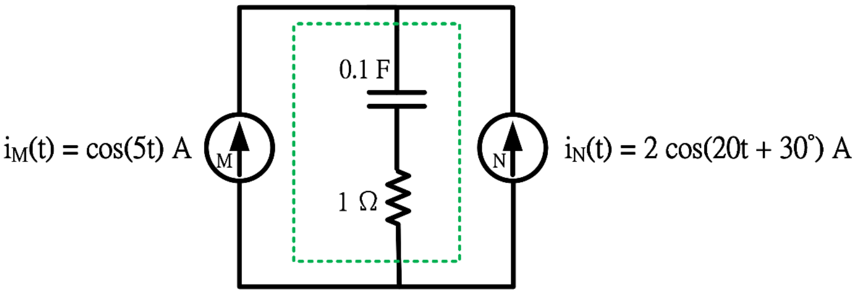
2024 Spring - Circuits – Quiz 3

1. (35 pts) The ac circuit below is in steady state.



A. (10 pts) Case 1: Let only current source M be on and the current source N be off (suppressed). Find the **complex impedance** of the elements in the box, written as a phasor.

N off, M on

$Z_R = 1$ (2)

$Z_C = \frac{1}{j\omega C} = \frac{1}{j5 \cdot 0.1} = -2j$ (3)

$Z = Z_R + Z_C = 1 - 2j = 2.2\angle -63.4^\circ$ (5)

B. (10 pts) Case 2: Let only the current source N be on and the current source M be off (suppressed). Find the **voltage over both elements** in the box, written as a phasor.

N on, M off

$Z = 1 - 0.5j = 1.1\angle -26.6^\circ$ (2)

$I = 2\angle 30^\circ$ (3)

$V = 2\angle 30^\circ \times 1.1\angle -26.6^\circ = 2.2\angle 3.4^\circ$ V (5)

C. (10 pts) Case 3: Let both current sources be on. Find the **voltage over the capacitor**, $v_C(t)$, written in the time domain.

$Z_{CM} = -j2$

$Z_{CN} = -j0.5$

$V_{CM} = 1\angle 0^\circ \cdot 2\angle -90^\circ = 2\angle -90^\circ$ (3)

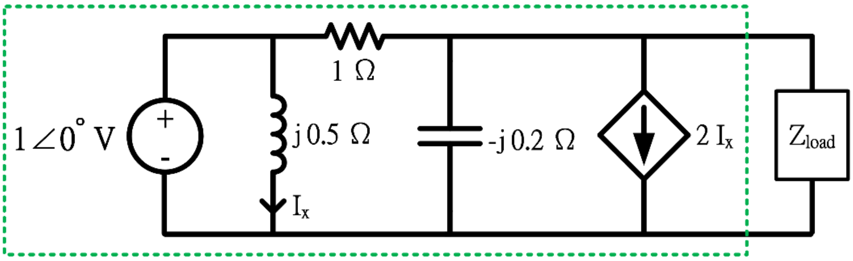
$V_{CN} = 2\angle 30^\circ \cdot 0.5\angle -90^\circ = 1\angle -60^\circ$ (3)

$V_{C\,Total} = 2\cos(5t - 90^\circ) + \cos(20t - 60^\circ)$ V (4)

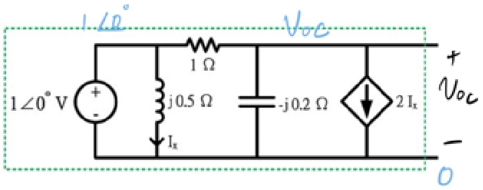
D. (5 pts) For the three cases (described above), in which case does the capacitor absorb the highest power? (Options: Case 1, Case 2, Case 3, or All The Same)

All the same. (0W)

2. (35 pts) The ac circuit below is in steady state. The source phasor value is the magnitude (not RMS). The circuit in the dashed box should be modeled as a Thevenin equivalent circuit.



