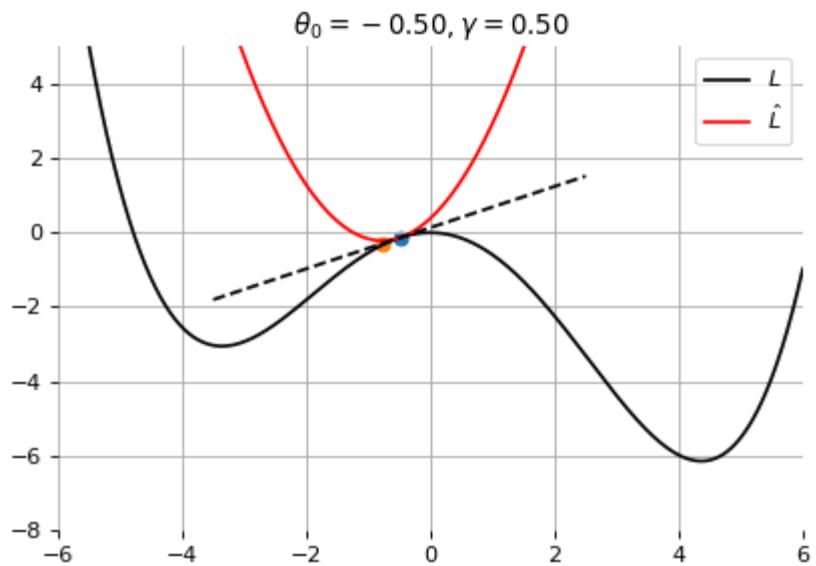
which results in the best improvement for the step  $\epsilon = -\gamma \nabla_{\theta} \mathcal{L}(\theta_0)$ .

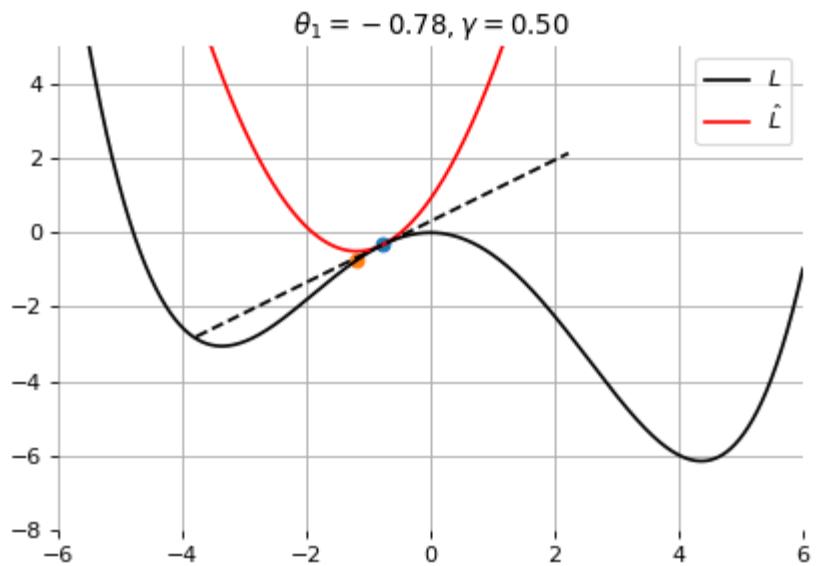
Therefore, model parameters can be updated iteratively using the update rule

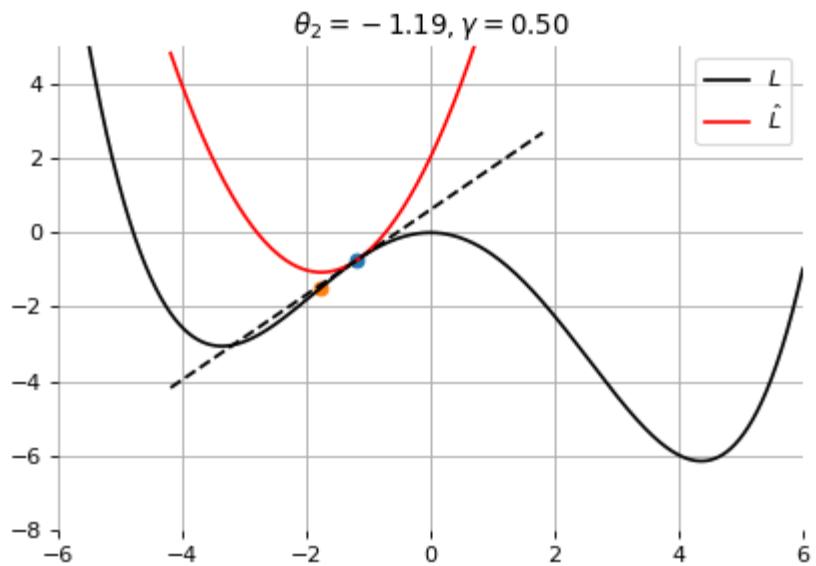
$$\theta_{t+1} = \theta_t - \gamma \nabla_{\theta} \mathcal{L}(\theta_t),$$

