

Deep Learning

Lecture 5: Recurrent neural networks

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Today

How to make sense of [sequential data](#)?

- Recurrent neural networks
- Applications
- Differentiable computers

Many real-world problems require to process a signal with a **sequence** structure.

- Sequence classification:

- sentiment analysis
- activity/action recognition
- DNA sequence classification
- action selection

- Sequence synthesis:

- text synthesis
- music synthesis
- motion synthesis

- Sequence-to-sequence translation:

- speech recognition
- text translation
- part-of-speech tagging

Given a set \mathcal{X} , if $S(\mathcal{X})$ denotes the set of sequences of elements from \mathcal{X} ,

$$S(\mathcal{X}) = \cup_{t=1}^{\infty} \mathcal{X}^t,$$

then we formally define:

Sequence classification

$$f : S(\mathcal{X}) \rightarrow \{1, \dots, C\}$$

Sequence synthesis

$$f : \mathbb{R}^d \rightarrow S(\mathcal{X})$$

Sequence-to-sequence translation

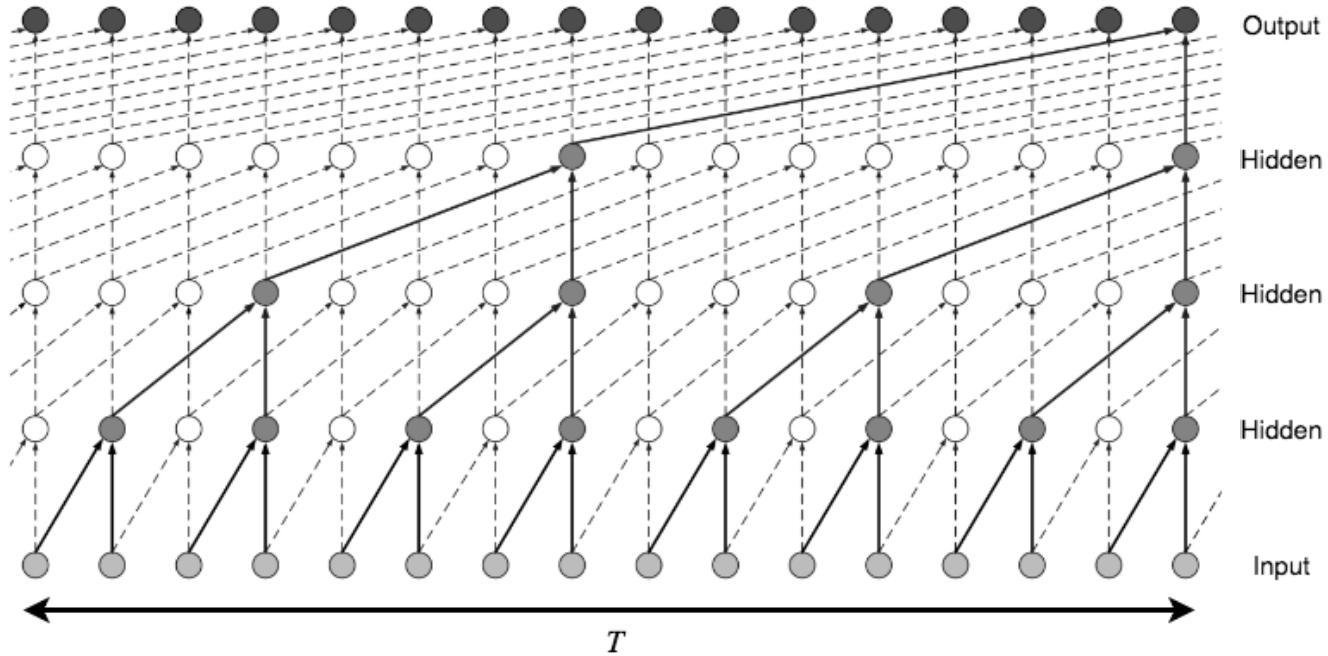
$$f : S(\mathcal{X}) \rightarrow S(\mathcal{Y})$$

In the rest of the slides, we consider only time-indexed signal, although it generalizes to arbitrary sequences.

Temporal convolutions

One of the simplest approach to sequence processing is to use **temporal convolutional networks** (TCNs).

- TCNs correspond to standard 1D convolutional networks.
- They process input sequences as sequences of the **maximum possible length**.



Complexity:

- Increasing the window size T makes the required number of layers grow as $O(\log T)$.
- Thanks to dilated convolutions, the model size is $O(\log T)$.
- The memory footprint and computation are $O(T \log T)$.

Table 1. Evaluation of TCNs and recurrent architectures on synthetic stress tests, polyphonic music modeling, character-level language modeling, and word-level language modeling. The generic TCN architecture outperforms canonical recurrent networks across a comprehensive suite of tasks and datasets. Current state-of-the-art results are listed in the supplement. ^h means that higher is better. ^ℓ means that lower is better.

Sequence Modeling Task	Model Size (\approx)	Models			
		LSTM	GRU	RNN	TCN
Seq. MNIST (accuracy ^h)	70K	87.2	96.2	21.5	99.0
Permuted MNIST (accuracy)	70K	85.7	87.3	25.3	97.2
Adding problem $T=600$ (loss ^ℓ)	70K	0.164	5.3e-5	0.177	5.8e-5
Copy memory $T=1000$ (loss)	16K	0.0204	0.0197	0.0202	3.5e-5
Music JSB Chorales (loss)	300K	8.45	8.43	8.91	8.10
Music Nottingham (loss)	1M	3.29	3.46	4.05	3.07
Word-level PTB (perplexity ^ℓ)	13M	78.93	92.48	114.50	88.68
Word-level Wiki-103 (perplexity)	-	48.4	-	-	45.19
Word-level LAMBADA (perplexity)	-	4186	-	14725	1279
Char-level PTB (bpc ^ℓ)	3M	1.36	1.37	1.48	1.31
Char-level text8 (bpc)	5M	1.50	1.53	1.69	1.45

Recurrent neural networks

When the input is a sequence $\mathbf{x} \in S(\mathbb{R}^p)$ of variable length $T(\mathbf{x})$, a standard approach is to use a recurrent model which maintains a **recurrent state** $\mathbf{h}_t \in \mathbb{R}^q$ updated at each time step t .

Formally, for $t = 1, \dots, T(\mathbf{x})$,

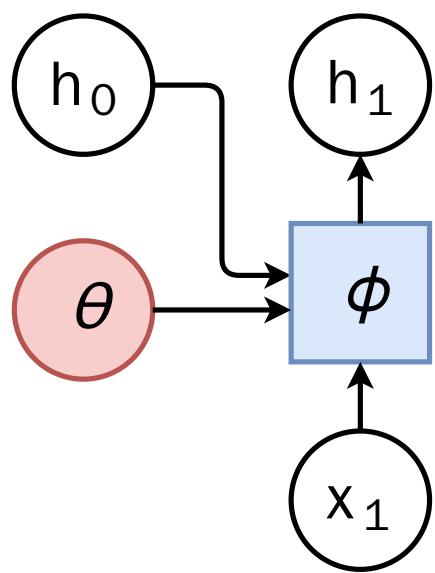
$$\mathbf{h}_t = \phi(\mathbf{x}_t, \mathbf{h}_{t-1}; \theta),$$

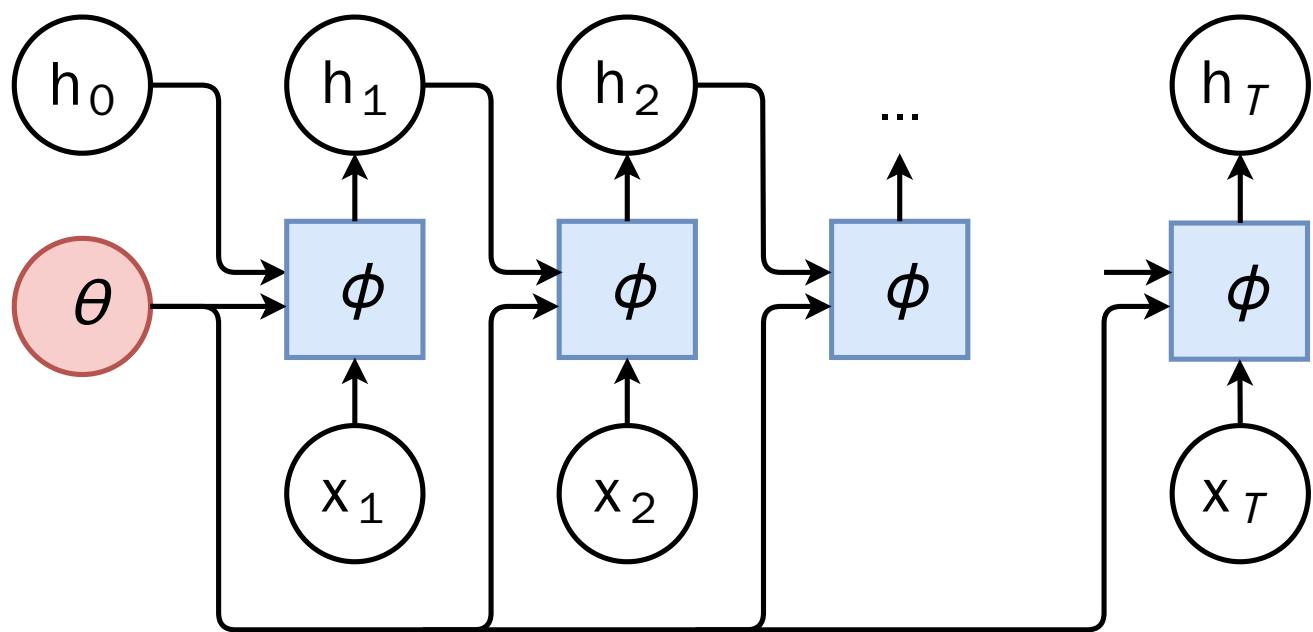
where $\phi : \mathbb{R}^p \times \mathbb{R}^q \rightarrow \mathbb{R}^q$ and $\mathbf{h}_0 \in \mathbb{R}^q$.

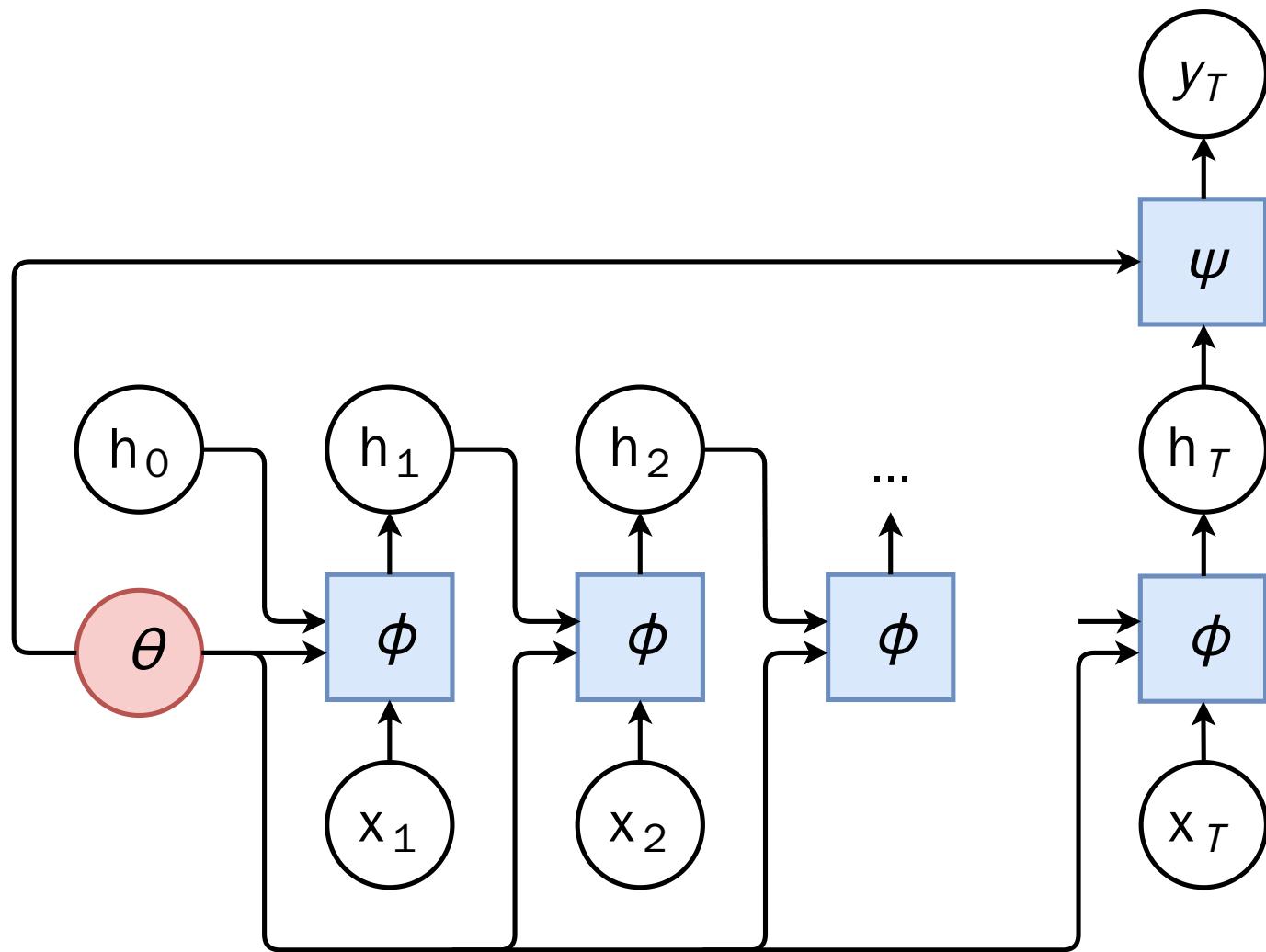
Predictions can be computed at any time step t from the recurrent state,

