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Theory of Quantum Transport in a Two-Dimensional Electron System under Magnetic Fields.* I. Characteristics of Level Broadening and Transport under Strong Fields

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Characteristics of level broadening and transverse conductivity of a two-dimensional electron system under extremely strong fields have been theoretically investigated in the simplest approximation without the difficulty of divergence, i e. in the so-called damping theoretical one. Those of various cases of short- and long-ranged scatterers have been obtained. To see the dependence on the range explicitly, numerical calculation has been performed for the system with scatterers with the Gaussian potential. Especially in case of short-ranged ones the peak value of the transverse conductivity has been shown to be $(N+1/2)e^2/\pi^2\hbar$ which depends only on the natural constants and the Landau level index. It has been argued from general point of view that this fact is approximately true without reference to kinds of approximations. Such characteristic of the conductivity was confirmed experimentally, but there still remain some problems as to the absolute value of the level width.

§ 1. Introduction

Recently, there has been much interest in the theory of two-dimensional systems, partly because of the relevance of such theories to the properties of surfaces and thin films. The inversion layers of semiconductor surfaces are a good example of two-dimensional systems. An n-type inversion layer is formed, when a sufficiently strong electric field is applied across the interface of an insulator and a p-type semiconductor, and when the conduction band edge is bent below the Fermi level in the bulk. The motion in the z-direction, the direction perpendicular to the surface is quantized and the energy levels of the electrons are grouped in electric sub-bands, each of which corresponds to a quantized level for motion in the z-direction, with a continuum for motion in the xy-plane parallel to the surface. It is possible for only the lowest electric sub-band to be occupied at sufficiently low temperatures, and one can regard the inversion layer essentially as a two-dimensional system.[1-5]

Typical natures of the two-dimensional system appear especially when a strong magnetic field is applied perpendicularly to the surface. The orbital motion of the electrons is perfectly quantized and the energy spectrum becomes discrete, each peak corresponding to the Landau level. Physical properties of such kind of a singular system are very interesting. Especially as for the transport phenomena, it is expected that characteristics of the quantum transport become clear, which are covered by the free motion in the direction of the applied magnetic field in the usual three dimensional system.

When discussing such a system, one has to take into account broadening effects self-consistently. Otherwise, one would be faced with the difficulty of divergence. At low temperatures, the broadening is caused by scatterers such as impurities, lattice defects, and also roughness of the surface. It is necessary to treat this system as a disordered system. On the other hand, if one looks at this system as an example of disordered systems from a purely theoretical point of view, one notices very interesting facts. Because of the nonexistence of the continuous spectrum, it becomes rather easy to solve the equation which determines the Green's function. Therefore, it becomes

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easy to discuss scatterers with a finite range and also to proceed to higher approximations than the single-site approximation (SSA) or the CPA.

Purposes of the series of our papers are to clarify various characteristics of the level broadening and the transport phenomena of this singular system, and to contribute to the theory of disordered system. Ohta[6] also calculated the level broadening and the conductivity of this system, assuming scatterers with a short-ranged δ-potential which are distributed randomly in the xy-plane and according to some distribution in the z-direction. However, he has obtained several unphysical results. For example, he argued that some of Landau levels are split into two peaks. These are due to the lack of the self-consistency in his calculation, which is very important as will be shown in this paper. Further, his discussion is strictly confined to his model and the characteristics of our singular system has not been clarified yet.

In this first paper, we will derive the important formulas which are necessary in the subsequent investigations and clarify the characteristic of the transport phenomena in this system under extremely strong magnetic fields, employing the simplest approximation without the difficulty of divergence. In §2, the problem is formulated in the language of the Green's function and the characteristic of the peak value of the conductivity in case of short-ranged scatterers is discussed from the general point of view. Various characteristics of the level broadening and the conductivity are considered in §3. Some discussion on the obtained results and comparison with experiments are given in §4. In this paper, we formulate the problem, confining ourselves to MOS inversion layers, but various characteristics obtained in this paper are expected to apply to other systems as well.