where

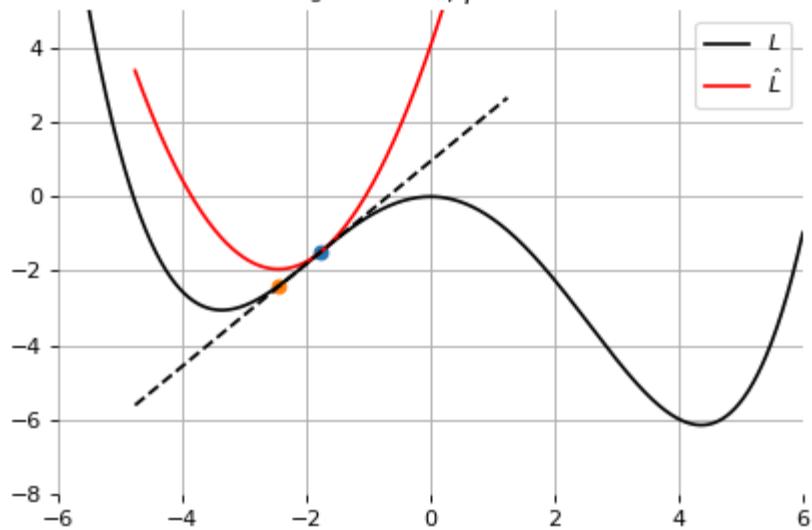
- $\theta_0$  are the initial parameters of the model,
- $\gamma$  is the learning rate.

