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Event-Based Visual Odometry
on Non-Holonomic Ground Vehicles

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Code is available:

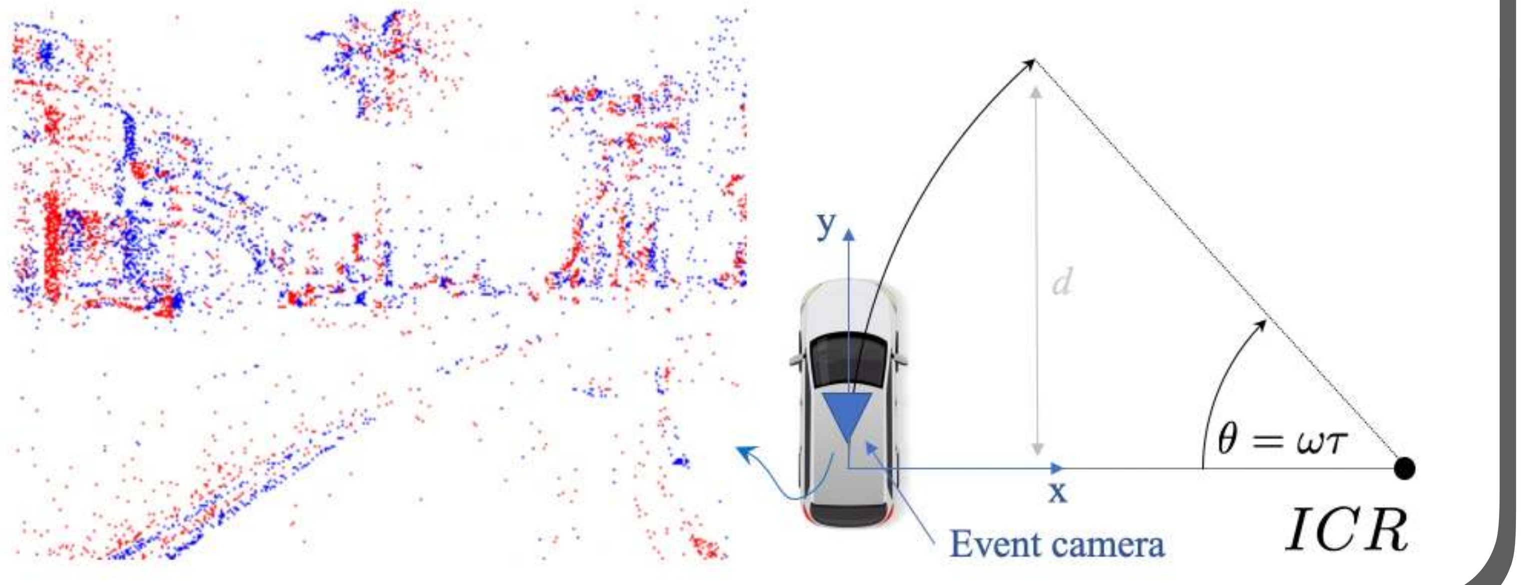
github.com/gowanting/NHEVO

Overview

Problem: Visual odometry with a forward-facing event camera mounted on a vehicle

Assumption: Ackermann motion; Constant rotational velocity

Compared: Traditional camera 1FPN [1]



The Ackermann Motion Model

Based on Ackermann motion model, a minimal parametrization of a relative displacement is given by

$$\mathbf{R} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \mathbf{t} = \frac{d}{\sin(\theta)} \begin{bmatrix} 1-\cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix}, \quad (1)$$

r: circle radius; θ: arc-angle; $d = r\sin(\theta)$

In event camera case, for each event $\mathbf{e}_i$ at t_i, we have

$$\mathbf{R}_i = \begin{bmatrix} \cos(\theta_i) & \sin(\theta_i) & 0 \\ -\sin(\theta_i) & \cos(\theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\omega\tau_i) & \sin(\omega\tau_i) & 0 \\ -\sin(\omega\tau_i) & \cos(\omega\tau_i) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$\mathbf{t}_i = \frac{d}{\sin(\theta)} \begin{bmatrix} 1-\cos(\theta_i) \\ \sin(\theta_i) \\ 0 \end{bmatrix} = \frac{d}{\sin(\omega\tau)} \begin{bmatrix} 1-\cos(\omega\tau_i) \\ \sin(\omega\tau_i) \\ 0 \end{bmatrix}. \quad (2)$$

