$c := 299792.458 \cdot km \cdot s^{-1}$ $\quad kb := 1.38 \cdot 10^{-23} \cdot J \cdot K^{-1}$

Define a band width $\quad BW := 250kHz$

Sampling rate $\quad Sr := 2 \cdot BW$

and sampling interval $\quad dt := Sr^{-1}$

$Nt := 8 \cdot 1024 \cdot 1024 \qquad Tspan := Nt \cdot dt \qquad Tspan = 16.777216\ s$

$jt := 0 .. Nt - 1 \qquad tt_{jt} := jt \cdot dt$

Rest frequency of a carrier line $\quad Fco := 8400.100 \cdot MHz \qquad Lam := c \cdot Fco^{-1} \qquad Lam = 0.035689153462459\ m$

And additional tones $\quad Fso1 := Fco - 10 \cdot kHz \qquad Fso1 = 8400090000\ s^{-1}$

$Fso2 := Fco + 20 \cdot kHz \qquad Fso2 = 8400120000\ s^{-1}$

$Fso3 := Fco - 50 \cdot kHz \qquad Fso3 = 8400050000\ s^{-1}$

Receiver LO $\quad FLO := 8400.100 \cdot MHz$

Amplitude of the carrier $\quad Amplc := \sqrt{0.1} \qquad Amplc = 0.316227766016838$

Amplitudes of tones $\quad Ampls1 := Amplc \cdot 0.1$

$Ampls2 := Amplc \cdot 0.5$

$Ampls3 := Amplc \cdot 0.3$

RMS of the additive thermal noise $\quad Anoise := 1.0$

Noise power per Hz of bandwidth $\quad Pnhz := \dfrac{Anoise^2}{BW} \qquad Pnhz = 0.000004\ s$

Signal power to noise power per Hz $\quad SNRhz := \dfrac{0.5 \cdot Amplc^2}{Pnhz} \qquad SNRhz = 12500.000000000002\ s^{-1}$

SNR in 1 Hz in dB $\quad 10 \cdot \log(SNRhz \cdot s) = 40.96910013008056$

add to get it in 1/16 Hz $\quad 10 \cdot \log(16) = 12.041199826559248$

Motion model

Initial distance $\quad Ro := 150 \cdot 10^6 \cdot km - 1.8cm$

Initial velocity $\quad Vo := 2.5 \cdot km \cdot s^{-1}$

Initial acceleration $\quad Ao := 15.0 \cdot m \cdot s^{-2}$

Initial third derivative $\quad Bo := -2.0 \cdot m \cdot s^{-3}$

Power balance between carrier line and noise is about what can be expected for 2.5 W Tx power, 1.5 m Tx antenna, 32 m Rx antenna with Tsys 35 K at X-band and distance 1 AU

$Rs := 150 \cdot 10^6 \cdot km \qquad Ta := 35K \qquad bw := 1Hz$

$Dt := 1.5m$

$Dr := 32m \qquad Aeff := 0.6$

$Pt := 2.5W$

$$Pr := \frac{Pt}{4\pi \cdot Rs^2} \cdot \frac{\pi}{4} \cdot \left(\frac{Dt}{Lam}\right)^2 \cdot \frac{\pi}{4} \cdot Dr^2 \cdot Aeff \qquad Pn := kb \cdot Ta \cdot bw$$

Power ratio in 1 Hz $\quad 10 \cdot \log\left(\dfrac{Pr}{Pn}\right) = 40.88341490205394$

$PN := kb \cdot Ta \cdot 16 \cdot MHz$

Power ratio in 16 MHz $\quad 10 \cdot \log\left(\dfrac{Pr}{PN}\right) = -31.157784924505307$

S/C spinning model with a period of 1.53s and TX antenna offset 2.5 cm

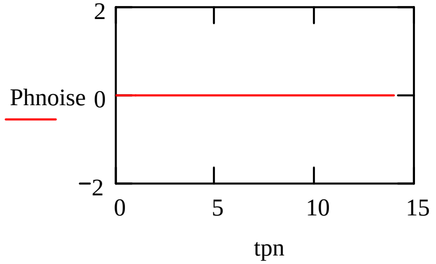
$\mathrm{Wspin} := \dfrac{2\pi}{1.53\mathrm{s}}$ $\quad$ $\mathrm{Aspin} := 0.0 \cdot \mathrm{cm}$ $\quad$ Spin frequency amplitude $\quad$ $\dfrac{\mathrm{Wspin} \cdot \mathrm{Aspin}}{\mathrm{c}} \cdot \mathrm{Fco} = 0$ $\quad$ Spin is disabled

