

BABYLONIAN Tonal System

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Abstract

Proposal for the BABYLONIAN Tonal System, based on consecutive heptachords.¹

Keywords. Ancient MESOPOTAMIAN music, Old BABYLONIAN music, MESOPOTAMIA, BABYLON, UR, SUMER, AKKADIAN, 1800 BCE to 500 BCE, Sexagesimal arithmetic and numbers, Musical cuneiform tablets, Seven diatonic Heptachords, Tuning system.

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[‡]Revised version.

¹Original research, based on [1] and an idea by composer CARL NIELSEN (1865-1931).

Contents

I Heptachords and Scales	3
1 Introduction	3
1.1 Heptachords	3
1.2 Scale Range	6
1.3 Scale Cycle	7
1.4 Tuning	8
2 <i>išartum</i>	10
2.1 Scale 1	10
2.2 Scale 2	12
3 <i>embūbum</i>	14
3.1 Scale 1	14
3.2 Scale 2	16
4 <i>kitnum</i>	18
4.1 Scale 1	18
4.2 Scale 2	20
5 <i>pītum</i>	22
5.1 Scale 1	22
5.2 Scale 2	24
6 <i>qablītum</i>	26
6.1 Scale 1	26
6.2 Scale 2	28
7 <i>nīš tuḥrim</i>	30
8 <i>nīd qablim</i>	34
References	38
A Appendix	39
A.1 Octatonic Scale	39
A.1.1 Scale 1	39
A.1.2 Scale 2	39
A.2 Plots	40
A.2.1 Heptachords	40
A.2.2 Octaves	41

Part I

Heptachords and Scales

1 Introduction

1.1 Heptachords

Table 1: Heptachords

	Heptachord	Pattern	Scales	Symmetric Center
1	<i>išartum</i>	$[1, \frac{1}{2}, 1, 1, 1, \frac{1}{2}]$	2	G
2	<i>embūbum</i>	$[\frac{1}{2}, 1, 1, 1, \frac{1}{2}, 1]$	2	A
3	<i>kitnum</i>	$[1, \frac{1}{2}, 1, 1, \frac{1}{2}, 1]$	2	D
4	<i>pītum</i>	$[\frac{1}{2}, 1, 1, \frac{1}{2}, 1, 1]$	2	E
5	<i>qablītum</i>	$[1, 1, \frac{1}{2}, 1, 1, \frac{1}{2}]$	2	C
6	<i>nīš tuhrim</i>	$[1, 1, \frac{1}{2}, 1, 1, 1]$	1	F
7	<i>nīd qablim</i>	$[1, 1, 1, \frac{1}{2}, 1, 1]$	1	B

²

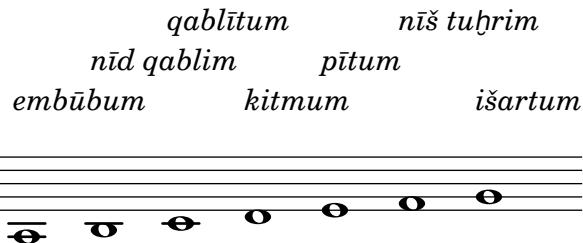


Figure 1: Symmetric Center

² Rising Scale.

Table 2: Heptachords II

	Heptachord	Keys per Scale	Key Signature Δ	Sum Σ
1	<i>išartum</i>	6	2	12
2	<i>embābum</i>	6	2	12
3	<i>kitnum</i>	6	2	12
4	<i>pītum</i>	6	2	12
5	<i>qablītum</i>	6	2	12
6	<i>nīš tuhrim</i>	12	5	60
7	<i>nīd qablim</i>	12	5	60

1: F

2: E

5: B

6: A

12: $\text{C}_\# \text{D} \text{F} \text{G} \text{A} \text{C} \text{E}$

60: $\text{F} \text{A} \text{C} \text{D} \text{E} \text{G} \text{B}$ ANU

Example. Consecutive Heptachords.^{3,4}

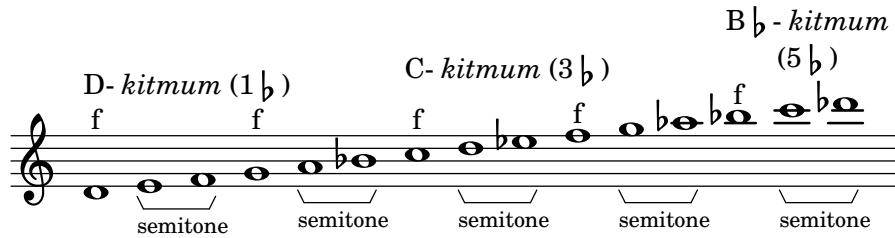


Figure 2: Consecutive *kitmum* Heptachords

³ From the main theme of CARL NIELSEN *Symphony No. 5* CNW 29 (1922).

⁴ Flatward shift for higher pitches.

1.2 Scale Range

Table 3: Scale Range

	Heptachord	Center	Semitones	Octaves	i , Cent	Cent per semitone
1	<i>išartum</i>	G	60	5	6000	100
2	<i>embābum</i>	A	60	5	6000	100
3	<i>kitmum</i>	D	60	5	6000	100
4	<i>pītum</i>	E	60	5	6000	100
5	<i>qablitum</i>	C	60	5	6000	100
6	<i>nīš tuḥrim</i>	F	120	10	12000	100
7	<i>nīd qablim</i>	B	120	10	12000	100

Definition. Cent per octave: 1200.

Definition. Cent per semitone: 100, Equal temperament (12-TET⁵ or 12-EDO⁶).

Definition. Scale Range: -3000 ... 0 (Center) ... +3000 Cent.⁷

Theorem. *The 7 Heptachords of the Babylonian Tonal System are based on an Equal Temperament system.*

⁵ Twelve-tone equal temperament.

⁶ Equal division of the octave.

⁷ Semitone: $i \leftarrow i + 100$, $i \leftarrow i - 100$. Whole tone: $i \leftarrow i + 200$, $i \leftarrow i - 200$.

1.3 Scale Cycle

Table 4: Scale Cycle

	Heptachord	Center	+3000 Cent	-3000 Cent
1	<i>išartum</i>	G	D \flat	C \sharp
2	<i>embūbum</i>	A	E \flat	D \sharp
3	<i>kitmum</i>	D	A \flat	G \sharp
4	<i>pītum</i>	E	B \flat	A \sharp
5	<i>qablītum</i>	C	G \flat	F \sharp

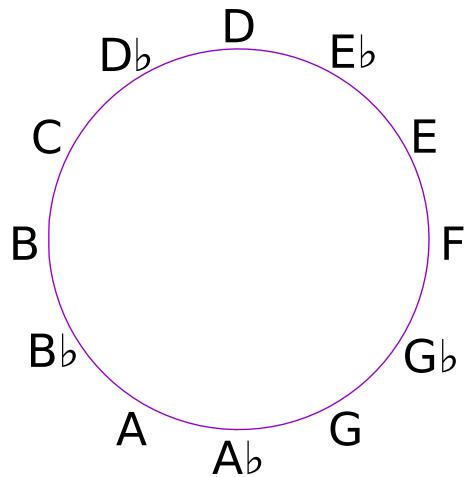


Figure 3: Tone Circle

1.4 Tuning

Conjecture. *Equal Temperament Tuning Procedure.*

Table 5: Tuning

	Heptachord	Center	i , Cent	f , Hz
1	<i>išartum</i>	G4	+500	
2	<i>embābum</i>	A3	-500	220
3	<i>kitmum</i>	D4	0	
4	<i>pītum</i>	E4	+200	
5	<i>qabhlītum</i>	C4	-200	
6	<i>nīš tuhrim</i>	F4	+300	
7	<i>nīd qablīm</i>	B3	-300	

8

Definition. Tuning Range: -500 ... 0 ... +500 Cent.

⁸ Center in Scientific Pitch Notation.