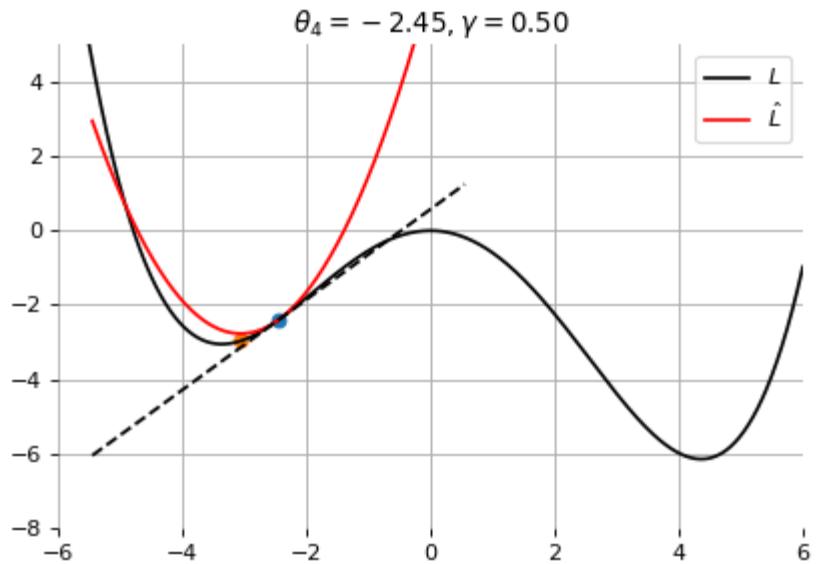


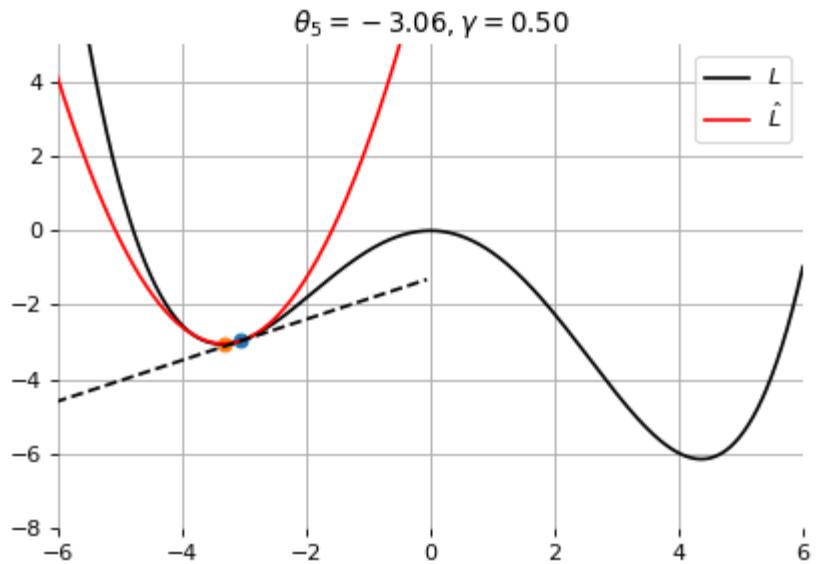


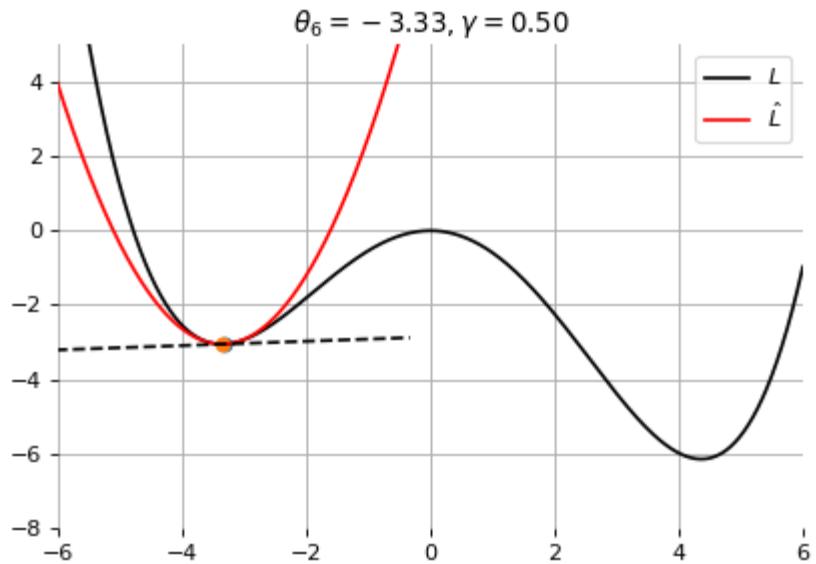


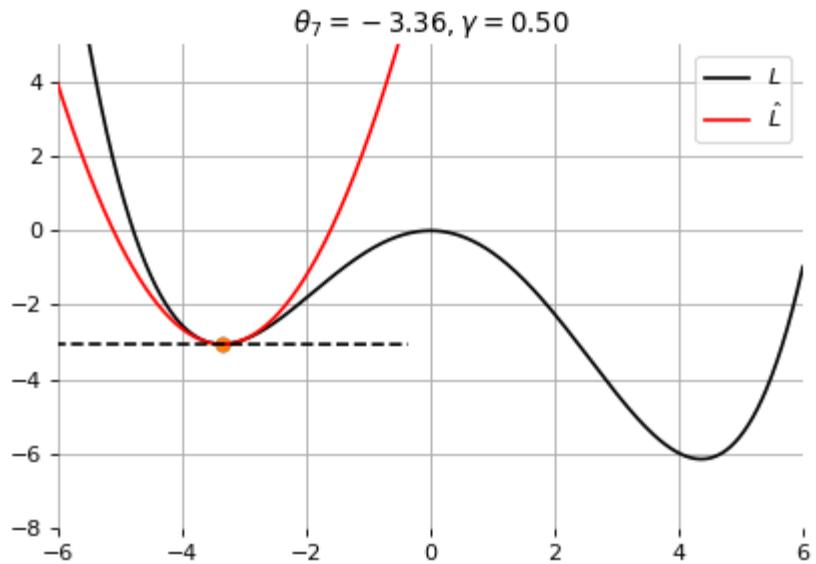
$$\theta_3 = -1.76, \gamma = 0.50$$











# Layers

So far we considered the logistic unit  $h = \sigma(\mathbf{w}^T \mathbf{x} + b)$ , where  $h \in \mathbb{R}$ ,  $\mathbf{x} \in \mathbb{R}^d$ ,  $\mathbf{w} \in \mathbb{R}^d$  and  $b \in \mathbb{R}$ .

These units can be composed **in parallel** to form a **layer** with  $q$  outputs:

$$\mathbf{h} = \sigma(\mathbf{W}^T \mathbf{x} + \mathbf{b})$$

where  $\mathbf{h} \in \mathbb{R}^q$ ,  $\mathbf{x} \in \mathbb{R}^d$ ,  $\mathbf{W} \in \mathbb{R}^{d \times q}$ ,  $\mathbf{b} \in \mathbb{R}^d$  and where  $\sigma(\cdot)$  is upgraded to the element-wise sigmoid function.

# Multi-layer perceptron

Similarly, layers can be composed **in series**, such that:

