Appendix A

Summary of Basic Formulae

Summary

Two sets of tables are provided for reference. The first records the definition, and the first two moments of the most common probability distributions used in this volume. The second records the basic elements of standard Bayesian inference processes for a number of special cases. In particular, it records the appropriate likelihood function, the sufficient statistics, the conjugate prior and corresponding posterior and predictive distributions, the reference prior and corresponding reference posterior and predictive distributions.

A.1 PROBABILITY DISTRIBUTIONS

The first section of this Appendix consists of a set of tables which record the notation, parameter range, variable range, definition, and first two moments of the probability distributions (discrete and continuous, univariate and multivariate) used in this volume.

Univariate Discrete Distributions

$\mathrm{Br}(x \mid \theta)$ *Bernoulli (p. 115)*	
$0 < \theta < 1$	$x = 0, 1$
$p(x) = \theta^x (1-\theta)^{1-x}$	
$E[x] = \theta$	$V[x] = \theta(1-\theta)$
$\mathrm{Bi}(x \mid \theta, n)$ *Binomial (p. 115)*	
$0 < \theta < 1, n = 1, 2, \ldots$	$x = 0, 1, \ldots, n$
$p(x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$	
$E[x] = n\theta$	$V[x] = n\theta(1-\theta)$
$\mathrm{Bb}(x \mid \alpha, \beta, n)$ *Binomial-Beta (p. 117)*	
$\alpha > 0, \beta > 0, n = 1, 2, \ldots$	$x = 0, 1, \ldots, n$
$p(x) = c \binom{n}{x} \Gamma(\alpha + x)\Gamma(\beta + n - x)$	$c = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+n)}$
$E[x] = n\dfrac{\alpha}{\alpha+\beta}$	$V[x] = \dfrac{n\alpha\beta}{(\alpha+\beta)^2} \dfrac{(\alpha+\beta+n)}{(\alpha+\beta+1)}$
$\mathrm{Hy}(x \mid N, M, n)$ *Hypergeometric (p. 115)*	
$N = 1, 2, \ldots$ $M = 1, 2, \ldots$ $n = 1, \ldots, N + M$	$x = a, a+1, \ldots, b$ $a = \max(0, n - M)$ $b = \min(n, N)$
$p(x) = c \binom{N}{x} \binom{M}{n-x}$	$c = \binom{N+M}{n}^{-1}$
$E[x] = n\dfrac{N}{N+M}$	$V[x] = \dfrac{nNM}{(N+M)^2} \dfrac{N+M-n}{N+M-1}$

Univariate Discrete Distributions (continued)

$\mathrm{Nb}(x \mid \theta, r)$ *Negative-Binomial (p. 116)*	
$0 < \theta < 1, r = 1, 2, \ldots$	$x = 0, 1, 2, \ldots$
$p(x) = c \binom{r+x-1}{r-1} (1-\theta)^x$	$c = \theta^r$
$E[x] = r\theta$	$V[x] = r\dfrac{1-\theta}{\theta^2}$
$\mathrm{Nbb}(x \mid \alpha, \beta, r)$ *Negative-Binomial-Beta (p. 118)*	
$\alpha > 0,\ \beta > 0,\ r = 1, 2 \ldots$	$x = 0, 1, 2, \ldots$
$p(x) = c \binom{r+x-1}{r-1} \dfrac{\Gamma(\beta+x)}{\Gamma(\alpha+\beta+r+x)}$	$c = \dfrac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\beta)}$
$E[x] = \dfrac{r\beta}{\alpha-1}$ $\quad V[x] = \dfrac{r\beta}{(\alpha-1)}\left[\dfrac{\alpha+\beta+r-1}{(\alpha-2)} + \dfrac{r\beta}{(\alpha-1)(\alpha-2)}\right]$	
$\mathrm{Pn}(x \mid \lambda)$ *Poisson (p. 116)*	
$\lambda > 0$	$x = 0, 1, 2, \ldots$
$p(x) = c\ \dfrac{\lambda^x}{x!}$	$c = e^{-\lambda}$
$E[x] = \lambda$	$V[x] = \lambda$
$\mathrm{Pg}(x \mid \alpha, \beta, n)$ *Poisson-Gamma (p. 119)*	
$\alpha > 0,\ \beta > 0,\ \nu > 0$	$x = 0, 1, 2, \ldots$
$p(x) = c\dfrac{\Gamma(\alpha+x)}{x!}\dfrac{\nu^x}{(\beta+\nu)^{\alpha+x}}$	$c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}$
$E[x] = \nu\dfrac{\alpha}{\beta}$	$V[x] = \dfrac{\nu\alpha}{\beta}\left[1 + \dfrac{\nu}{\beta}\right]$

Univariate Continuous Distributions

$\mathrm{Be}(x \mid \alpha, \beta)$ *Beta (p. 116)*	
$\alpha > 0, \beta > 0$	$0 < x < 1$
$p(x) = c\ x^{\alpha-1}(1-x)^{\beta-1}$	$c = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$
$E[x] = \dfrac{\alpha}{\alpha+\beta}$	$V[x] = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
$\mathrm{Un}(x \mid a, b)$ *Uniform (p. 117)*	
$b > a$	$a < x < b$
$p(x) = c$	$c = (b-a)^{-1}$
$E[x] = \frac{1}{2}(a+b)$	$V[x] = \frac{1}{12}(b-a)^2$
$\mathrm{Ga}(x \mid \alpha, \beta)$ *Gamma (p. 118)*	
$\alpha > 0, \beta > 0$	$x > 0$
$p(x) = c\ x^{\alpha-1}e^{-\beta x}$	$c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}$
$E[x] = \alpha\beta^{-1}$	$V[x] = \alpha\beta^{-2}$
$\mathrm{Ex}(x \mid \theta)$ *Exponential (p. 118)*	
$\theta > 0$	$x > 0$
$p(x) = c\, e^{-\theta x}$	$c = \theta$
$E[x] = 1/\theta$	$V[x] = 1/\theta^2$
$\mathrm{Gg}(x \mid \alpha, \beta, n)$ *Gamma-Gamma (p. 120)*	
$\alpha > 0, \beta > 0, n > 0$	$x > 0$
$p(x) = c\, \dfrac{x^{n-1}}{(\beta+x)^{\alpha+n}}$	$c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}\ \dfrac{\Gamma(\alpha+n)}{\Gamma(n)}$
$E[x] = n\dfrac{\beta}{\alpha-1}$	$V[x] = \dfrac{\beta^2(n^2+n(\alpha-1))}{(\alpha-1)^2(\alpha-2)}$