Make a reasonable phase noise of onboard LO, 1.0 radian sigma on a time scale ~1 s

$\mathrm{Npn} := 15$ $\quad$ $\mathrm{Phnoise} := \mathrm{rnorm}(\mathrm{Npn}, 0, 1.0)$ $\quad$ $\mathrm{jpn} := 0 .. \mathrm{Npn} - 1$ $\quad$ $\mathrm{tpn}_{\mathrm{jpn}} := \mathrm{jpn} \cdot 1\mathrm{s}$

$\mathrm{Phnoise} := \mathrm{han}(\mathrm{Phnoise})$ $\quad$ $\mathrm{rmsPhnoise} := \mathrm{stdev}(\mathrm{Phnoise})$ $\quad$ $\mathrm{Phnoise} := 0 \cdot \mathrm{Phnoise} \cdot \mathrm{rmsPhnoise}^{-1}$ $\quad$ LO phase noise is disabled

$\mathrm{Phnc} := \mathrm{Spline}(\mathrm{tpn}, \mathrm{Phnoise}, \mathrm{tt})$



Phase noise in radians

Compute "Doppler" parameters of the motion

$dFco := Fco \cdot \frac{Vo}{c}$ $\quad dFco = 70049.29390184993\ s^{-1}$ $\quad dFso1 := Fso1 \cdot \frac{Vo}{c}$ $\quad dFso1 = 70049.21051082613\ s^{-1}$

$vFc := Fco + dFco - FLO$ $\quad vFc = 70049.29390144348\ s^{-1}$ $\quad vFs1 := Fso1 + dFso1 - FLO$ $\quad vFs1 = 60049.21051120758\ s^{-1}$

$aFc := Fco \cdot \frac{Ao}{c}$ $\quad aFc = 420.29576341109953\ s^{-2}$ $\quad aFs1 := Fso1 \cdot \frac{Ao}{c}$ $\quad aFs1 = 420.2952630649567\ s^{-2}$

$bFc := Fco \cdot \frac{Bo}{c}$ $\quad bFc = -56.03943512147994\ s^{-3}$ $\quad bFs1 := Fso1 \cdot \frac{Bo}{c}$ $\quad bFs1 = -56.0393684086609\ s^{-3}$

$dFso2 := Fso2 \cdot \frac{Vo}{c}$ $\quad dFso2 = 70049.46068389753\ s^{-1}$ $\quad dFso3 := Fso3 \cdot \frac{Vo}{c}$ $\quad dFso3 = 70048.87694673093\ s^{-1}$

$vFs2 := Fso2 + dFso2 - FLO$ $\quad vFs2 = 90049.46068382263\ s^{-1}$ $\quad vFs3 := Fso3 + dFso3 - FLO$ $\quad vFs3 = 20048.87694644928\ s^{-1}$

$aFs2 := Fso2 \cdot \frac{Ao}{c}$ $\quad aFs2 = 420.2967641033851\ s^{-2}$ $\quad aFs3 := Fso3 \cdot \frac{Ao}{c}$ $\quad aFs3 = 420.29326168038557\ s^{-2}$

$bFs2 := Fso2 \cdot \frac{Bo}{c}$ $\quad bFs2 = -56.039568547118016\ s^{-3}$ $\quad bFs3 := Fso3 \cdot \frac{Bo}{c}$ $\quad bFs3 = -56.03910155738474\ s^{-3}$

And initial phases

$PLamco := Ro \cdot Fco \cdot c^{-1}$ $\quad Phc0 := 2\pi \cdot (PLamco - floor(PLamco))$ $\quad Phc0 = 3.083301383650139$

$PLamso1 := Ro \cdot Fso1 \cdot c^{-1}$ $\quad Phs01 := 2\pi \cdot (PLamso1 - floor(PLamso1))$ $\quad Phs01 = 0.395767043274495$

