Course Project 1
Nondeterministic Finite Automata

CSE 30151 Spring 2024

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We've studied the theory of nondeterministic finite automata, and now it's time to implement them. The interesting challenge is that NFAs are nondeterministic, but real computers are deterministic – how do we simulate nondeterminism?

One option is backtracking: when two transitions are possible, try one, and if it fails, try the other. But this will lead to a $O(2^n)$ time algorithm (where n is the input length). The theory provides another option: convert the NFA to an equivalent DFA. That gives a $O(n)$ algorithm, but the conversion could take $O(2^{|Q|})$ time and space (where $|Q|$ is the number of states).

In this project, you'll implement a third solution, one that runs in $O(|\delta|n)$ time, where $|\delta|$ is the number of transitions. You can write your implementation in C++ or Python (or another language with permission of the instructor).

Getting started

You should have been given access on GitHub to a repository named after your team. Please clone this repository to wherever you plan to work on the project:

```
git clone https://github.com/ND-CSE-30151/team
cd team
```

If you're the first team member to do this, your repository is empty. In that case, run the commands:

```
git pull https://github.com/ND-CSE-30151/regexp-skeleton
git push
```

If one of your teammates already did this, there's no need for you to repeat it. Whenever we make an update to `regexp-skeleton`, we'll send out an announcement, and one of you will need to repeat the pull/push (resolving any merge conflicts if necessary) to get the update.

Now your directory should include the following files (among others):

```
bin.{linux,darwin}/
  nfa_path
  re_to_nfa
examples/
  cycle.nfa
  sipser-n1.nfa
  ...
```

```
tests/
  test-cp1.sh
cp1/
```

- The `bin.linux` and `bin.darwin` contain binaries for Linux and Mac, respectively. They contain reference implementations for the tools you will implement and tools used by the test scripts.
- The `examples` directory contains examples of NFAs that you will use for testing. See below for a description of the file format.
- The `tests` directory contains test scripts. The script `tests/test-cp1.sh` tests your code for correctness and speed. Your code needs to pass all tests in order to get full credit.
- Please place the programs that you write into the `cp1/` subdirectory.

1 NFAs

Design a data structure for representing a NFA M, and write the following functions to read and write NFAs. (For all projects, the names of functions and the way that they are called are just suggestions; if you prefer a different style, that's fine.)

read_nfa(*file*)

- *file*: File containing definition of NFA M
- Returns: The NFA M

write_nfa(M, *file*)

- M: The NFA to write
- *file*: File to write to
- Effect: Writes definition of M to file

NFA file format The NFA definition must have the following format. It begins with a four-line header:

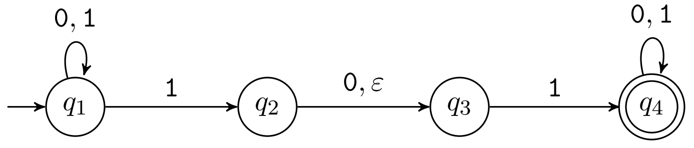
1. A whitespace-separated list of states, Q.
2. A whitespace-separated list of input symbols, Σ. Each symbol in Σ is a single character, and Σ is disjoint with $Q \cup \{\texttt{\&}\}$.
3. The start state, $s \in Q$.
4. A whitespace-separated list of accept states, $F \subseteq Q$.

The rest of the lines list the transitions, one transition per line. Each line has three fields, separated by whitespace:

1. The state $q \in Q$ that the transition leaves from.
2. The symbol $a \in \Sigma$ that the transition reads, or `&` for the empty string.

3. The state $r \in Q$ that the transition goes to.

For example, the following NFA (N_1 in the book):



is specified by the file (`examples/sipser-n1.nfa`):

```
q1 q2 q3 q4
0 1
q1
q4
q1 0 q1
q1 1 q1
q1 1 q2
q2 0 q3
q2 & q3
q3 1 q4
q4 0 q4
q4 1 q4
```

2 Matcher

Write a function that tests whether a NFA M accepts a string w:

match(M, w)

- M: An NFA to run
- w: The string to run on
- Returns: A pair (*flag*, *path*), where
  - *flag*: True if M accepts w; false otherwise.
  - *path*: List of the transitions on an accepting path. If there is more than one, an arbitrary path is returned.

This function must run in $O(|M| \cdot |w|)$ time, where $|M|$ is the number of states plus transitions in M.