§2. Formulation of the Problem

2.1 Preliminaries

The effective Hamiltonian of the two-dimensional electrons in the inversion layer can be written as

$$H=H_0+H_1\ , \tag{2.1}$$

with

$$H_0=\frac{1}{2m}\left(p+\frac{e}{c}A\right)^2\ , \tag{2.2}$$

and

$$H_1=\sum_{\mu}\sum_{i} v^{(\mu)}(r-R_i, Z_i)\ . \tag{2.3}$$

The perturbing Hamiltonian H_1 represents effects of the scatterers and

$$v^{(\mu)}(r-R_i, Z_i)=\int dz|\xi_0(z)|^2 V^{(\mu)}(r-R_i, z-Z_i)\ , \tag{2.4}$$

where $\xi_0(z)$ is the wave function of the ground electric sub-band in the z-direction, $V^{(\mu)}(r-R_i, z-Z_i)$ is the potential of the scatterer of μ-th kind, the position of which is (R_i, Z_i), and $r=(x, y)$ is the coordinate parallel to the surface. In our system roughness of the surface also contributes to scattering, and this Hamiltonian does not include effects of the surface roughness exactly. In case that the surface roughness is sufficiently weak and that higher electric subbands can be neglected, it can be shown that the roughness is expressed in terms of the potential strongly dependent on the applied electric field.[7] Therefore, one can include the effect of the surface roughness to a great extent by this Hamiltonian in such a case. The surface roughness has come to be known to be important only at very high gate voltages or very high concentrations of electrons.[7] Then, the above Hamiltonian will be satisfactory in the present consideration.

In the following, we employ the technique of the Green's function, which has worked well for other usual disordered systems. We use the following artifice in summing up graphs. When an electron interacts with a scatterer, we use the representation in which the angular momentum L_z around the position of the scatterer is diagonalized, and when the electron moves freely, we use that in which one of the center coordinates X is diagonal. This artifice enables us to sum up series of graphs easily especially when we deal with scatterers with a finite range, which will be clear in the subsequent paper. In the symmetric gauge $A=(-Hy/2, Hx/2)$, the wave function of the latter representation is

$$\varphi_{NX}(r)=\frac{1}{\sqrt{L}}\exp\left[i\frac{xy}{2l^2}-i\frac{Xy}{l^2}\right]\chi_N(x-X)\ , \tag{2.5}$$

where L^2 is the area of the system, l is the radius of the ground cyclotron orbit given by $l^2=c\hbar/eH$, and

$$\chi_N(x)=\sqrt{\frac{1}{2^N N!\sqrt{\pi}\,l}}\exp\left[-\frac{x^2}{2l^2}\right]H_N\left(\frac{x}{l}\right)\ . \tag{2.6}$$

The wave function of the former representation is

$$\varphi_{Nm;\,R}(\boldsymbol{r})=\varphi_{Nm}(\boldsymbol{r}-\boldsymbol{R})\exp\left[i\frac{\boldsymbol{r}\times\boldsymbol{R}}{2l^2}\right], \tag{2.7}$$

with

$$\varphi_{Nm}(\boldsymbol{r})=\sqrt{\frac{N!}{2\pi l^2(N+m)!}}\exp\left[-im\phi-\frac{r^2}{4l^2}\right]\left(\frac{r^2}{2l^2}\right)^{m/2}L_N^{\ m}\left(\frac{r^2}{2l^2}\right), \tag{2.8}$$

where $\phi=\tan^{-1}y/x$ and $\boldsymbol{r}_1\times\boldsymbol{r}_2=x_1y_2-x_2y_1$. These wave functions are connected with each other by

$$\varphi_{NX}(\boldsymbol{r})=\sum_{m=-N}^{\infty}\varphi_{Nm;\,R}(\boldsymbol{r})\sqrt{2\pi l^2}(-1)^m\varphi_{N+mX}(R)\,. \tag{2.9}$$ [8]