$PLamso2 := Ro \cdot Fso2 \cdot c^{-1}$ $\quad Phs02 := 2\pi \cdot (PLamso2 - floor(PLamso2))$ $\quad Phs02 = 2.17825271879761$

$PLamso3 := Ro \cdot Fso3 \cdot c^{-1}$ $\quad Phs03 := 2\pi \cdot (PLamso3 - floor(PLamso3))$ $\quad Phs03 = 2.205864372979552$

Carrier line phase polynomial coefficients

$Cpp_0 := Phc0$

$Cpp_1 := 2\pi \cdot vFc \cdot s$

$Cpp_2 := 2\pi \cdot \frac{1}{2} \cdot aFc \cdot s^2$

$Cpp_3 := 2\pi \cdot \frac{1}{6} \cdot bFc \cdot s^3$

Write them down for bookkeeping

WxRITEPRN("Data3.Model.PhaseCoeffs.tone0.txt") := Cpp

Compute the phases

$$Phc_{jt} := 2\pi \cdot \left[vFc \cdot tt_{jt} + \frac{1}{2} \cdot aFc \cdot \left(tt_{jt}\right)^2 + \frac{1}{6} \cdot bFc \cdot \left(tt_{jt}\right)^3 \right] + Phc0 + Phnc_{jt} + 2\pi Aspin \cdot \frac{Fco}{c} \cdot \cos\left(Wspin \cdot tt_{jt}\right)$$

$vFc = 70049.29390144348\ s^{-1}$

$$Phs1_{jt} := 2\pi \cdot \left[vFs1 \cdot tt_{jt} + \frac{1}{2} \cdot aFs1 \cdot \left(tt_{jt}\right)^2 + \frac{1}{6} \cdot bFs1 \cdot \left(tt_{jt}\right)^3 \right] + Phs01 + Phnc_{jt} \cdot \frac{Fso1}{Fco} + 2 \cdot \pi \cdot Aspin \cdot \frac{Fso1}{c} \cdot \cos\left(Wspin \cdot tt_{jt}\right)$$

$$Phs2_{jt} := 2\pi \cdot \left[vFs2 \cdot tt_{jt} + \frac{1}{2} \cdot aFs2 \cdot \left(tt_{jt}\right)^2 + \frac{1}{6} \cdot bFs2 \cdot \left(tt_{jt}\right)^3 \right] + Phs02 + Phnc_{jt} \cdot \frac{Fso2}{Fco} + 2 \cdot \pi \cdot Aspin \cdot \frac{Fso2}{c} \cdot \cos\left(Wspin \cdot tt_{jt}\right)$$

$$Phs3_{jt} := 2\pi \cdot \left[vFs3 \cdot tt_{jt} + \frac{1}{2} \cdot aFs3 \cdot \left(tt_{jt}\right)^2 + \frac{1}{6} \cdot bFs3 \cdot \left(tt_{jt}\right)^3 \right] + Phs03 + Phnc_{jt} \cdot \frac{Fso3}{Fco} + 2 \cdot \pi \cdot Aspin \cdot \frac{Fso3}{c} \cdot \cos\left(Wspin \cdot tt_{jt}\right)$$

Make a signal

$$signal_{jt} := Amplc \cdot \cos\left(Phc_{jt}\right) + Ampls1 \cdot \cos\left(Phs1_{jt}\right) + Ampls2 \cdot \cos\left(Phs2_{jt}\right) + Ampls3 \cdot \cos\left(Phs3_{jt}\right)$$

$stdev(signal) = 0.259807648226341$

Make the additive thermal noise

$noise := rnorm(Nt, 0, Anoise)$

$stdev(noise) = 0.999948073723099$

$data := noise + signal$ $\quad rsd := stdev(data)$ $\quad rsd = 1.032995969114083$

Automatic Gain Control is set to put +/- 4 sigma range into 256 levels of 8 bit ADC