Univariate Continuous Distributions (continued)

$\chi^2(x \mid \nu) = \chi^2_\nu$ *Chi-squared (p. 120)*	
$\nu > 0$	$x > 0$
$p(x) = c\, x^{(\nu/2)-1} e^{-x/2}$	$c = \dfrac{(1/2)^{\nu/2}}{\Gamma(\nu/2)}$
$E[x] = \nu$	$V[x] = 2\nu$

$\chi^2(x \mid \nu, \lambda)$ *Non-central Chi-squared (p. 121)*	
$\nu > 0, \lambda > 0$	$x > 0$
$p(x) = \sum_{i=0}^{\infty} \mathrm{Pn}\left(i \,\Big\vert\, \dfrac{\lambda}{2}\right) \chi^2(x \mid \nu + 2i)$	
$E[x] = \nu + \lambda$	$V[x] = 2(\nu + 2\lambda)$

$\mathrm{Ig}(x \mid \alpha, \beta)$ *Inverted-Gamma (p. 119)*	
$\alpha > 0, \beta > 0$	$x > 0$
$p(x) = c\, x^{-(\alpha+1)} e^{-\beta/x}$	$c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}$
$E[x] = \dfrac{\beta}{\alpha - 1}$	$V[x] = \dfrac{\beta^2}{(\alpha-1)^2(\alpha-2)}$

$\chi^{-1}(x \mid \nu)$ *Inverted-Chi-squared (p. 119)*	
$\nu > 0$	$x > 0$
$p(x) = c\ x^{-(\nu/2+1)} e^{-1/2x^2}$	$c = \dfrac{(1/2)^{\nu/2}}{\Gamma(\nu/2)}$
$E[x] = \dfrac{1}{\nu - 2}$	$V[x] = \dfrac{2}{(\nu-2)^2(\nu-4)}$

$\mathrm{Ga}^{-1/2}(x \mid \alpha, \beta)$ *Square-root Inverted-Gamma (p. 119)*	
$\alpha > 0, \beta > 0$	$x > 0$
$p(x) = c\ x^{-(2\alpha+1)} e^{-\beta/x^2}$	$c = \dfrac{2\beta^\alpha}{\Gamma(\alpha)}$
$E[x] = \dfrac{\sqrt{\beta}\,\Gamma(\alpha - 1/2)}{\Gamma(\alpha)}$	$V[x] = \dfrac{\beta}{\alpha - 1} - E[x]^2$

Univariate Continuous Distributions (continued)

$\mathrm{Pa}(x \mid \alpha, \beta)$ *Pareto (p. 120)*	
$\alpha > 0, \beta > 0$	$\beta \le x < +\infty$
$p(x) = c\ x^{-(\alpha+1)}$	$c = \alpha\beta^{\alpha}$
$E[x] = \frac{\beta\alpha}{\alpha-1}\ , \quad \alpha > 1$	$V[x] = \frac{\beta^2\alpha}{(\alpha-1)^2(\alpha-2)}\ , \quad \alpha > 2$
$\mathrm{Ip}(x \mid \alpha, \beta)$ *Inverted-Pareto (p. 120)*	
$\alpha > 0, \quad \beta > 0$	$0 < x < \beta^{-1}$
$p(x) = c\, x^{\alpha-1}$	$c = \alpha\beta^{\alpha}$
$E[x] = \beta^{-1}\alpha(\alpha+1)^{-1}$	$V[x] = \beta^{-2}\alpha(\alpha+1)^{-2}(\alpha+2)^{-1}$
$\mathrm{N}(x \mid \mu, \lambda)$ *Normal (p. 121)*	
$-\infty < \mu < +\infty, \lambda > 0$	$-\infty < x < +\infty$
$p(x) = c\ \exp\left\{-\frac{1}{2}\lambda(x-\mu)^2\right\}$	$c = \lambda^{1/2}(2\pi)^{-1/2}$
$E[x] = \mu$	$V[x] = \lambda^{-1}$
$\mathrm{St}(x \mid \mu, \lambda, \alpha)$ *Student t (p. 122)*	
$-\infty < \mu < +\infty, \lambda > 0, \alpha > 0$	$-\infty < x < +\infty$
$p(x) = c\ \left[1 + \alpha^{-1}\lambda(x-\mu)^2\right]^{-(\alpha+1)/2}$	$c = \dfrac{\Gamma\left(\frac{1}{2}(\alpha+1)\right)}{\Gamma(\frac{1}{2}\alpha)} \left(\dfrac{\lambda}{\alpha\pi}\right)^{1/2}$
$E[x] = \mu$	$V[x] = \lambda^{-1}\alpha(\alpha-2)^{-1}$
$\mathrm{F}(x \mid \alpha, \beta) = \mathrm{F}_{\alpha,\beta}$ *Snedecor F (p. 123)*	
$\alpha > 0, \beta > 0$	$x > 0$
$p(x) = c\ \dfrac{x^{\alpha/2-1}}{(\beta+\alpha x)^{(\alpha+\beta)/2}}$	$c = \dfrac{\Gamma\left(\frac{1}{2}(\alpha+\beta)\right)\alpha^{\alpha/2}\beta^{\beta/2}}{\Gamma(\frac{1}{2}\alpha)\Gamma(\frac{1}{2}\beta)}$
$E[x] = \dfrac{\beta}{\beta-2}\ , \quad \beta > 2$	$V[x] = \dfrac{2\beta^2(\alpha+\beta-2)}{\alpha(\beta-2)^2(\beta-4)}\ , \beta > 4$

Univariate Continuous Distributions (continued)

$\mathrm{Lo}(x \mid \alpha, \beta)$ *Logistic (p. 122)*	
$-\infty < \alpha < +\infty, \beta > 0$	$-\infty < x < +\infty$
$p(x) = \beta^{-1} \exp\left\{-\beta^{-1}(x-\alpha)\right\} \left[1 + \exp\left\{-\beta^{-1}(x-\alpha)\right\}\right]^{-2}$	
$E[x] = \alpha$	$V[x] = \beta^2\pi^2/3$

