MOOC Econometrics: Test Exercise #5

Greg Faletto

Problem (a)

Consider again the application in lecture 5.5, where we have analyzed response to a direct mailing using the following logit specification:

$$\Pr[resp_i = 1] = \frac{\exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

for $i = 1, \ldots, 925$. The maximum likelihood estimates of the parameters are given by

Variable	Coefficient	Std. Error	t-value	p-value
Intercept	-2.488	0.890	-2.796	0.005
Male	0.954	0.158	6.029	0.000
Active	0.914	0.185	4.945	0.000
Age	0.070	0.036	1.964	0.050
$(Age/10)^2$	-0.069	0.034	-2.015	0.044

The marginal effect of activity status is defined as

$$\frac{\partial \Pr[resp_i = 1]}{\partial active_i} = \Pr[resp_i = 1] \Pr[resp_i = 0]\beta_2.$$

We could use this result to construct an activity status elasticity

$$\frac{\partial \Pr[resp_i = 1]}{\partial active_i} \frac{active_i}{\Pr[resp_i = 1]} = \Pr[resp_i = 0] active_i \beta_2.$$

Use this result to compute the elasticity effect of active status for a 50-year-old active male customer. Do the same for a 50-year-old inactive male customer.

Solution

$$\Pr[resp_i = 0] = 1 - \Pr[resp_i = 1]$$

$$= 1 - \frac{\exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$= \frac{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2) - \exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$= \frac{1}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

For a 50-year-old active male customer, $male_i = 1, active_i = 1, age_i = 50, (age_i/10)^2 = (50/10)^2 = 25$.

Therefore

$$elasticity = \Pr[resp_i = 0] active_i \beta_2$$

$$= \frac{1}{1 + \exp(\beta_0 + 1\beta_1 + 1\beta_2 + 50\beta_3 + 25\beta_4} \cdot 1 \cdot \beta_2$$

Problem (a) continued on next page. . .

Plugging in the given coefficients, we have

$$elasticity = \frac{1}{1 + \exp(-2.488 + 1 \cdot 0.954 + 1 \cdot 0.914 + 50 \cdot 0.070 + 25 \cdot -0.069} \cdot 1 \cdot 0.914$$

$$\boxed{\approx \mathbf{0.2189734}}$$

For a 50-year-old inactive male customer, $male_i = 1, active_i = 0, age_i = 50, (age_i/10)^2 = (50/10)^2 = 25$. Therefore

$$elasticity = \Pr[resp_i = 0]active_i\beta_2$$

$$= \frac{1}{1 + \exp(\beta_0 + 1\beta_1 + 1\beta_2 + 50\beta_3 + 25\beta_4} \cdot 0 \cdot \beta_2$$

$$\boxed{= \mathbf{0}}$$

Problem (b)

The activity status variable is only a dummy variable and hence it can take only two values. It is therefore better to define the elasticity as

$$\frac{\Pr[resp_i = 1|active_i = 1] - \Pr[resp_i = 1|active_i = 0]}{\Pr[resp_i = 1|active_i = 0]}$$

Show that you can simplify the expression for the elasticity as

$$(\exp(\beta_2) - 1)\Pr[resp_i = 0|active_i = 1].$$

Solution

$$\Pr[resp_i = 1|active_i = 1] = \frac{\exp(\beta_0 + \beta_1 male_i + \beta_2 \cdot 1 + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 \cdot 1 + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$\Pr[resp_i = 1|active_i = 0] = \frac{\exp(\beta_0 + \beta_1 male_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$\Pr[resp_i = 0|active_i = 1] = 1 - \Pr[resp_i = 1|active_i = 1]$$

$$= 1 - \frac{\exp(\beta_0 + \beta_1 male_i + \beta_2 \cdot 1 + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 \cdot 1 + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$= \frac{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 \cdot 1 + \beta_3 age_i + \beta_4 (age_i/10)^2) - \exp(\beta_0 + \beta_1 male_i + \beta_2 \cdot 1 + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$= \frac{1}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

