$$\dot{v} = k_g v - k_{dv} v^2 - k_{da} a^2$$

$$v_{free} = \frac{k_g}{2k_{dv}}$$

If $v = v_{free} + \Delta v$

$$\dot{v} = k_g\left(\frac{k_g}{2k_{dv}} + \Delta v\right) - k_{dv}\left(\frac{k_g}{2k_{dv}} + \Delta v\right)^2 - k_{da} a^2$$

$$= \frac{k_g^2}{2k_{dv}} + k_g \Delta v - k_{dv}\left(\frac{k_g^2}{4k_{dv}^2} + \frac{k_g}{k_{dv}}\Delta v + \Delta v^2\right) - k_{da} a^2$$

$$= \frac{k_g^2}{2k_{dv}} + \cancel{k_g \Delta v} - \frac{k_g^2}{4k_{dv}} - \cancel{k_g \Delta v} - k_{dv}\Delta v^2 - k_{da} a^2$$

$$= \frac{k_g^2}{\quad} - k_{dv}\Delta v^2 - k_{da} a^2$$

But $\Delta v_p = \cancel{\Delta t} \cdot T_p \cdot \frac{a}{2}$ (Assumes $\Delta t = T_p$)

with $\Delta v = \Delta v_0 + \Delta v_p$

i.e.:

$$V = \frac{k_g^2}{4k_{dv}} - k_{dv}\underbrace{\left(\Delta v_0 + \frac{T_p a}{2}\right)^2}_{\left(\Delta v_0^2 + \Delta v_0 T_p a + \frac{T_p^2 a^2}{4}\right)} - k_{da} a^2$$

$$\frac{dV}{da} = -k_{dv}\left(\Delta v_0 T_p + \frac{2T_p^2 a}{4}\right) - 2k_{da} a$$

$$= -k_{dv}\Delta v_0 T_p - \frac{k_{dv} T_p^2 a}{2} - 2k_{da} a$$

$$= -k_{dv}\Delta v_0 T_p - a\left(\frac{k_{dv} T_p^2}{2} + 2k_{da}\right)$$

$$= 0$$

$$\Leftrightarrow \quad a = -\frac{k_{dv}\Delta v_0 T_p}{\frac{k_{dv} T_p^2}{2} + 2k_{da}}$$