Multivariate Discrete Distributions

$\mathrm{Mu}_k(\boldsymbol{x} \mid \boldsymbol{\theta}, n)$ *Multinomial (p. 133)*		
$\boldsymbol{\theta} = (\theta_1, \ldots, \theta_k)$		$\boldsymbol{x} = (x_1, \ldots, x_k)$
$0 < \theta_i < 1, \quad \sum_{\ell=1}^k \theta_\ell \le 1$		$\sum_{\ell=1}^k x_\ell \le n$
$n = 1, 2, \ldots$		$x_i = 0, 1, 2, \ldots$
$p(\boldsymbol{x}) = \dfrac{n!}{\prod_{\ell=1}^{k+1} x_\ell!} \prod_{\ell=1}^{k+1} \theta^{x_\ell}, \qquad \theta_{k+1} = 1 - \sum_{\ell=1}^{k} \theta_\ell, \quad x_{k+1} = n - \sum_{\ell=1}^{k} x_\ell$		
$E[x_i] = n\theta_i$	$V[x_i] = n\theta_i(1-\theta_i)$	$C[x_i, x_j] = -n\theta_i\theta_j$

$\mathrm{Md}_k(\boldsymbol{x} \mid \boldsymbol{\theta}, n)$ *Multinomial-Dirichlet (p. 135)*	
$\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_{k+1})$	$\boldsymbol{x} = (x_1, \ldots, x_k)$
$\alpha_i > 0$	$x_i = 0, 1, 2, \ldots$
$n = 1, 2, \ldots$	$\sum_{\ell=1}^n x_l \le n$
$p(\boldsymbol{x}) = c \prod_{\ell=1}^{k+1} \dfrac{\alpha_\ell^{[x_\ell]}}{x_\ell!}$	$c = \dfrac{n!}{\left(\sum_{\ell=1}^{k+1} \alpha_\ell\right)^{[n]}}$
$\alpha^{[s]} = \prod_{\ell=1}^s (\alpha + \ell - 1)$	$x_{k+1} = n - \sum_{\ell=1}^k x_\ell$
$E[x_i] = np_i$	$V[x_i] = \dfrac{n + \sum_{\ell=1}^{k+1} \alpha_\ell}{1 + \sum_{\ell=1}^{k+1} \alpha_\ell}\, np_i(1-p_i)$
$p_i = \dfrac{\alpha_i}{\sum_{\ell=1}^{k+1} \alpha_\ell}$	$C[x_i, x_j] = -\dfrac{n + \sum_{\ell=1}^{k+1} \alpha_\ell}{1 + \sum_{\ell=1}^{k+1} \alpha_\ell}\, np_ip_j$

Multivariate Continuous Distributions

$\mathrm{Di}_k(\boldsymbol{x} \mid \boldsymbol{\alpha})$ *Dirichlet (p. 134)*

$\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_{k+1})$ $\quad$ $\boldsymbol{x} = (x_1, \ldots, x_k)$

$\alpha_i > 0$ $\quad$ $0 < x_i < 1, \quad \sum_{\ell=1}^{k} x_\ell \leq 1$

$$p(\boldsymbol{x}) = c\,\left(1 - \sum_{\ell=1}^{k} x_\ell\right)^{\alpha_{k+1}-1} \prod_{\ell=1}^{k} x_\ell^{\alpha_\ell - 1} \qquad c = \frac{\Gamma(\sum_{\ell=1}^{k+1} \alpha_\ell)}{\prod_{\ell=1}^{k+1} \Gamma(\alpha_\ell)}$$

$$E[x_i] = \frac{\alpha_i}{\sum_{\ell=1}^{k+1} \alpha_\ell} \quad V[x_i] = \frac{E[x_i](1 - E[x_i])}{1 + \sum_{\ell=1}^{k+1} \alpha_\ell} \quad C[x_i, x_j] = \frac{-E[x_i]E[x_j]}{1 + \sum_{\ell=1}^{k+1} \alpha_\ell}$$

$\mathrm{Ng}(x, y \mid \mu, \lambda, \alpha, \beta)$ *Normal-Gamma (p. 136)*

$\mu \in \Re,\ \lambda > 0,\ \alpha > 0,\ \beta > 0,$ $\quad$ $x \in \Re,\ y > 0$

$p(x, y) = \mathrm{N}(x \mid \mu, \lambda y)\ \mathrm{Ga}(y \mid \alpha, \beta)$

$E[x] = \mu \qquad E[y] = \alpha\beta^{-1} \qquad V[x] = \beta\lambda^{-1}(\alpha - 1)^{-1} \qquad V[y] = \alpha\beta^{-2}$

$p(x) = \mathrm{St}(x \mid \mu, \alpha\beta^{-1}\lambda, 2\alpha)$

$\mathrm{N}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\lambda})$ *Multivariate Normal (p. 136)*

$\boldsymbol{\mu} = (\mu_1, \ldots, \mu_k) \in \Re^k$ $\quad$ $\boldsymbol{x} = (x_1, \ldots, x_k) \in \Re^k$

$\boldsymbol{\lambda}$ symmetric positive-definite

$p(\boldsymbol{x}) = c\ \exp\left\{-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^t \boldsymbol{\lambda} (\boldsymbol{x} - \boldsymbol{\mu})\right\}$ $\quad$ $c = |\,\boldsymbol{\lambda}\,|^{1/2} (2\pi)^{-k/2}$

$E[\boldsymbol{x}] = \boldsymbol{\mu}$ $\quad$ $V[\boldsymbol{x}] = \boldsymbol{\lambda}^{-1}$

$\mathrm{Pa}_2(x, y \mid \alpha, \beta_0, \beta_1)$ *Bilateral Pareto (p. 141)*

$(\beta_0, \beta_1) \in \Re^2,\ \beta_0 < \beta_1,\ \alpha > 0$ $\quad$ $(x, y) \in \Re^2,\ x < \beta_0,\ y > \beta_1$

$p(x, y) = c\ (y - x)^{-(\alpha+2)}$ $\quad$ $c = \alpha(\alpha + 1)(\beta_1 - \beta_0)^\alpha$

$$E[x] = \frac{\alpha\beta_0 - \beta_1}{\alpha - 1} \quad E[y] = \frac{\alpha\beta_1 - \beta_0}{\alpha - 1} \qquad V[x] = V[y] = \frac{\alpha(\beta_1 - \beta_0)^2}{(\alpha - 1)^2(\alpha - 2)}$$