$$\mathbf{h}_0 = \mathbf{x}$$

$$\mathbf{h}_1 = \sigma(\mathbf{W}_1^T \mathbf{h}_0 + \mathbf{b}_1)$$

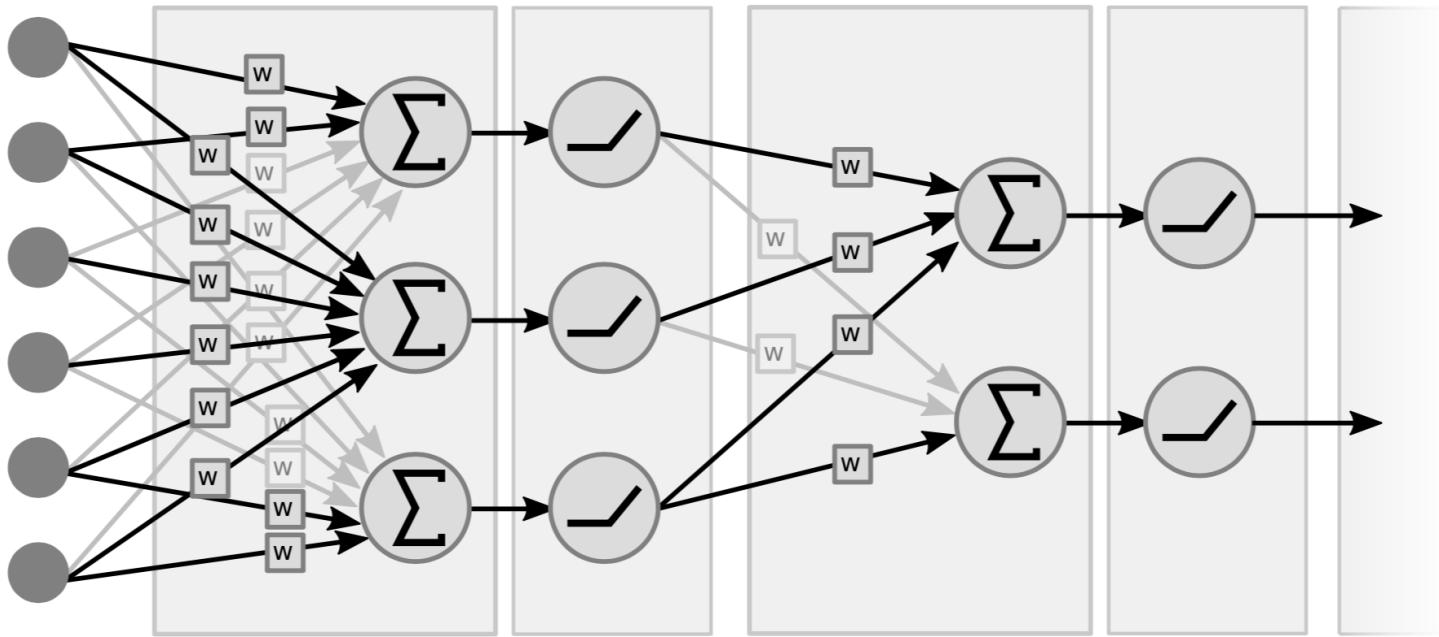
...

$$\mathbf{h}_L = \sigma(\mathbf{W}_L^T \mathbf{h}_{L-1} + \mathbf{b}_L)$$

$$f(\mathbf{x}; \theta) = \mathbf{h}_L$$

where  $\theta$  denotes the model parameters  $\{\mathbf{W}_k, \mathbf{b}_k, \dots | k = 1, \dots, L\}$  and can be determined through gradient descent.

- This model is the **multi-layer perceptron**, also known as the **fully connected feedforward network**.
- Optionally, the last activation  $\sigma$  can be skipped to produce unbounded output values  $\hat{y} \in \mathbb{R}$ .

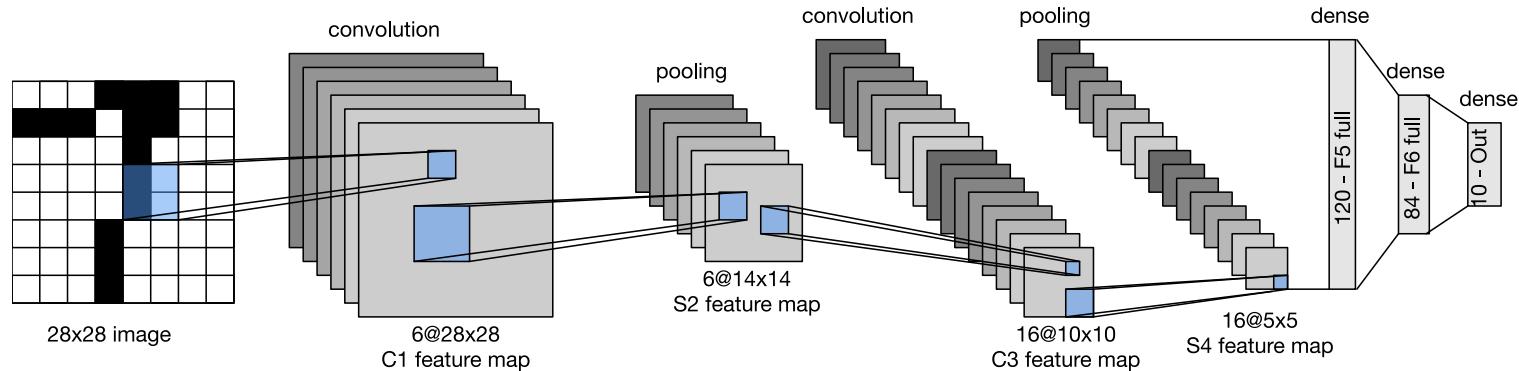


# Convolutional networks

Convolutional neural networks extend fully connected architectures with

- convolutional layers: cross-correlation of the input through learnable kernels.
- pooling layers: reduce the input dimension by pooling (e.g., averaging) clusters of input values.

