電路學 2024 Spring Final

1. **(15 pts)** The circuit network shown in Figure 1 has a DC independent current source and a DC dependent voltage source. Please determine:
   (a) its Thevenin’s equivalent circuit looking into terminal A-B; **(5 pts)**
   (b) its Norton’s equivalent circuit looking into terminal A-B; **(5 pts)**
   (c) the maximum power that can be consumed by the load resistor, R_L. **(5 pts)**

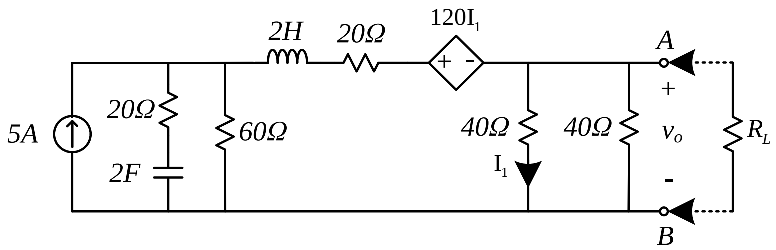
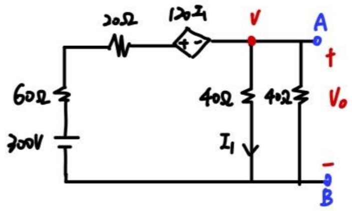


Figure 1

(a) its Thevenin’s equivalent circuit look into terminal A-B; [5]

(b) its Norton’s equivalent circuit look into terminal A-B; [5]

L short , C open



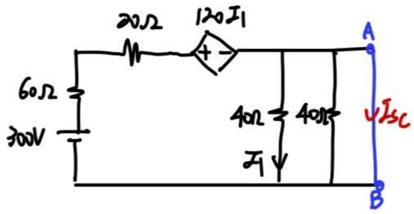
$I_1 = \frac{V}{40}$

$\frac{V-300+120I_1}{80} + \frac{V}{20} = 0$

$\Rightarrow \frac{V-300+3V}{80} + \frac{V}{20} = 0$

$\Rightarrow 4V - 300 + 4V = 0$

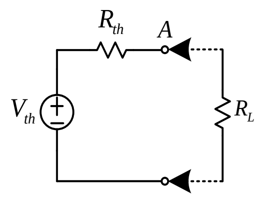
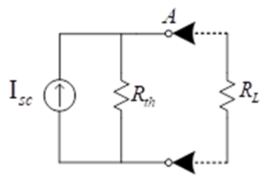
$\Rightarrow 8V = 300 \Rightarrow V = 37.5\,V$ ※ (3)



$I_1 = 0A$ $(\Rightarrow 120 I_1 = 0)$

$I_{sc} = \frac{300}{80} = 3.75A$ ※ (3)

$\Rightarrow R_{th} = \frac{V_{th}}{I_{sc}} = \frac{37.5}{3.75} = 10\,\Omega$ ※ (2)

	
Thevenin’s equivalent circuit(1)	Norton’s equivalent circuit(1)

(c) the maximum power that can be consumed by the load resistor, RL. [5]

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{37.5^2}{4\times 10} \fallingdotseq 35.16\,W \text{ ※}$$

2. **(8 pts)** The AC circuit in Figure 2 is in steady state. Find the <u>time domain</u> expression of the output voltage $\mathbf{V_o}$.