Multivariate Continuous Distributions (continued)

$\mathrm{Ng}_k(\boldsymbol{x}, y \mid \boldsymbol{\mu}, \boldsymbol{\lambda}, \alpha, \beta)$ *Multivariate Normal-Gamma (p. 140)*

$-\infty < \mu_i < +\infty,\ \alpha > 0,\ \beta > 0$ $\quad (\boldsymbol{x}, y) = (x_1, \ldots, x_k, y)$
$\boldsymbol{\lambda}$ symmetric positive-definite $\quad -\infty < x_i < \infty, \quad y > 0$

$p(\boldsymbol{x}, y) = \mathrm{N}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\lambda} y)\ \mathrm{Ga}(y \mid \alpha, \beta)$

$E[\boldsymbol{x}, y] = (\boldsymbol{\mu},\ \alpha\beta^{-1}), \qquad V[\boldsymbol{x}] = (\alpha - 1)^{-1}\beta\boldsymbol{\lambda}^{-1}, \qquad V[y] = \alpha\beta^{-2}$

$p(\boldsymbol{x}) = \mathrm{St}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\lambda}\alpha\beta^{-1}, 2\alpha)$ $\quad p(y) = Ga(y \mid \alpha, \beta)$

$\mathrm{Nw}_k(\boldsymbol{x}, \boldsymbol{y} \mid \boldsymbol{\mu}, \lambda, \alpha, \boldsymbol{\beta})$ *Multivariate Normal-Wishart (p. 140)*

$-\infty < \mu_i < +\infty,\ \lambda > 0$ $\quad \boldsymbol{x} = (x_1, \ldots, x_k)$
$2\alpha > k - 1$ $\quad -\infty < x_i < +\infty$
$\boldsymbol{\beta}$ symmetric non-singular $\quad \boldsymbol{y}$ symmetric positive-definite

$p(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{N}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, \lambda\boldsymbol{y})\ \mathrm{Wi}_k(\boldsymbol{y} \mid \alpha, \boldsymbol{\beta})$

$E[\boldsymbol{x}, \boldsymbol{y}] = \{\boldsymbol{\mu}, \alpha\boldsymbol{\beta}^{-1}\}$ $\quad V[\boldsymbol{x}] = (\alpha - 1)^{-1}\boldsymbol{\beta}\lambda^{-1}$

$p(\boldsymbol{x}) = \mathrm{St}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, \lambda\alpha\boldsymbol{\beta}^{-1}, 2\alpha)$ $\quad p(\boldsymbol{y}) = \mathrm{Wi}_k(\boldsymbol{y} \mid \alpha, \boldsymbol{\beta})$

$\mathrm{St}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\lambda}, \alpha)$ *Multivariate Student (p. 139)*

$-\infty < \mu_i < +\infty,\ \alpha > 0$ $\quad \boldsymbol{x} = (x_1, \ldots, x_k)$
$\boldsymbol{\lambda}$ symmetric positive-definite $\quad -\infty < x_i < +\infty$

$$p(\boldsymbol{x}) = c\left[1 + \frac{1}{\alpha}(\boldsymbol{x} - \boldsymbol{\mu})^t\boldsymbol{\lambda}(\boldsymbol{x} - \boldsymbol{\mu})\right]^{-(\alpha+k)/2} \qquad c = \frac{\Gamma\left(\frac{1}{2}(\alpha + k)\right)}{\Gamma(\frac{1}{2}\alpha)(\alpha\pi)^{k/2}}|\boldsymbol{\lambda}|^{1/2}$$

$E[\boldsymbol{x}] = \boldsymbol{\mu}, \quad V[\boldsymbol{x}] = \boldsymbol{\lambda}^{-1}(\alpha - 2)^{-1}\alpha$

$\mathrm{Wi}_k(\boldsymbol{x} \mid \alpha, \boldsymbol{\beta})$ *Wishart (p. 138)*

$2\alpha > k - 1$ $\quad \boldsymbol{x}$ symmetric positive-definite
$\boldsymbol{\beta}$ symmetric non-singular

$$p(\boldsymbol{x}) = c\,|\boldsymbol{x}|^{\alpha-(k+1)/2}\exp\{-\operatorname{tr}(\boldsymbol{\beta}\boldsymbol{x})\} \qquad c = \frac{\pi^{-k(k-1)/4}|\boldsymbol{\beta}|^{\alpha}}{\prod_{\ell=1}^{k}\Gamma(\frac{1}{2}(2\alpha + 1 - \ell))}$$

$E[\boldsymbol{x}] = \alpha\boldsymbol{\beta}^{-1}, \quad E[\boldsymbol{x}^{-1}] = (\alpha - \frac{k+1}{2})^{-1}\boldsymbol{\beta}$

A.2 INFERENTIAL PROCESSES

The second section of this Appendix records the basic elements of the Bayesian learning processes for many commonly used statistical models.

For each of these models, we provide, in separate sections of the table, the following: the sufficient statistic and its sampling distribution; the conjugate family, the conjugate prior predictives for a single observable and for the sufficient statistic, the conjugate posterior and the conjugate posterior predictive for a single observable.

When clearly defined, we also provide, in a final section, the reference prior and the corresponding reference posterior and posterior predictive for a single observable. In the case of uniparameter models this can always be done. We recall, however, from Section 5.2.4 that, in multiparameter problems, the reference prior is only defined *relative to an ordered parametrisation*. In the univariate normal model (Example 5.17), the reference prior for (μ, λ) happens to be the same as that for (λ, μ), namely $\pi(\mu, \lambda) = \pi(\lambda, \mu) \propto \lambda^{-1}$, and we provide the corresponding reference posteriors for μ and λ, together with the reference predictive distribution for a future observation.

In the multinomial, multivariate normal and linear regression models, however, there are very many different reference priors, corresponding to different inference problems, and specified by different ordered parametrisations. These are not reproduced in this Appendix.