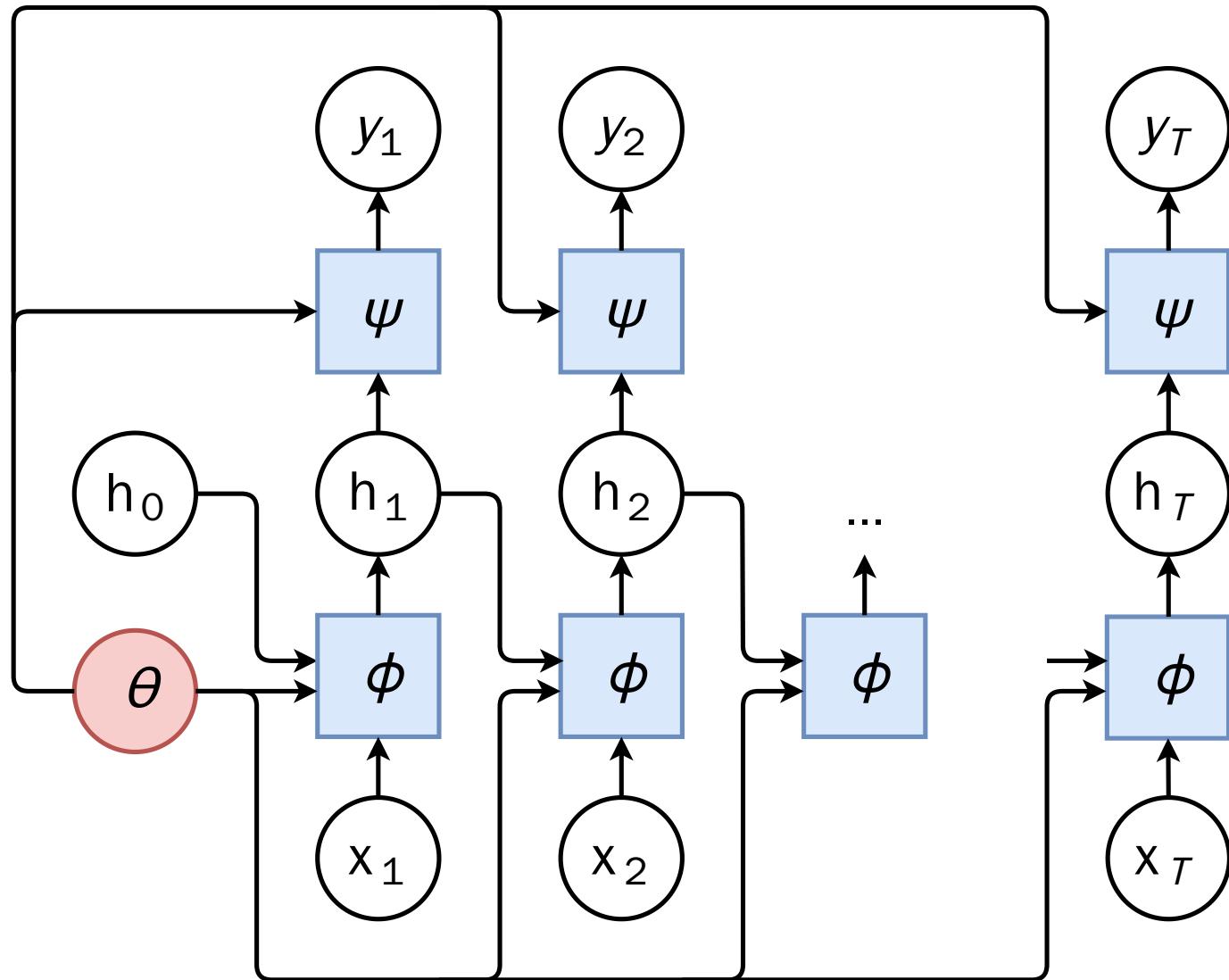
$$y_t = \psi(\mathbf{h}_t; \theta),$$

with $\psi : \mathbb{R}^q \rightarrow \mathbb{R}^C$.



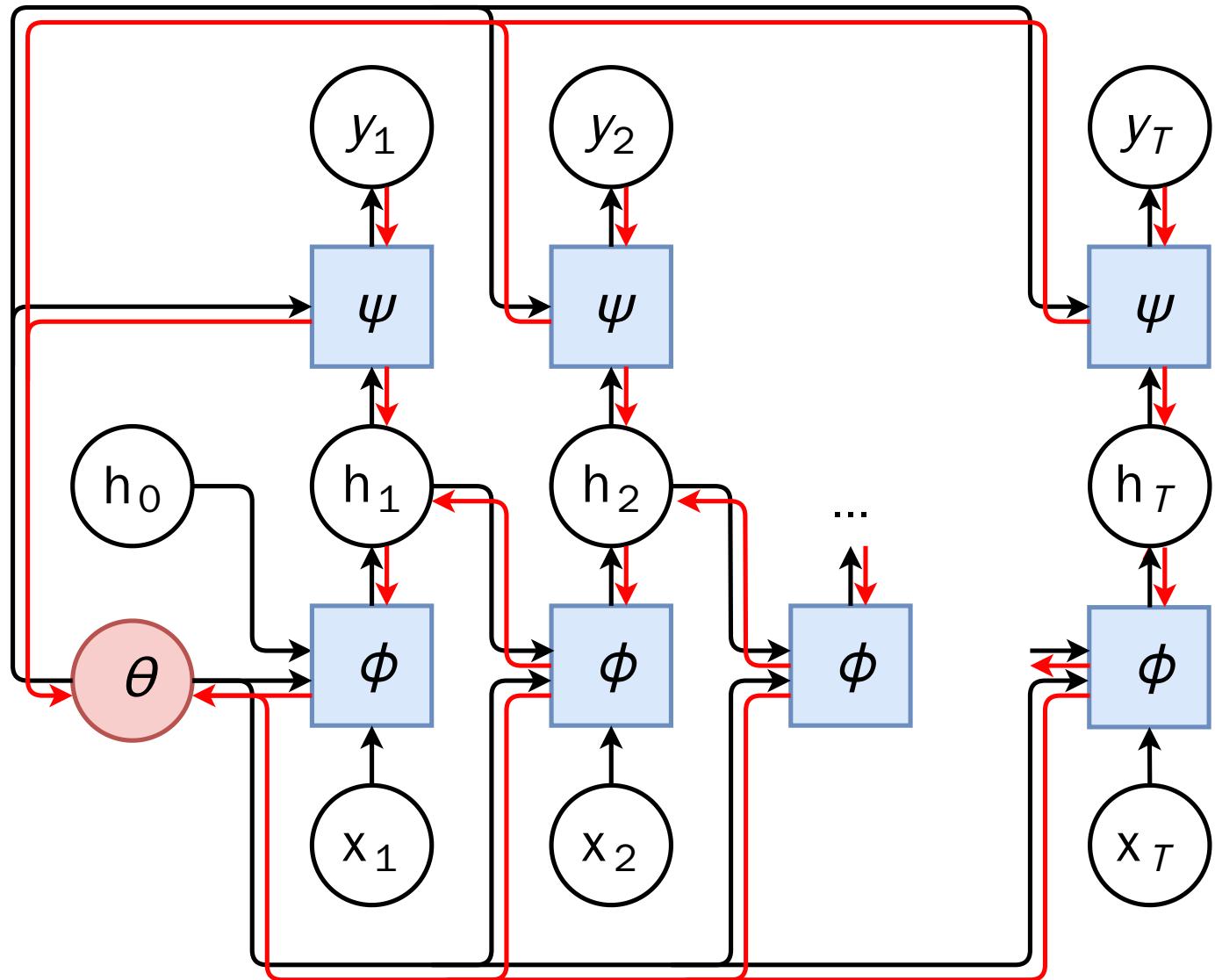






Even though the number of steps T depends on \mathbf{x} , this is a standard computational graph, and automatic differentiation can deal with it as usual.

In the case of recurrent neural networks, this is referred to as **backpropagation through time**.



Elman networks

Elman networks consist of ϕ and ψ defined as primitive neuron units, such as logistic regression units. That is,

$$\mathbf{h}_t = \sigma_h (\mathbf{W}_{xh}^T \mathbf{x}_t + \mathbf{W}_{hh}^T \mathbf{h}_{t-1} + \mathbf{b}_h)$$

$$y_t = \sigma_y (\mathbf{W}_y^T \mathbf{h}_t + b_y)$$

$$\mathbf{W}_{xh}^T \in \mathbb{R}^{p \times q}, \mathbf{W}_{hh}^T \in \mathbb{R}^{q \times q}, \mathbf{b}_h \in \mathbb{R}^q, b_y \in \mathbb{R}, \mathbf{h}_0 = 0$$

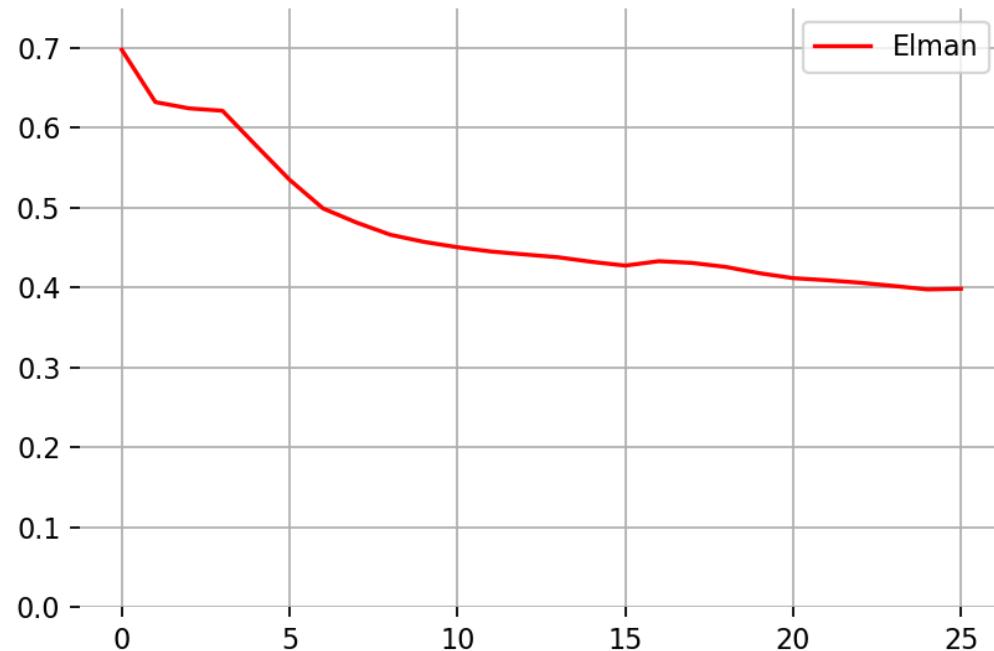
where σ_h and σ_y are non-linear activation functions, such as the sigmoid function, tanh or ReLU.

Example

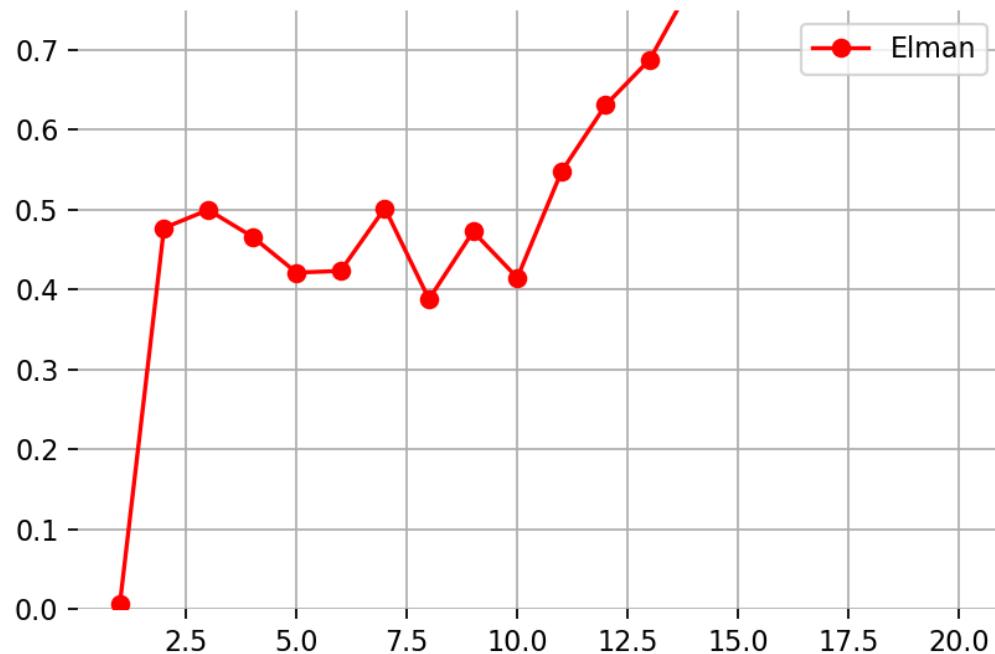
Can a recurrent network learn to tell whether a variable-length sequence is a **palindrome**?

x	y
(1, 2, 3, 2, 1)	1
(2, 1, 2)	1
(3, 4, 1, 2)	0
(0)	1
(1, 4)	0

For training, we will use sequences of random sizes, from 1 to 10.