The Hamiltonian becomes

$$H=\sum_{NX}E_N a^+_{NX}a_{NX}+\sum_{\mu}\sum_{i}\sum_{NX}\sum_{N'X'}\sum_{m}\sum_{m'}2\pi l^2\varphi^*_{N+mX}(R_i)(-1)^m$$
$$\times(Nm|v^{(\mu)}(Z_i)|N'm')(-1)^{m'}\varphi_{N'+m'X'}(R_i)a^+_{NX}a_{N'X'}\,, \tag{2.10}$$

where $E_N=(N+1/2)\hbar\omega_c$, and

$$(Nm|v^{(\mu)}(Z_i)|N'm')=\int\varphi^*_{Nm}(\boldsymbol{r})v^{(\mu)}(\boldsymbol{r},Z_i)\varphi_{N'm'}(\boldsymbol{r})d^2r\,, \tag{2.11}$$

which is proportional to $\delta_{mm'}$ if the potential is cylindrically symmetric. For simplicity, we will confine ourselves to this case and also neglect the spin.

2.2 *Transverse conductivity*

The Kubo formula[9] for the electric conductivity gives

$$\sigma_{\mu\nu}(\omega)=(i\omega)^{-1}[K_{\mu\nu}(\hbar\omega+i0)-K_{\mu\nu}(0)]\,, \tag{2.12}$$

where the thermal Green's function $K_{\mu\nu}(i\omega_m)$ can be written by the use of the averaged single-particle Green's function and of the current vertex part J as

$$K_{\mu\nu}(i\omega_m)=-\frac{1}{\beta}\sum_{i\varepsilon_n}\sum_{N_1\pm,X}(N_1\pm1|j_\mu|N_1)G_{N_1}(i\varepsilon_n+i\omega_m)G_{N_1\pm1}(i\varepsilon_n)J^{\nu}_{N_1N_1\pm1}(i\varepsilon_n+i\omega_m,i\varepsilon_n)\,. \tag{2.13}$$

The vertex part satisfies the following equation:

$$J^{\nu}_{N_1N_1\pm1}(E,E')=(N_1|j_\nu|N_1\pm1)+\sum_{N_1'}\gamma_{N_1N_1\pm1\,N_1'N_1'\pm1}(E,E')$$
$$\times G_{N_1'}(E)G_{N_1'\pm1}(E')J^{\nu}_{N_1'N_1'\pm1}(E,E')\,, \tag{2.14}$$

where one has introduced the proper vertex part

$$\gamma_{N_1N_1\pm1\,N_1'N_1'\pm1}(E,E')=\sum_{X'}\gamma_{N_1X\,N_1\pm1X\,N_1'X'N_1'\pm1X'}(E,E')\,. \tag{2.15}$$

By the analytic continuation which is the same as in standard textbooks, the static transverse conductivity becomes

$$\sigma_{xx}=-\frac{\hbar}{4\pi^2l^2}\mathrm{Re}\int\left(-\frac{\partial f}{\partial E}\right)dE\sum_{N_1\pm}(N_1|j_x|N_1\pm1)\,G_{N_1\pm1}(+)$$
$$\times\{G_{N_1}(+)J^x_{N_1\pm1N_1}(++)-G_{N_1}(-)J^x_{N_1\pm1N_1}(+-)\}\,, \tag{2.16}$$

where for simplicity, $G_N(\pm)=G_N(E\pm i0)$, etc. This formula is suited especially when we deal with systems with scatterers with δ-potentials, because the proper vertex part $\gamma_{N_1N_1\pm1\,N_2N_2\pm1}$ vanishes if we employ the so-called single-site approximation (SSA) and lower ones. In this case, the conductivity is reduced to the formula already obtained by Ohta.[8]

Next, let us obtain the expression of the conductivity under extremely strong magnetic fields in which quantities of the order of $O(\Gamma/\hbar\omega_c)$ are neglected (Γ is of the order of the level width). Assume that the Fermi level lies in the N-th Landau level. It is easily seen that graphs which include more than two proper vertex parts can be neglected and that only necessary graphs are those shown in Fig. 1. We can also expand the Green's function of other Landau levels than the N-th one with respect to $1/\hbar\omega_c$. We have the following formula.