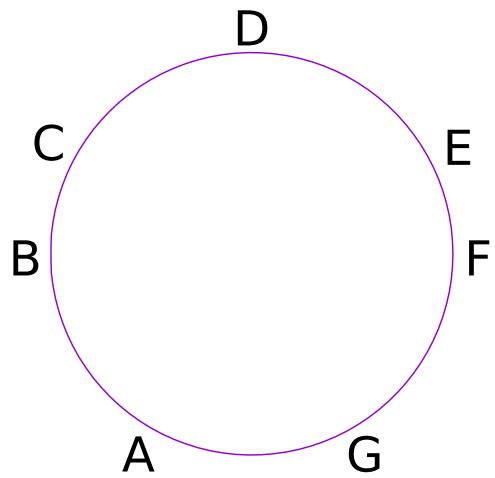


Figure 4: Tuning

2 *išartum*

$[1, \frac{1}{2}, 1, 1, 1, \frac{1}{2}]$

2.1 Scale 1

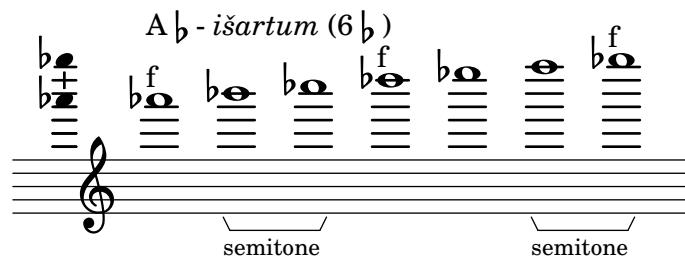


Figure 5: Scale I

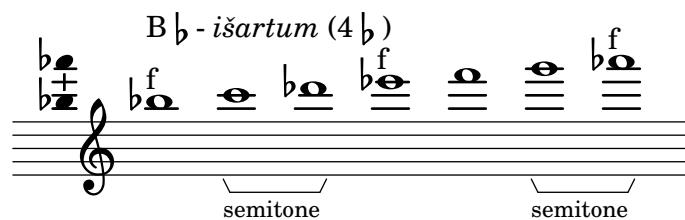


Figure 6: Scale II

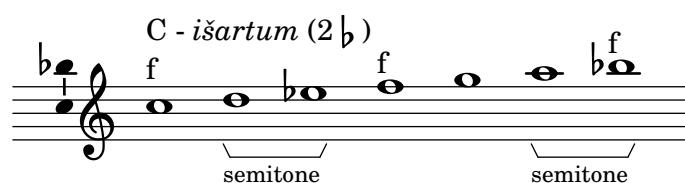


Figure 7: Scale III

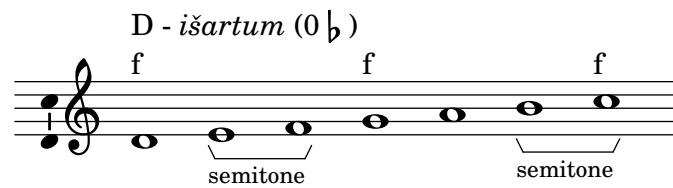


Figure 8: Scale IV

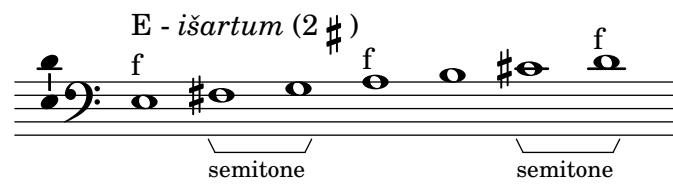


Figure 9: Scale V

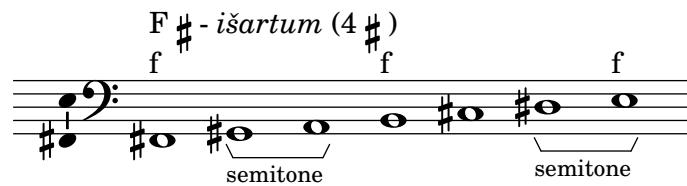


Figure 10: Scale VI

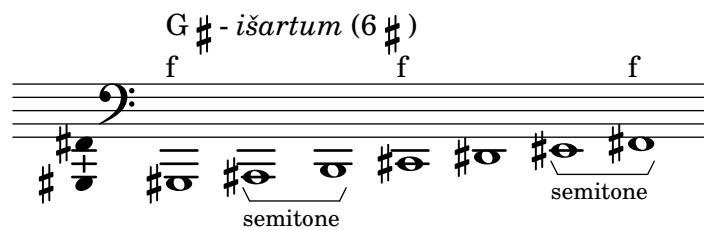


Figure 11: Scale VII

2.2 Scale 2

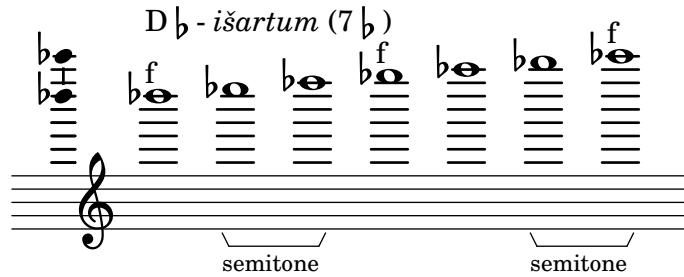


Figure 12: Scale I

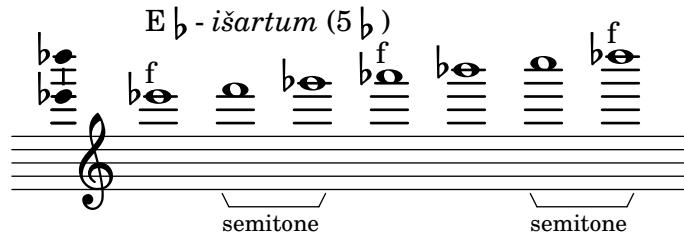


Figure 13: Scale II

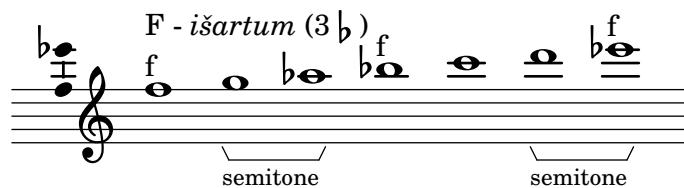


Figure 14: Scale III

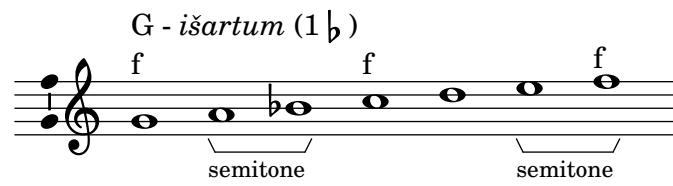


Figure 15: Scale IV

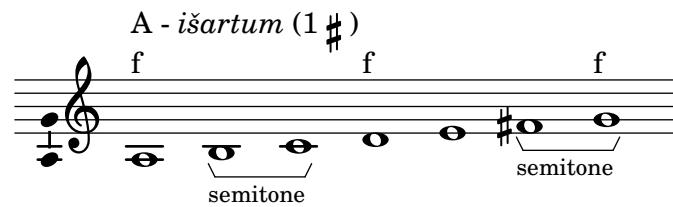


Figure 16: Scale V

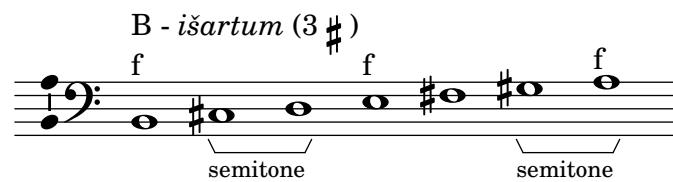


Figure 17: Scale VI

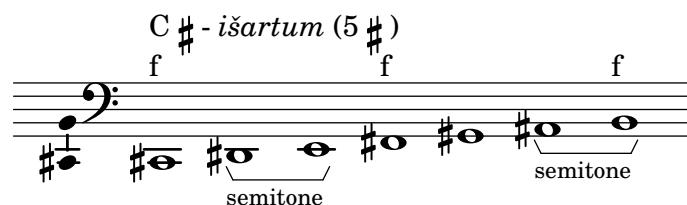


Figure 18: Scale VII

3 *embūbum*

$[\frac{1}{2}, 1, 1, 1, \frac{1}{2}, 1]$