$sigma := \frac{128}{4}$ $\quad sigma = 32$ $\quad agn := \frac{sigma}{rsd}$

$data := data \cdot agn$ $\quad stdev(data) = 32.00000000000153$

Sampler

$$data_{jt} := if\left(data_{jt} \cdot sign\left(data_{jt}\right) \le 127, data_{jt}, 127 \cdot sign\left(data_{jt}\right)\right)$$

$$data_{jt} := floor\left(data_{jt} + 127 + 0.5\right)$$

$min(data) = 0$
$max(data) = 254$
$mean(data) = 127.0034967660904$

WxRITEBIN("Data3.NoSpin.NoPhNoise.byte" , "byte" , 0) := data

$Nd := 1000 \qquad Ntd := floor\left(\frac{Nt}{Nd}\right) \qquad jtd := 0..Ntd-1 \qquad jtd1 := 0..Ntd-2$

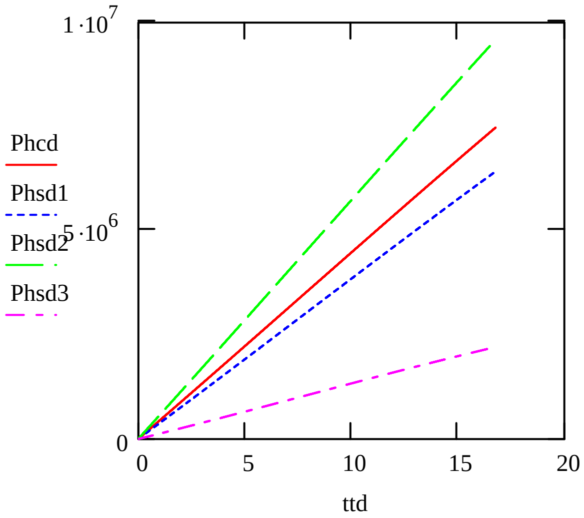
$ttd_{jtd} := tt_{jtd \cdot Nd} \qquad ttd_1 - ttd_0 = 0.002\ s$

$Phcd_{jtd} := Phc_{jtd \cdot Nd} \qquad Phsd1_{jtd} := Phs1_{jtd \cdot Nd} \qquad Phsd2_{jtd} := Phs2_{jtd \cdot Nd} \qquad Phsd3_{jtd} := Phs3_{jtd \cdot Nd} \qquad Fhcd_{jtd1} := \frac{Phcd_{jtd1+1} - Phcd_{jtd1}}{2\pi \cdot (dt \cdot Nd)} \qquad Fhcd_{Ntd-1} := Fhcd_{Ntd-2}$

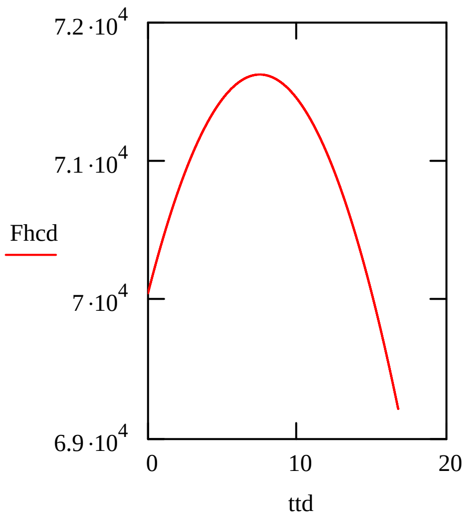
$Phmdat := augment\left(ttd \cdot s^{-1}, Fhcd \cdot s, Phcd, Phsd1, Phsd2, Phsd3\right)$

WxRITEPRN("Data3.NoSpin.NoPhNoise.PhaseModel.txt") := Phmdat

Phases (in video band) for all tones



Frequency of the carrier in video band



Briefly check the spectrum on the sub-sample of the full data

$Ns := 16 \cdot 2048 \quad js := 0 .. Ns - 1 \qquad Ts := Ns \cdot dt \qquad dfs := Ts^{-1} \qquad fs_{js} := js \cdot dfs$

$Ds := \mathrm{submatrix}(data, 0, Ns - 1, 0, 0) \qquad Ds := Ds - 127$

$Ss := \mathrm{cfft}(Ds)$

$Ps_{js} := \mathrm{Re}\left(Ss_{js}\right)^2 + \mathrm{Im}\left(Ss_{js}\right)^2$