$$\begin{aligned}\sigma_{xx}=\frac{e^2}{\pi^2\hbar}\int\Big(-\frac{\partial f}{\partial E}\Big)\,\mathrm{d}E\,\Big\{&\Big(N+\frac{1}{2}\Big)\mathrm{Im}\,G_N(+)\,\mathrm{Im}\,\Sigma_N(+)\\
&+\frac{1}{2}\sum_{\pm}\Big[\Big(N+\frac{1}{2}\pm\frac{1}{2}\Big)\mathrm{Im}\,G_N(+)\,\mathrm{Im}\,(\Sigma_{N\pm1}(+)-\Sigma_N(+))\\
&-\Big(N+\frac{1}{2}\pm\frac{1}{2}\Big)\frac{1}{2}\mathrm{Re}\,G_N(+)^2(\gamma_{NN\pm1\,NN\pm1}(++)-\gamma_{NN\pm1\,NN\pm1}(+-))\\
&+\sqrt{N(N+1)}\,\frac{1}{2}\mathrm{Re}\,G_N(+)(G_N(+)\gamma_{N\pm1N\,NN\mp1}(++)\\
&-G_N(-)\gamma_{N\pm1N\,NN\mp1}(+-))\Big]\Big\}\,, \qquad (2.17)\end{aligned}$$

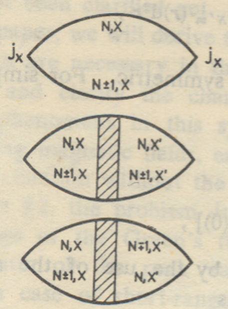

Fig. 1. The graphs which should be taken into account in the calculation of the conductivity under extremely strong fields. The shaded part represents the proper vertex part.

where all the internal electron lines in the self-energy and vertex part should be replaced by the Green's function of the N-th Landau level.

From the above expression, we find a noticeable characteristic of the transverse conductivity in our system. The first term of eq. (2.17) is rewritten as

$$\frac{e^2}{\pi^2\hbar}\int\Big(-\frac{\partial f}{\partial E}\Big)\mathrm{d}E\,\frac{G_N''^2}{G_N'^2+G_N''^2}\,, \qquad (2.18)$$

where $G_N=G_N'+iG_N''$. It has the maximum value $(N+1/2)e^2/\pi^2\hbar$ at zero of temperature in case G_N'' is finite at $G_N'=0$, which is the case usually. This maximum value depends only on the natural constants and the Landau level index N, and is independent of the magnetic field, the mass of an electron, and of the strength and the density of the scatterers. If the scatterers are of short-range, the terms except the first in eq. (2.17) are expected to be small compared with the first especially in the neighbourhood of the energy at which the conductivity has a peak. Except in the vicinity of the spectral edges, the simplest approximation in this paper and the SSA are considered to work well. In such approximations, the vertex part vanishes, the self-energy is independent of N, and only the first term remains finite. Therefore, the peak value of the transverse conductivity is expected to be very close to the above value, if the scatterers are of short range. This fact was confirmed experimentally by Kobayashi and Komatsubara in the MOS inversion layer made on the (100) surface of silicon.[10]

In case that the scatterers are of finite range, on the other hand, the other terms than the first remain finite and play an important role in reducing the peak value. As will be shown in the following, the vertex part is indispensable to give the same result as that obtained by another representation of the Kubo formula[8] which is expressed by the correlation function of the center coordinates.

§3. Characteristics of Level Broadening and the Transverse Conductivity under Extremely Strong Fields

In this section, we will discuss the characteristics of the transport in our system in the so-called damping theoretical approximation.[8] In this approximation effects of scattering from impurities are taken into account in the lowest Born one, while the broadening effect is included in a self-consistent way. It works sufficiently well except in the energy region of the vicinity of spectral edges in case of sufficiently high concentration of scatterers as will be shown in subsequent papers. Therefore, it is sufficient to see the characteristics of the transport.