Here's how to do this. Define a *configuration* of M on input string w to be a pair (q, i), where $q \in Q$ and $0 \leq i \leq |w|$. These configurations can be thought of as nodes in a graph. For example, if the NFA is N_1 above and $w =$ `010110`, then the graph of configurations is:

$$\begin{tikzcd}
{} \arrow[d] & & & \\
\boxed{q_1, 0} \arrow[d] & & & \\
\boxed{q_1, 1} \arrow[d] \arrow[dr] & & & \\
\boxed{q_1, 2} \arrow[d] & \boxed{q_2, 2} \arrow[r] \arrow[dr] & \boxed{q_3, 2} & \\
\boxed{q_1, 3} \arrow[d] \arrow[dr] & & \boxed{q_3, 3} \arrow[dr] & \\
\boxed{q_1, 4} \arrow[d] \arrow[dr] & \boxed{q_2, 4} \arrow[r] & \boxed{q_3, 4} \arrow[dr] & \boxed{q_4, 4} \arrow[d] \\
\boxed{q_1, 5} \arrow[d] & \boxed{q_2, 5} \arrow[r] \arrow[dr] & \boxed{q_3, 5} & \boxed{q_4, 5} \arrow[d] \\
\boxed{q_1, 6} & & \boxed{q_3, 6} & \boxed{\boxed{q_4, 6}}
\end{tikzcd}$$

This is similar to Sipser's Figure 1.29, but there are several differences here. The most important difference is that configuration $(q_4, 5)$ appears only once with two incoming edges, instead of twice. In general, each configuration appears at most once in the graph. As a result, the graph has at most $|Q| \cdot |w| + 1$ nodes and $|\delta| \cdot |w|$ edges.

Then, deciding whether N_1 accepts w amounts to searching for a path from the start configuration (in this case, $(q_1, 0)$) to an accept configuration (in this case, $(q_4, 6)$). You can use any graph search algorithm. Depth-first search is most similar to how the real Unix tools work, but breadth-first search is probably the least hassle. Beware that if you use recursion, you could get an error for exceeding the maximum recursion depth. Another thing to watch out for is cycles of ε-transitions, as in `examples/cycle.nfa`. Make sure your matcher doesn't hang when it encounters one.

When you studied graph search algorithms, you may not have seen how to reconstruct the found path. Graph searches maintain a set that keeps track of which configurations have been visited. If you change this to a data structure that records, for each configuration, which edge led to that configuration, then after the search finishes, you can use that information to reconstruct the path.

3 Putting it together

Package the above into a command-line tool called `nfa_path`:

`nfa_path` *nfafile string*

- *nfafile*: name of file defining an NFA M
- *string*: string to run M on
- Output:
  - If M accepts *string*, prints `accept` on one line, followed by an accepting path on subsequent lines
  - Otherwise, prints `reject`

The path must be printed with one transition per line, in the same format as the NFA file format. For example:

```
$ nfa_path examples/sipser-n1.nfa 11
accept
q1 1 q2
q2 & q3
q3 1 q4
$ nfa_path examples/sipser-n1.nfa 1
reject
```

Test your program by running `tests/test-cp1.sh`. This script runs `nfa_path` on several NFAs and several test strings, and it also produces a graph of the running time of `nfa_path` on NFAs of various sizes. The sizes are chosen so that if the running time is $\Theta(n^2)$, then the graph will look roughly linear, like this:

```
n=100:      *
n=142:        *
n=174:          *
n=200:             *
n=224:                *
n=245:                  *
n=265:                     *
n=283:                        *
n=300:                          *
n=317:                             *
n=332:                               *
n=347:                                 *
n=361:                                   *
n=375:                                       *
n=388:                                            *
n=400:                                            *
```

Submission instructions

Your code should build and run on `student`*nn*`.cse.nd.edu`. The automatic tester will clone your repository, run `make -C cp1`, and then run `tests/test-cp1.sh`. You're advised to try all of the above steps and ensure that all tests pass.

To submit your work, please push your repository to GitHub and then create a new release with tag version `cp1` (note that the tag version is not the same thing as the release title). If you are making a partial submission, then use tag version `cp1-part1` for part 1, `cp1-part2` for part 2, and so on.

Rubric

Part 1	
data structure	3
read_nfa	3
write_nfa	3
Part 2 (match)	
correct algorithm	6
handling ε	3
reconstructing path	6
Part 3 (`nfa_path`)	
correctness	3
time complexity	3
Total	30