3.1 Scale 1

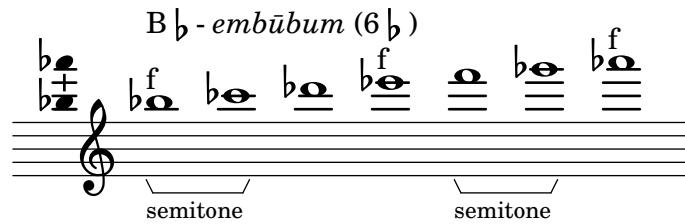


Figure 19: Scale I

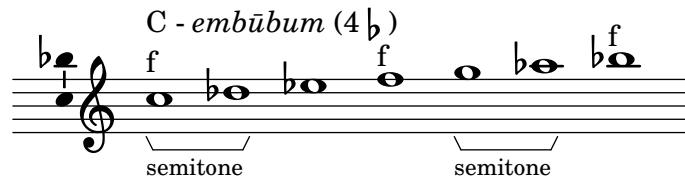


Figure 20: Scale II

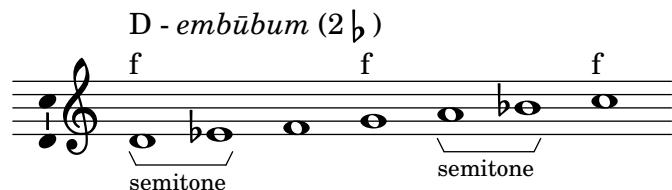


Figure 21: Scale III

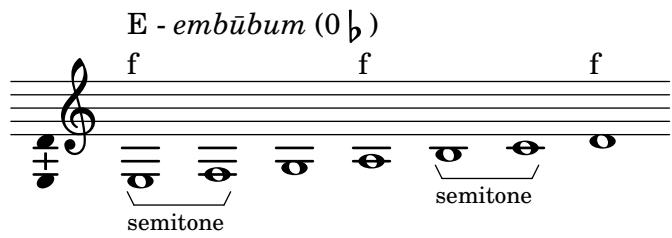


Figure 22: Scale IV

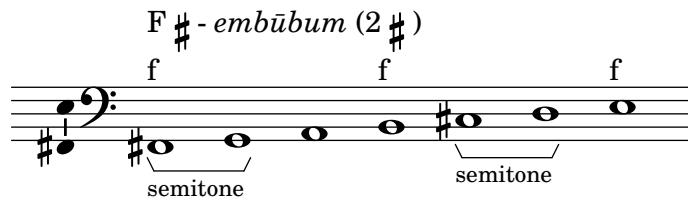


Figure 23: Scale V

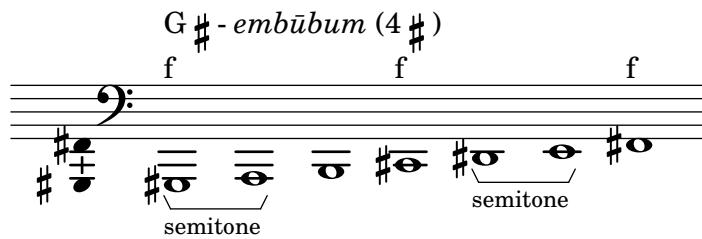


Figure 24: Scale VI

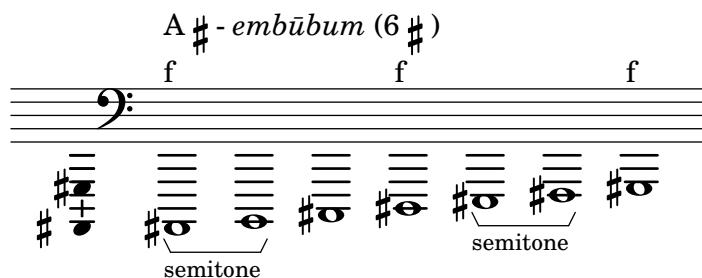


Figure 25: Scale VII

3.2 Scale 2

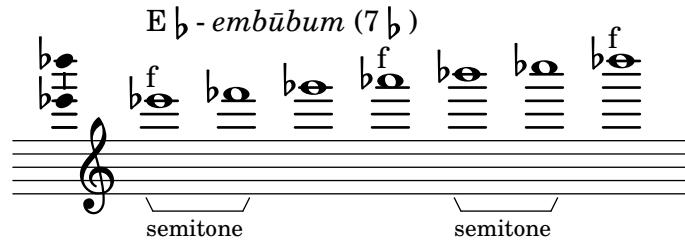


Figure 26: Scale I

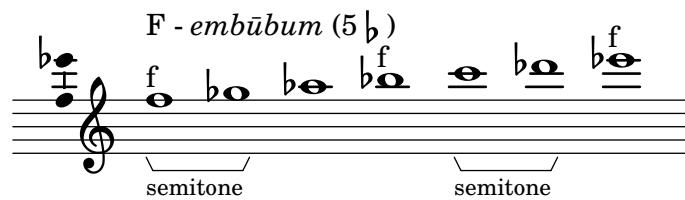


Figure 27: Scale II

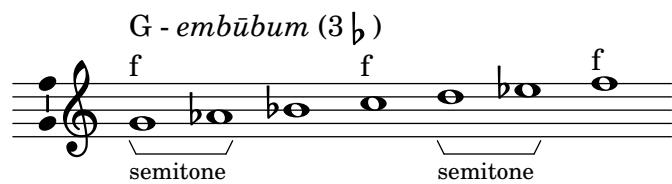


Figure 28: Scale III

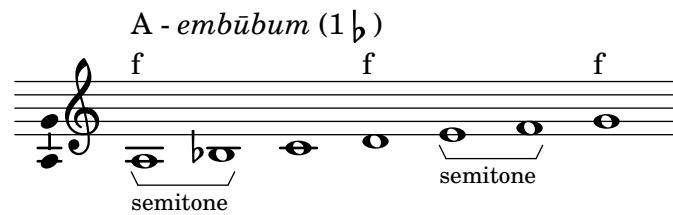


Figure 29: Scale IV

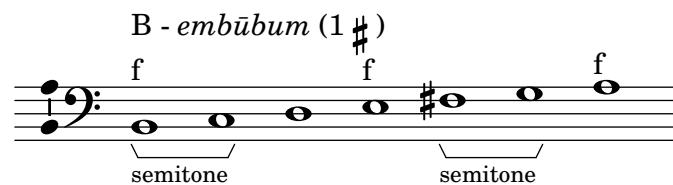


Figure 30: Scale V

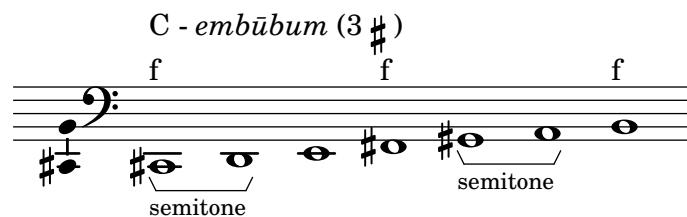


Figure 31: Scale VI

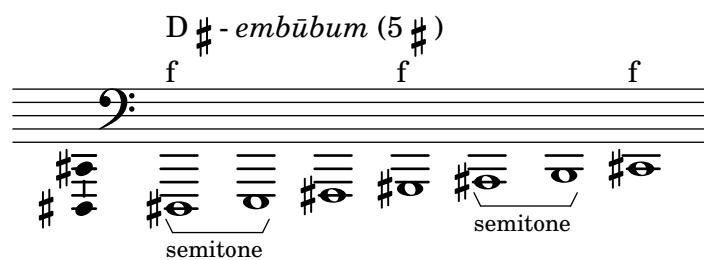


Figure 32: Scale VII

4 *kitmum*

$[1, \frac{1}{2}, 1, 1, \frac{1}{2}, 1]$

4.1 Scale 1

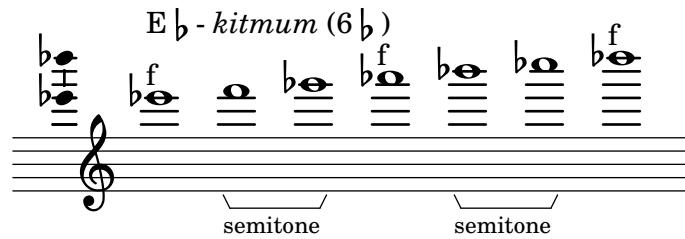


Figure 33: Scale I