Graphically, we should take graphs shown in Fig. 2 as the self-energy and have

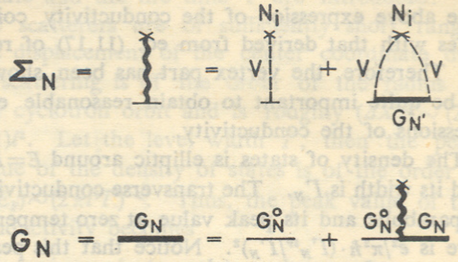

Fig. 2. The self-energy in the simplest approximation without the difficulty of divergence.

$$\Sigma_N(E)=2\pi l^2 \sum_{\mu} \int dZ N_i^{(\mu)}(Z) \sum_{N'} \sum_{m} |(Nm|v^{(\mu)}(Z)|N'm)|^2 G_{N'}(E)\,, \tag{3.1}$$

where the first term in Fig. 2 has been neglected since it only shifts the origin of the energy, and $N_i^{(\mu)}(Z)\, d R_x\, d R_y\, d Z$ is the mean number of scatterers of the μ-th kind in the volume $d R_x\, d R_y\, d Z$. The proper vertex part is obtained by cutting off the internal electron line of the self-energy and becomes

$$\gamma_{N_1N_2N_3N_4}=\delta_{N_1-N_2,\, N_3-N_4} \sum_{m} \sum_{m'} \delta_{m+N_1,\, m'+N_2} \sum_{\mu} \int d Z N_i^{(\mu)}(Z) \times (N_1 m|v^{(\mu)}(Z)|N_3 m)\, (N_4 m'|v^{(\mu)}(Z)|N_2 m')\,. \tag{3.2}$$

Let us consider the energy region close to the N-th Landau level and the case under extremely strong fields such as interactions between different Landau levels are neglected. The self-energy becomes

$$\Sigma_N(E)=\frac{1}{4}\Gamma_N^2 G_N(E)\,, \tag{3.3}$$

where

$$\Gamma_N^2=4\cdot 2\pi l^2 \sum_{\mu} \int d Z N_i^{(\mu)}(Z) \sum_{m} |(Nm|v^{(\mu)}(Z)|Nm)|^2\,, \tag{3.4}$$

and the self-consistency equation is easily solved to give

$$D(E)=\frac{1}{2\pi l^2}\left(-\frac{1}{\pi}\right)\operatorname{Im} G_N(E)=\frac{1}{2\pi l^2}\frac{2}{\pi\Gamma_N}\left[1-\left(\frac{E-E_N}{\Gamma_N}\right)^2\right]^{1/2}. \tag{3.5}$$

Similarly, the self-energy of other Landau levels and the proper vertex part are obtained, and the conductivity is reduced to

$$\sigma_{xx}=\int\left(-\frac{\partial f}{\partial E}\right) d E \frac{e^2}{\pi^2\hbar}\left(\frac{\Gamma_N^{tr}}{\Gamma_N}\right)^2\left[1-\left(\frac{E-E_N}{\Gamma_N}\right)^2\right], \tag{3.6}$$

with

$$\begin{aligned}(\Gamma_N^{tr})^2&=4\cdot 2\pi l^2 \sum_{\mu}\int d Z N_i^{(\mu)}(Z)\frac{1}{2}\sum_{m} [|\sqrt{N+1}\,(N+1\, m-1|v^{(\mu)}(Z)|Nm-1)\\&\quad -\sqrt{N}\,(N-1\, m|v^{(\mu)}(Z)|Nm)|]^2\\&=4\cdot 2\pi l^2 \sum_{\mu}\int d Z N_i^{(\mu)}(Z)\sum_{m} |(Nm|l\frac{\partial v^{(\mu)}(Z)}{\partial y}|Nm+1)|^2\,,\end{aligned} \tag{3.7}$$

where use has been made of the recurrence formulas of Laguerre's polynomial:[11)]

$$\frac{d}{dx}L_N^m(x)=-L_{N-1}^{m+1}(x)\,,\quad (N+1)L_{N+1}^{m-1}(x)=x\frac{d}{dx}L_N^m(x)+(m-x)L_N^m(x)\,. \tag{3.8}$$

The above expression of the conductivity coincides with that derived from eq. (11.17) of ref. 8. Therefore, the vertex part has been shown to be quite important to obtain reasonable expressions of the conductivity.

