電路學第四次小考

2024/05/23

1. (30%) For the filter network shown in Fig. A

   (a) (15%) Derive the voltage transfer function $H(S) = V_O(S)/V_S(S)$. What type of filter is it? Verify your answer.

   (b) (15%) If $R_S = 1\ k\Omega, R_L = 9\ k\Omega, L = 1$ mH, what is the cutoff frequency?

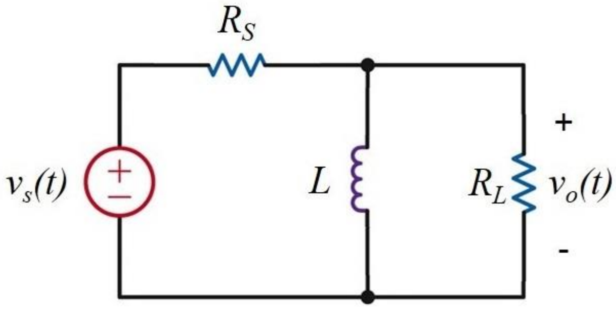


Fig. A

(a)

Derive the voltage transfer function (10%)

To derive the voltage transfer function $H(s) = \frac{V_0(s)}{V_S(s)}$ for the given circuit, we need to analyze the impedance of each component and use voltage division.

The components are:

R_s: series resistance

L: inductance

R_L: parallel resistance

First, let's write down the impedances:

Impedance of R_s: R_s **(1%)**

Impedance of L: sL(where s is the complex frequency) **(1%)**

Impedance of R_L: R_L**(1%)**

Since L and $R_L$are in parallel, their combined impedance $Z_{parallel}$is: **(2%)**

$$Z_{parallel} = \frac{R_L sL}{R_L + sL}$$

The total impedance Z_{total} of the circuit is the sum of R_s and $Z_{parallel}$: **(2%)**

$$Z_{total} = R_s + \frac{R_L sL}{R_L + sL}$$

Now, the transfer function $H(s)$ can be found using the voltage division rule: **(3%)**

$$H(s) = \frac{Z_{parallel}}{Z_{total}} = \frac{\frac{R_L sL}{R_L + sL}}{R_s + \frac{R_L sL}{R_L + sL}} = \frac{sLR_L}{R_sR_L + sL(R_L + R_s)} = \frac{j\omega LR_L}{R_sR_L + j\omega L(R_L + R_s)}$$

What type of filter is it? (5%)

To determine the type of filter, analyze the transfer function at different frequency limits:

At low frequencies $s \to 0$, $H(s) \to 0$ **(2%)**

At high frequencies $s \to \infty$, $H(s) \to 1$ **(2%)**

This suggests that the circuit is a high-pass filter. **(3%)**

(b)

Cutoff frequency (15%)

The cutoff frequency $\omega_c$is where the magnitude of the transfer function is $\frac{1}{\sqrt{2}}$ of its maximum value.

This occurs when the imaginary part of the denominator equals the real part:

$$\omega_c L(R_L + R_s) = R_L R_s \quad \textbf{(7\%)}$$

Solving for ω_c:

$$\omega_c = \frac{R_L R_s}{L(R_L + R_s)} \quad \textbf{(8\%)}$$

Or the cutoff frequency f_c in hertz is:

$$f_c = \frac{\omega_c}{2\pi} = \frac{R_L R_s}{2\pi L(R_L + R_s)}$$

2. (30%) For the following transfer function

$$H(j\omega) = \frac{-1000(j\omega + 10)}{(j\omega + 100)(j\omega + 200)}$$

(a) (15%) Construct its Bode plot of the magnitude (label the important values and features in the plot, such as break frequencies, slopes, and actual curve).

(b) (15%) Construct its asymptotic Bode plot of phase (label the important values and features in the plot, such as break frequencies and slopes).

1. Identify Zeros and Poles

Zero:

jω = -10

Poles:

jω = -100

jω = -200

2. Calculate Initial Gain and Phase

Initial Gain (Magnitude)

For very low frequencies (ω → 0):

H(jω) ≈ -1000(0 + 10) / ((0 + 100)(0 + 200)) = -10000 / 20000 = -0.5

Convert to dB:

20 log10(0.5) = 20 log10(1/2) = 20 × (-0.3010) = -6.02 dB

Initial Phase

The constant term -1000 contributes -180° to the phase.

The zero and poles at low frequencies contribute negligible phase shift.

So, the initial phase is:

-180° + 0° - 0° - 0° = -180°

3. Bode Plot: Magnitude

Frequency Range ω < 10:

- Initial magnitude is -6.02 dB.
- Slope: 0 dB/decade.