Epoch vs. cross-entropy.



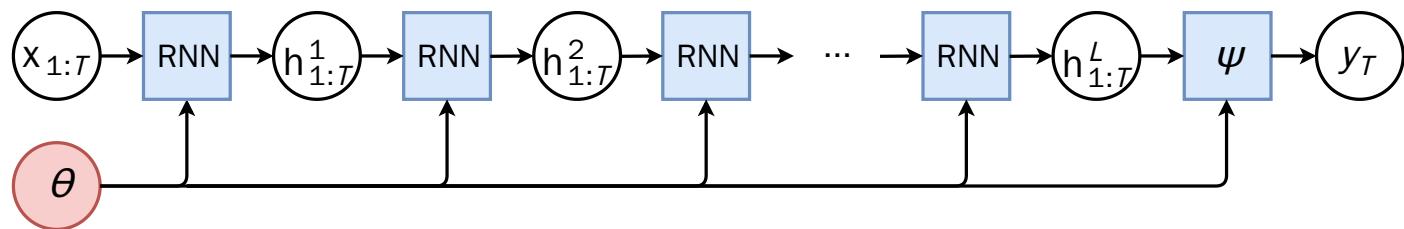
Sequence length vs. cross-entropy.

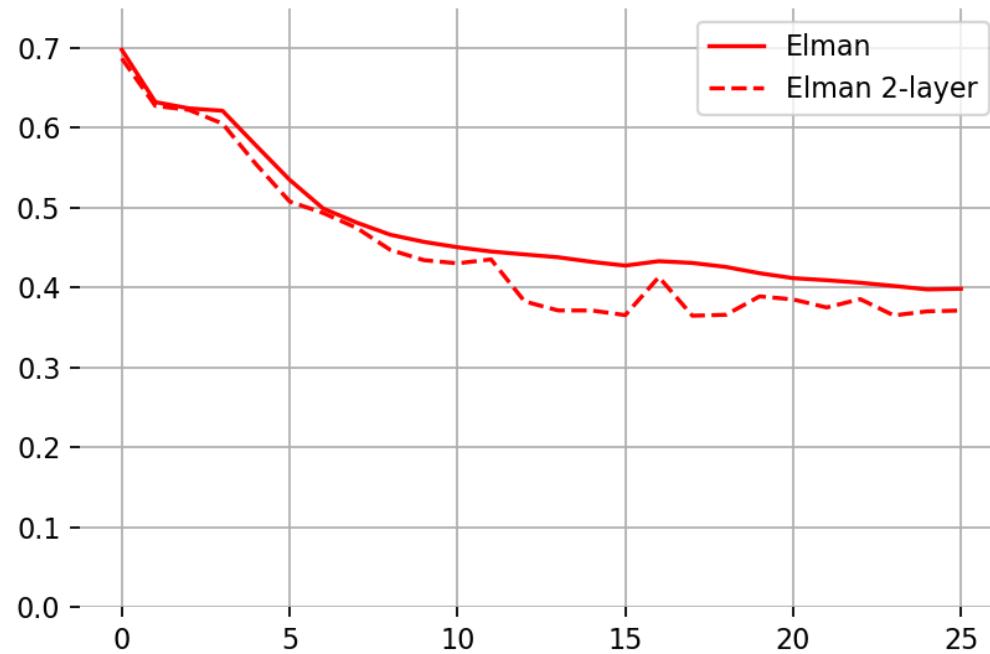
Note that the network was trained on sequences of size 10 or lower.
It does not appear to generalize outside the range.

Stacked RNNs

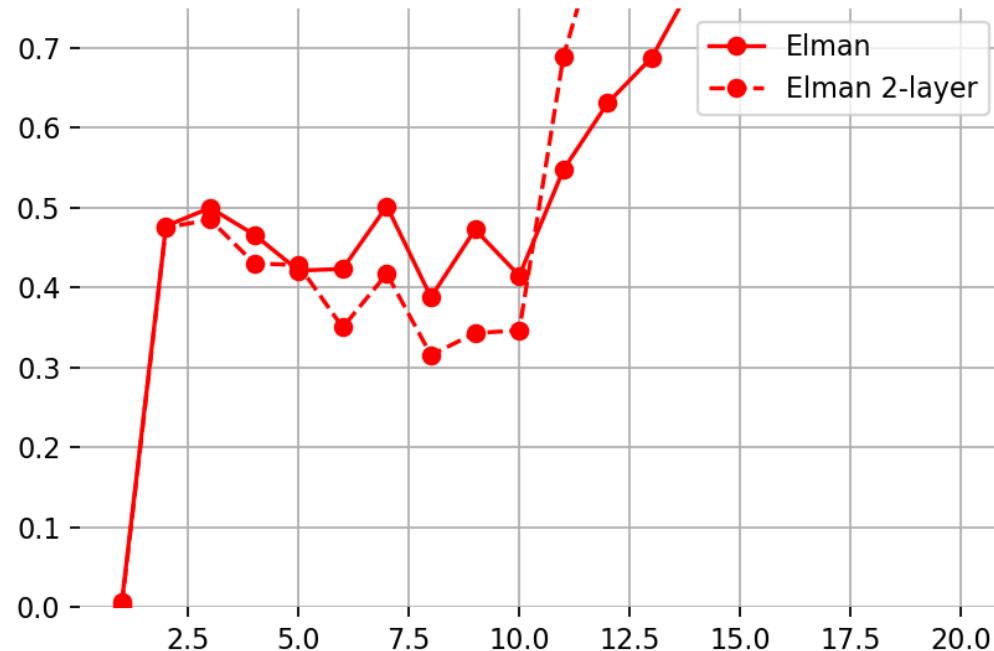
Recurrent networks can be viewed as layers producing sequences $\mathbf{h}_{1:T}^l$ of activations.

As for multi-perceptron layers, recurrent layers can be composed in series to form a **stack** of recurrent networks.





Epoch vs. cross-entropy.



Sequence length vs. cross-entropy.

Bidirectional RNNs

Computing the recurrent states forward in time does not make use of future input values $\mathbf{x}_{t+1:T}$, even though there are known.

- RNNs can be made **bidirectional** by consuming the sequence in both directions.
- Effectively, this amounts to run the same (single direction) RNN twice:
 - once over the original sequence $\mathbf{x}_{1:T}$,
 - once over the reversed sequence $\mathbf{x}_{T:1}$.
- The resulting recurrent states of the bidirectional RNN is the concatenation of two resulting sequences of recurrent states.

Gating

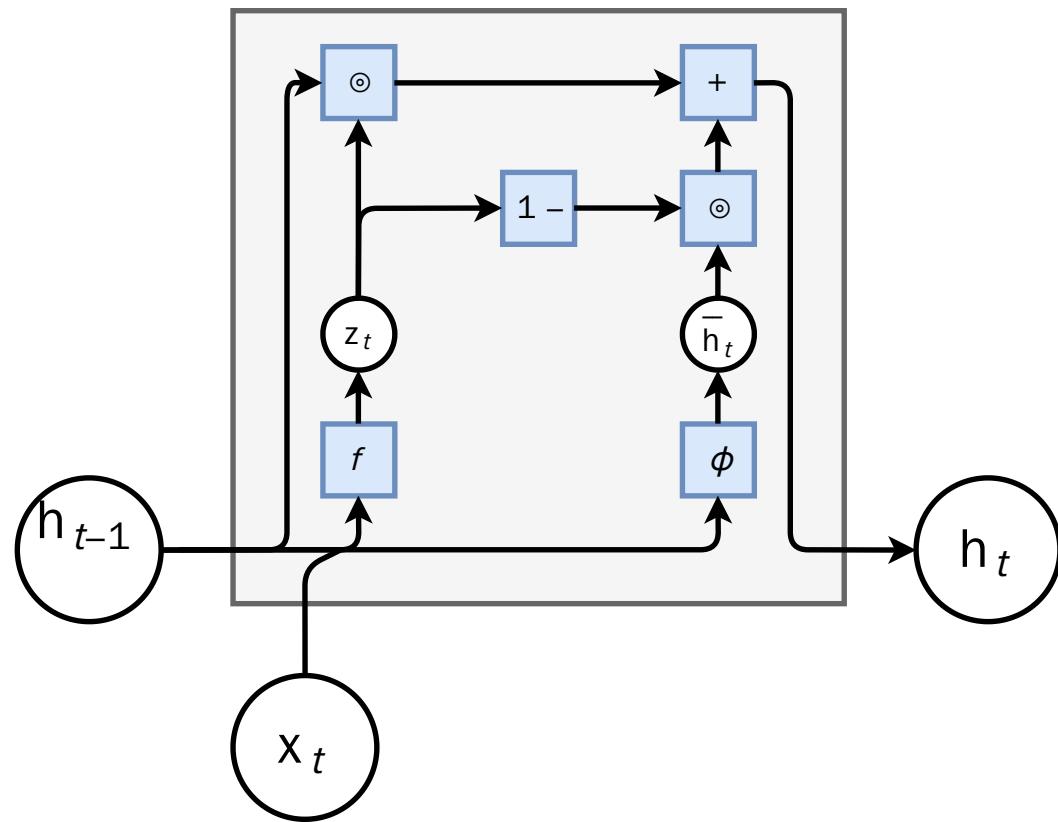
When unfolded through time, the resulting network can grow very deep, and training it involves dealing with **vanishing gradients**.

- A critical component in the design of RNN cells is to add in a **pass-through**, or additive paths, so that the recurrent state does not go repeatedly through a squashing non-linearity.
- This is very similar to skip connections in ResNets.

For instance, the recurrent state update can be a per-component weighted average of its previous value \mathbf{h}_{t-1} and a full update $\bar{\mathbf{h}}_t$, with the weighting \mathbf{z}_t depending on the input and the recurrent state, hence acting as a **forget gate**.

Formally,

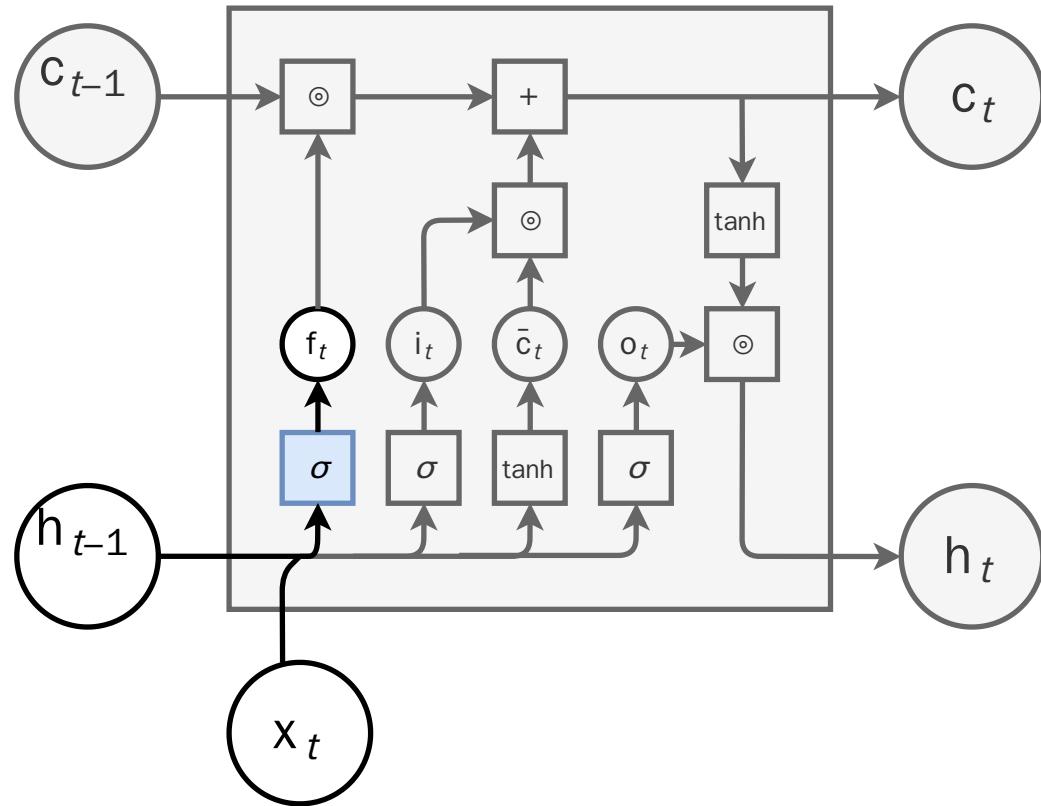
$$\begin{aligned}\bar{\mathbf{h}}_t &= \phi(\mathbf{x}_t, \mathbf{h}_{t-1}; \theta) \\ \mathbf{z}_t &= f(\mathbf{x}_t, \mathbf{h}_{t-1}; \theta) \\ \mathbf{h}_t &= \mathbf{z}_t \odot \mathbf{h}_{t-1} + (1 - \mathbf{z}_t) \odot \bar{\mathbf{h}}_t.\end{aligned}$$