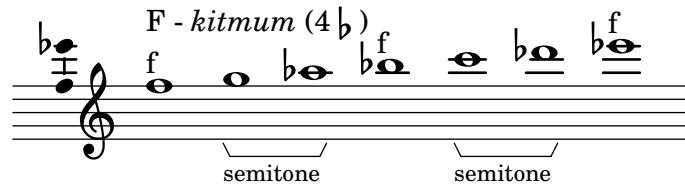


Figure 34: Scale II

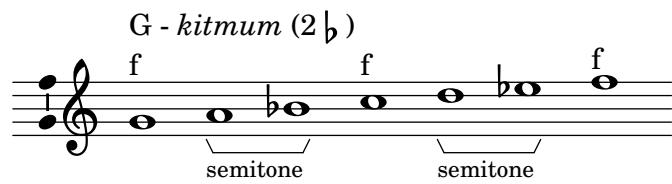


Figure 35: Scale III

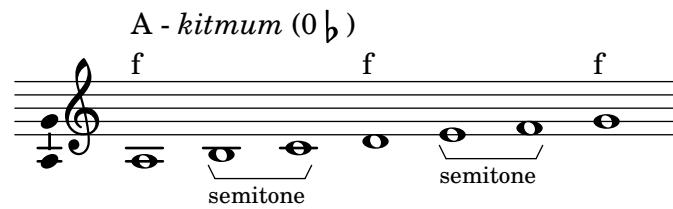


Figure 36: Scale IV

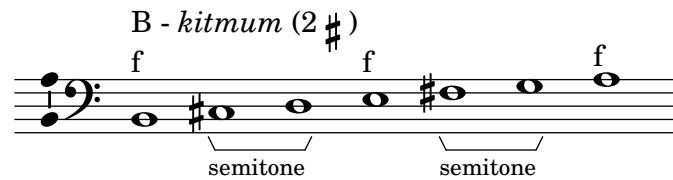


Figure 37: Scale V

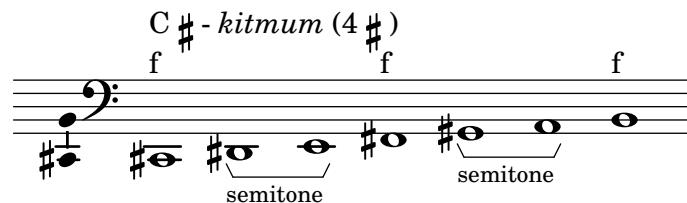


Figure 38: Scale VI

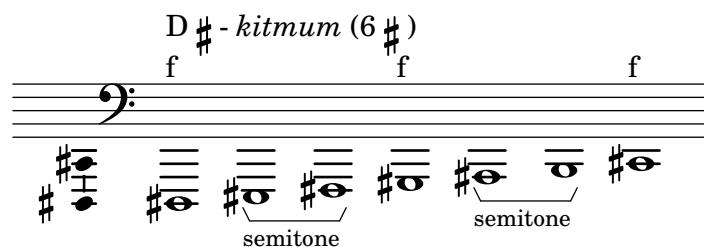


Figure 39: Scale VII

4.2 Scale 2

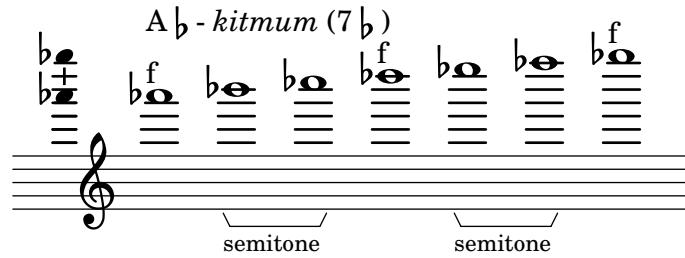


Figure 40: Scale I

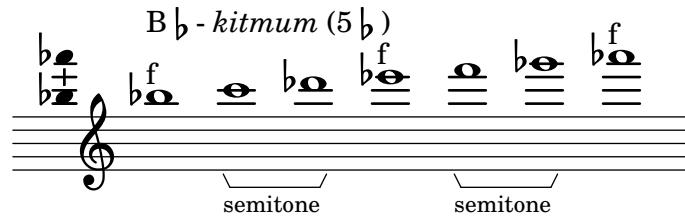


Figure 41: Scale II

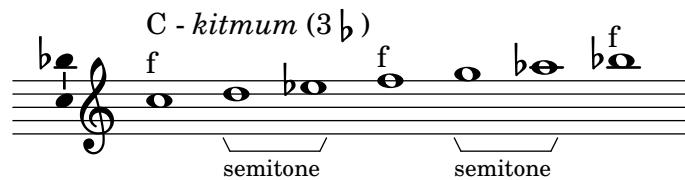


Figure 42: Scale III

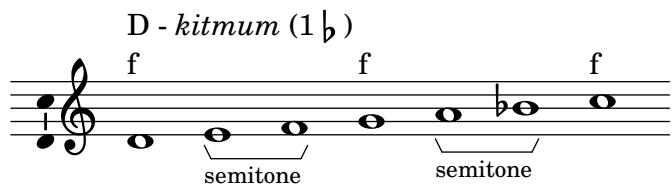


Figure 43: Scale IV

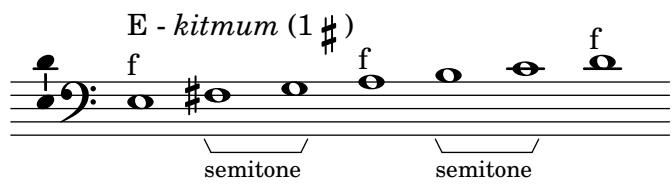


Figure 44: Scale V

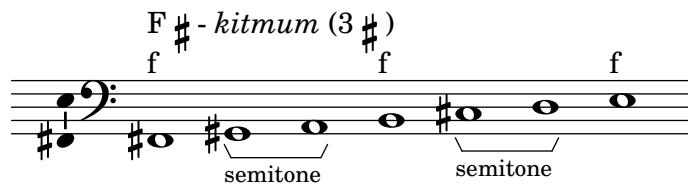


Figure 45: Scale VI

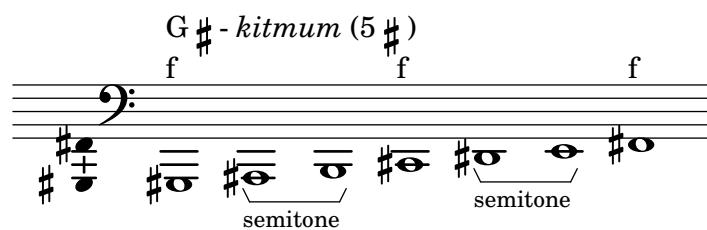


Figure 46: Scale VII

5 *pītum*

$[\frac{1}{2}, 1, 1, \frac{1}{2}, 1, 1]$

5.1 Scale 1

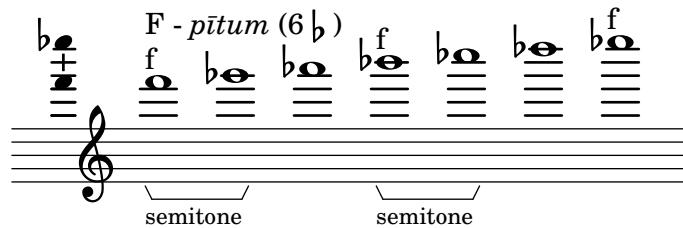


Figure 47: Scale I

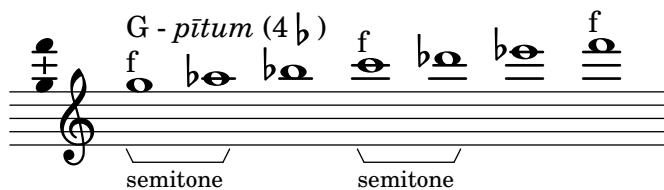


Figure 48: Scale II

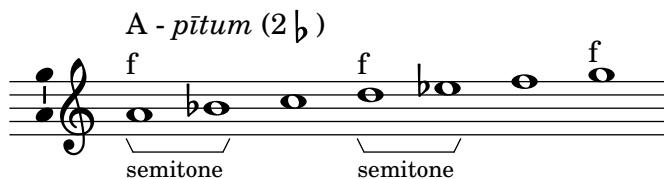