Frequency Range 10 < ω < 100:

- Zero at ω = 10 adds +20 dB/decade.
- At ω = 100:

  -6.02 dB + 20 dB/decade × 1 = 13.98 dB

Frequency Range 100 < ω < 200:

- Pole at ω = 100 adds -20 dB/decade.
- Slope: 0 dB/decade.

- Magnitude at $\omega = 200$:

  13.98 dB

Frequency Range $\omega > 200$:

- Pole at $\omega = 200$ adds -20 dB/decade.
- Slope: -20 dB/decade.

4. Bode Plot: Phase

Frequency Range $\omega < 10$:

- Initial phase is -180°.

Frequency Range $10 < \omega < 100$:

- Zero at $\omega = 10$ adds +45°.
- Phase at $\omega = 100$:

  $-180° + 45° = -135°$

Frequency Range $100 < \omega < 200$:

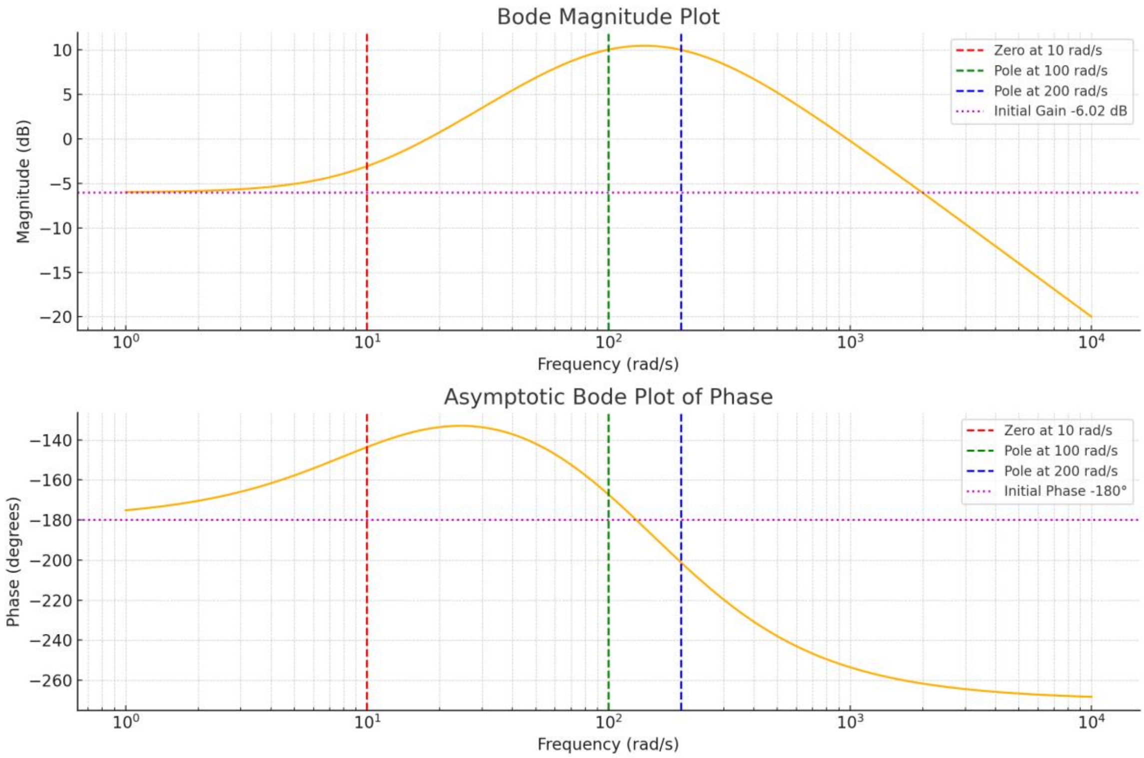
- Pole at $\omega = 100$ adds -45°.
- Phase at $\omega = 200$:

  $-135° - 45° = -180°$

Frequency Range $\omega > 200$:

- Pole at $\omega = 200$ adds -45°.
- Phase:

  $-180° - 45° = -225°$



配分

(a) Bode Magnitude Plot (15%)

1. Initial Gain Marking (3%): Correctly mark the initial gain as -6.02 dB.
2. Zero Marking (3%): Correctly identify and mark the zero at ω = 10.
3. First Pole Marking (3%): Correctly identify and mark the first pole at ω = 100.
4. Second Pole Marking (3%): Correctly identify and mark the second pole at ω = 200.
5. Plot Accuracy (3%): Correctly plot the magnitude response curve with appropriate slopes and transitions.

b) Asymptotic Bode Phase Plot (15%)

1. Initial Phase Marking (3%): Correctly mark the initial phase as -180°.
2. Zero Phase Shift (3%): Correctly identify and mark the phase shift due to the zero at ω=10.
3. First Pole Phase Shift (3%): Correctly identify and mark the phase shift due to the first pole at ω = 100.
4. Second Pole Phase Shift (3%): Correctly identify and mark the phase shift due to the second pole at ω = 200.
5. Plot Accuracy (3%): Correctly plot the phase response curve with appropriate phase shifts and transitions.