The density of states is elliptic around $E=E_N$ and its width is Γ_N. The transverse conductivity is parabolic and its peak value at zero temperature is $e^2/\pi^2\hbar\cdot(\Gamma_N{}^{tr}/\Gamma_N)^2$. Notice that this peak value is almost independent of the concentrations and strengths of scatterers. As a matter of fact, they exactly cancel out if the potential of the scatterers are separable into $V^{(\mu)}(Z)v(\boldsymbol{r})$. This fact can be thought of as a characteristic of the transverse conductivity of our system.

If the scatterers are of sufficiently short range, one can put $v^{(\mu)}(\boldsymbol{r}, Z)=V^{(\mu)}(Z)\delta^{(2)}(\boldsymbol{r})$ and has the simple result

$$\Gamma_N{}^2=\Gamma^2=4\sum_\mu\int \mathrm{d}Z\,N_i{}^{(\mu)}(Z)\frac{|V^{(\mu)}(Z)|^2}{2\pi l^2}, \tag{3.9}$$

and

$$(\Gamma_N{}^{tr})^2=\left(N+\frac{1}{2}\right)\Gamma^2. \tag{3.10}$$

The peak value of the conductivity is $(N+1/2)$ $e^2/\pi^2\hbar$ as has already been observed in the previous section. Notice that the level width Γ can be written in terms of the mobility μ or the relaxation time τ calculated in the Born approximation under no magnetic field as follows.

$$\Gamma=\sqrt{4\cdot\frac{1}{2\pi l^2}\cdot\frac{e\hbar^3}{m^2\mu}}=\sqrt{\frac{2}{\pi}\hbar\,\omega_c\frac{\hbar}{\tau}}. \tag{3.11}$$

Therefore, the level width is proportional to $(H/\mu)^{1/2}$ and independent of the Landau level.

When the scatterers are of sufficiently slowly varying type, the matrix elements which appear in eqs. (3.4) and (3.7) can be put equal to the value of the potential energy at the center coordinate R which is given by $\pi R^2=2\pi l^2(N+m+1/2)$ and the summation can be replaced by the integration. Thus, the level widths Γ_N and $\Gamma_N{}^{tr}$ take the form

$$\begin{aligned}\Gamma_N{}^2&\approx4\sum_\mu\int\mathrm{d}Z\,N_i{}^{(\mu)}(Z)\int\mathrm{d}^2\boldsymbol{r}\,v^{(\mu)}(\boldsymbol{r}, Z)^2\\&=4\langle(V(\boldsymbol{r})-\langle V(\boldsymbol{r})\rangle)^2\rangle,\end{aligned} \tag{3.12}$$

and

$$\begin{aligned}(\Gamma_N{}^{tr})^2&\approx4\sum_\mu\int\mathrm{d}Z\,N_i{}^{(\mu)}(Z)\int\mathrm{d}^2\boldsymbol{r}\left(l\frac{\partial v^{(\mu)}(\boldsymbol{r}, Z)}{\partial y}\right)^2\\&=4\left\langle\left(l\frac{\partial V(\boldsymbol{r})}{\partial y}\right)^2\right\rangle,\end{aligned} \tag{3.13}$$

where $V(\boldsymbol{r})$ is the local potential due to the scatterers and $\langle\cdots\rangle$ means the average over their configurations. Both of those are independent of the Landau level, and are expressed by the fluctuation of the local potential energy and its derivative.

In order to see explicitly the range dependence of the level width and the transverse conductivity, take for instance scatterers with the Gaussian potential $v^{(\mu)}(\boldsymbol{r}, Z)=V^{(\mu)}(Z)\exp(-r^2/d^2)/\pi d^2$, which approaches to the δ-potential of the strength $V^{(\mu)}(Z)$ as the range d goes to zero. After a little manipulation, for the ground Landau level $N=0$, one has

$$\Gamma_0{}^2=\frac{\Gamma^2}{1+\alpha^2}, \qquad \left(\frac{\Gamma_0{}^{tr}}{\Gamma_0}\right)^2=\frac{1}{2(1+\alpha^2)},$$

with $\alpha=d/l$. The absolute value of the level width depends not only on the functional form of the potential, i.e. its range, but also on the multiplicative factor which appears in the potential. The above level width decreases as the range increases, because the model potential which becomes weaker as d larger has been assumed. The peak value of the transverse conductivity, on the other hand, depends only on the range. The level width and the peak value of the conductivity for several N have been calculated numerically, which are shown in Figs. 3 and 4. It is seen that the level width is generally smaller for higher Landau levels and also that it is the same for every Landau level in the both limits $\alpha\to0$ and infinity. The difference of the level width of different Landau