Figure 49: Scale III

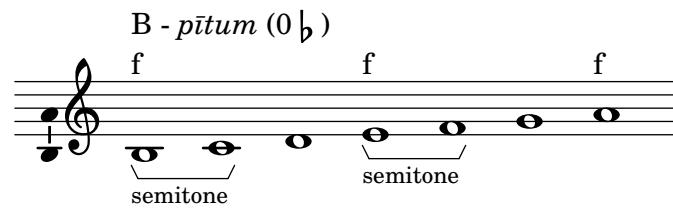


Figure 50: Scale IV

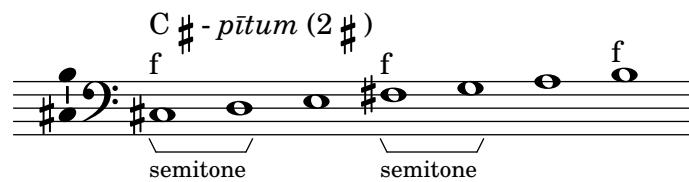


Figure 51: Scale V

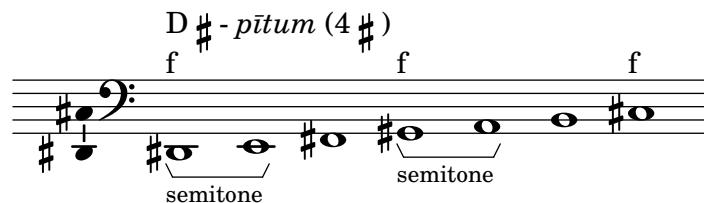


Figure 52: Scale VI

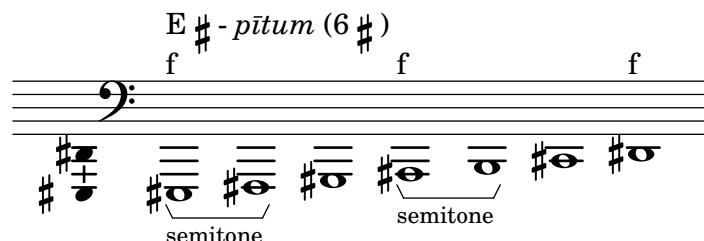


Figure 53: Scale VII

5.2 Scale 2

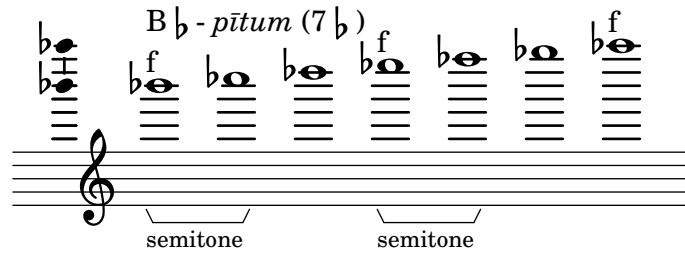


Figure 54: Scale II

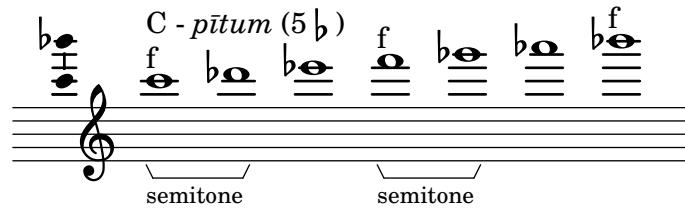


Figure 55: Scale II

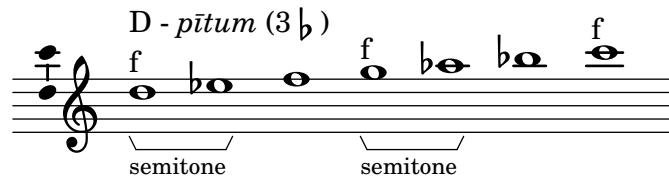


Figure 56: Scale III

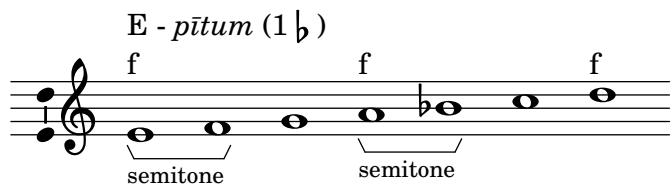


Figure 57: Scale IV

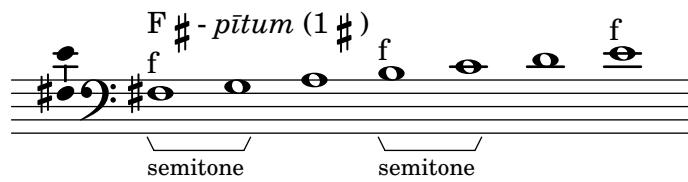


Figure 58: Scale V

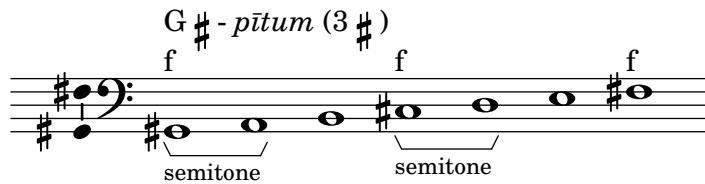


Figure 59: Scale VI

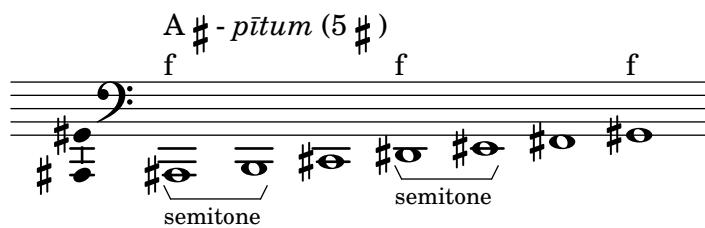


Figure 60: Scale VII

6 *qablītum*

$[1, 1, \frac{1}{2}, 1, 1, \frac{1}{2}]$

6.1 Scale 1

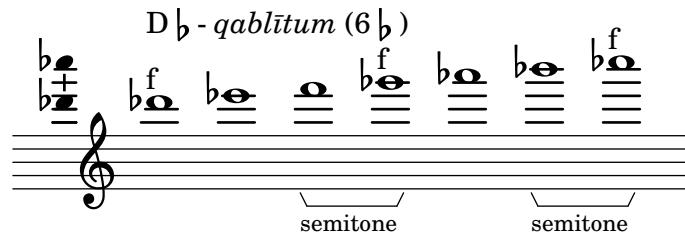


Figure 61: Scale I

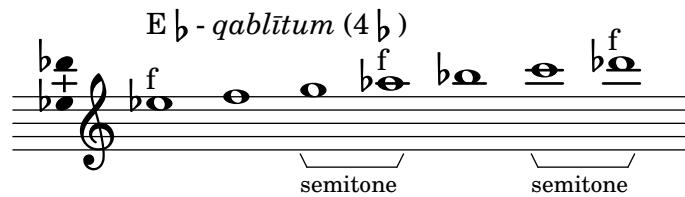


Figure 62: Scale II

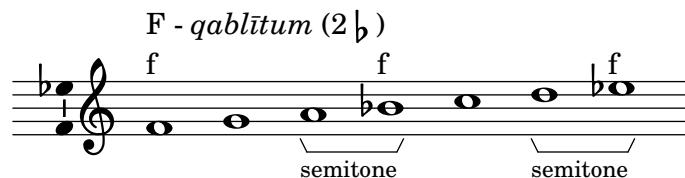


Figure 63: Scale III

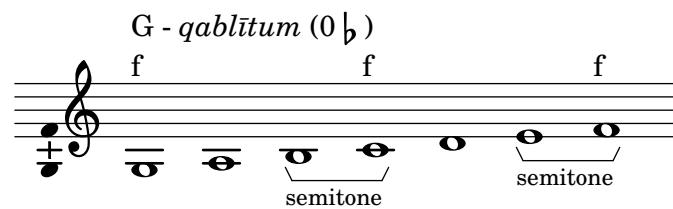