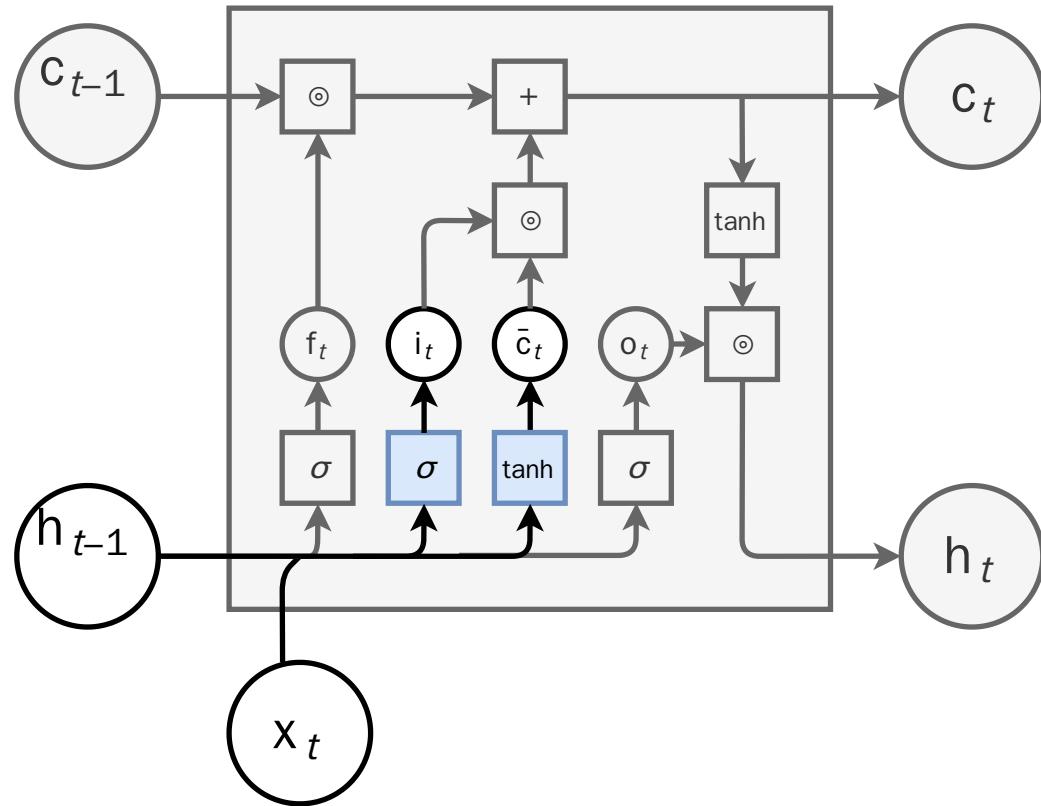
LSTM

The **long short-term memory model** (Hochreiter and Schmidhuber, 1997) is an instance of the previous gated recurrent cell, with the following changes:

- The recurrent state is split into two parts \mathbf{c}_t and \mathbf{h}_t , where
 - \mathbf{c}_t is the cell state and
 - \mathbf{h}_t is output state.
- A forget gate \mathbf{f}_t selects the cell state information to erase.
- An input gate \mathbf{i}_t selects the cell state information to update.
- An output gate \mathbf{o}_t selects the cell state information to output.

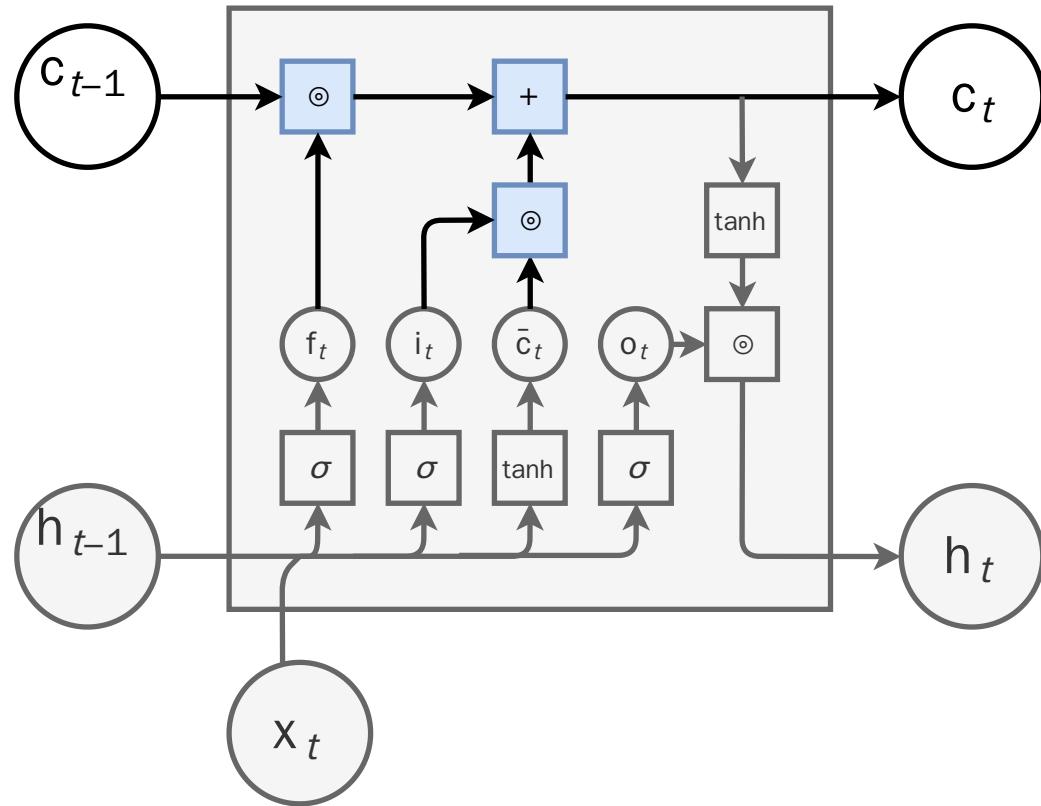


$$\mathbf{f}_t = \sigma (\mathbf{W}_f^T [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_f)$$

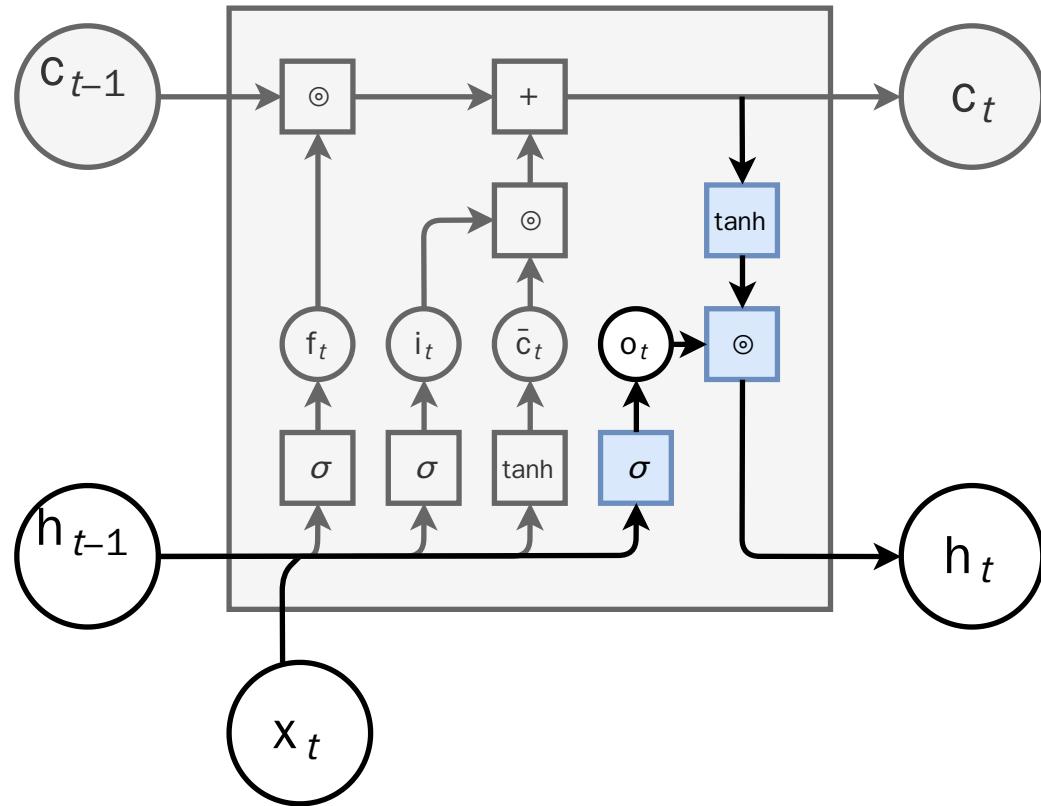


$$\mathbf{i}_t = \sigma(\mathbf{W}_i^T[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_i)$$

$$\bar{\mathbf{c}}_t = \tanh(\mathbf{W}_c^T[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_c)$$

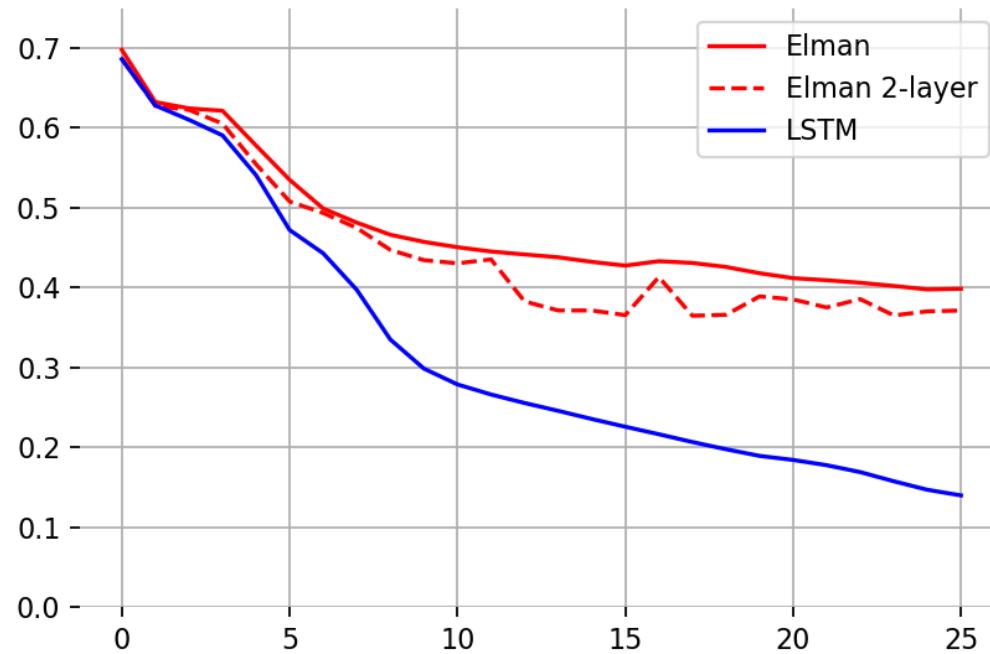


$$\mathbf{c}_t = \mathbf{f}_t \odot \mathbf{c}_{t-1} + \mathbf{i}_t \odot \bar{\mathbf{c}}_t$$

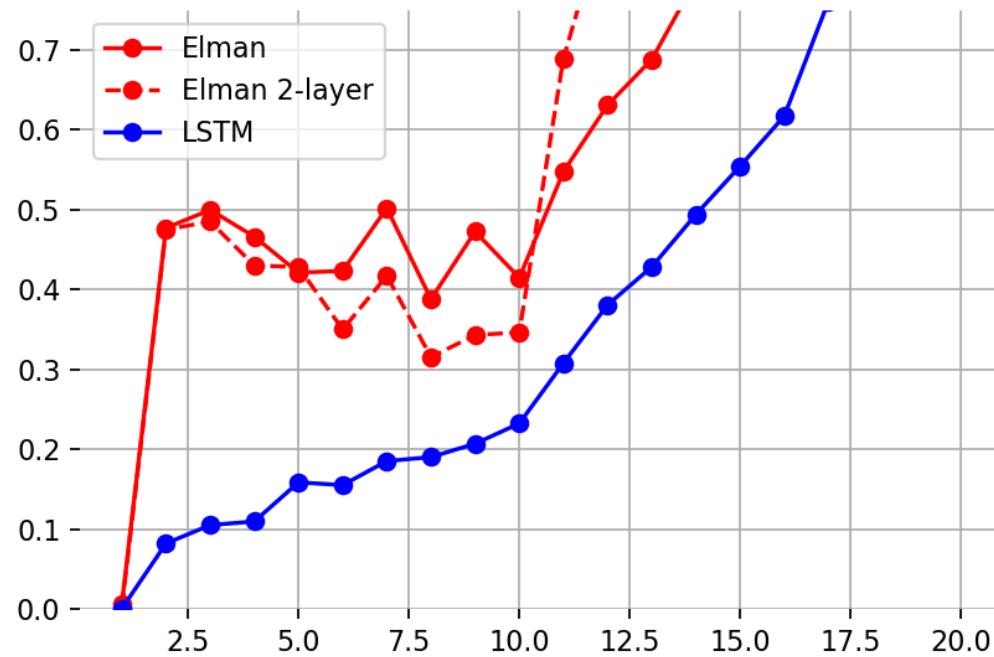


$$\mathbf{o}_t = \sigma (\mathbf{W}_o^T [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_o)$$

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{c}_t)$$



Epoch vs. cross-entropy.