ω: rotational velocity; $\tau_i = t_i - t_0$; τ: fixed time interval

t_0: start time of the considered time interval

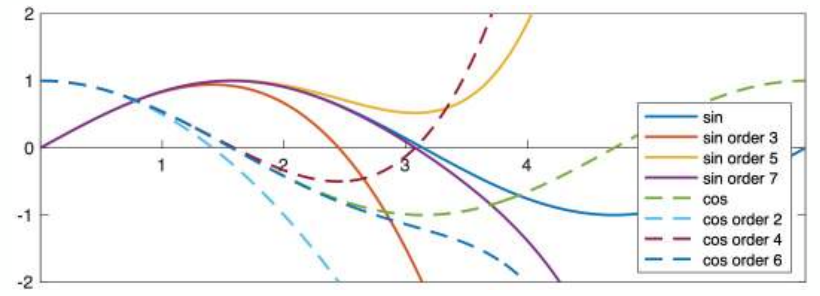
Event-based Non-holonomic Solver

Given points transformation $\mathbf{p}_i = \mathbf{R}_i^\top(\mathbf{p}_0 - \mathbf{t}_i)$ between frame at t_i and frame at t_0, with $\mathbf{p}_i = [p_i^x, p_i^y, p_i^z]$, we get

$$\begin{bmatrix} a_{i1} & a_{i2} & a_{i3} \end{bmatrix} \begin{bmatrix} p_0^x \\ p_0^y \\ d \end{bmatrix} = 0, \text{ where } \begin{aligned} a_{i1} &= -x_i\sin(\omega\tau_i) + \cos(\omega\tau_i), \\ a_{i2} &= -x_i\cos(\omega\tau_i) - \sin(\omega\tau_i), \\ a_{i3} &= \frac{x_i\sin(\omega\tau_i) - \cos(\omega\tau_i) + 1}{\sin(\omega\tau)}. \end{aligned} \quad (3)$$

However, the solution is challenging as the left-hand matrix remains a highly non-linear, trigonometric function of ω. We consider Taylor series expansions to approximate trigonometric functions, and make it a simple, uni-variate polynomial

$$\sin(\theta) \approx \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots + \frac{(-1)^n\theta^{2n+1}}{(2n+1)!} + \cdots,$$
$$\cos(\theta) \approx 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots + \frac{(-1)^n\theta^{2n}}{(2n)!} + \cdots. \quad (4)$$



s3c2: sin order 3 cos order 2. ***s5c4*** and ***s7c6*** defined similarly. We take ***s3c2*** as an example, substituting (4) into (3), we get

$$\begin{aligned} a_{i1} &\approx \widetilde{a}_{i1} = x_i\left(\frac{(\omega\tau_i)^3}{6} - \omega\tau_i\right) - \frac{(\omega\tau_i)^2}{2} + 1, \\ a_{i2} &\approx \widetilde{a}_{i2} = x_i\left(\frac{(\omega\tau_i)^2}{2} - 1\right) + \frac{(\omega\tau_i)^3}{6} - \omega\tau_i, \\ a_{i3} &\approx \widetilde{a}_{i3} = \frac{-(\tau_i(-x_i\tau_i^2\omega^2 + 3\tau_i\omega + 6x_i))}{\tau(\tau^2\omega^2 - 6)}. \end{aligned} \quad (5)$$

After eliminating the denominator, we obtain

$$\begin{bmatrix} b_{i1} & b_{i2} & b_{i3} \end{bmatrix} \begin{bmatrix} p_0^x \\ p_0^y \\ d \end{bmatrix} \approx 0, \text{ where } b_{ij} = c\widetilde{a}_{ij},\ j = 1,2,3,\ c = \tau(\tau^2\omega^2 - 6), \quad (6)$$