Bernoulli model

$\boldsymbol{z} = \{x_1, \ldots, x_n\}, \qquad x_i \in \{0, 1\}$ $p(x_i \mid \theta) = \mathrm{Br}(x_i \mid \theta), \qquad 0 < \theta < 1$
$t(\boldsymbol{z}) = r = \sum_{i=1}^{n} x_i$ $p(r \mid \theta) = \mathrm{Bi}(r \mid \theta, n)$
$p(\theta) = \mathrm{Be}(\theta \mid \alpha, \beta)$ $p(x) = \mathrm{Bb}(r \mid \alpha, \beta, 1)$ $p(r) = \mathrm{Bb}(r \mid \alpha, \beta, n)$ $p(\theta \mid \boldsymbol{z}) = \mathrm{Be}(\theta \mid \alpha + r, \beta + n - r)$ $p(x \mid \boldsymbol{z}) = \mathrm{Bb}(x \mid \alpha + r, \beta + n - r, 1)$
$\pi(\theta) = \mathrm{Be}(\theta \mid \frac{1}{2}, \frac{1}{2})$ $\pi(\theta \mid \boldsymbol{z}) = \mathrm{Be}(\theta \mid \frac{1}{2} + r, \frac{1}{2} + n - r)$ $\pi(x \mid \boldsymbol{z}) = \mathrm{Bb}(x \mid \frac{1}{2} + r, \frac{1}{2} + n - r, 1)$

Poisson Model

$\boldsymbol{z} = \{x_1, \ldots, x_n\}, \qquad x_i = 0, 1, 2, \ldots$
$p(x_i \mid \lambda) = \text{Pn}(x_i \mid \lambda), \qquad \lambda \geq 0$

$t(\boldsymbol{z}) = r = \sum_{i=1}^{n} x_i$
$p(r \mid \lambda) = \text{Pn}(r \mid n\lambda)$

$p(\lambda) = \text{Ga}(\lambda \mid \alpha, \beta)$
$p(x) = \text{Pg}(x \mid \alpha, \beta, 1)$
$p(r) = \text{Pg}(x \mid \alpha, \beta, n)$
$p(\lambda \mid \boldsymbol{z}) = \text{Ga}(\lambda \mid \alpha + r, \beta + n)$
$p(x \mid \boldsymbol{z}) = \text{Pg}(x \mid \alpha + r, \beta + n, 1)$

$\pi(\lambda) \propto \lambda^{-1/2}$
$\pi(\lambda \mid \boldsymbol{z}) = \text{Ga}(\lambda \mid r + \frac{1}{2}, n)$
$\pi(x \mid \boldsymbol{z}) = \text{Pg}(x \mid r + \frac{1}{2}, n, 1)$

Negative-Binomial model

$\boldsymbol{z} = (x_1, \ldots, x_n), \qquad x_i = 0, 1, 2, \ldots$
$p(x_i \mid \theta) = \text{Nb}(x_i \mid \theta, r), \qquad 0 < \theta < 1$

$t(\boldsymbol{z}) = s = \sum_{i=1}^{n} x_i$
$p(s \mid \theta) = \text{Nb}(s \mid \theta, nr)$

$p(\theta) = \text{Be}(\theta \mid \alpha, \beta)$
$p(x) = \text{Nbb}(x \mid \alpha, \beta, r)$
$p(s) = \text{Nbb}(s \mid \alpha, \beta, nr)$
$p(\theta \mid \boldsymbol{z}) = \text{Be}(\theta \mid \alpha + nr, \beta + s)$
$p(x \mid \boldsymbol{z}) = \text{Nbb}(x \mid \alpha + nr, \beta + s, r)$

$\pi(\theta) \propto \theta^{-1}(1 - \theta)^{-1/2}$
$\pi(\theta \mid \boldsymbol{z}) = \text{Be}(\theta \mid nr, s + \frac{1}{2})$
$\pi(x \mid \boldsymbol{z}) = \text{Nbb}(x \mid nr, s + \frac{1}{2}, r)$

Exponential Model

$\boldsymbol{z} = \{x_1, \ldots, x_n\}, \qquad 0 < x_i < \infty$
$p(x_i \,|\, \theta) = \mathrm{Ex}(x_i \,|\, \theta), \qquad \theta > 0$

$t(\boldsymbol{z}) = t = \sum_{i=1}^{n} x_i$
$p(t \,|\, \theta) = \mathrm{Ga}(t \,|\, n, \theta)$

$p(\theta) = \mathrm{Ga}(\theta \,|\, \alpha, \beta)$
$p(x) = \mathrm{Gg}(x \,|\, \alpha, \beta, 1)$
$p(t) = \mathrm{Gg}(t \,|\, \alpha, \beta, n)$
$p(\theta \,|\, \boldsymbol{z}) = \mathrm{Ga}(\theta \,|\, \alpha + n, \beta + t)$
$p(x \,|\, \boldsymbol{z}) = \mathrm{Gg}(x \,|\, \alpha + n, \beta + t, 1)$

$\pi(\theta) \propto \theta^{-1}$
$\pi(\theta \,|\, \boldsymbol{z}) = \mathrm{Ga}(\theta \,|\, n, t)$
$\pi(x \,|\, \boldsymbol{z}) = \mathrm{Gg}(x \,|\, n, t, 1)$

Uniform Model

$\boldsymbol{z} = \{x_1, \ldots, x_n\}, \qquad 0 < x_i < \theta$
$p(x_i \,|\, \theta) = \mathrm{Un}(x_i \,|\, 0, \theta), \qquad \theta > 0$

$t(\boldsymbol{z}) = t = \max\{x_1, \ldots, x_n\}$
$p(t \,|\, \theta) = \mathrm{Ip}(t \,|\, n, \theta^{-1})$

$p(\theta) = \mathrm{Pa}(\theta \,|\, \alpha, \beta)$
$p(x) = \frac{\alpha}{\alpha+1}\mathrm{Un}(x \,|\, 0, \beta), \text{ if } x \le \beta, \quad \frac{1}{\alpha+1}\mathrm{Pa}(x \,|\, \alpha, \beta), \text{ if } x > \beta$
$p(t) = \frac{\alpha}{\alpha+n}\mathrm{Ip}(t \,|\, n, \beta^{-1}), \text{ if } t \le \beta, \quad \frac{n}{\alpha+n}\mathrm{Pa}(t \,|\, \alpha, \beta), \text{ if } t > \beta$
$p(\theta \,|\, \boldsymbol{z}) = \mathrm{Pa}(\theta \,|\, \alpha + n, \beta_n), \quad \beta_n = \max\{\beta, t\}$
$p(x \,|\, \boldsymbol{z}) = \frac{\alpha+n}{\alpha+n+1}\mathrm{Un}(x \,|\, 0, \beta_n), \text{ if } x \le \beta_n, \quad \frac{1}{\alpha+n+1}\mathrm{Pa}(x \,|\, \alpha, \beta_n), \text{ if } x > \beta_n$