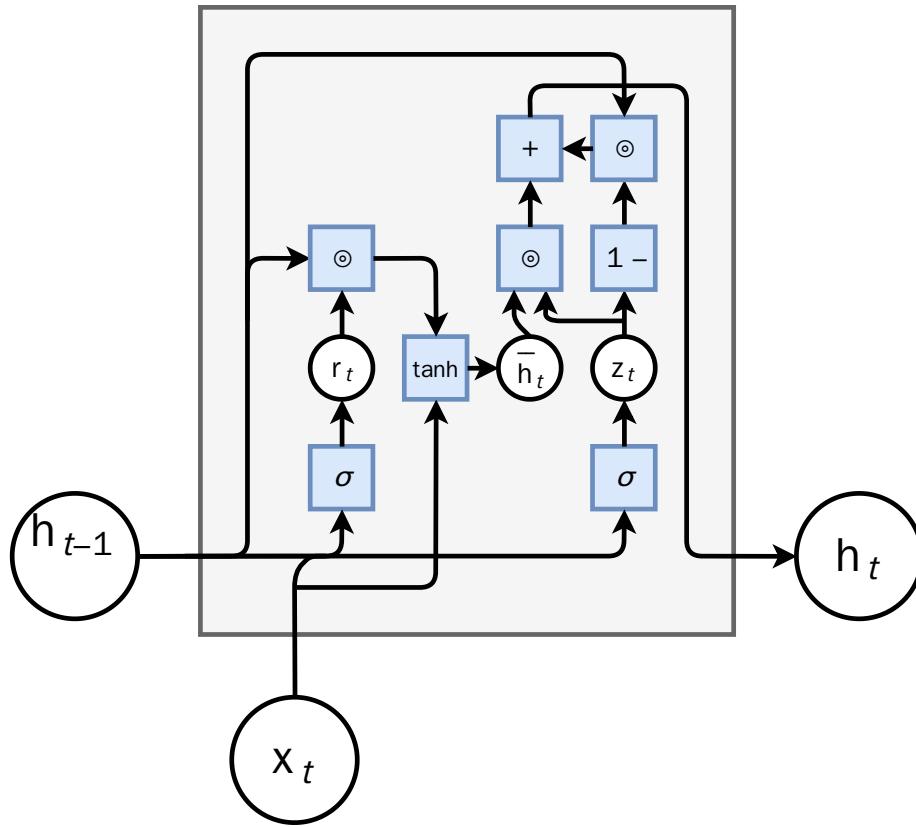


Sequence length vs. cross-entropy.

GRU

The **gated recurrent unit** (Cho et al, 2014) is another gated recurrent cell.

- It is based on two gates instead of three: an update gate \mathbf{z}_t and a reset gate \mathbf{r}_t .
- GRUs perform similarly as LSTMs for language or speech modeling sequences, but with fewer parameters.
- However, LSTMs remain strictly stronger than GRUs.

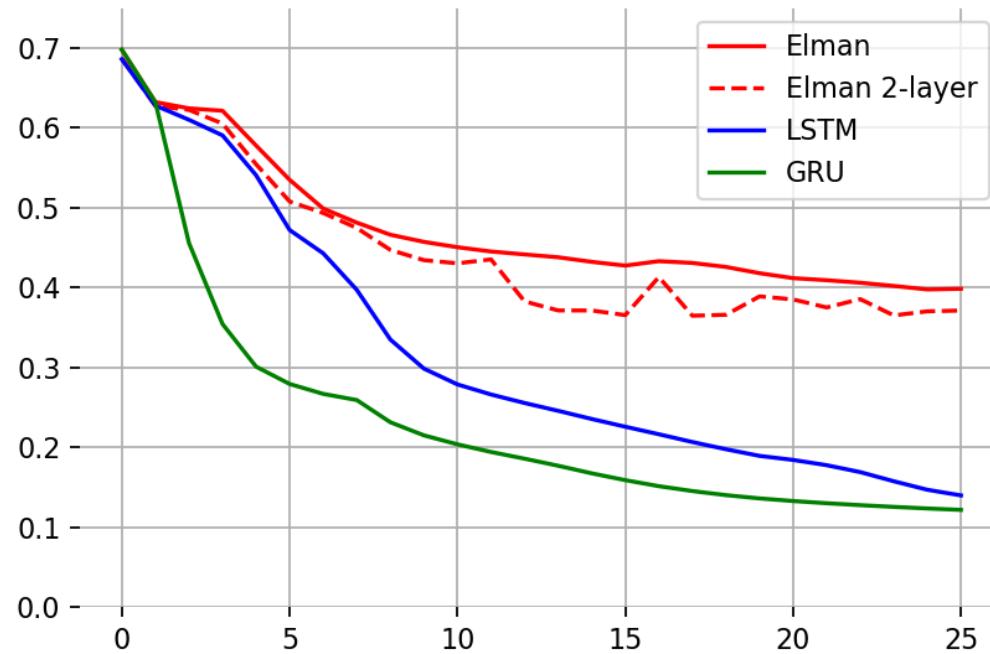


$$\mathbf{z}_t = \sigma(\mathbf{W}_z^T[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_z)$$

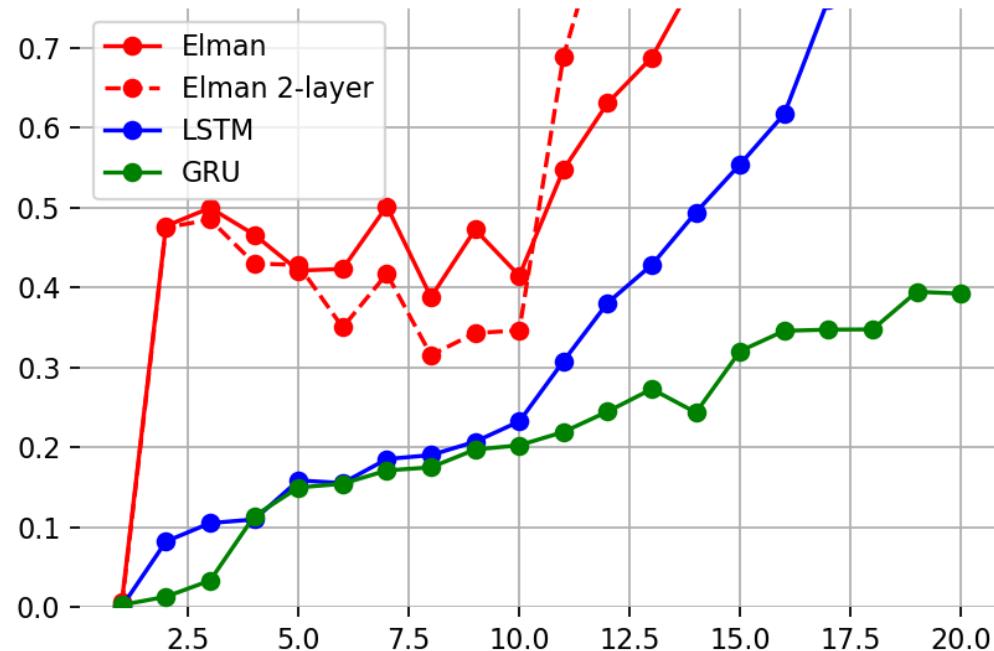
$$\mathbf{r}_t = \sigma(\mathbf{W}_r^T[\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_r)$$

$$\bar{\mathbf{h}}_t = \tanh(\mathbf{W}_h^T[\mathbf{r}_t \odot \mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_h)$$

$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \bar{\mathbf{h}}_t$$



Epoch vs. cross-entropy.



Sequence length vs. cross-entropy.

Gradient clipping

Gated units prevent gradients from vanishing, but not from exploding.

The standard strategy to solve this issue is gradient norm clipping, which rescales the norm of the gradient to a fixed threshold δ when it is above:

$$\tilde{\nabla}f = \frac{\nabla f}{\|\nabla f\|} \min(\|\nabla f\|, \delta).$$

Orthogonal initialization

Let us consider a simplified RNN, with no inputs, no bias, an identity activation function σ (as in the positive part of a ReLU) and the initial recurrent state \mathbf{h}_0 set to the identity matrix.

We have,

$$\begin{aligned}\mathbf{h}_t &= \sigma(\mathbf{W}_{xh}^T \mathbf{x}_t + \mathbf{W}_{hh}^T \mathbf{h}_{t-1} + \mathbf{b}_h) \\ &= \mathbf{W}_{hh}^T \mathbf{h}_{t-1} \\ &= \mathbf{W}^T \mathbf{h}_{t-1}.\end{aligned}$$

For a sequence of size n , it comes

$$\mathbf{h}_n = \mathbf{W}(\mathbf{W}(\mathbf{W}(\dots(\mathbf{W}\mathbf{h}_0)\dots))) = \mathbf{W}^n \mathbf{h}_0 = \mathbf{W}^n I = \mathbf{W}^n.$$

Ideally, we would like \mathbf{W}^n to neither vanish nor explode as n increases.

Fibonacci digression

The Fibonacci sequence is

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...

It grows fast! But how fast?

In matrix form, the Fibonacci sequence is equivalently expressed as

$$\begin{pmatrix} f_{k+2} \\ f_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} f_{k+1} \\ f_k \end{pmatrix}.$$

With $\mathbf{f}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, we have

$$\mathbf{f}_{k+1} = \mathbf{A}\mathbf{f}_k = \mathbf{A}^{k+1}\mathbf{f}_0.$$

The matrix \mathbf{A} can be diagonalized as

$$\mathbf{A} = \mathbf{S}\Lambda\mathbf{S}^{-1},$$

where

$$\begin{aligned}\Lambda &= \begin{pmatrix} \varphi & 0 \\ 0 & -\varphi^{-1} \end{pmatrix} \\ \mathbf{S} &= \begin{pmatrix} \varphi & -\varphi^{-1} \\ 1 & 1 \end{pmatrix}.\end{aligned}$$

In particular,

$$\mathbf{A}^n = \mathbf{S}\Lambda^n\mathbf{S}^{-1}.$$

Therefore, the Fibonacci sequence grows **exponentially fast** with the golden ratio φ .

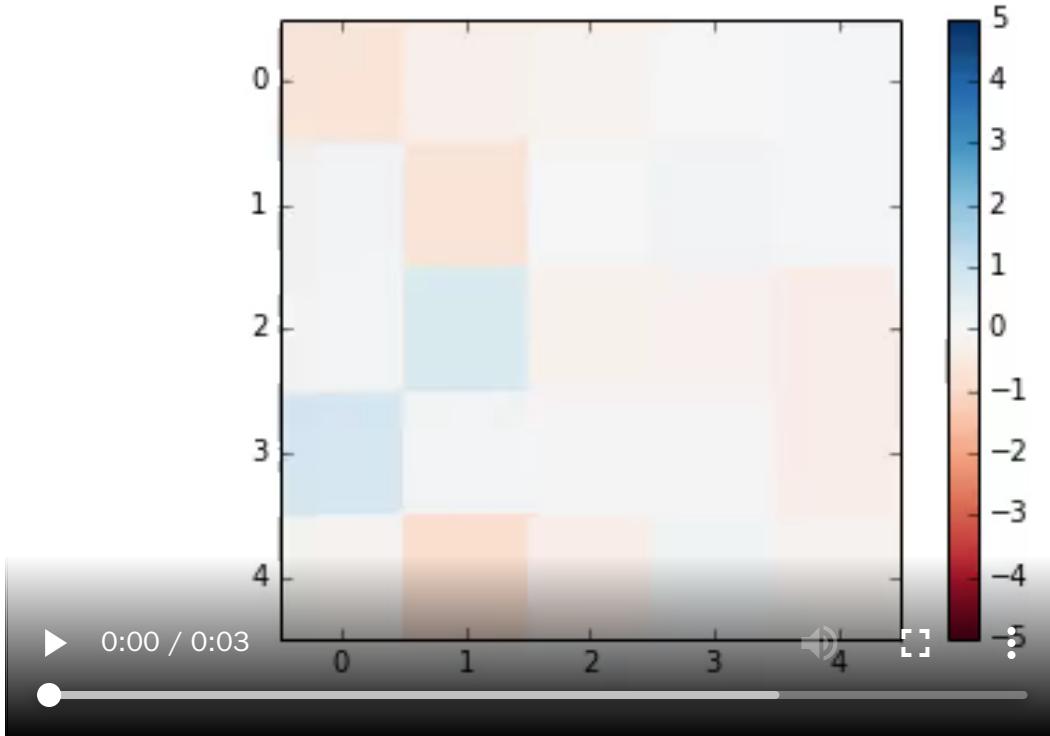
Theorem

Let $\rho(\mathbf{A})$ be the spectral radius of the matrix \mathbf{A} , defined as

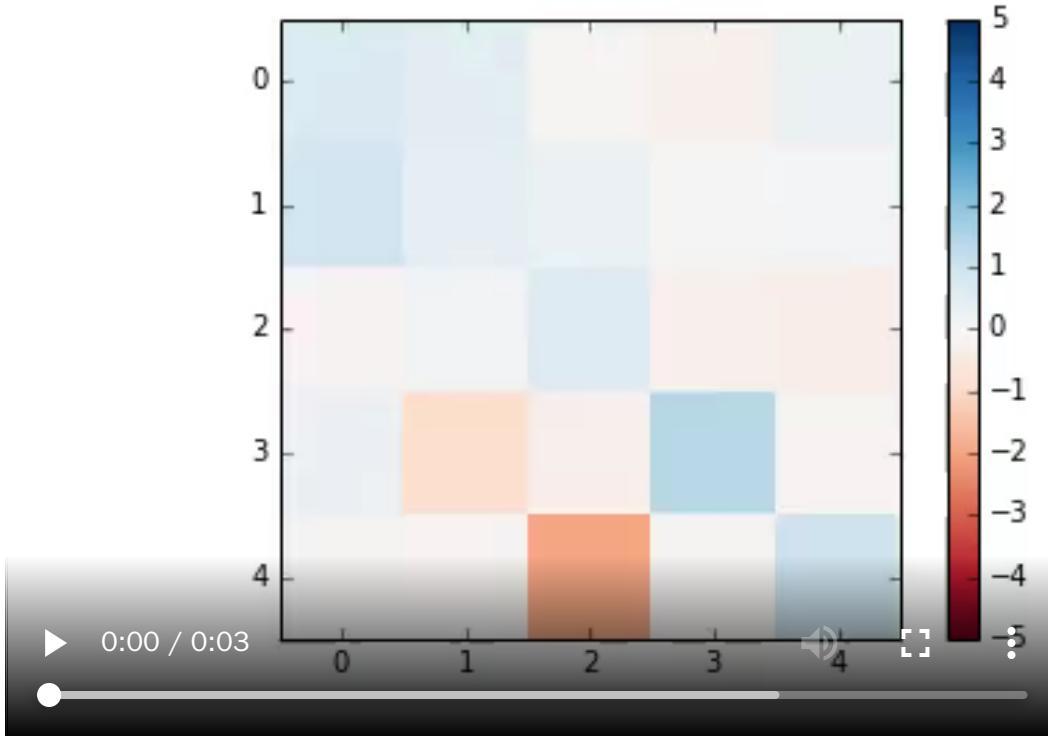
$$\rho(\mathbf{A}) = \max\{|\lambda_1|, \dots, |\lambda_d|\}.$$

We have:

- if $\rho(\mathbf{A}) < 1$ then $\lim_{n \rightarrow \infty} \|\mathbf{A}^n\| = \mathbf{0}$ (= vanishing activations),
- if $\rho(\mathbf{A}) > 1$ then $\lim_{n \rightarrow \infty} \|\mathbf{A}^n\| = \infty$ (= exploding activations).