Let $\beta_{-2} = [\beta_0 \ \beta_1 \ \beta_3 \ \beta_4]'$. Let $X_{-2} = [1 \ male_i \ age_i \ (age_i/10)^2]'$. Then

Problem (b) continued on next page...

$$\beta_{-2}{}'X_{-2} = \beta_0 + \beta_1 male_i + \beta_3 age_i + \beta_4 (age_i/10)^2$$

and

$$\beta_{-2}{}'X_{-2} + \beta_2 = \beta_0 + \beta_1 male_i + \beta_2 + \beta_3 age_i + \beta_4 (age_i/10)^2$$

so

$$\Pr[resp_i = 1 | active_i = 1] = \frac{\exp(\beta_{-2}{}'X_{-2} + \beta_2)}{(1 + \exp(\beta_{-2}{}'X_{-2} + \beta_2)} = \frac{\exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)}{1 + \exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)}$$

and

$$\Pr[resp_i = 1 | active_i = 0] = \frac{\exp(\beta_{-2}{}'X_{-2})}{1 + \exp(\beta_{-2}{}'X_{-2})}$$

Therefore

$$elasticity = \frac{\frac{\exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)}{1+\exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)} - \frac{\exp(\beta_{-2}{}'X_{-2})}{1+\exp(\beta_{-2}{}'X_{-2})}}{\frac{\exp(\beta_{-2}{}'X_{-2})}{1+\exp(\beta_{-2}{}'X_{-2})}}$$

$$= \left(\frac{\exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)}{1 + \exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)} - \frac{\exp(\beta_{-2}{}'X_{-2})}{1 + \exp(\beta_{-2}{}'X_{-2})} \right) \cdot \frac{1 + \exp(\beta_{-2}{}'X_{-2})}{\exp(\beta_{-2}{}'X_{-2})}$$

$$= \left(\frac{\exp(\beta_2)}{1 + \exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)} - \frac{1}{1 + \exp(\beta_{-2}{}'X_{-2})} \right) \cdot \left[1 + \exp(\beta_{-2}{}'X_{-2}) \right]$$

$$= \frac{\exp(\beta_2)(1 + \exp(\beta_{-2}{}'X_{-2}))}{1 + \exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)} - 1$$

$$= \frac{\exp(\beta_2) + \exp(\beta_2)\exp(\beta_{-2}{}'X_{-2})) - 1 - \exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)}{1 + \exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)}$$

$$= \frac{\exp(\beta_2) - 1}{1 + \exp(\beta_{-2}{}'X_{-2})\exp(\beta_2)}$$

$$= \big(\exp(\beta_2) - 1\big) \cdot \frac{1}{1 + \exp(\beta_{-2}{}'X_{-2} + \beta_2)}$$

$$= \big(\exp(\beta_2) - 1\big) \cdot \frac{1}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$\boxed{= \big(\exp(\boldsymbol{\beta_2}) - \mathbf{1}\big) \cdot \Pr[\mathbf{resp_i} = \mathbf{0} | \mathbf{active_i} = \mathbf{1}]}$$

Problem (c)

Use the formula in (b) to compute the activity elasticity of a 50-year-old male active customer.

Solution

$$elasticity = \big(\exp(\beta_2) - 1\big) \cdot \frac{1}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

For a 50-year-old male active customer, $male_i = 1, active_i = 1, age_i = 50, (age_i/10)^2 = (50/10)^2 = 25$.

Plugging in our variable values and the given coefficients, we have

$$elasticity = \big(\exp(0.914) - 1\big) \cdot \frac{1}{1 + \exp(-2.488 + 0.954 \cdot 1 + 0.914 + 0.070 \cdot 50 + -0.069 \cdot 25)}$$

$$\boxed{\approx \mathbf{0.3579951}}$$