Figure 64: Scale IV

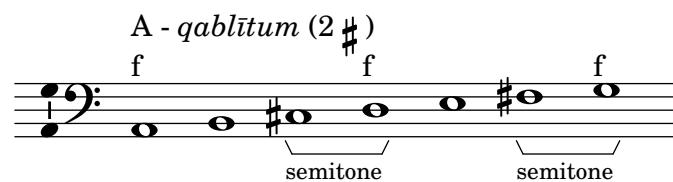


Figure 65: Scale V

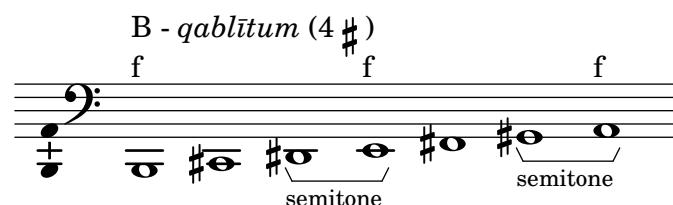


Figure 66: Scale VI

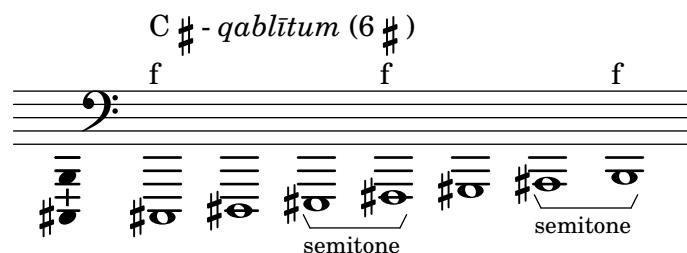


Figure 67: Scale VII

6.2 Scale 2

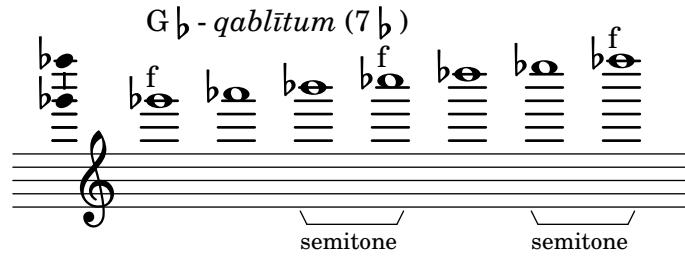


Figure 68: Scale I

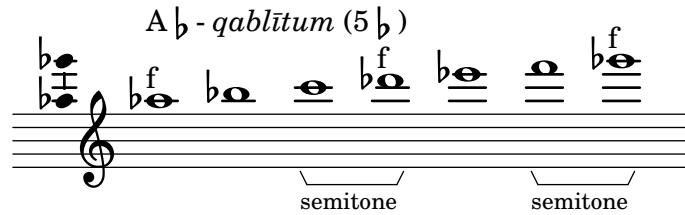


Figure 69: Scale II

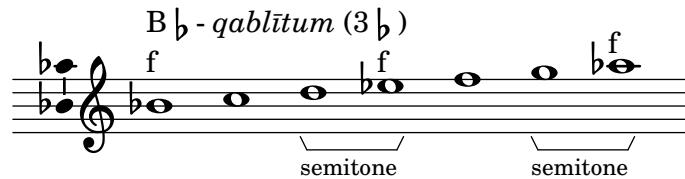


Figure 70: Scale III

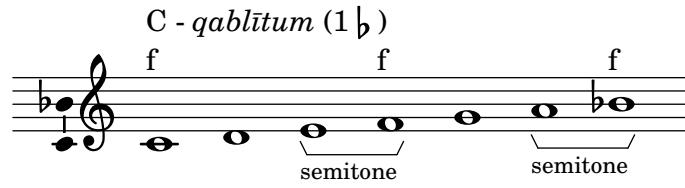


Figure 71: Scale IV

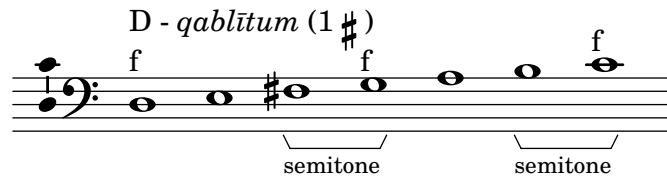


Figure 72: Scale V

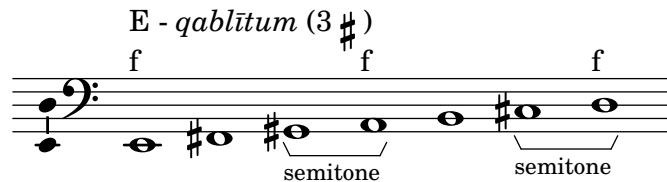


Figure 73: Scale VI

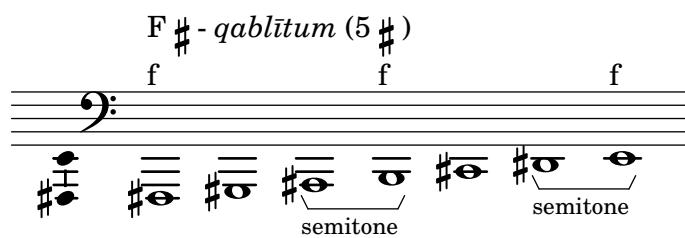


Figure 74: Scale VII

7 *nīš tuḥrim*

$[1, 1, \frac{1}{2}, 1, 1, 1]$

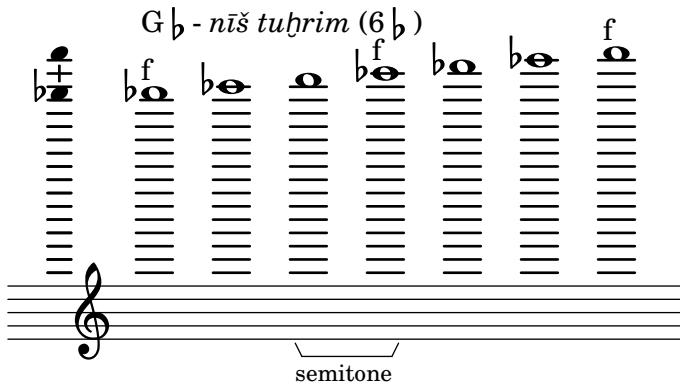


Figure 75: Scale I

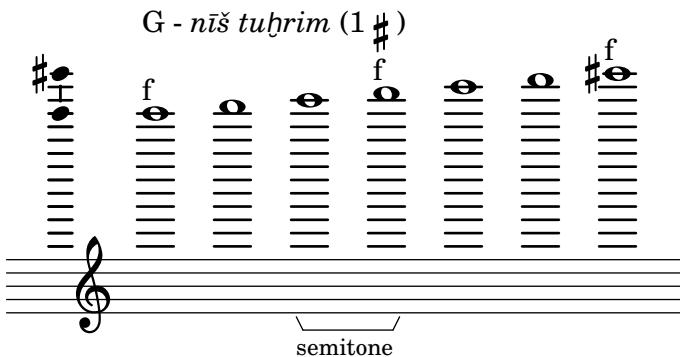


Figure 76: Scale II

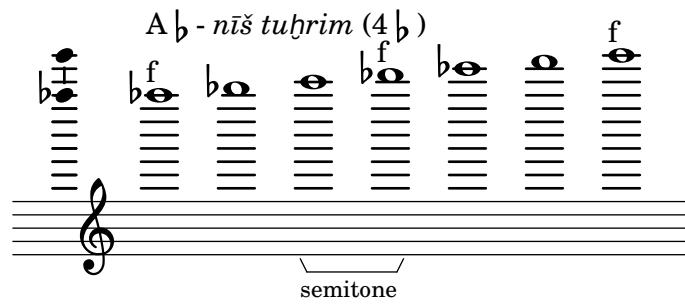


Figure 77: Scale III

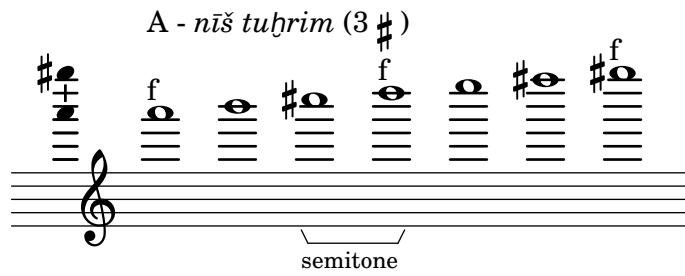


Figure 78: Scale IV

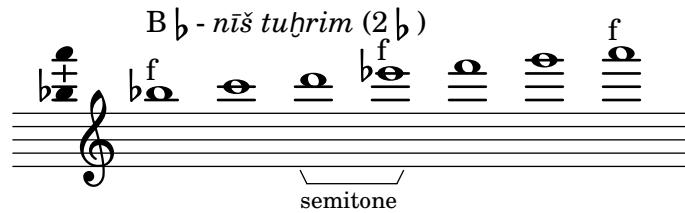


Figure 79: Scale V

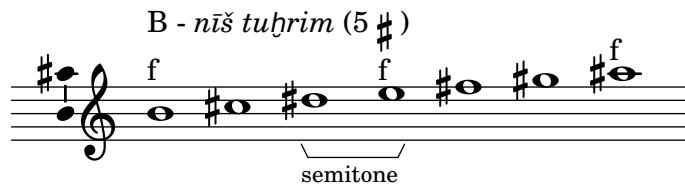