$\rho(\mathbf{A}) < 1, \mathbf{A}^n$ vanish.



$\rho(\mathbf{A}) > 1, \mathbf{A}^n$ explode.

Orthogonal initialization

If \mathbf{A} is orthogonal, then it is diagonalizable and all its eigenvalues are equal to -1 or 1 . In this case, the norm of

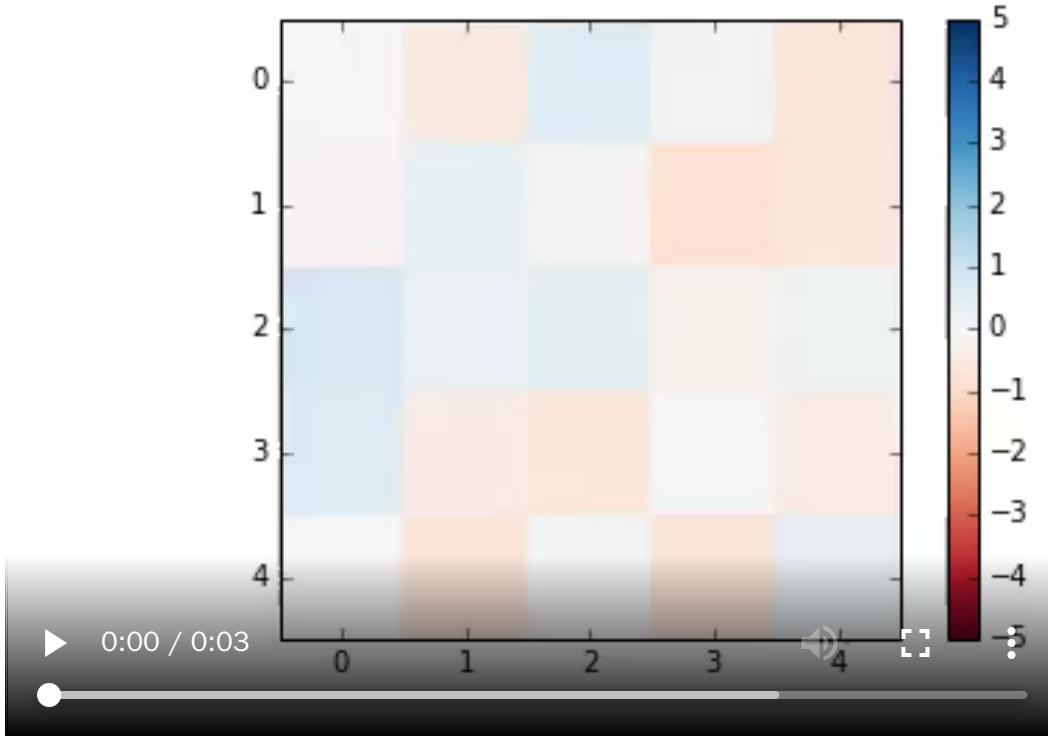
$$\mathbf{A}^n = \mathbf{S}\Lambda^n\mathbf{S}^{-1}$$

remains bounded.

- Therefore, initializing \mathbf{W} as a random orthogonal matrix will guarantee that activations will neither vanish nor explode.
- In practice, a random orthogonal matrix can be found through the SVD decomposition or the QR factorization of a random matrix.
- This initialization strategy is known as **orthogonal initialization**.

In Tensorflow's Orthogonal initializer:

```
# Generate a random matrix
a = random_ops.random_normal(flat_shape, dtype=dtype, seed=self.seed)
# Compute the qr factorization
q, r = gen_linalg_ops.qr(a, full_matrices=False)
# Make Q uniform
d = array_ops.diag_part(r)
q *= math_ops.sign(d)
if num_rows < num_cols:
    q = array_ops.matrix_transpose(q)
return self.gain * array_ops.reshape(q, shape)
```



\mathbf{A} is orthogonal.

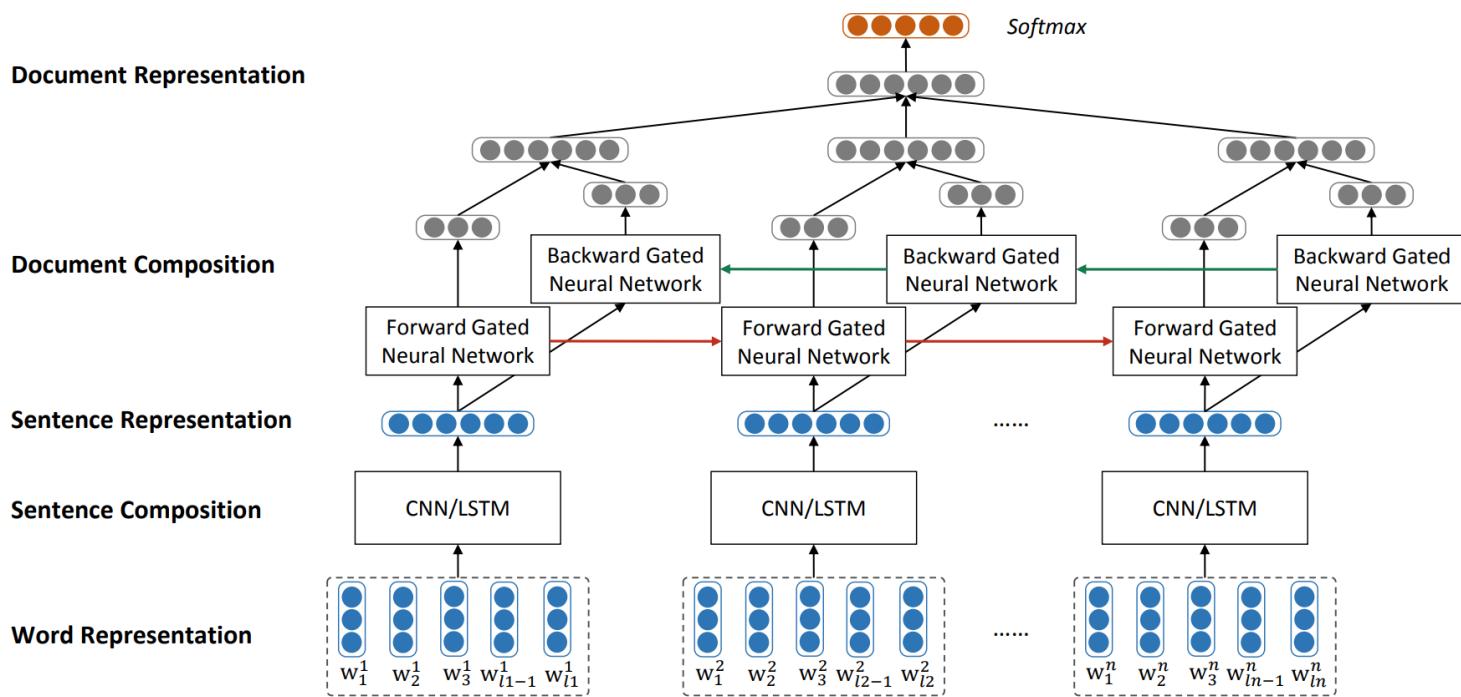
Finally, let us note that exploding activations are also the reason why squashing non-linearity functions (such as \tanh) are preferred in RNNs.

- They avoid recurrent states from exploding by upper bounding $\|\mathbf{h}_t\|$.
- (At least when running the network forward.)

Applications

(some)

Sentiment analysis



Document-level modeling for sentiment analysis (= text classification), with stacked, bidirectional and gated recurrent networks.

Language models

Model language as a Markov chain, such that sentences are sequences of words $\mathbf{w}_{1:T}$ drawn repeatedly from

$$p(\mathbf{w}_t | \mathbf{w}_{1:t-1}).$$

This is an instance of sequence synthesis, for which predictions are computed at all time steps t .

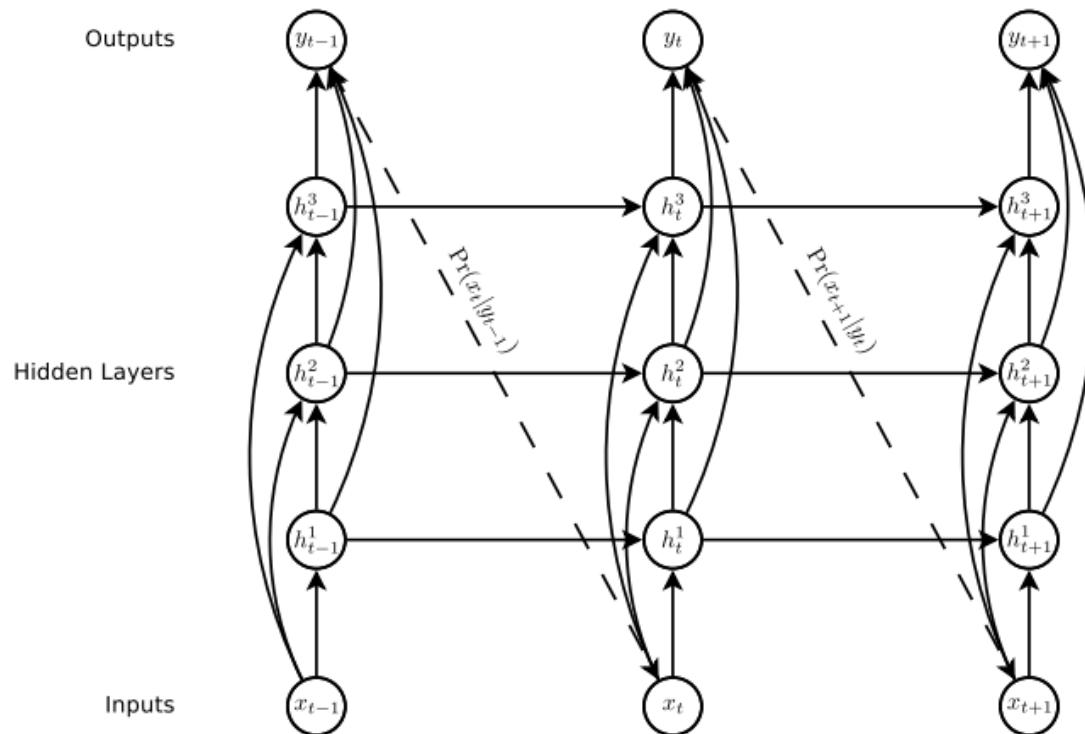
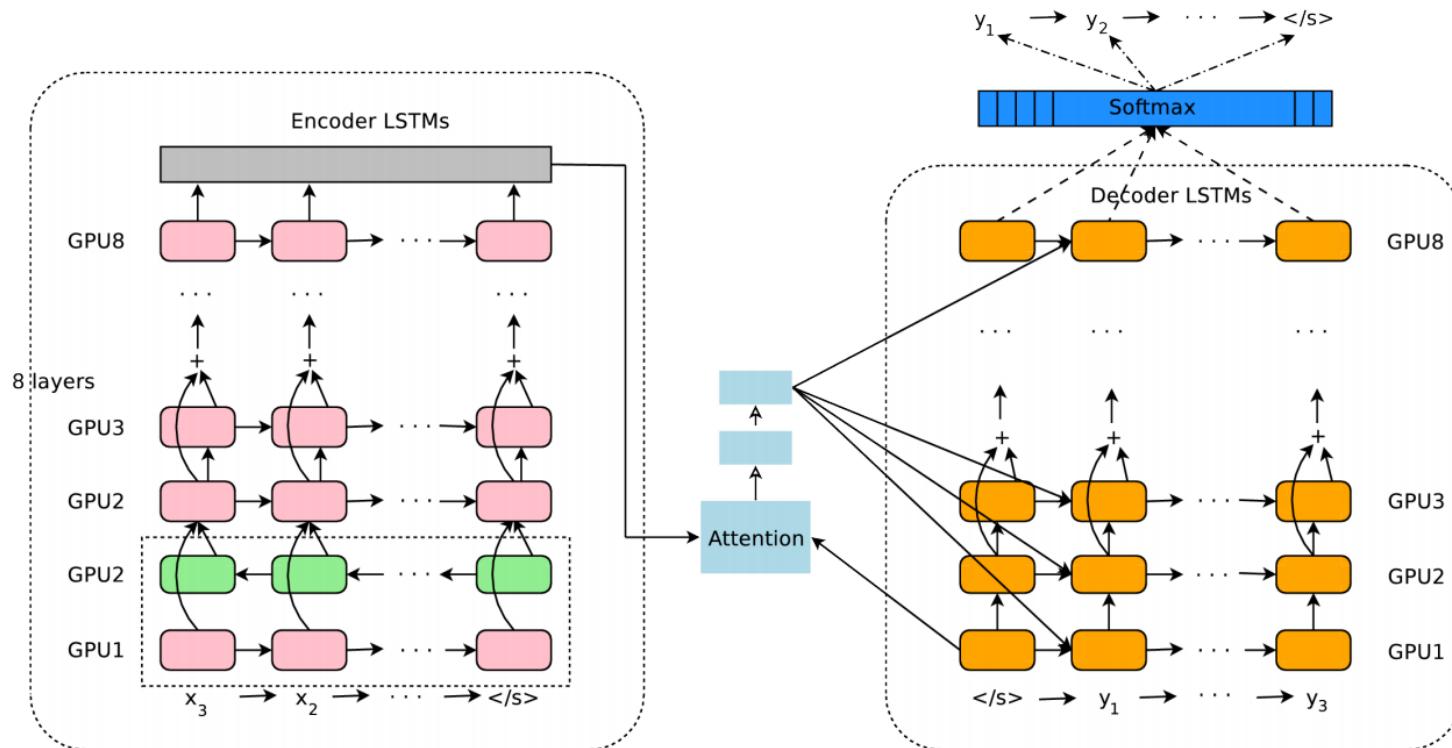


