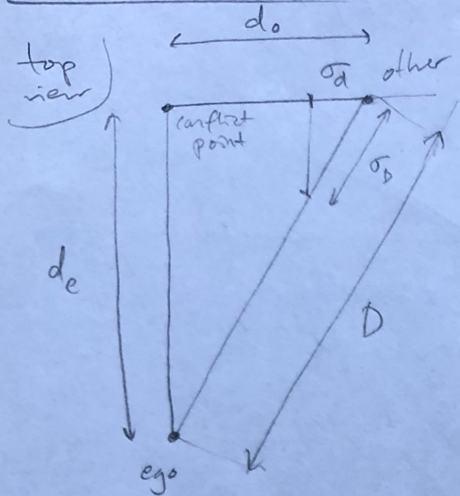
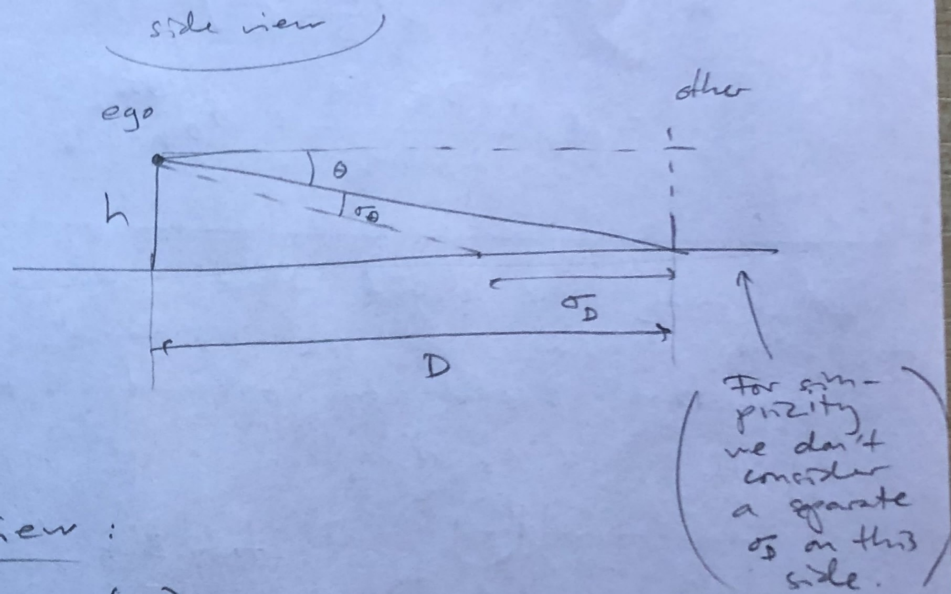
2021-11-02

Uncertainty in distance to conflict point, from uncertainty in angle over horizon





From the top view:

$$D = \sqrt{d_e^2 + d_o^2} \qquad (1)$$

$$\frac{\sigma_d}{\sigma_D} = \frac{d_o}{D} \qquad (2)$$

From the side view:

$$\theta = \arctan \frac{h}{D} \qquad (3)$$

$$\tan(\theta + \sigma_\theta) = \frac{h}{D - \sigma_D}$$

$$\Leftrightarrow \quad D - \sigma_D = \frac{h}{\tan(\theta + \sigma_\theta)}$$

$$\Leftrightarrow \quad \sigma_D = D - \frac{h}{\tan(\theta + \sigma_\theta)} \qquad (4)$$

$$= \{(3)\} = D - \frac{h}{\tan\left(\arctan \frac{h}{D} + \sigma_\theta\right)} \qquad (5)$$

$$\overset{(2)}{\Leftrightarrow} \quad \sigma_d = \frac{d_o}{D} \sigma_D = \{(5)\} = \frac{d_o}{D}\left(D - \frac{h}{\tan\left(\arctan\frac{h}{D} + \sigma_\theta\right)}\right)$$

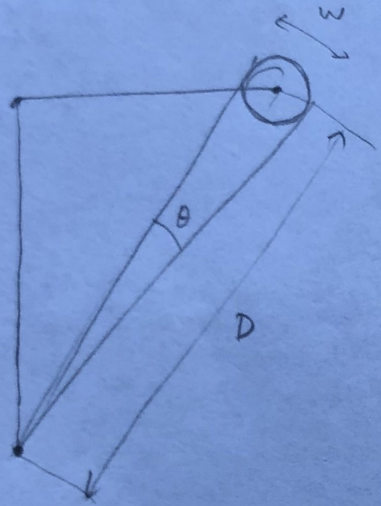
$$\boxed{= d_o\left(1 - \frac{h}{D \tan\left(\arctan\frac{h}{D} + \sigma_\theta\right)}\right)}$$

For small angles; $h << D$ and σ_θ small:

$$\sigma_d \simeq d_0\left(1 - \frac{h}{D\left(\frac{h}{D} + \sigma_\theta\right)}\right) = d_0\left(1 - \frac{h}{h + D\sigma_\theta}\right)$$

$$= d_0 \frac{h + D\sigma_\theta - h}{h + D\sigma_\theta} = d_0 \frac{D\sigma_\theta}{h + D\sigma_\theta}$$

It may be noted that we get something very similar if we instead estimate positions based on optical size:



$$\tan\frac{\theta}{2} = \frac{w/2}{D}$$

If we now instead observe an optical angle $\theta + \sigma_\theta$, indicative of a smaller distance $D - \sigma_D$, we get:

$$\tan\frac{\theta + \sigma_\theta}{2} = \frac{w/2}{D - \sigma_D}$$

$$\Leftrightarrow D - \sigma_D = \frac{w/2}{\tan\frac{\theta + \sigma_\theta}{2}}$$

$$\Leftrightarrow \sigma_D = D - \frac{w/2}{\tan\frac{\theta + \sigma_\theta}{2}},$$

which is identical in form to (4) above.