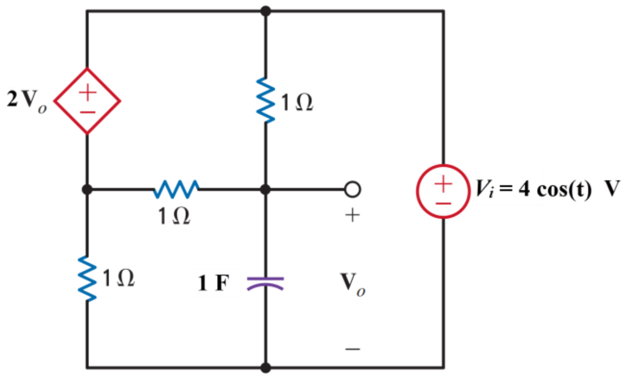
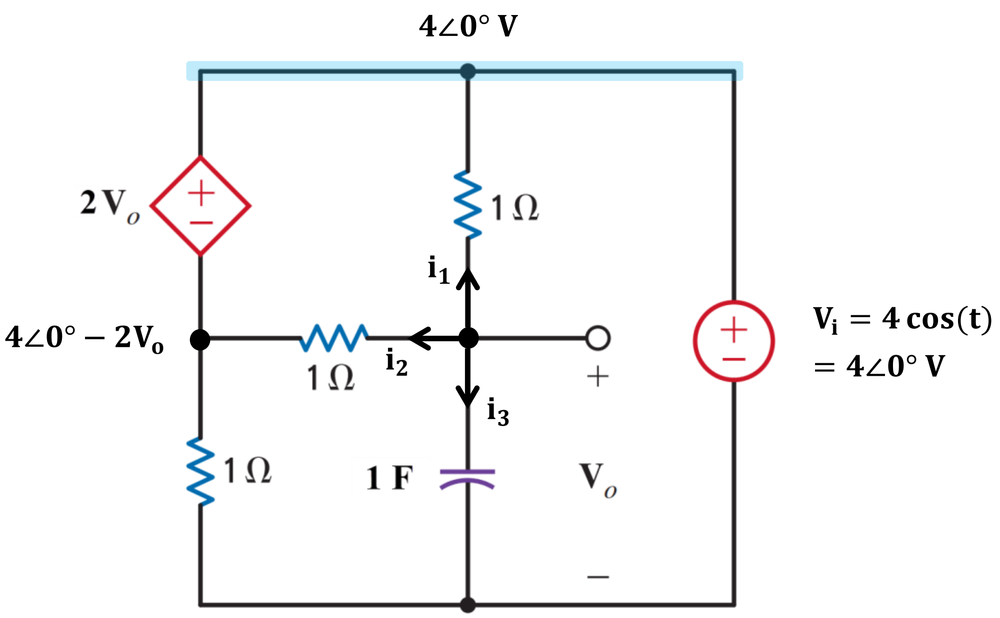


Figure 2



$i_1 + i_2 + i_3 = 0 \rightarrow \frac{V_o - 4\angle 0^\circ}{1\Omega} + \frac{V_o - (4\angle 0^\circ - 2V_o)}{1\Omega} + \frac{V_o}{\frac{1}{jw}\Omega} = 0$ (4pts)

$w = 1;\ V_o(1 + 3 + j) - 8\angle 0^\circ = 0$

$\rightarrow V_o = \frac{8\angle 0^\circ}{4+j} = \frac{8\angle 0^\circ}{4.12\angle 14.03^\circ} = 1.942\angle -14.03^\circ\ \mathrm{V}$ (2pts)

$V_o(t) = 1.942\cos(t - 14.03^\circ)\ \mathrm{V}$ (2pts)

3. **(7 pts)** The circuit below operates at 50 Hz and in steady state. The phasor value is the RMS (not magnitude). Voltage source $\mathbf{V_1}$ supplies a complex power of $\mathbf{S_1}$ = 1000∠-30° VA. Find $\mathbf{V_2}$ as a phasor.

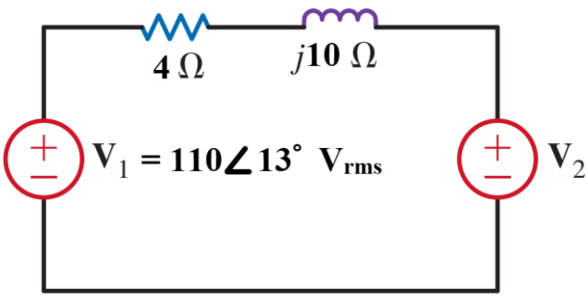


Figure 3

$S_1 = V_1 I_1^* \rightarrow I_1^* = \frac{S_1}{V_1} = \frac{1000\angle{-30}}{110\angle 13} = 9.091\angle -43°\ A_{rms}$

$\rightarrow I_1 = 9.091\angle 43°\ A_{rms}$ (3pts)

$Z_{total} = 4 + j10 = 10.77\angle 68.2°\ \Omega$ (1pt)

$V_z = I_1 Z_{total} = (9.091\angle 43°)(10.77\angle 68.2°) = 97.91\angle 111.2°\ V_{rms}$ (1pt)

$V_1 - V_2 = V_z \rightarrow V_2 = V_1 - V_z = 110\angle 13° - 97.91\angle 111.2°$
$= 142.59 - j66.54 = 157.35\angle -25°\ V_{rms}$ (2pts)

4. **(15 pts)** Consider the filter circuit shown in Figure 4.