A. (10pts) Find the **open-circuit voltage** of the Thevenin equivalent circuit, written as a <u>phasor</u>.



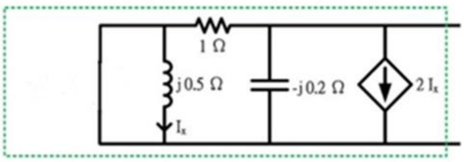
$$I_x = \frac{1\angle 0^\circ}{j\,0.5} = -j2 \quad 4$$

"Node Voc": $\dfrac{V_{oc} - 1\angle 0^\circ}{1} + \dfrac{V_{oc}}{-j0.2} + 2(-j2) = 0$ 4

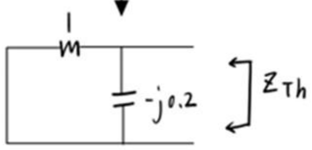
$$(1+j5)V_{oc} = 1+j4$$

$$\Rightarrow V_{oc} = \frac{21}{26} - j\frac{1}{26} = 0.81\angle -2.73^\circ \text{ V} \quad 2$$

B. (10pts) Find the **series impedance** of the Thevenin equivalent circuit, written as a <u>phasor</u>.



short circuit $\Rightarrow I_x = 0$ 4



4 $Z_{Th} = 1 \parallel (-j0.2) = \dfrac{-j0.2}{1-j0.2}$

$$= \frac{1}{26} - j\frac{5}{26} = 0.2\angle -78.7^\circ\ \Omega \quad 2$$

C. (5pts) Find the **load impedance** $\boldsymbol{Z_{load}}$ that maximizes average output power, written as a <u>phasor</u>.

max. output power $\Rightarrow$ conjugate 3

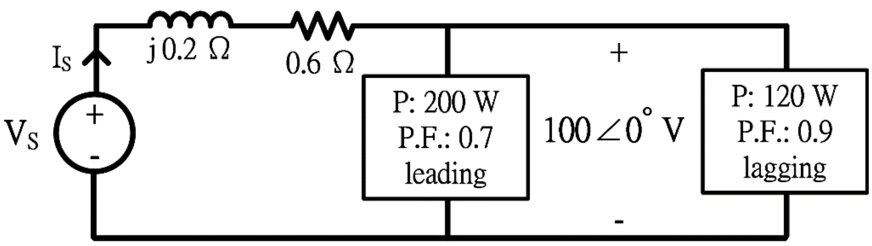
$$Z_{load} = Z_{Th}^{*} = 0.2\angle 78.7^\circ\ \Omega \quad 2$$

D. (10pts) Assuming the correct load impedance to maximize average output power, find the **maximum average output power**.

$$\text{max. } P_{avg} = \frac{|V_{oc}|^2}{8R_{Th}} = \frac{0.81^2}{8\cdot\frac{1}{26}} = 2.13 \text{ W} \quad 2$$

8

3. (30 pts) The circuit below operates at 50 Hz and in steady state. The phasor value is the RMS (not magnitude).



A. (5pts) Find the **reactive power** of the combined load.

For load 1, P=200W PF=0.7 leading:

$Q_{L1} = -200 \cdot tan(cos^{-1} 0.7) = -204.04VAR$

For load 2, P=120W PF=0.9 lagging:

$Q_{L2} = 120 \cdot tan(cos^{-1} 0.9) = 58.12VAR$

Total reactive power:

$Q\_\{eq\} = Q\_\{L1\} + Q\_\{L2\} = -145.92VAR$

(3pts for equation and 2pts for calculation)

B. (10pts) Find the **source current** I_s, written as a phasor in RMS.

(4pts) For load 1, P=200W PF=0.7 leading:

$$I_1 = \left[\frac{S}{V}\right]^* = \left[\frac{\frac{200}{0.7}\angle cos^{-1}(-0.7)}{100\angle 0°}\right]^* = 2.85\angle 45.57°\ A$$

(4pts) For load 2, P=120W PF=0.9 lagging:

$$I_1 = \left[\frac{S}{V}\right]^* = \left[\frac{\frac{200}{0.9}\angle cos^{-1}(0.9)}{100\angle 0°}\right]^* = 1.33\angle -25.84°\ A$$

(2pts) Source current:

$I_S = I_1 + I_2 = 3.51\angle 24.51°A$

C. (10pts) Find the **source voltage** V_s, written as a phasor in RMS.

$V_S = 100\angle 0 + I_S(j0.2\Omega + 0.6\Omega) = 101.63\angle 0.852°V$

(5pts for equation and 5pts for calculation)

D. (5pts) Which component can be connected in parallel with the combined loads to make the power factor unity? (Options: resistor, capacitor, inductor, none)

Ans: inductor