Distributed Control Systems

Course Project #3

Distributed Task Assignment for Robotic Networks

The project deals with modeling and control of a network of robots that need to self-assign a set of target locations. The objective is to choose an assignment that optimizes the total path length.

Task 1

Suppose we have N agents that want to cooperatively solve the constraint-coupled linear program

$$\begin{aligned} \min_{z_1,\ldots,z_N} \quad & \sum_{i=1}^{N} c_i^\top z_i \\ \text{subj. to} \quad & \sum_{i=1}^{N} H_i z_i = b \\ & z_i \in P_i, \qquad \forall\, i \in \{1,\ldots,N\}, \end{aligned} \tag{1}$$

where $c_i \in \mathbb{R}^{n_i}$, $H_i \in \mathbb{R}^{S\times n_i}$, $b \in \mathbb{R}^S$ and P_i is a compact polyhedron described by linear equality and inequality constraints, i.e., $P_i \triangleq \{z_i \in \mathbb{R}^{n_i} \mid D_i z_i \leq d_i, G_i z_i = g_i\}, \quad \forall\, i \in \{1,\ldots,N\}$.

Design a software, written in Matlab/Python, implementing the Distributed Dual Subgradient algorithm given in [1], to solve problem (1). Perform (Montecarlo) simulations and provide plots to show the convergence of the algorithm while varying the number of agents and the problem size.

Task 2

Consider a team of N robots that want to self-assign a set of N task scattered in the environment. Each robot has to serve a task and each task must be served by at most one robot. Robots want to minimize the total travelled distance. The assignment problem can be illustrated with a bipartite assignment graph $G_A = \{V_A, U_A; E_A\}$, where the node sets $V_A = \{1,\ldots,N\}$ and $U_A = \{1,\ldots,N\}$ represent the set of agents and tasks respectively. An edge $(i,k) \in E_A$ exists if and only if agent i can be assigned to the task k. The cost c_{ik} for agent i to perform the task k is a weight on the edge. An illustrative example is depicted in Figure 1.

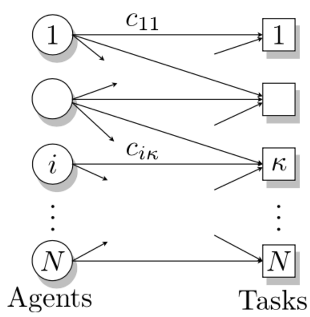

Figure 1: Example of bipartite graph for the task assignment problem.

This problem can be cast as the following linear optimization program

$$
\begin{aligned}
\min_{x} \quad & \sum_{(i,\kappa)\in E_A} c_{i\kappa} x_{i\kappa} \\
\text{subj. to} \quad & 0 \le x \le 1 \\
& \sum_{\{\kappa \mid (i,\kappa)\in E_A\}} x_{i\kappa} = 1 \qquad \forall\, i \in \{1,\dots,N\} \\
& \sum_{\{i \mid (i,\kappa)\in E_A\}} x_{i\kappa} = 1 \qquad \forall\, \kappa \in \{1,\dots,N\},
\end{aligned}
\tag{2}
$$

where x is the variable stacking all $x_{i\kappa}$ for all i, κ. Problem (2) can be cast in the form (1) by defining z_i and c_i as the vectors stacking all $x_{i\kappa}$ and $c_{i\kappa}$ (respectively) for all κ such that $(i,\kappa) \in E_A$, and by defining the local polyhedra P_i as

$$
P_i = \Big\{ x_i \mid 0 \le x_i \le 1 \text{ and } \sum_{\{\kappa \mid (i,\kappa)\in E_A\}} x_{i\kappa} = 1 \Big\}, \qquad i \in \{1,\dots,N\},
$$

and, finally, by defining suitably the H_i matrices and the b vector.

Robots are scattered in the environment and, as soon as tasks appear, they evaluate the c_i vectors, start the optimization and move towards the assigned tasks. An example is depicted in Figure 2.

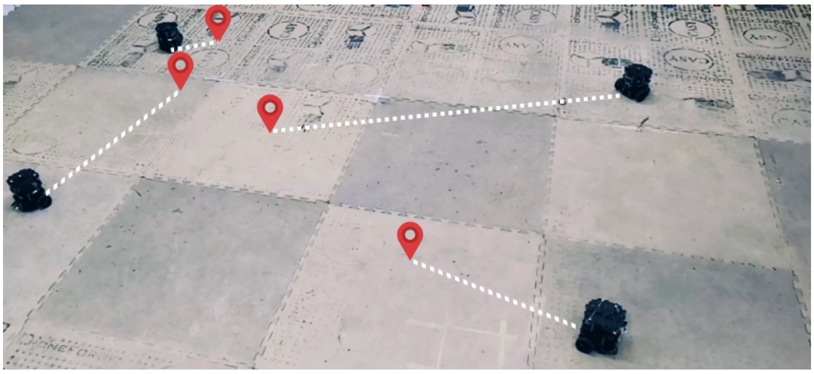

Figure 2: Snapshot from a task assignment experiment. Robots move to reach the assigned tasks after the distributed optimization procedure.

Use the developed software to solve an instance of the problem and show the results.

Note: task positions can be generated randomly, but it is necessary to make sure the optimal solution is unique. To this end, add a random, small perturbation to the c_i vectors.

Notes

1. Any other information and material necessary for the project development will be given during project "meetings".
2. The project report must be written in Latex and follow the main structure of the attached template.
3. Any email for project support must have the subject:

   "[DCS2021]-Group X: rest of the subject".

References

[1] G. Notarstefano, I. Notarnicola, and A. Camisa, "Distributed optimization for smart cyber-physical networks," *Foundations and Trends in Systems and Control*, vol. 7, no. 3, pp. 253–383, 2019. [Online]. Available: http://dx.doi.org/10.1561/2600000020