(a) The frequency response can be expressed as $\mathrm{H}(j\omega) = \mathbf{V_0}(j\omega)/\mathbf{V_i}(j\omega) = \frac{D\times(j\omega)^2+E\times(j\omega)+F}{(j\omega)^2+A\times(j\omega)+B}$. Determine A, B, D, E, F in terms of $R_S, R_o, L,$ and C. **(5 pts)**

(b) Let $R_S = R_o = 50\ \Omega$, C = 1 μF, and L = 10 mH. Determine the half-power frequency (frequencies) and the quality factor. **(5 pts)**

(c) Plot the Bode magnitude diagram. Label important values and features in the plot. **(5 pts)**

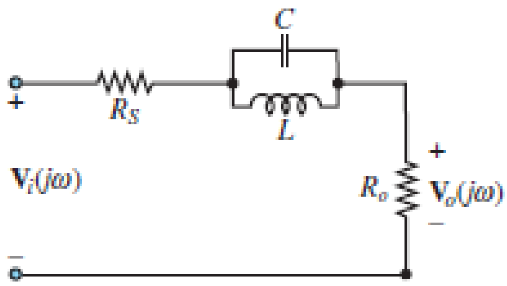


Figure 4

(a)

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{R_0}{R_S + R_0 + \frac{sL}{s^2LC+1}} \ (2)$$

$$= \frac{s^2R_0LC + R_0}{s^2LC(R_S+R_0) + sL(R_S+R_0)} = \frac{\frac{R_0}{R_S+R_0}s^2 + \frac{R_0}{LC(R_S+R_0)}}{s^2 + \frac{1}{C(R_S+R_0)}s + \frac{1}{LC}}$$

ANS:

$$A = \frac{1}{C(R_S+R_0)} \quad B = \frac{1}{LC} \quad D = \frac{R_0}{R_S+R_0} \quad E = 0 \quad F = \frac{R_0}{LC(R_0+R_S)} \ (3)$$

(b)

$R_S = R_0 = 50\Omega \ \ C = 1\times10^{-6}F.\ \ L = 10\times10^{-3}H$

$$H(j\omega) = \frac{0.5(j\omega)^2+0.5\times10^8}{(j\omega)^2+10^4j\omega+10^8} \ (1)$$

$$|H(j\omega)| = \frac{0.5(10^8-\omega^2)}{\sqrt{(10^8-\omega^2)^2+10^8\omega^2}} = \frac{0.5}{\sqrt{1+\frac{10^8\omega^2}{(10^8-\omega^2)^2}}}$$