These are specifically designed for processing **spatially structured** data (e.g., images, sequences) with known shift invariance.





## ConvNet forward pass demo



À regarder ...



Partager



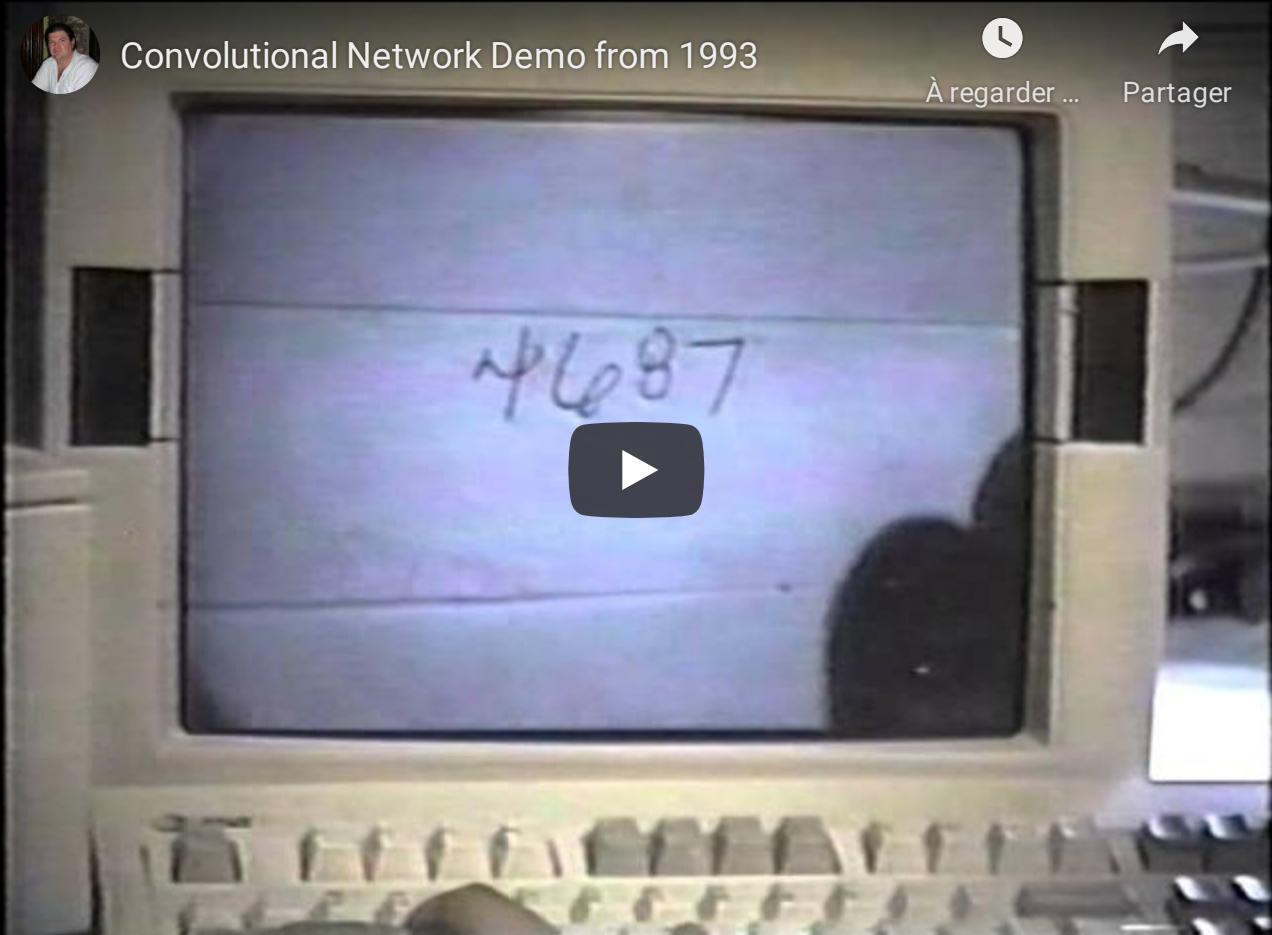
Deep neural networks learn a hierarchical composition of features.



Convolutional Network Demo from 1993



À regarder ... Partager



LeNet-1, LeCun et al, 1993.