Figure 1: Deep recurrent neural network prediction architecture. The circles represent network layers, the solid lines represent weighted connections and the dashed lines represent predictions.

Demos

- [Obama-RNN](#): machine generated political speeches
- [Harry Potter](#) (written by AI)
- [@lexiconjure](#): artificial word definitions

Neural machine translation



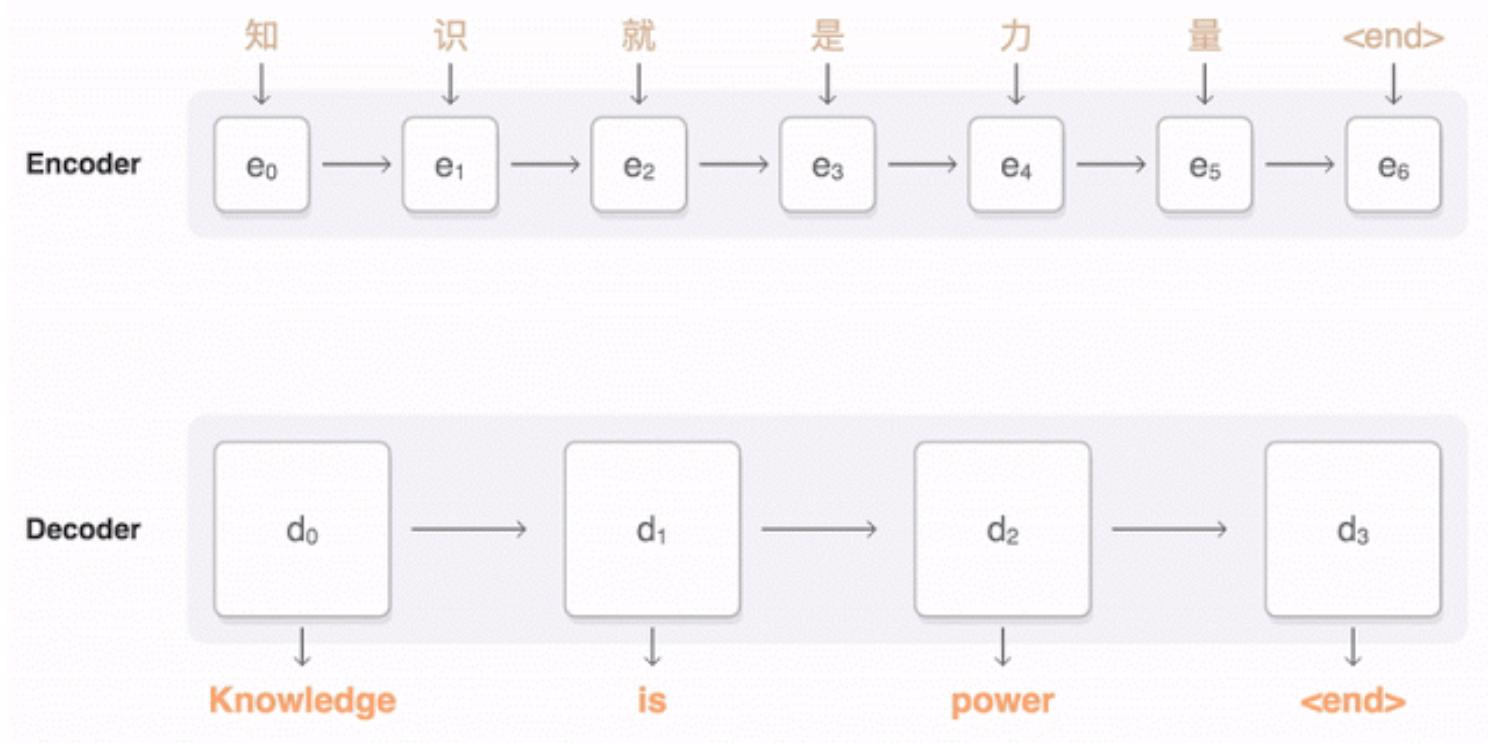
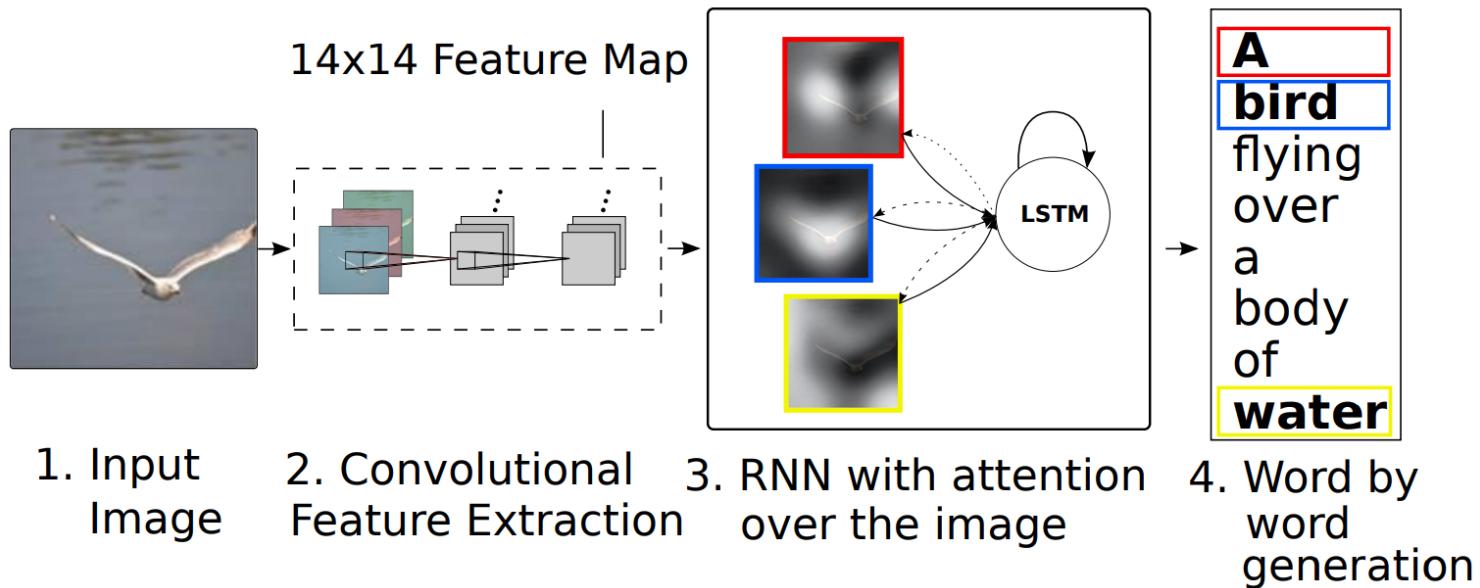
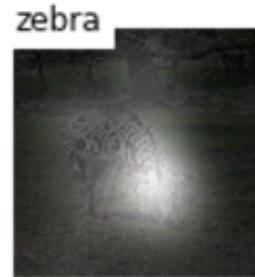
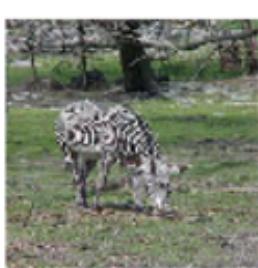
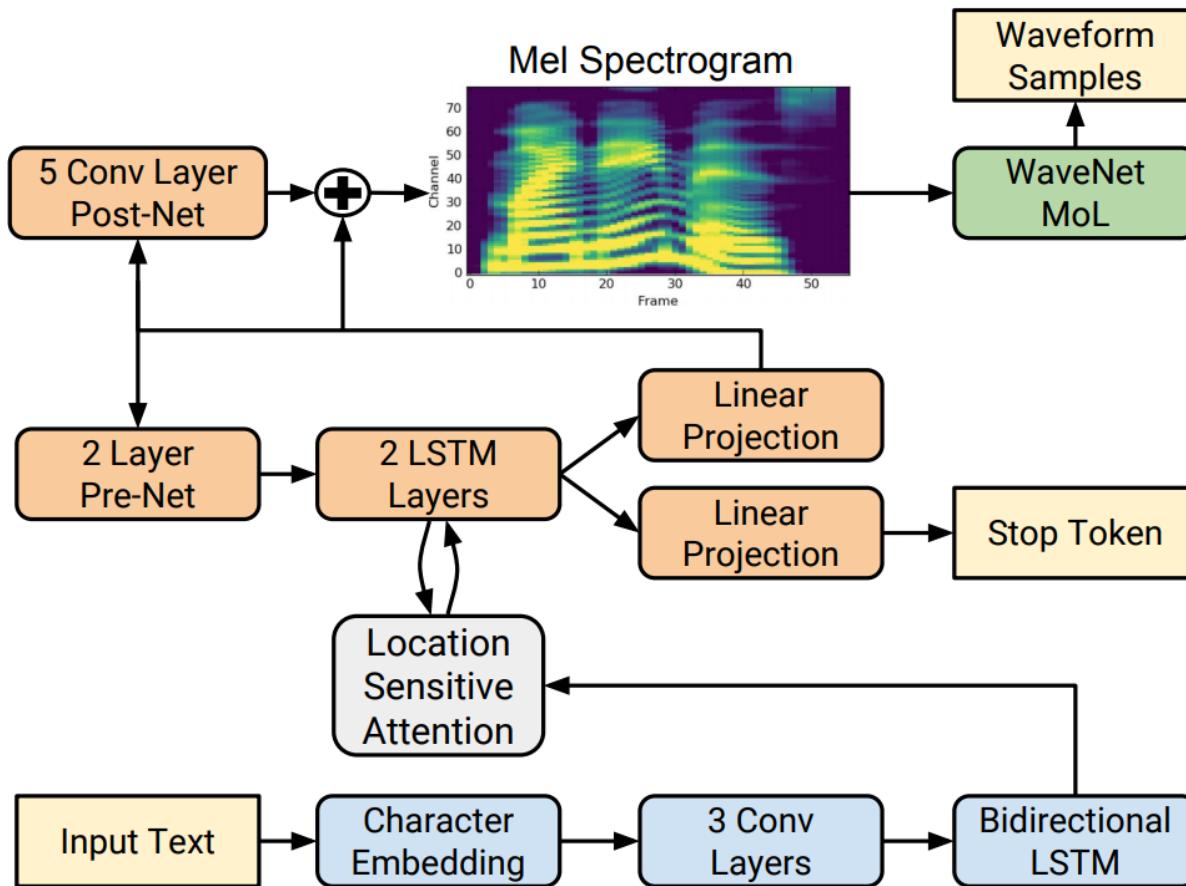


Image captioning





Text-to-speech synthesis



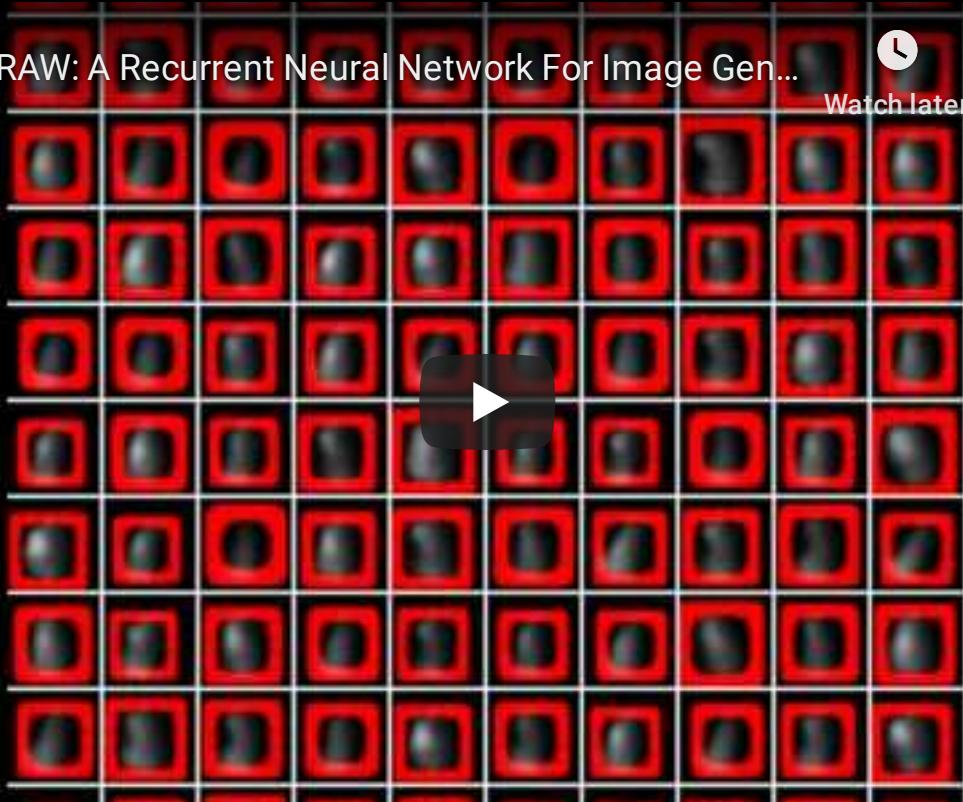


DRAW: A Recurrent Neural Network For Image Gen...