Half power frequency：

$$let\ \frac{10^8\omega^2}{(10^8-\omega^2)^2}=1$$

$$10^8\omega^2=(10^8-\omega^2)^2$$

$$10^8\omega^2=10^{16}-2\times10^8\omega^2+\omega^4$$

$$\omega^4-3\times10^8\omega^2+10^{16}=0$$

$$\omega^2=\frac{3\pm\sqrt{5}}{2}\times10^8$$

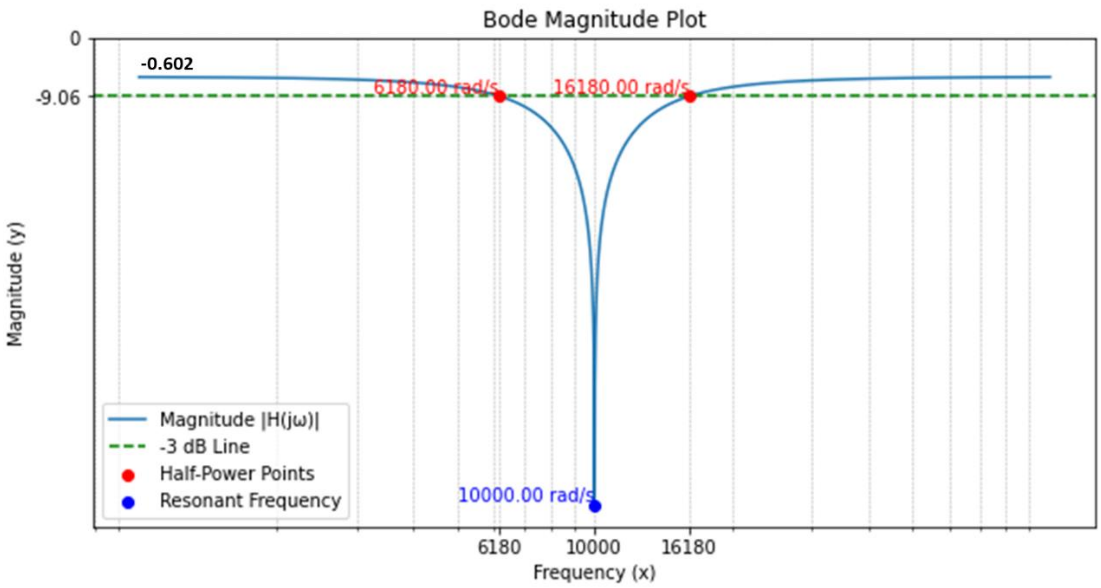
$$\omega_{hp1}=0.618\times\frac{10^4\,rad}{s}$$

$$\omega_{hp2}=1.618\times\frac{10^4\,rad}{s}$$ （2）

Q-factor

$$Q=\frac{1}{2\zeta}=\frac{1}{2\times0.5}=1$$ （2）

(c)



1. Draw the Curve with the correct Trend and List the Two Half-Power Frequencies. （3）
2. List the magnitude of the Half Power Frequencies and the Maximum value. （2）

5. **(42 pts)** Consider the dynamic circuit in Figure 5. Use the Laplace Transform method to calculate the complete circuit response under different conditions. If the response contains certain sinusoidal function, be sure to express it as the standard-form cosine function $A\cos(Bt+\phi)$.

(a) Calculate $i_L(t)$ for $t \geq 0$, given L=1 H, R = 2 Ω, C= 1/16 F, $v_s(t)$= unit step function u(t). (Be sure to show the s-domain circuit, how you get $I_L(s)$, and how you do Inverse Laplace Transform in your analysis.) **(16 pts)**

(b) Calculate $v_C(t)$ for $t \geq 0$, given L=0.5 H, R = 5 Ω, C= 1/40 F, $v_s(t)$=20 V for t<0 and -20 V for t≥0. (Be sure to show the s-domain circuit, how you get $V_C(s)$, and how you do Inverse Laplace Transform in your analysis.) **(26 pts)**

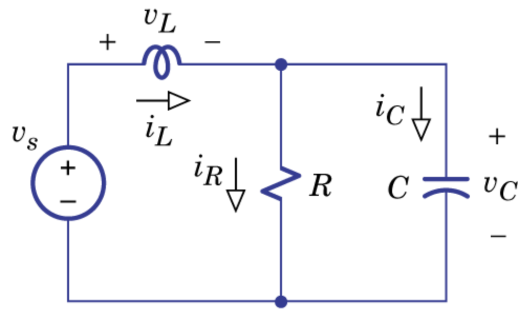
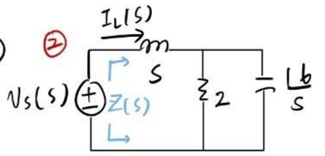


Figure 5

1.

(a) 2



$V_s(s) = \frac{1}{s}$ 2

$$\frac{I_L(s)}{V_s(s)} = \frac{1}{Z(s)} = \frac{1}{sL+(R\|\frac{1}{sC})} = \frac{1}{s+(2\|\frac{16}{s})} = \frac{s+8}{s^2+8s+16} \quad 4$$

$$\Rightarrow I_L(s) = V_s(s)\cdot\frac{s+8}{s^2+8s+16} = \frac{1}{s}\cdot\frac{s+8}{s^2+8s+16} = \frac{s+8}{s^3+8s^2+16s} \quad 2$$

$$I_L(s) = \frac{(s+8)}{s\cdot(s+4)^2} = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{(s+4)^2} \quad 1$$