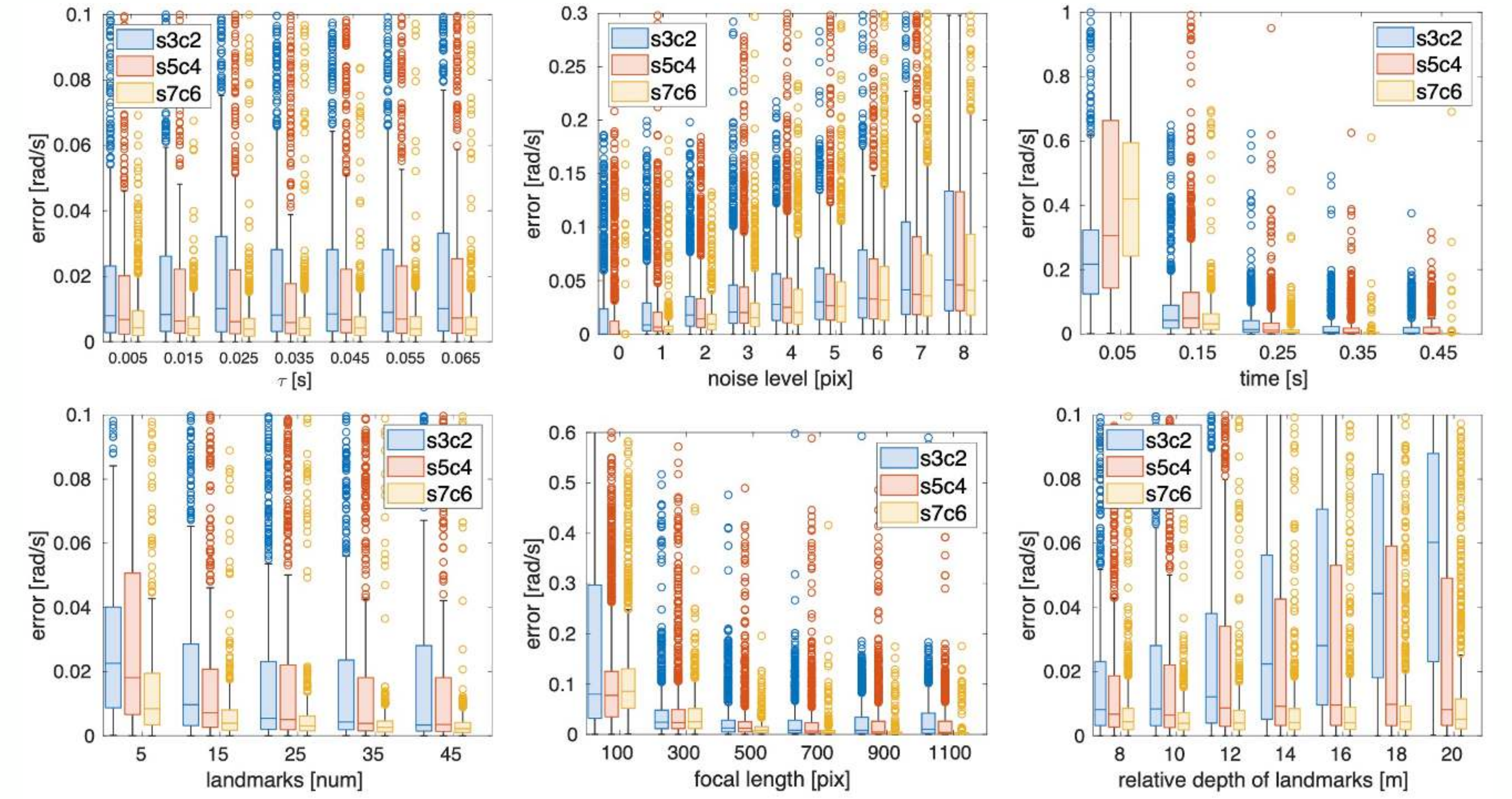
Given n corresponding events

$$\begin{bmatrix} b_{01} & b_{02} & b_{03} \\ \vdots & \vdots & \vdots \\ b_{i1} & b_{i2} & b_{i3} \\ \vdots & \vdots & \vdots \\ b_{(n-1)1} & b_{(n-1)2} & b_{(n-1)3} \end{bmatrix} \begin{bmatrix} p_0^x \\ p_0^y \\ d \end{bmatrix} = \mathbf{B}[k](\omega) \begin{bmatrix} p_0^x \\ p_0^y \\ d \end{bmatrix} \approx \mathbf{0}, \quad (7)$$

where $\mathbf{B}[k](\omega)$ represents a degree-k matrix in ω

For a significant ω solution in (7), $\mathbf{B}$ must be rank-deficient, which translates into a rank minimization problem. Finally use Sturm's root bracketing methodology to solve a univariate polynomial in ω.