Figure 80: Scale VI

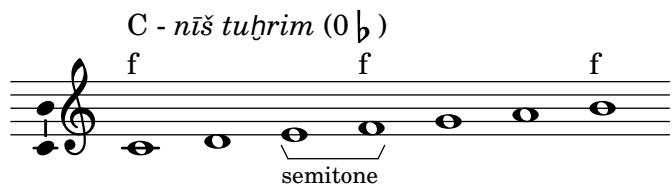


Figure 81: Scale VII

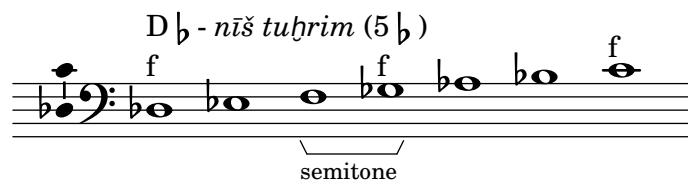


Figure 82: Scale VIII

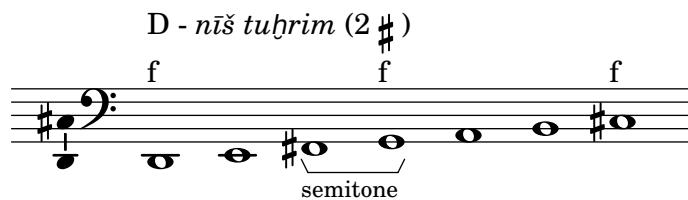


Figure 83: Scale IX

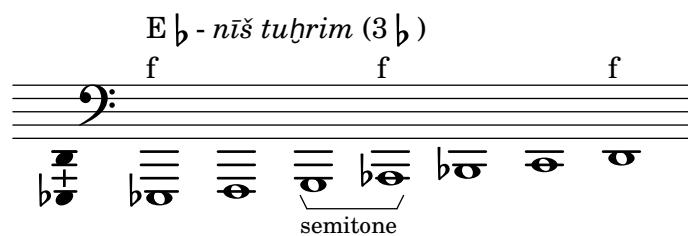


Figure 84: Scale X

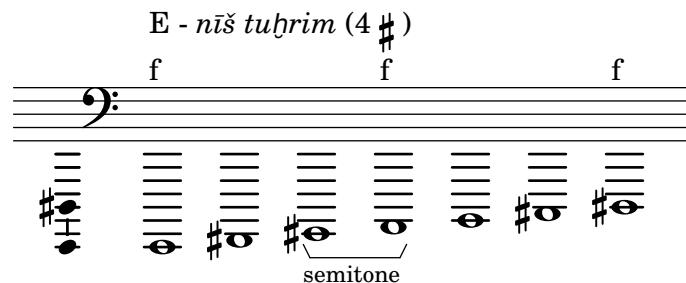


Figure 85: Scale XI

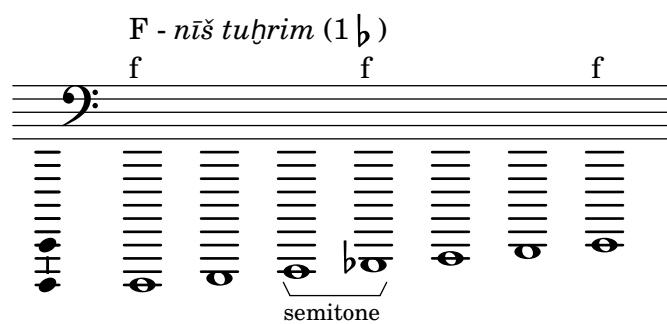


Figure 86: Scale XII

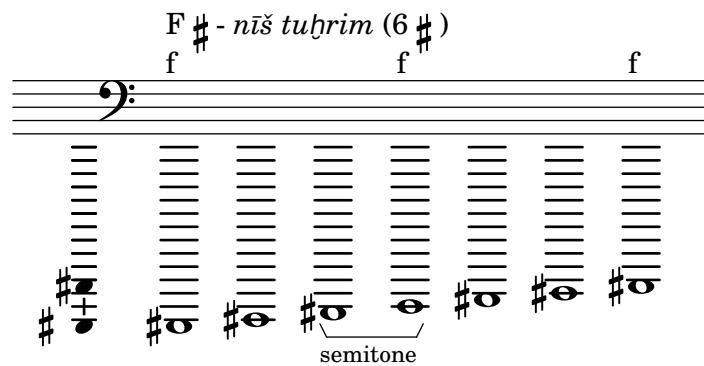


Figure 87: Scale XIII

8 *nīd qablim*

$[1, 1, 1, \frac{1}{2}, 1, 1]$

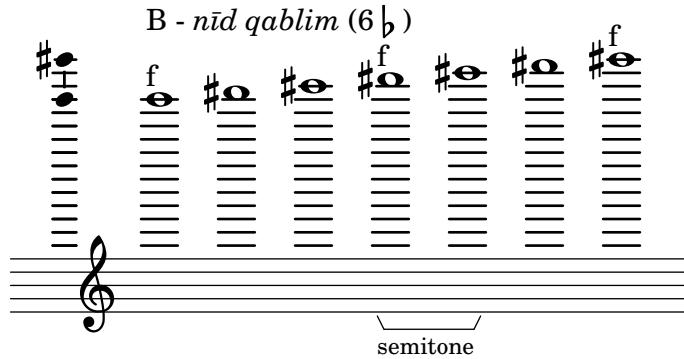


Figure 88: Scale I

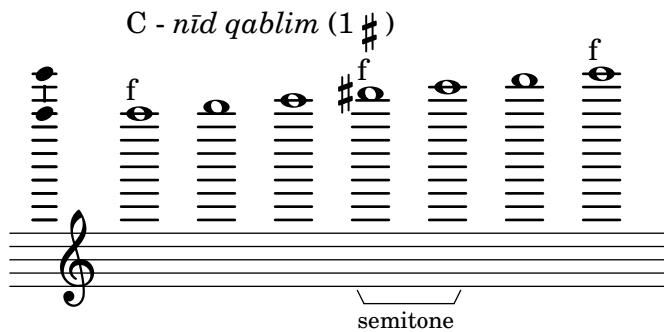


Figure 89: Scale II

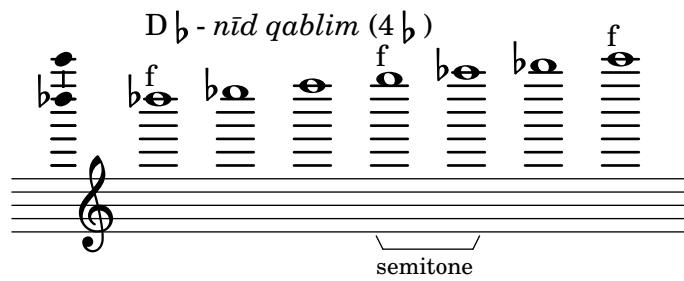


Figure 90: Scale III

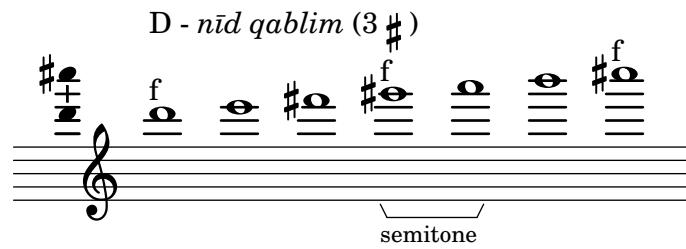


Figure 91: Scale IV

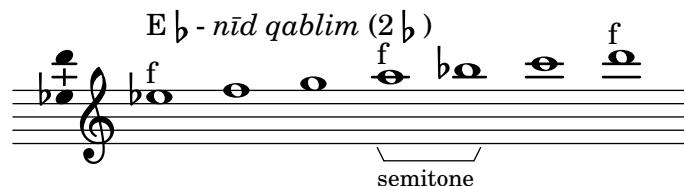


Figure 92: Scale V

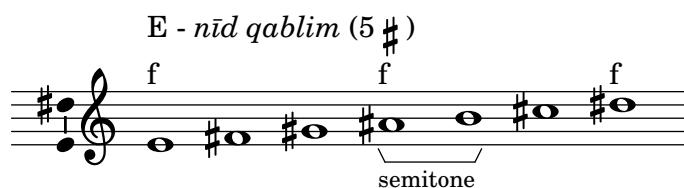


Figure 93: Scale VI

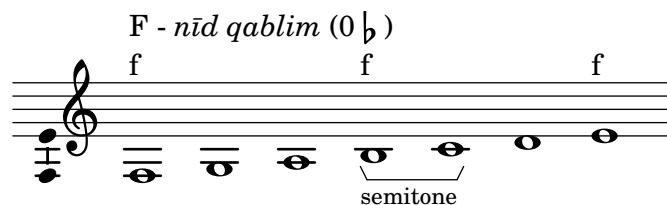


Figure 94: Scale VII

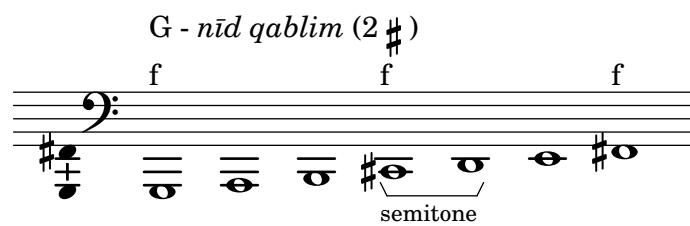
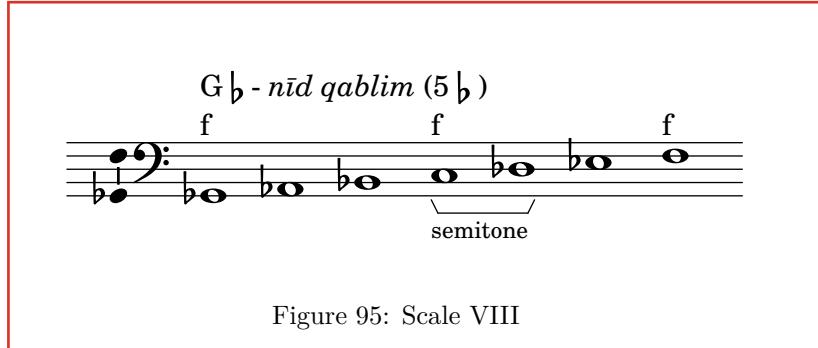