$\pi(\theta) \propto \theta^{-1}$
$\pi(\theta \,|\, \boldsymbol{z}) = \mathrm{Pa}(\theta \,|\, n, t)$
$\pi(x \,|\, \boldsymbol{z}) = \frac{n}{n+1}\mathrm{Un}(x \,|\, 0, t), \text{ if } x \le t, \quad \frac{1}{n+1}\mathrm{Pa}(x \,|\, n, t), \text{ if } x > t$

Normal Model (known precision λ)

$\boldsymbol{z} = \{x_1, \ldots, x_n\}, \qquad -\infty < x_i < \infty$ $p(x_i \mid \mu, \lambda) = \mathrm{N}(x_i \mid \mu, \lambda), \qquad -\infty < \mu < \infty$
$t(\boldsymbol{z}) = \bar{x} = n^{-1} \sum_{i=1}^{n} x_i$ $p(\bar{x} \mid \mu, \lambda) = \mathrm{N}(x \mid \mu, n\lambda)$
$p(\mu) = \mathrm{N}(\mu \mid \mu_0, \lambda_0)$ $p(x) = \mathrm{N}\left(x \mid \mu_0, \lambda\, \lambda_0(\lambda_0 + \lambda)^{-1}\right)$ $p(\overline{x}) = \mathrm{N}\left(\overline{x} \mid \mu_0, n\lambda\, \lambda_0 \lambda_n^{-1}\right), \qquad \lambda_n = \lambda_0 + n\lambda,$ $p(\mu \mid \boldsymbol{z}) = \mathrm{N}(\mu \mid \mu_n, \lambda_n), \qquad \mu_n = \lambda_n^{-1}(\lambda_0\mu_0 + n\lambda\overline{x})$ $p(x \mid \boldsymbol{z}) = \mathrm{N}\left(x \mid \mu_n, \lambda\, \lambda_n(\lambda_n + \lambda)^{-1}\right)$
$\pi(\mu) = \text{constant}$ $\pi(\mu \mid \boldsymbol{z}) = \mathrm{N}(\mu \mid \bar{x}, n\lambda)$ $\pi(x \mid \boldsymbol{z}) = \mathrm{N}(x \mid \bar{x}, \lambda\, n(n+1)^{-1})$

Normal Model (known mean μ)

$\boldsymbol{z} = \{x_1, \ldots, x_n\}, \qquad -\infty < x_i < \infty$ $p(x_i \mid \mu, \lambda) = \mathrm{N}(x_i \mid \mu, \lambda), \qquad \lambda > 0$
$t(\boldsymbol{z}) = t = \sum_{i=1}^{n} (x_i - \mu)^2$ $p(t \mid \mu, \lambda) = \mathrm{Ga}(t \mid \tfrac{1}{2}n, \tfrac{1}{2}\lambda), \qquad p(\lambda t) = \chi^2(\lambda t \mid n)$
$p(\lambda) = \mathrm{Ga}(\lambda \mid \alpha, \beta)$ $p(x) = \mathrm{St}(x \mid \mu, \alpha\beta^{-1}, 2\alpha)$ $p(t) = \mathrm{Gg}(x \mid \alpha, 2\beta, \tfrac{1}{2}n)$ $p(\lambda \mid \boldsymbol{z}) = \mathrm{Ga}(\lambda \mid \alpha + \tfrac{1}{2}n, \beta + \tfrac{1}{2}t)$ $p(x \mid \boldsymbol{z}) = \mathrm{St}(x \mid \mu, (\alpha + \tfrac{1}{2}n)(\beta + \tfrac{1}{2}t)^{-1}, 2\alpha + n)$
$\pi(\lambda) \propto \lambda^{-1}$ $\pi(\lambda \mid \boldsymbol{z}) = \mathrm{Ga}(\lambda \mid \tfrac{1}{2}n, \tfrac{1}{2}t)$ $\pi(x \mid \boldsymbol{z}) = \mathrm{St}(x \mid \mu, nt^{-1}, n)$

Normal Model (both parameters unknown)

$$z = \{x_1, \ldots, x_n\}, \qquad -\infty < x_i < \infty$$
$$p(x_i \mid \mu, \lambda) = \mathrm{N}(x_i \mid \mu, \lambda), \qquad -\infty < \mu < \infty, \quad \lambda > 0$$

$$t(z) = (\bar{x}, s), \qquad n\bar{x} = \sum_{i=1}^n x_i, \qquad ns^2 = \sum_{i=1}^n (x_i - \bar{x})^2$$
$$p(\bar{x} \mid \mu, \lambda) = \mathrm{N}(\bar{x} \mid \mu, n\lambda)$$
$$p(ns^2 \mid \mu, \lambda) = \mathrm{Ga}(ns^2 \mid \tfrac{1}{2}(n-1), \tfrac{1}{2}\lambda), \qquad p(\lambda ns^2) = \chi^2(\lambda ns^2 \mid n-1)$$

$$p(\mu, \lambda) = \mathrm{Ng}(\mu, \lambda \mid \mu_0, n_0, \alpha, \beta) = \mathrm{N}(\mu \mid \mu_0, n_0\lambda)\ \mathrm{Ga}(\lambda \mid \alpha, \beta)$$
$$p(\mu) = \mathrm{St}(\mu \mid \mu_0, n_0\alpha\beta^{-1}, 2\alpha)$$
$$p(\lambda) = \mathrm{Ga}(\lambda \mid \alpha, \beta)$$
$$p(x) = \mathrm{St}(x \mid \mu_0, n_0(n_0+1)^{-1}\alpha\beta^{-1}, 2\alpha)$$
$$p(\overline{x}) = \mathrm{St}(\overline{x} \mid \mu_0, n_0 n(n_0+n)^{-1}\alpha\beta^{-1}, 2\alpha)$$
$$p(ns^2) = \mathrm{Gg}(ns^2 \mid \alpha, 2\beta, \tfrac{1}{2}(n-1))$$
$$p(\mu \mid z) = \mathrm{St}(\mu \mid \mu_n, (n+n_0)(\alpha + \tfrac{1}{2}n)\beta_n^{-1}, 2\alpha + n),$$
$$\mu_n = (n_0+n)^{-1}(n_0\mu_0 + n\bar{x}),$$
$$\beta_n = \beta + \tfrac{1}{2}ns^2 + \tfrac{1}{2}(n_0+n)^{-1}n_0 n(\mu_0 - \overline{x})^2$$
$$p(\lambda \mid z) = \mathrm{Ga}(\lambda \mid \alpha + \tfrac{1}{2}n, \beta_n)$$
$$p(x \mid z) = \mathrm{St}\big(x \mid \mu_n, (n+n_0)(n+n_0+1)^{-1}(\alpha + \tfrac{1}{2}n)\beta_n^{-1}, 2\alpha + n\big)$$