$$K_1 = s\cdot I_L(s)\Big|_{s=0} = \frac{8}{16} = \frac{1}{2} \quad 1$$

$$K_3 = (s+4)^2 I_L(s)\Big|_{s=-4} = \frac{4}{-4} = -1 \quad 1$$

$$K_2 = \frac{d}{ds}\left[(s+4)^2\cdot I_L(s)\right]\Big|_{s=-4} = \frac{-8}{s^2}\Big|_{s=-4} = -\frac{1}{2} \quad 1$$

$$I_L(s) = \frac{1/2}{s} - \frac{1/2}{s+4} - \frac{1}{(s+4)^2}$$

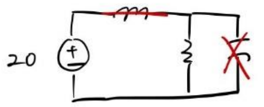
$$i_L(t) = \mathcal{L}^{-1}[I_L(s)] = \frac{1}{2}u(t) - \frac{1}{2}\cdot e^{-4t}u(t) - t\cdot e^{-4t}u(t)$$

$$= \left(\frac{1}{2} - \frac{1}{2}e^{-4t} - te^{-4t}\right)u(t) \quad 2$$

for $t \geq 0$: $i_L(t) = \frac{1}{2} - \frac{1}{2}e^{-4t} - te^{-4t}$

(b)

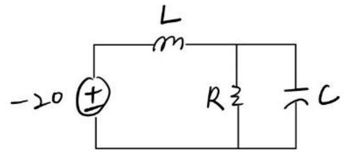
1° for $t<0$:



$i_L(0^-) = \frac{20}{5} = 4A$ 1

$V_c(0^-) = 20V$ 1

2° for $t \geq 0$:

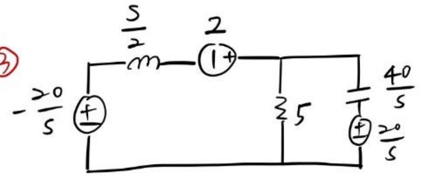


$V_s(s) = -20u(t) = -\frac{20}{s}$ 1

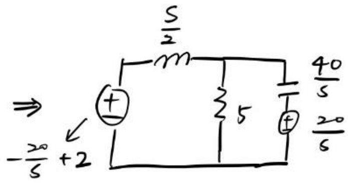
$v_c(t) = \frac{1}{C}\int_0^t i_c(t)\,dt + V_c(0^-) \xRightarrow{\mathcal{L}} V_c(s) = 40\cdot\frac{I_c(s)}{s} + \frac{20}{s}$ 1

$V_L(t) = L\frac{di_L(t)}{dt} \xRightarrow{\mathcal{L}} V_L(s) = L\cdot(sI_L(s) - 4) = \frac{s}{2}\cdot I_L(s) - 2$ 1

3



$\Rightarrow$



superposition:

"$2-\frac{20}{s}$": $V_{c1}(s) = \frac{2s-20}{s}\cdot\frac{(5\,||\,\frac{40}{s})}{\frac{s}{2}+(5\,||\,\frac{40}{s})} = \frac{160(s-10)}{s(s^2+8s+80)}$ 4 (分開寫各2分)

"$\frac{20}{s}$": $V_{c2}(s) = \frac{20}{s}\cdot\frac{(\frac{s}{2}\,||\,5)}{\frac{40}{s}+(\frac{s}{2}\,||\,5)} = \frac{20s^2}{s(s^2+8s+80)}$ 2

$V_c(s) = V_{c1}(s) + V_{c2}(s) = \frac{20s^2+160s-1600}{s(s^2+8s+80)}$ 2

$V_c(s) = \frac{K_1}{s} + \frac{K_2}{s-(-4+j8)} + \frac{K_2^*}{s-(-4-j8)}$ 2

$K_1 = s\cdot V_c(s)\Big|_{s=0} = \frac{-1600}{80} = -20$ 2

$K_2 = \frac{20s^2+160s-1600}{s\cdot(s+4+j8)}\Big|_{s=-4+j8} = 20-j10 = 22.36\angle-26.57°$ 2

$\Rightarrow K_2^* = 22.36\angle 26.57°$ 2

$V_c(s) = \frac{-20}{s} + \frac{22.36\angle-26.57°}{s+4-j8} + \frac{22.36\angle 26.57°}{s+4+j8}$