Figure 96: Scale IX

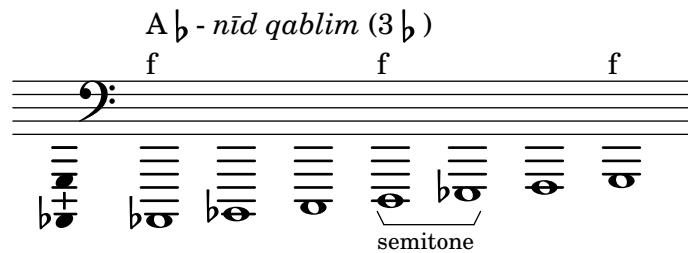


Figure 97: Scale X

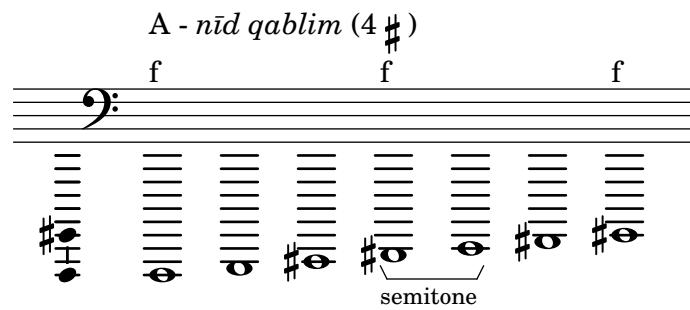


Figure 98: Scale XI

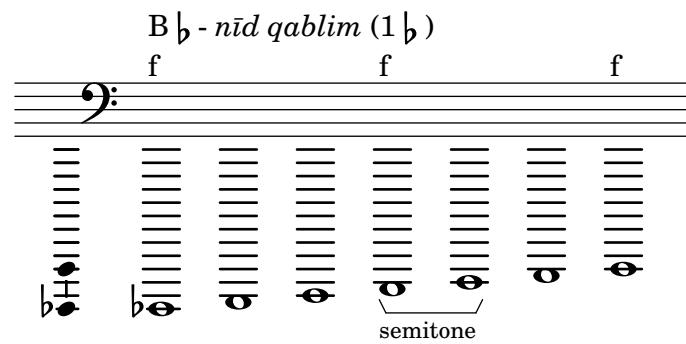


Figure 99: Scale XII

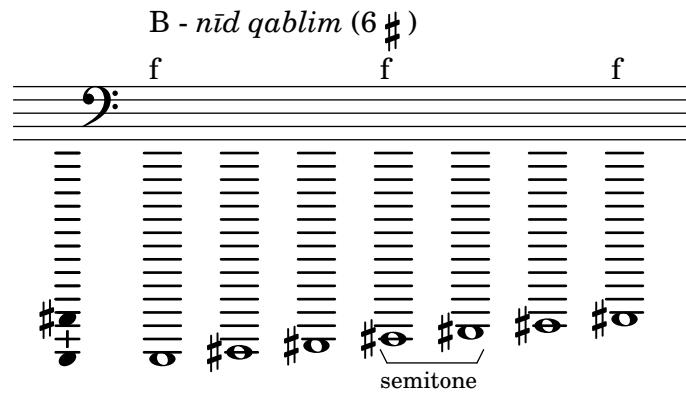


Figure 100: Scale XIII

References

- [1] Leon Crickmore. New Light on the Babylonian Tonal System. In Richard Dumbrill and Irving Finkel, editors, *ICONEA 2008: Proceedings of the International Conference of Near Eastern Archaeomusicology*, volume 24, pages 11–22. Lulu, 2008. Held at the British Museum, December 4-6. [1](#)

A Appendix

A.1 Octatonic Scale

⁹

A.1.1 Scale 1

$$[1, \frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{2}]^{10}$$

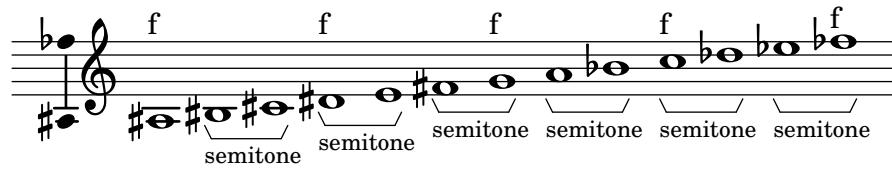


Figure 101: Scale 1, Center G

A.1.2 Scale 2

$$[\frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{2}, 1]$$

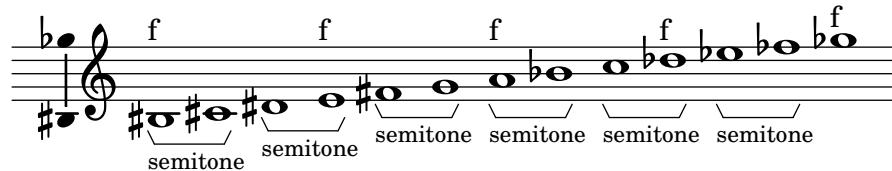


Figure 102: Scale 2, Center A

⁹ Derived from CARL NIELSEN *Symphony No. 5* CNW 29.

¹⁰ OLIVIER MESSIAEN's *second mode* of limited transposition.

A.2 Plots

A.2.1 Heptachords

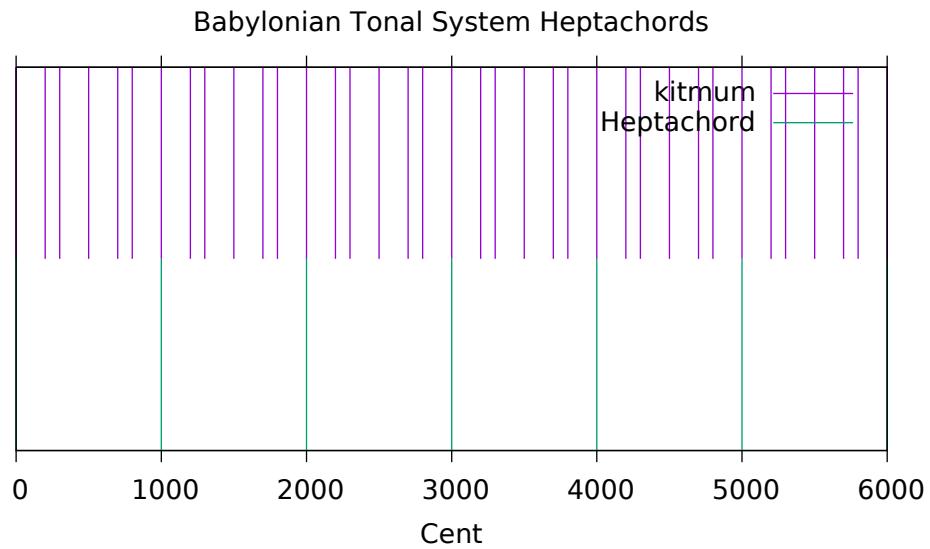


Figure 103: *kitmum* vs. Heptachord

A.2.2 Octaves

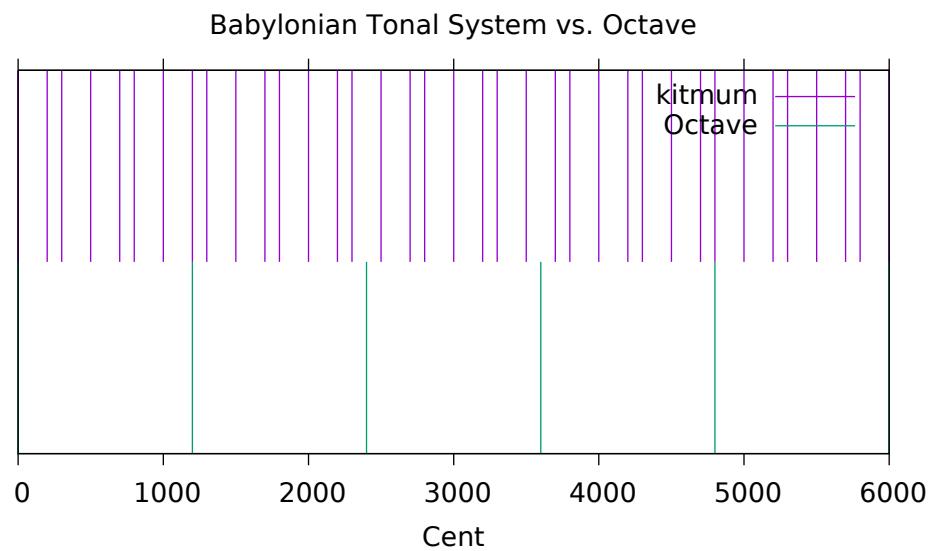


Figure 104: *kitmum* vs. Octave