$$\pi(\mu, \lambda) = \pi(\lambda, \mu) \propto \lambda^{-1}, \qquad n > 1$$
$$\pi(\mu \mid z) = \mathrm{St}\big(\mu \mid \bar{x}, (n-1)s^{-2}, n-1\big)$$
$$\pi(\lambda \mid z) = \mathrm{Ga}\left(\lambda \mid \tfrac{1}{2}(n-1), \tfrac{1}{2}ns^2\right)$$
$$\pi(x \mid z) = \mathrm{St}\left(x \mid \bar{x}, (n-1)(n+1)^{-1}s^{-2}, n-1\right)$$

Multinomial Model

$$\boldsymbol{z} = \{r_1, \ldots, r_k, n\}, \qquad r_i = 0, 1, 2, \ldots, \qquad \textstyle\sum_{\ell=1}^{k} r_i \leq n$$
$$p(\boldsymbol{z} \,|\, \boldsymbol{\theta}) = \mathrm{Mu}_k(\boldsymbol{z} \,|\, \boldsymbol{\theta}, n), \qquad 0 < \theta_i < 1, \quad \textstyle\sum_{i=1}^{k} \theta_i \leq 1$$

$$t(\boldsymbol{z}) = (\boldsymbol{r}, n), \quad \boldsymbol{r} = (r_1, \ldots, r_k)$$
$$p(\boldsymbol{r} \,|\, \boldsymbol{\theta}) = \mathrm{Mu}_k(\boldsymbol{r} \,|\, \boldsymbol{\theta}, n)$$

$$p(\boldsymbol{\theta}) = \mathrm{Di}_k(\boldsymbol{\theta} \,|\, \boldsymbol{\alpha}), \quad \boldsymbol{\alpha} = \{\alpha_1, \ldots, \alpha_{k+1}\}$$
$$p(\boldsymbol{r}) = \mathrm{Md}_k(\boldsymbol{r} \,|\, \boldsymbol{\alpha}, n)$$
$$p(\boldsymbol{\theta} \,|\, \boldsymbol{z}) = \mathrm{Di}_k\left(\boldsymbol{\theta} \,|\, \alpha_1 + r_1, \ldots, \alpha_k + r_k, \alpha_{k+1} + n - \textstyle\sum_{\ell=1}^{k} r_\ell\right)$$
$$p(\boldsymbol{x} \,|\, \boldsymbol{z}) = \mathrm{Md}_k\left(\boldsymbol{x} \,|\, \alpha_1 + r_1, \ldots, \alpha_k + r_k, \alpha_{k+1} + n - \textstyle\sum_{\ell=1}^{k} r_\ell, n\right)$$

Multivariate Normal Model

$$\boldsymbol{z} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}, \qquad \boldsymbol{x}_i \in \Re^k$$
$$p(\boldsymbol{x}_i \,|\, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \mathrm{N}_k(\boldsymbol{x}_i \,|\, \boldsymbol{\mu}, \boldsymbol{\lambda}), \qquad \boldsymbol{\mu} \in \Re^k, \quad \boldsymbol{\lambda} \;\; k \times k \text{ positive-definite}$$

$$t(\boldsymbol{z}) = (\bar{\boldsymbol{x}}, \boldsymbol{S}), \qquad \overline{\boldsymbol{x}} = n^{-1} \textstyle\sum_{i=1}^{n} \boldsymbol{x}_i, \qquad \boldsymbol{S} = \textstyle\sum_{i=1}^{n} (\boldsymbol{x}_i - \bar{\boldsymbol{x}})(\boldsymbol{x}_i - \bar{\boldsymbol{x}})^t$$
$$p(\bar{\boldsymbol{x}} \,|\, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \mathrm{N}_k(\bar{\boldsymbol{x}} \,|\, \boldsymbol{\mu}, n\boldsymbol{\lambda})$$
$$p(\boldsymbol{S} \,|\, \boldsymbol{\lambda}) = \mathrm{Wi}_k\left(\boldsymbol{S} \,|\, \tfrac{1}{2}(n-1), \tfrac{1}{2}\boldsymbol{\lambda}\right)$$

$$p(\boldsymbol{\mu}, \boldsymbol{\lambda}) = \mathrm{Nw}_k(\boldsymbol{\mu}, \boldsymbol{\lambda} \,|\, \boldsymbol{\mu}_0, n_0, \alpha, \boldsymbol{\beta}) = \mathrm{N}_k(\boldsymbol{\mu} \,|\, \mu_0, n_0\boldsymbol{\lambda})\, \mathrm{Wi}_k(\boldsymbol{\lambda} \,|\, \alpha, \boldsymbol{\beta})$$
$$p(\boldsymbol{x}) = \mathrm{St}_k\left(\boldsymbol{x} \,|\, \boldsymbol{\mu}_0, (n_0+1)^{-1} n_0 (\alpha - \tfrac{1}{2}(k-1))\boldsymbol{\beta}^{-1}, 2\alpha - k + 1\right)$$
$$p(\boldsymbol{\mu} \,|\, \boldsymbol{z}) = \mathrm{St}_k\left(\boldsymbol{\mu} \,|\, \boldsymbol{\mu}_n, (n+n_0)\alpha_n \boldsymbol{\beta}_n^{-1}, 2\alpha_n\right),$$
$$\boldsymbol{\mu}_n = (n_0 + n)^{-1}(n_0 \boldsymbol{\mu}_0 + n\bar{\boldsymbol{x}}),$$
$$\boldsymbol{\beta}_n = \boldsymbol{\beta} + \tfrac{1}{2}\boldsymbol{S} + \tfrac{1}{2}(n+n_0)^{-1} n n_0 (\boldsymbol{\mu}_0 - \overline{\boldsymbol{x}})(\boldsymbol{\mu}_0 - \overline{\boldsymbol{x}})^t$$
$$p(\boldsymbol{\lambda} \,|\, \boldsymbol{z}) = \mathrm{Wi}_k(\boldsymbol{\lambda} \,|\, \alpha + \tfrac{1}{2}n, \boldsymbol{\beta}_n)$$
$$p(\boldsymbol{x} \,|\, \boldsymbol{z}) = \mathrm{St}_k(\boldsymbol{x} \,|\, \boldsymbol{\mu}_n, (n_0 + n + 1)^{-1}(n_0 + n)\alpha_n \boldsymbol{\beta}_n^{-1}, 2\alpha_n),$$
$$\alpha_n = \alpha + \tfrac{1}{2}n - \tfrac{1}{2}(k-1)$$