3. (40%) Consider the circuit in Fig. B.

(a) (10%) Derive the frequency response of the gain $G_v(j\omega) = V_O(j\omega)/V_S(j\omega)$ in terms of R_S(source resistance), R_L(load resistance), L(inductor) and C(capacitor), where $V_S(j\omega)$ and $V_O(j\omega)$ are the phasors of $v_S(t)$ and $v_o(t)$, respectively.

(b) (10%) Determine the quality factor (Q) in terms of R_S, R_L, L and C.

(c) (10%) Determine the bandwidth (BW) in terms of R_S, R_L, L and C.

(d) (10%) If $R_S = 30\ \Omega, R_L = 150\ \Omega, L = 2 \times 10^{-4}$ H and $C = 2 \times 10^{-6}$ F, calculate the half-power frequencies.

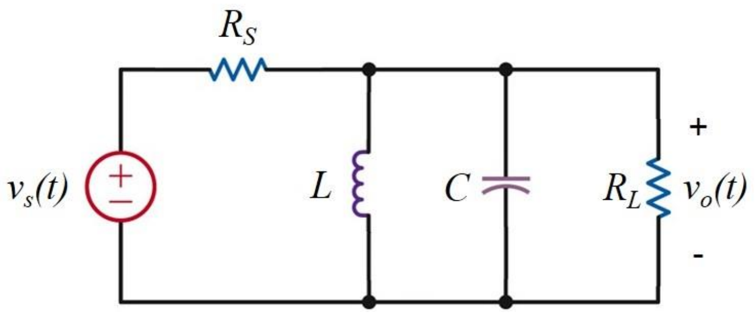


Fig. B

Sol:

(a)

$$G_v(j\omega) = G_v(S) = \frac{v_o(S)}{v_s(S)} = \frac{SL // \frac{1}{SC} // R_L}{R_S + (SL // \frac{1}{SC} // R_L)} \quad (2\%)$$

$$= \frac{S\frac{1}{R_S C}}{S^2 + S\left(\frac{R_s + R_L}{R_S R_L C}\right) + \frac{1}{LC}} \quad (2\%)$$

Let S = jw

$$= \frac{j\omega\frac{1}{R_S C}}{(j\omega)^2 + j\omega\left(\frac{R_s + R_L}{R_S R_L C}\right) + \frac{1}{LC}}$$

$$= \frac{j}{(\frac{R_S}{\omega L} - \omega R_S C) + j\frac{R_S + R_L}{R_L}} \quad (6\%)$$

(c)

At resonant, $\omega = \omega_0$, the network become pure resistive: (2%)

$$\frac{R_S}{\omega_0 L} - \omega_0 R_S C = 0$$

Yield:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Next, find ω_{Lo} and ω_{Hi} by letting:

$$(\frac{R_S}{\omega L} - \omega R_S C)^2 = (\frac{R_S + R_L}{R_L})^2$$

$$=> \omega^2 \pm \omega \frac{R_S + R_L}{R_S R_L C} - \frac{1}{LC} = 0 \text{ (2\%)}$$

Solve for ω_{Hi} and ω_{Lo}

$$\omega_{Hi} = \frac{\frac{R_S + R_L}{R_S R_L C} \pm \sqrt{(\frac{R_S + R_L}{R_S R_L C})^2 + \frac{4}{LC}}}{2} \text{ (2\%)}$$

$$\omega_{Lo} = \frac{-\frac{R_S + R_L}{R_S R_L C} \pm \sqrt{(\frac{R_S + R_L}{R_S R_L C})^2 + \frac{4}{LC}}}{2} \text{ (2\%)}$$

The bandwidth is defined as ω_{Hi} - ω_{Lo}:

$$BW = \frac{R_S + R_L}{R_S R_L C} \text{ (2\%)}$$

(b) Q is defined as:

$$Q = \frac{\omega_0}{BW} \text{ (3\%)}$$

$$= \frac{R_S R_L C}{(R_S + R_L)\sqrt{LC}} \text{ (7\%)}$$

(d)

Half-power frequencies:

ω_{Hi} = 9703Hz(60966rad/s) (5%)

ω_{Lo} = 6527Hz(41010rad/s) (5%)

BW = 3176Hz(19956rad/s)

ω_0=7960Hz(50014rad/s)

Q = $\frac{\omega_0}{BW}$ = 2.5