$= [-20 + 2\cdot 22.36\cdot e^{-4t}\cdot\cos(8t-26.57°)]\,u(t)$

for $t \geq 0$: $v_c(t) = -20 + 44.72e^{-4t}\cos(8t-26.57°)$ 2

6. **(6 pts)** Please find the Laplace transform *Y*(s) for the following function *y*(*t*), where: $y'(t) + 6y(t) = 10te^{-2t}u(t)$

$$\mathcal{L}\{y'(t)\} + 6\mathcal{L}\{y(t)\} = 10\mathcal{L}\{te^{-2t}u(t)\}$$

$$sY(s) - y(0) + 6Y(s) = \frac{10}{(s+2)^2} \quad (4)$$

$$(s+6)Y(s) = \frac{10}{(s+2)^2} + y(0)$$

$$Y(s) = \frac{10}{(s+6)(s+2)^2} + \frac{y(0)}{(s+6)} \quad (2)$$

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7. **(7 pts)** Please find the inverse Laplace transform *f*(*t*) for the following s-domain function *F*(s) = $\frac{3s^3+9s^2+5s+16}{s(s+1)(s^2+9)}$

$$\frac{3s^3+9s^2+5s+16}{s(s+1)(s^2+9)} = \frac{K_0}{s} + \frac{K_1}{s+1} + \frac{K_2}{s-3i} + \frac{K_2^*}{s+3i} \quad (1)$$

$$K_0 = sY(s)\Big|_{s=0} = \frac{16}{9}$$

$$K_1 = (s+1)Y(s)\Big|_{s=-1} = \frac{-3+9-5+16}{-10} = \frac{-17}{10}$$

$$K_2 = (s-3i)Y(s)\Big|_{s=3i} = \frac{-65-66i}{-18-54i} = \frac{92.63\angle 225.44}{56.92\angle 251.57} = 1.63\angle -26.13 \quad (3)$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{16}{9s}\right\} = \frac{16}{9}, \quad \mathcal{L}^{-1}\left\{\frac{-17}{10}\frac{1}{s+1}\right\} = \frac{-17}{10}e^{-t}$$

$$\Rightarrow \frac{K_2}{s-3i} + \frac{K_2^*}{s+3i} = 2|K_2|\cos(3t+\phi) = 3.26\cos(3t-26.13^\circ)$$

$$\Rightarrow f(t) = \left(\frac{16}{9} - \frac{17}{10}e^{-t} + 3.26\cos(3t-26.13^\circ)\right)\mathrm{u(t)} \quad (3) \ \#$$

TABLE 13.1 Short table of Laplace transform pairs

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{t^n e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$

TABLE 13.2 Some useful properties of the Laplace transform

PROPERTY NUMBER	$f(t)$	$\mathbf{F}(s)$
1. Magnitude scaling	$Af(t)$	$A\mathbf{F}(s)$
2. Addition/subtraction	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(s) \pm \mathbf{F}_2(s)$
3. Time scaling	$f(at)$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right),\ a > 0$
4. Time shifting	$f(t-t_0)u(t-t_0),\ t \geq 0$	$e^{-t_0 s}\mathbf{F}(s)$
	$f(t)u(t-t_0)$	$e^{-t_0 s}\mathcal{L}[f(t+t_0)]$
5. Frequency shifting	$e^{-at}f(t)$	$\mathbf{F}(s+a)$
6. Differentiation	$\frac{d^n f(t)}{dt^n}$	$s^n\mathbf{F}(s) - s^{n-1}f(0) - s^{n-2}f^1(0) \cdots - s^0 f^{n-1}(0)$
7. Multiplication by t	$tf(t)$	$-\frac{d\mathbf{F}(s)}{ds}$
	$t^n f(t)$	$(-1)^n \frac{d^n\mathbf{F}(s)}{ds^n}$
8. Division by t	$\frac{f(t)}{t}$	$\int_s^\infty \mathbf{F}(\lambda)\,d\lambda$
9. Integration	$\int_0^t f(\lambda)\,d\lambda$	$\frac{1}{s}\mathbf{F}(s)$
10. Convolution	$\int_0^t f_1(\lambda)f_2(t-\lambda)\,d\lambda$	$\mathbf{F}_1(s)\mathbf{F}_2(s)$