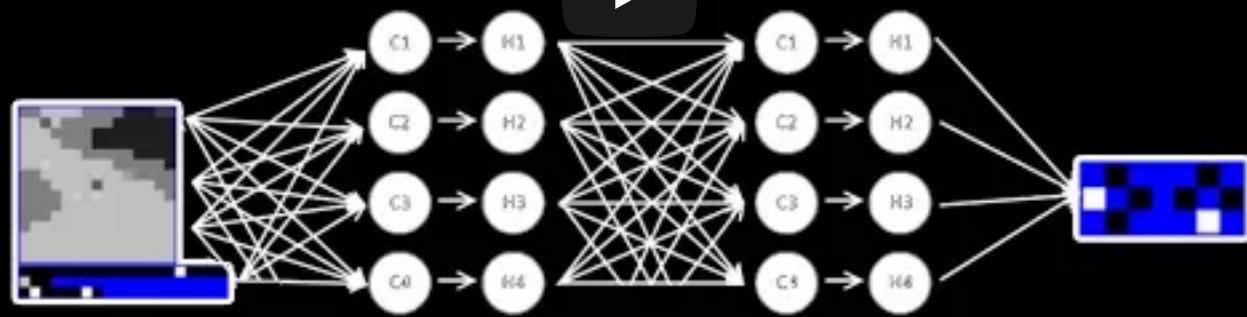


Watch later

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DRAW: A Recurrent Neural Network For Image Generation



A recurrent network playing Mario Kart.

Differentiable computers



People are now building a new kind of software by assembling networks of parameterized functional blocks and by training them from examples using some form of gradient-based optimization.

*An increasingly large number of **people are defining the networks procedurally in a data-dependent way (with loops and conditionals)**, allowing them to change dynamically as a function of the input data fed to them. It's really **very much like a regular program, except it's parameterized**.*

Yann LeCun (Director of AI Research, Facebook, 2018)

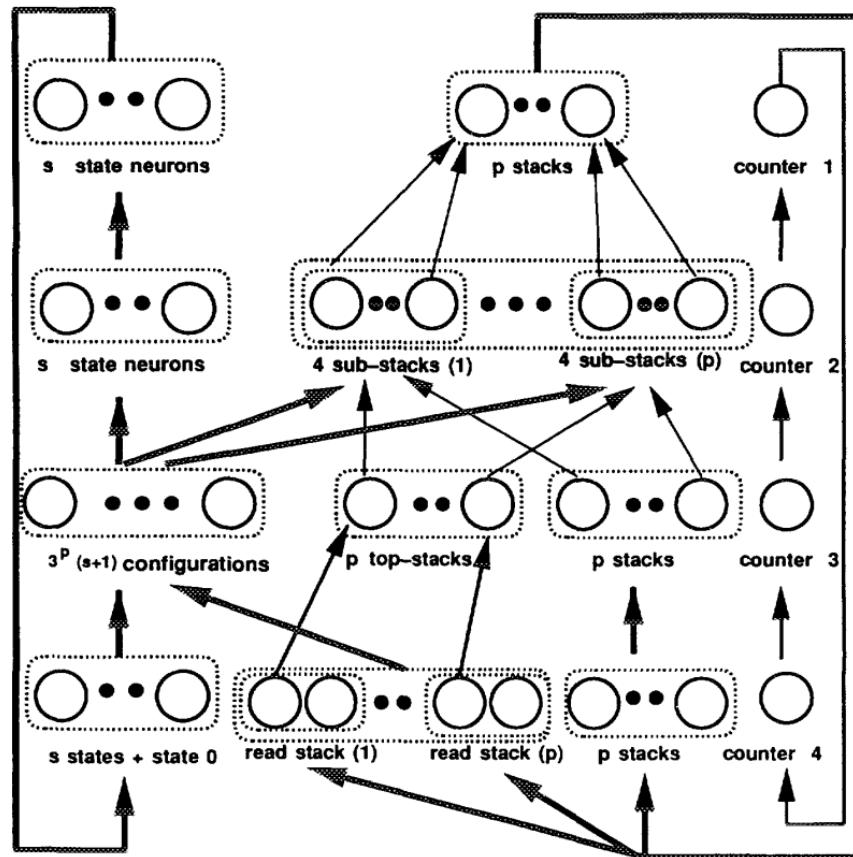
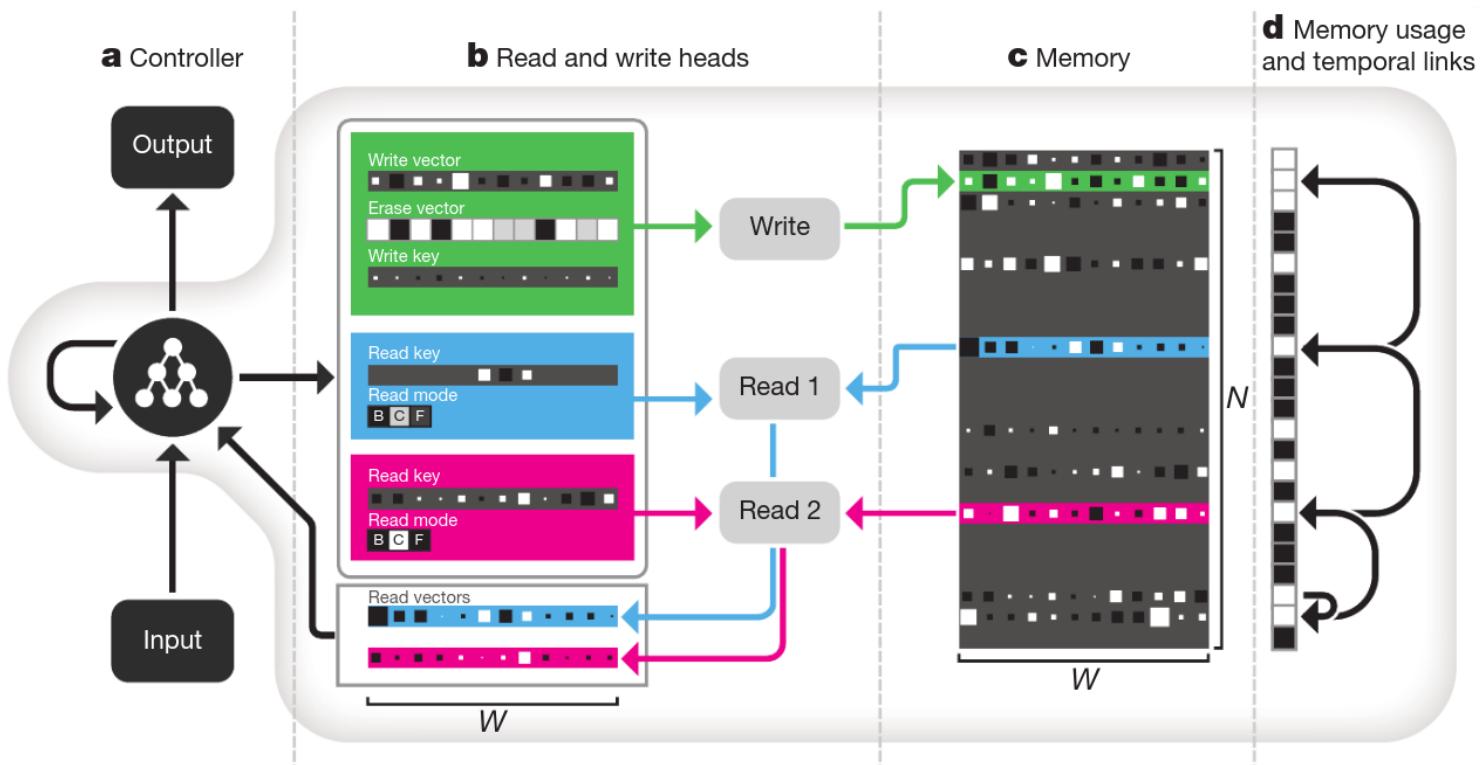
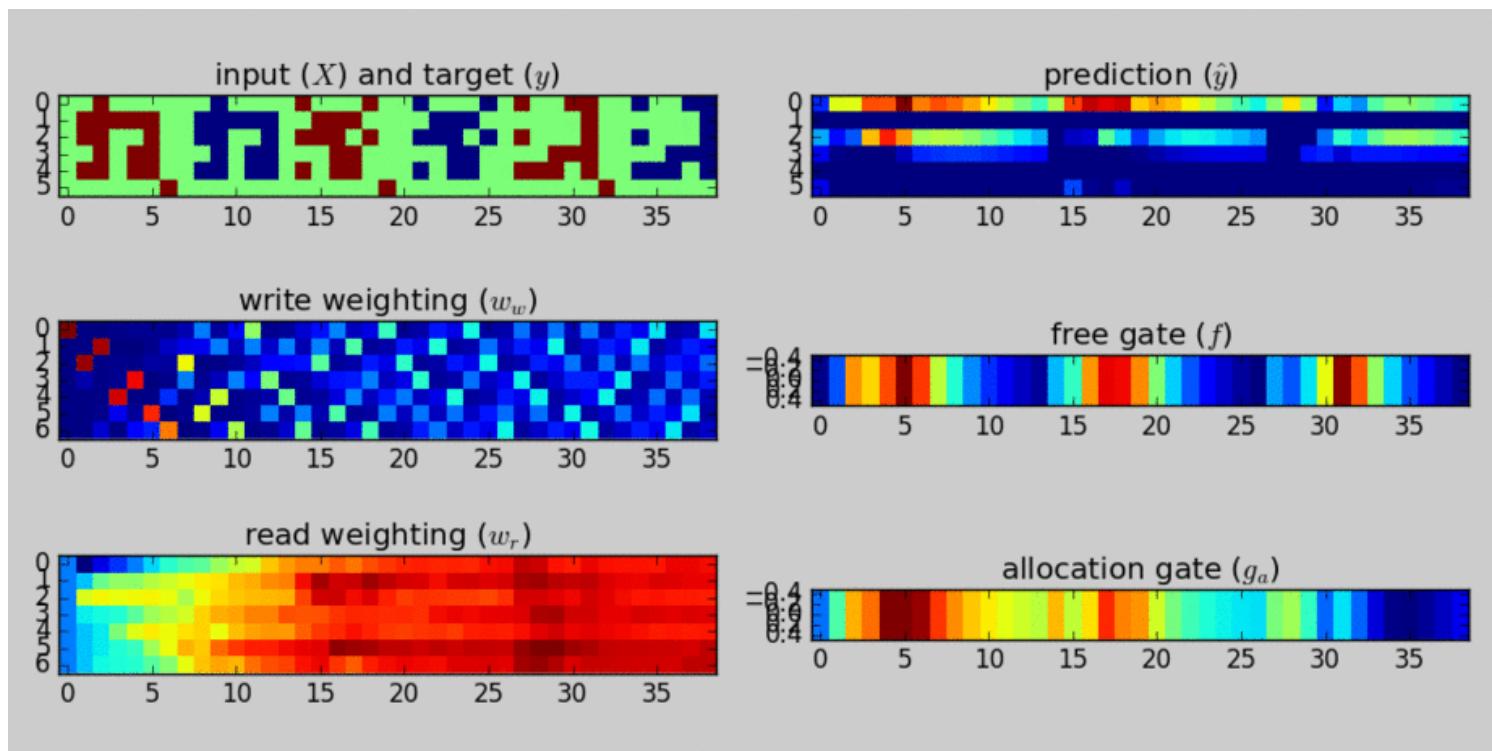


FIG. 1. The universal network.

Any Turing machine can be simulated by a recurrent neural network
 (Siegelmann and Sontag, 1995)



Differentiable Neural Computer (Graves et al, 2016)



A differentiable neural computer being trained to store and recall dense binary numbers. Upper left: the input (red) and target (blue), as 5-bit words and a 1 bit interrupt signal. Upper right: the model's output

The end.

References

- Kyunghyun Cho, "Natural Language Understanding with Distributed Representation", 2015.