

## Week 5 Test Exercise

Test Exercise 5

(a) Elasticity of active status ( $50\text{-year-old male}$ )

$$\text{elasticity} = \Pr[\text{resp}_i = 0 | \text{active}_i; \beta_2]$$

$$= \frac{\exp(\beta_0 + \beta_1(\text{age}_i) + \beta_2(\text{active}_i) + \beta_3(\text{age}_i)^2)}{1 + \exp(\beta_0 + \beta_1(\text{age}_i) + \beta_2(\text{active}_i) + \beta_3(\text{age}_i)^2)}$$

$$\approx \frac{b_2}{1 + \exp(b_0 + b_1 + b_2 + b_3 + 25b_4)} = \frac{0.914}{1 + \exp(-2.488 + 0.954 + \dots)} = 0.2189734$$

$$\text{elasticity} = \Pr[\text{resp}_i = 0 | \text{active}_i; \beta_2] = \Pr[\text{resp}_i = 0] \cdot 0 \cdot \beta_2 = 0$$

(b)  $\Pr[\text{resp}_i = 1 | \text{active}_i = 1]$

$$= \frac{\exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}$$

Let  $\beta_2 = [\beta_0, \beta_1, \beta_3, \beta_4]'$

Let  $X_2 = [1, \text{male}_i, \text{age}_i, (\text{age}_i/10)^2]'$

$$\Rightarrow \Pr[\text{resp}_i = 1 | \text{active}_i = 1] = \frac{\exp(\beta_2' X_2 + \beta_2)}{1 + \exp(\beta_2' X_2 + \beta_2)} = \frac{\exp(\beta_2' X_2) \exp(\beta_2)}{1 + \exp(\beta_2' X_2) \exp(\beta_2)}$$

$\Pr[\text{resp}_i = 1 | \text{active}_i = 0] = \frac{\exp(\beta_2' X_2)}{1 + \exp(\beta_2' X_2)}$

$$\Rightarrow \text{elasticity} = \frac{\exp(\beta_2' X_2) \exp(\beta_2)}{1 + \exp(\beta_2' X_2)} \cdot \frac{\exp(\beta_2' X_2)}{1 + \exp(\beta_2' X_2)} \cdot \frac{\exp(\beta_2' X_2)}{1 + \exp(\beta_2' X_2)} = \frac{\exp(\beta_2' X_2)}{1 + \exp(\beta_2' X_2)}$$

$$\begin{aligned}
&= \left[ \frac{\exp(\beta_2)}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)} - \frac{1}{1 + \exp(\beta_2' X_{-2})} \right] [1 + \exp'(\beta_2' X_{-2})] \\
&= \left[ \frac{(1 + \exp(\beta_2' X_{-2})) \exp(\beta_2) - 1 \cdot (1 + \exp(\beta_2' X_{-2}) \exp(\beta_2))}{(1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)) \cdot (1 + \exp(\beta_2' X_{-2}))} \right] [1 + \exp'(\beta_2' X_{-2})] \\
&= \frac{\exp(\beta_2) + \exp(\beta_2) \exp(\beta_2' X_{-2}) - 1 - \exp(\beta_2' X_{-2}) \exp(\beta_2)}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)} \\
&= \frac{\exp(\beta_2) \cdot \frac{1}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)} - \frac{1}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)}}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)}
\end{aligned}$$

$$\begin{aligned}
&\Pr[\text{resp}_i = 0 | \text{active}_i = 1] = 1 - \Pr[\text{resp}_i = 1 | \text{active}_i = 1] \\
&= 1 - \frac{\exp(\beta_2' X_{-2}) \exp(\beta_2)}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)} \quad \text{← from last page} \\
&= \frac{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)} - \frac{\exp(\beta_2' X_{-2}) \exp(\beta_2)}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)} \\
&= \boxed{\frac{1}{1 + \exp(\beta_2' X_{-2}) \exp(\beta_2)}} \\
&\quad \downarrow \\
&= [\exp(\beta_2) - 1] \cdot \Pr[\text{resp}_i = 0 | \text{active}_i = 1] = \boxed{\text{elasticity}}
\end{aligned}$$

(c) elasticity of 50-year-old male active customer?

$$(\exp(B_2) - 1) \cdot \Pr[\text{resp}_i = 0 \mid \text{active}_i = 1]$$

$$= (\exp(B_2) - 1) \cdot \frac{1}{1 + \exp(B_0 + B_1(1) + B_2(1) + B_3(50) + B_4(50/10)^2)}$$

$$\frac{\exp(b_2) - 1}{1 + \exp(b_0 + b_1 + b_2 + 50b_3 + 25b_4)}$$

$$= \frac{\exp(0.914) - 1}{1 + \exp(-2.488 + 0.954 + 0.914 + 50 \cdot 0.070 + -0.069 \cdot 25)}$$

$$\approx \boxed{0.3579951}$$