# Recurrent networks

When the input is a sequence  $\mathbf{x}_{1:T}$ , the feedforward network can be made **recurrent** by computing a sequence  $\mathbf{h}_{1:T}$  of hidden states, where  $\mathbf{h}_t$  is a function of both  $\mathbf{x}_t$  and the previous hidden states in the sequence.

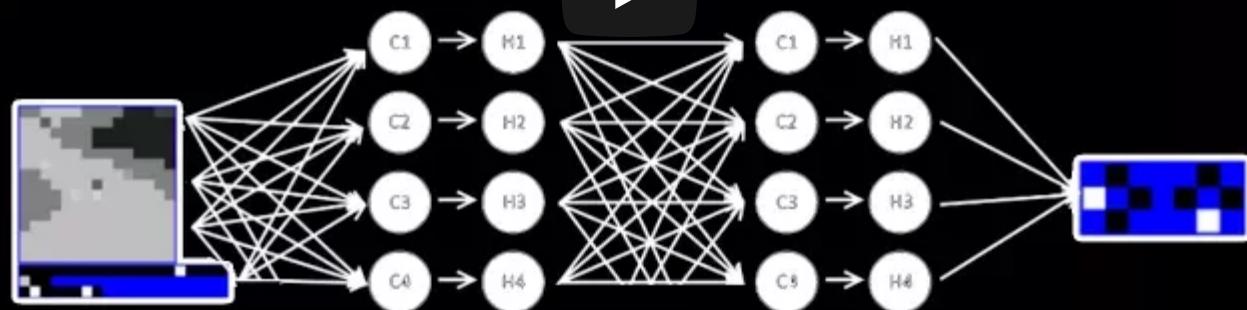
For example,

$$\mathbf{h}_t = \sigma(\mathbf{W}_{xh}^T \mathbf{x} + \mathbf{W}_{hh}^T \mathbf{h}_{t-1} + \mathbf{b}),$$

where  $\mathbf{h}_{t-1}$  is the previous hidden state in the sequence.

Notice how this is similar to filtering and dynamic decision networks:

- $\mathbf{h}_t$  can be viewed as some current belief state;
- $\mathbf{x}_{1:T}$  is a sequence of observations;
- $\mathbf{h}_{t+1}$  is computed from the current belief state  $\mathbf{h}_t$  and the latest evidence  $\mathbf{x}_t$  through some fixed computation (in this case a neural network, instead of being inferred from the assumed dynamics).
- $\mathbf{h}_t$  can also be used to decide on some action, through another network  $f$  such that  $a_t = f(\mathbf{h}_t; \theta)$ .



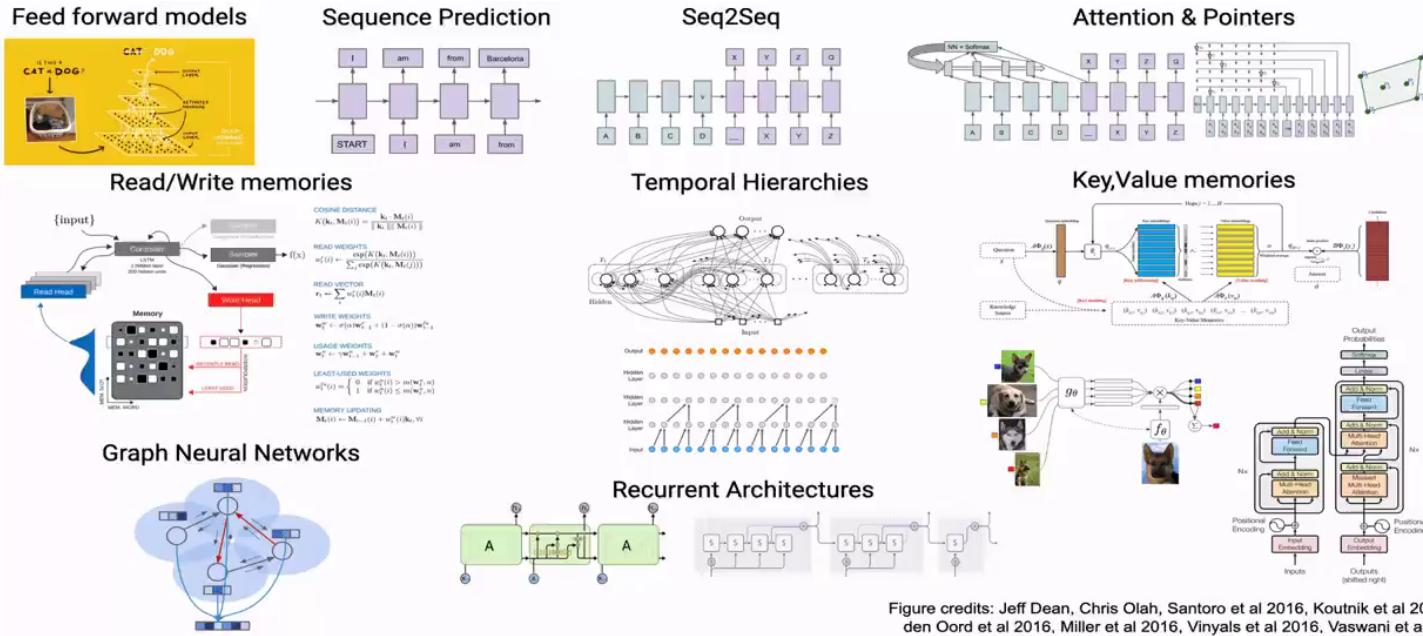
A recurrent network playing Mario Kart.

