Analysis of generative modeling methods based on Schrödinger bridges

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Research

The problem of generating pictures using Schrödinger bridge problem is investigated.

Research objective —

suggest a method of improving diffusion models using Schrödinger bridges.

Required to suggest

1. theory to connect generative problem with Schrödinger bridge problem,
2. method of fitting model using Schrödinger bridges.

Problem statement

It is given

1. $\{x_i\}_{i=0}^N \in \mathbb{R}^d$ – dataset,
2. $p_{prior} = \mathcal{N}(0, \mathbf{I})$,
3. $d\mathbf{X}_t = f(t, \mathbf{X}_t)dt + g(t)d\mathbf{W}_t, X_0 \sim p_{data}$ – forward process,

We want to find reverse process, that goes from p_{prior} to p_{data}. It solves using diffusion models minimizing sophisticated loss

$$d\mathbf{X}t = [f(t, \mathbf{X}_t) - g^2(t)s(t, \mathbf{X}_t; \theta)]dt + g(t)d\mathbf{W}_t, X_T \sim p_{prior},$$

where $t \in [0, T]$ However, when we solve this problem, we do not enforce the constraint on $X_T \sim p_{prior}$, thus it is practical task to find a good T to get good approximation of p_{prior}

Suggested Method

Instead of considering the time-reversal of a forward noising process, let's build bridges (solve Schrödinger bridge problem) between the two boundary distributions and learn a mimicking diffusion process. It is given

1. $\mathbb{Q} \in \mathcal{P}(p_{data}, p_{prior})$ – path measure of desired process,
2. $\mathbb{P}$ – path measure of forward process

$$\min_{\mathbb{Q} \in \mathcal{P}(p_{data}, p_{prior})} D_{KL}(\mathbb{Q}||\mathbb{P})$$

The solution to the optimization can be expressed by the path measure of the following forward, or equivalently backward, SDE

$$d\mathbf{X}_t = [f(t, \mathbf{X}_t) + g^2(t)\nabla_x \log \Psi(t, \mathbf{X}_t)]dt + g(t)d\mathbf{W}_t, \mathbf{X}_0 \sim p_{data}$$

$$d\mathbf{X}_t = [f(t, \mathbf{X}_t) - g^2(t)\nabla_x \log \hat{\Psi}(t, \mathbf{X}_t)]dt + g(t)d\mathbf{W}_t, \mathbf{X}_T \sim p_{prior}$$

$\Psi(t, \mathbf{X}_t)$ and $\hat{\Psi}(t, \mathbf{X}_t)$ are solution of duality problem of SBP

Related papers

1. *Likelihood Training of Schrödinger Bridge using Forward-Backward SDEs Theory*[1]: solving SB using duality problem
2. *Diffusion Schrödinger Bridge with Applications to Score-Based Generative Modeling*[2]: solving SB using IPF algorithm on Dynamic Schrödinger Bridge problem
3. *Diffusion Schrödinger Bridge Matching*[3]: solving SB using proposed algorithm IMF on Dynamic Schrödinger Bridge problem

[1] https://arxiv.org/pdf/2110.11291.pdf
[2] https://arxiv.org/pdf/2106.01357.pdf
[3] https://arxiv.org/pdf/2303.16852.pdf

Future work plan

1. Make a theory to connect SB and diffusion models
2. Investigate D_{KL} minimization, generalize to f-divergence.