Linear Regression

$$\boldsymbol{z} = (\boldsymbol{y}, \boldsymbol{X}), \quad \boldsymbol{y} = (y_1, \ldots, y_n)^t \in \Re^n,\ \boldsymbol{x}_i = (x_{i1}, \ldots, x_{ik}) \in \Re^k,\ \boldsymbol{X} = (x_{ij})$$
$$p(\boldsymbol{y} \mid \boldsymbol{X}, \boldsymbol{\theta}, \lambda) = \mathrm{N}_n(\boldsymbol{y} \mid \boldsymbol{X}\boldsymbol{\theta}, \lambda \boldsymbol{I}_n), \qquad \boldsymbol{\theta} \in \Re^k, \quad \lambda > 0$$

$$t(\boldsymbol{z}) = (\boldsymbol{X}^t\boldsymbol{X}, \boldsymbol{X}^t\boldsymbol{y})$$

$$p(\boldsymbol{\theta}, \lambda) = \mathrm{Ng}(\boldsymbol{\theta}, \lambda \mid \boldsymbol{\theta}_0, \boldsymbol{n}_0, \alpha, \beta) = \mathrm{N}_k(\boldsymbol{\theta} \mid \boldsymbol{\theta}_0, \boldsymbol{n}_0\lambda)\,\mathrm{Ga}(\lambda \mid \alpha, \beta)$$
$$p(\boldsymbol{\theta}) = \mathrm{St}_k(\boldsymbol{\theta} \mid \boldsymbol{\theta}_0,\ \boldsymbol{n}_0\alpha\beta^{-1},\ 2\alpha)$$
$$p(\lambda) = \mathrm{Ga}(\lambda \mid \alpha, \beta)$$
$$p(y \mid \boldsymbol{x}) = \mathrm{St}\left(y \mid \boldsymbol{x}\boldsymbol{\theta}_0,\ f(\boldsymbol{x})\alpha\beta^{-1},\ 2\alpha\right)$$
$$f(\boldsymbol{x}) = 1 - \boldsymbol{x}(\boldsymbol{x}^t\boldsymbol{x} + \boldsymbol{n}_0)^{-1}\boldsymbol{x}^t,$$
$$p(\boldsymbol{\theta} \mid \boldsymbol{z}) = \mathrm{St}_k\left(\boldsymbol{\theta} \mid \boldsymbol{\theta}_n,\ (\boldsymbol{n}_0 + \boldsymbol{X}^t\boldsymbol{X})(\alpha + \tfrac{1}{2}n)\beta_n^{-1},\ 2\alpha + n\right),$$
$$\boldsymbol{\theta}_n = (\boldsymbol{n}_0 + \boldsymbol{X}^t\boldsymbol{X})^{-1}(\boldsymbol{n}_0\boldsymbol{\theta}_0 + \boldsymbol{X}^t\boldsymbol{y}),$$
$$\beta_n = \beta + \tfrac{1}{2}(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\theta}_n)^t\boldsymbol{y} + \tfrac{1}{2}(\boldsymbol{\theta}_0 - \boldsymbol{\theta}_n)^t\boldsymbol{n}_0\boldsymbol{\theta}_0$$
$$p(\lambda \mid \boldsymbol{z}) = \mathrm{Ga}(\lambda \mid \alpha + \tfrac{1}{2}n, \beta_n)$$
$$p(y \mid \boldsymbol{x}, \boldsymbol{z}) = \mathrm{St}(y \mid \boldsymbol{x}\boldsymbol{\theta}_n,\ f_n(\boldsymbol{x})(\alpha + \tfrac{1}{2}n)\beta_n^{-1},\ 2\alpha + n),$$
$$f_n(\boldsymbol{x}) = 1 - \boldsymbol{x}(\boldsymbol{x}^t\boldsymbol{x} + \boldsymbol{n}_0 + \boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{x}^t$$

$$\pi(\boldsymbol{\theta}, \lambda) = \pi(\lambda, \boldsymbol{\theta}) \propto \lambda^{-(k+1)/2} \quad \text{(for all reorderings of the } \theta_j\text{)}$$
$$\pi(\boldsymbol{\theta} \mid \boldsymbol{z}) = \mathrm{St}_k\left(\boldsymbol{\theta} \mid \hat{\boldsymbol{\theta}}_n,\ \tfrac{1}{2}\boldsymbol{X}^t\boldsymbol{X}(n-k)\hat{\beta}_n^{-1},\ n-k\right),$$
$$\hat{\boldsymbol{\theta}}_n = (\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X}^t\boldsymbol{y},$$
$$\hat{\beta}_n = \tfrac{1}{2}(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\theta}}_n)^t\boldsymbol{y}$$
$$\pi(\lambda \mid \boldsymbol{z}) = \mathrm{Ga}(\lambda \mid \tfrac{1}{2}(n-k), \hat{\beta}_n)$$
$$\pi(y \mid \boldsymbol{x}, \boldsymbol{z}) = \mathrm{St}(y \mid \boldsymbol{x}\hat{\boldsymbol{\theta}}_n,\ \tfrac{1}{2}f_n(\boldsymbol{x})(n-k)\hat{\beta}_n^{-1},\ n-k),$$
$$f_n(\boldsymbol{x}) = 1 - \boldsymbol{x}(\boldsymbol{x}^t\boldsymbol{x} + \boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{x}^t$$