# Deep Learning as an architectural language



*People are now building a new kind of software by **assembling networks of parameterized functional blocks** and by **training them from examples using some form of gradient-based optimization**.*

Yann LeCun, 2018.



## The toolbox

# AI beyond Pacman

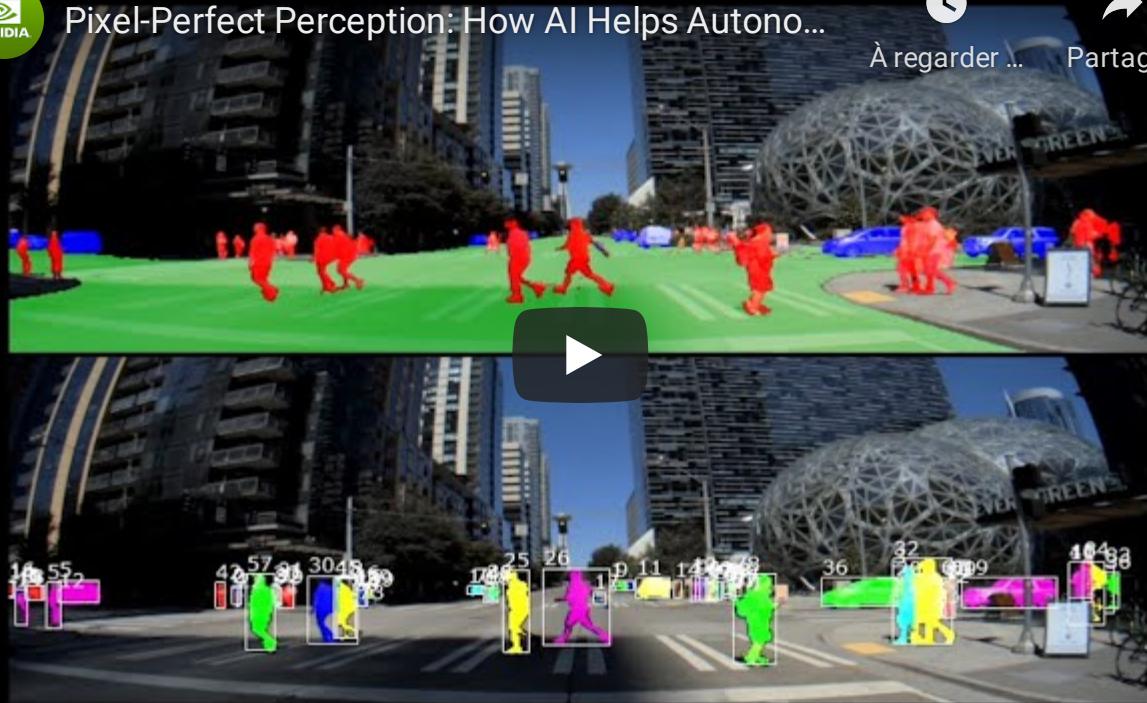


## Pixel-Perfect Perception: How AI Helps Autono...

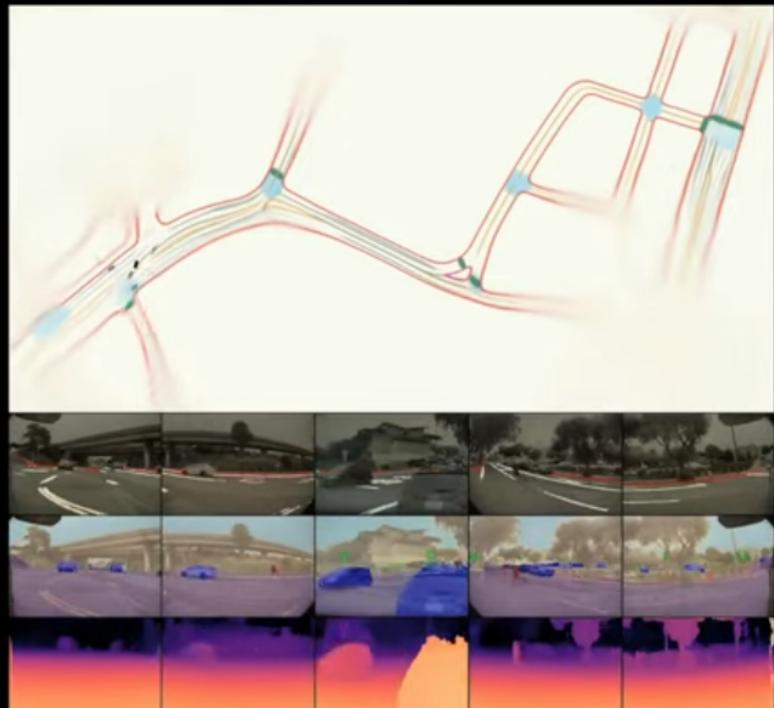
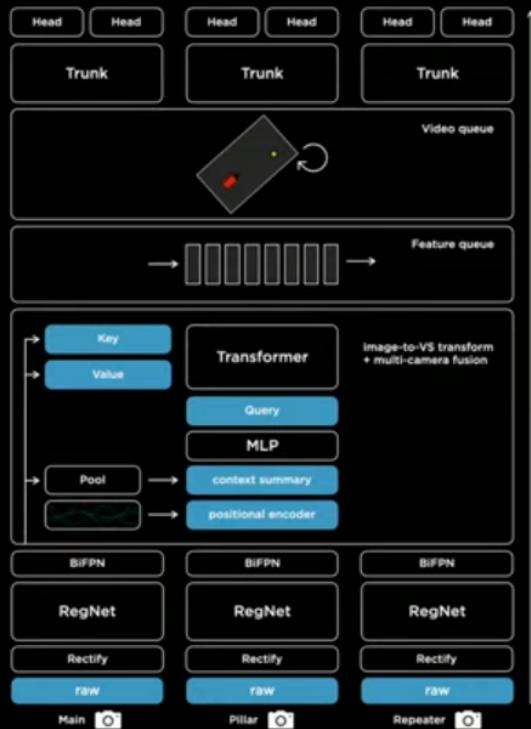


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How AI Helps Autonomous Vehicles See Outside the Box  
(See also other episodes from NVIDIA DRIVE Labs)



Hydranet (Tesla, 2021)



Solving impactful and challenging problems



## Improving Tuberculosis Monitoring with Deep Learning

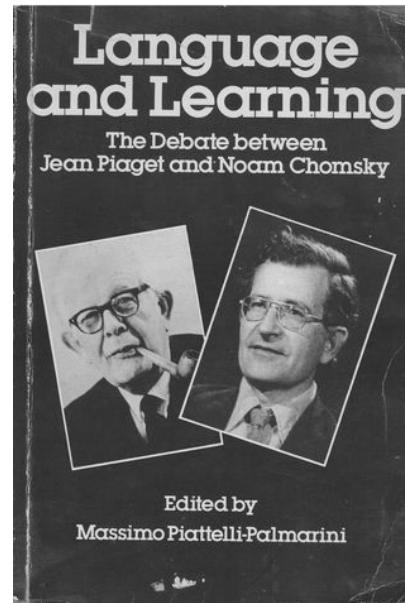
# Summary

- Learning is (supposedly) a key element of intelligence.
- Statistical learning aims at learning probabilistic models (their parameters or structures) automatically from data.
- Supervised learning is used to learn functions from a set of training examples.
  - Linear models are simple predictive models, effective on some tasks but usually insufficiently expressive.
  - Neural networks are defined as a composition of squashed linear models.

The end.

# Chomsky vs. Piaget

- Noam Chomsky's **innatism**:
  - State that humans possess a genetically determined faculty for thought and language.
  - The structures of language and thought are set in motion through interaction with the environment.
- Jean Piaget's **constructivism**:
  - Deny the existence of innate cognitive structure specific for thought and language.
  - Postulate instead all cognitive acquisitions, including language, to be the outcome of a gradual process of construction, i.e., a learning procedure.



## What about AI?

- Should it be a pre-wired efficient machine?
- Or a machine that can learn and improve?
- or maybe a bit of both?



## Artificial Intelligence Debate - Yann LeCun vs. G...



À regarder ...

Partager



The debate continues...

# Unsupervised learning

- Most of the learning performed by animals and humans is **unsupervised**.
  - Without labeled examples nor rewards.
- We learn how the world works through observation:
  - We learn that the world is 3-dimensional.
  - We learn that objects can move independently of each other.
  - We learn **object permanence**.
  - We learn to predict what the world will look one second or one hour from now.



## OBJECT PERMANENCE

Object permanence in infants (part 1)



Object permanence in infants (part 2)



In animals

## Common sense

We build a model of the world through **predictive unsupervised learning**.

- This predictive model gives us **common sense**.
- Unsupervised learning discovers regularities in the world.

If I say: "Bernard picks up his bag and leaves the room".

You can **infer**:

- Bernard stood up, extended his arm to pick the bag, walked towards the door, opened the door, walked out.
- He and his bag are not in the room anymore.
- He probably did not dematerialized or flied out.



How do we do that?

We have no clue! (mostly)