Matrix calculus cheat sheet-simple stuff

$$
\begin{aligned}
\partial \mathbf{A} &= 0 & (\mathbf{A} \text{ is a constant}) \\
\partial(\alpha \mathbf{X}) &= \alpha \partial \mathbf{X} \\
\partial(\mathbf{X} + \mathbf{Y}) &= \partial \mathbf{X} + \partial \mathbf{Y} \\
\partial(\text{Tr}(\mathbf{X})) &= \text{Tr}(\partial \mathbf{X}) \\
\partial(\mathbf{X}\mathbf{Y}) &= (\partial \mathbf{X})\mathbf{Y} + \mathbf{X}(\partial \mathbf{Y}) \\
\partial(\mathbf{X} \circ \mathbf{Y}) &= (\partial \mathbf{X}) \circ \mathbf{Y} + \mathbf{X} \circ (\partial \mathbf{Y}) \\
\partial(\mathbf{X} \otimes \mathbf{Y}) &= (\partial \mathbf{X}) \otimes \mathbf{Y} + \mathbf{X} \otimes (\partial \mathbf{Y}) \\
\partial(\mathbf{X}^{-1}) &= -\mathbf{X}^{-1}(\partial \mathbf{X})\mathbf{X}^{-1} \\
\partial(\det(\mathbf{X})) &= \text{Tr}(\text{adj}(\mathbf{X})\partial \mathbf{X}) \\
\partial(\det(\mathbf{X})) &= \det(\mathbf{X})\text{Tr}(\mathbf{X}^{-1}\partial \mathbf{X}) \\
\partial(\ln(\det(\mathbf{X}))) &= \text{Tr}(\mathbf{X}^{-1}\partial \mathbf{X}) \\
\partial \mathbf{X}^{T} &= (\partial \mathbf{X})^{T} \\
\partial \mathbf{X}^{H} &= (\partial \mathbf{X})^{H}
\end{aligned}
$$

Matrix calculus cheat sheet-Derivatives of determinant

$$\frac{\partial \det(\mathbf{Y})}{\partial x} = \det(\mathbf{Y})\operatorname{Tr}\left[\mathbf{Y}^{-1}\frac{\partial \mathbf{Y}}{\partial x}\right]$$

$$\sum_k \frac{\partial \det(\mathbf{X})}{\partial X_{ik}}X_{jk} = \delta_{ij}\det(\mathbf{X})$$

$$\frac{\partial^2 \det(\mathbf{Y})}{\partial x^2} = \det(\mathbf{Y})\left[\operatorname{Tr}\left[\mathbf{Y}^{-1}\frac{\partial \frac{\partial \mathbf{Y}}{\partial x}}{\partial x}\right]\right.$$

$$+\operatorname{Tr}\left[\mathbf{Y}^{-1}\frac{\partial \mathbf{Y}}{\partial x}\right]\operatorname{Tr}\left[\mathbf{Y}^{-1}\frac{\partial \mathbf{Y}}{\partial x}\right]$$

$$\left.-\operatorname{Tr}\left[\left(\mathbf{Y}^{-1}\frac{\partial \mathbf{Y}}{\partial x}\right)\left(\mathbf{Y}^{-1}\frac{\partial \mathbf{Y}}{\partial x}\right)\right]\right]$$

Matrix calculus cheat sheet-grad and hessians

$$\begin{aligned} f &= \mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{b}^T \mathbf{x} \\ \nabla_{\mathbf{x}} f = \frac{\partial f}{\partial \mathbf{x}} &= (\mathbf{A} + \mathbf{A}^T)\mathbf{x} + \mathbf{b} \\ \frac{\partial^2 f}{\partial \mathbf{x} \partial \mathbf{x}^T} &= \mathbf{A} + \mathbf{A}^T \end{aligned}$$

Matrix calculus cheat sheet-first order

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{X}) = \mathbf{I}$$

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{X}\mathbf{A}) = \mathbf{A}^T$$

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{A}\mathbf{X}\mathbf{B}) = \mathbf{A}^T\mathbf{B}^T$$

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{A}\mathbf{X}^T\mathbf{B}) = \mathbf{B}\mathbf{A}$$

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{X}^T\mathbf{A}) = \mathbf{A}$$

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{A}\mathbf{X}^T) = \mathbf{A}$$

$$\frac{\partial}{\partial \mathbf{X}} \text{Tr}(\mathbf{A} \otimes \mathbf{X}) = \text{Tr}(\mathbf{A})\mathbf{I}$$