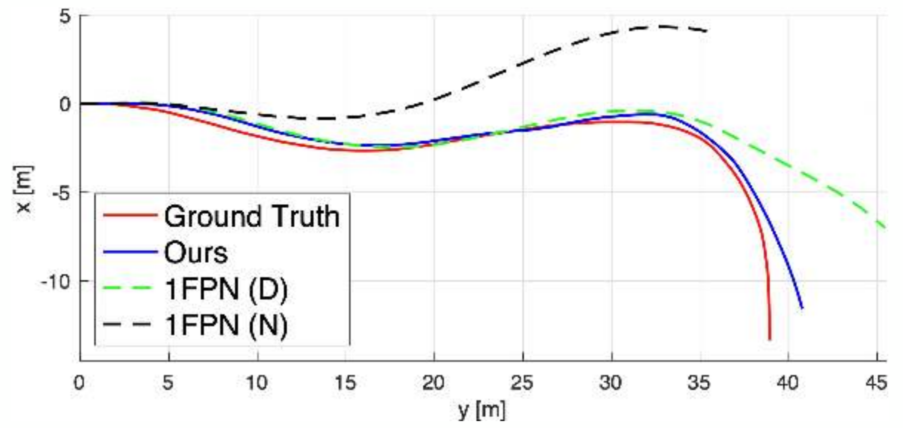
Experiments on Synthetic Data



Impact of different factors on the accuracy

Experiments on Real-World Data

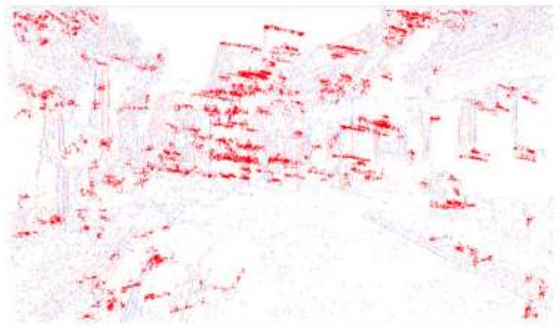
(a) Raw Image (KITTI)



Results on KITTI sequence

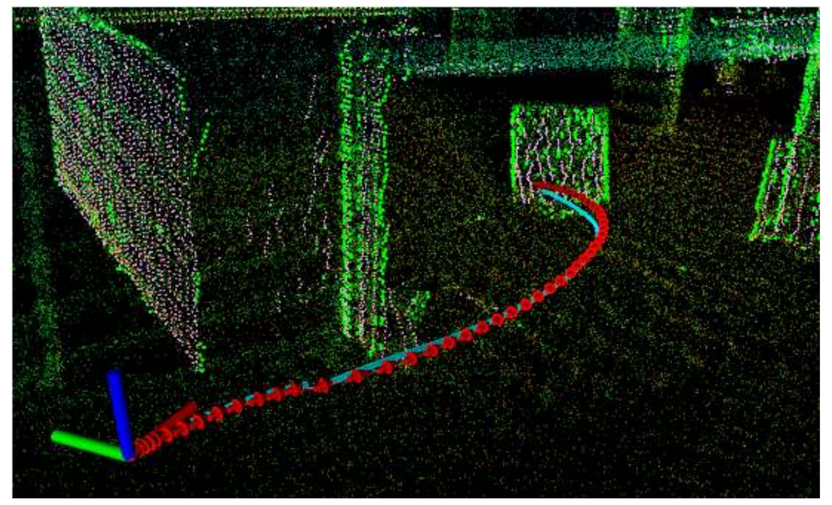
(b) Cropped Image (KITTI)

(c) Dark Night Image (KITTI)



(d) Events and Corner Tracking (KITTI)

(e) Raw Image (SCD)



Results on Self-Collected Data (SCD)
Note that 1FPN does not work at night
Ours (in blue) GT (in red)

[1] Huang, Kun, Yifu Wang, and Laurent Kneip. Motion estimation of non-holonomic ground vehicles from a single feature correspondence measured over n views. CVPR 2019.