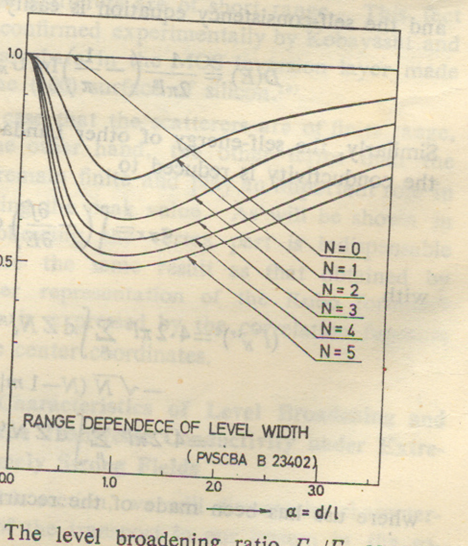

Fig. 3. The level broadening ratio Γ_N/Γ_0 as a function of the range $\alpha=d/l$ of the Gaussian potential.

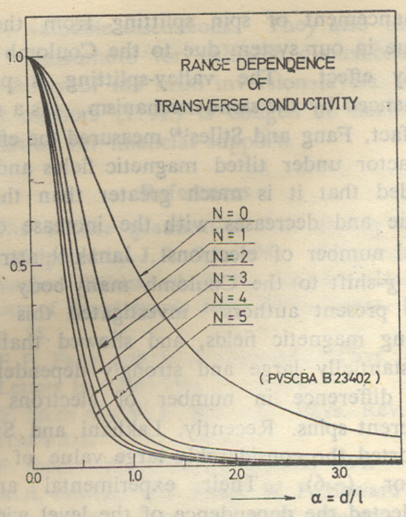

Fig. 4. The peak value of the conductivity $(\Gamma_N^{tr}/\Gamma_N)^2/(N+1/2)$ as a function of the range $\alpha=d/l$ of the Gaussian potential.

levels is remarkable when the range is comparable with the cyclotron radius l. The peak value of the conductivity decreases rather sharply with the increase of the range, which is remarkable for higher Landau levels. In the limit of large range, it decreases according to $l^2/2d^2$, irrespective of the Landau level, as has been observed previously. Such range dependence of the level width is closely related to the de Broglie wave length of each Landau level, which is of the order of $l/(2N+1)^{1/2}$ and smaller for higher Landau levels. An electron of higher Landau level feels potentials of the scatterers more weakly and the level width becomes smaller for higher Landau levels. In the limit of short- and long-range, on the other hand, such difference disappears.

§4. Discussion and Comparison with Experiments

In the first part of this section, we will show that the various characteristic features of the transverse conductivity obtained before are physically understood by the simple diffusion picture similar to the case of the theory of the impurity band conduction.[12,13] The conductivity at zero temperature is written as

$$\sigma_{xx}=e^2 D(E_F)\, D^* , \tag{4.1}$$

where $D(E_F)$ is the density of states at the Fermi energy and D^* is the diffusion constant, which is roughly given by $D^*\sim(\Delta X)^2/\tau$, if the mean jumping distance ΔX of the center coordinate and the life time τ are introduced. If the scatterers are of sufficiently short range, the displacement of the center coordinate due to scattering is of the order of the radius of the cyclotron orbit and is roughly $(\Delta X)^2\sim(2N+1)l^2$. Let the level width Γ, then the peak value of the density of states is of the order of $D(E_F)\sim(2\pi l^2\Gamma)^{-1}$. Thus, the peak value of the conductivity becomes

$$\sigma_{xx}\sim\frac{e^2}{\Gamma\tau}\left(N+\frac{1}{2}\right)\sim\frac{e^2}{\hbar}\left(N+\frac{1}{2}\right),$$

where the uncertainty relation $\Gamma\tau\sim 2\pi\hbar$ has been used. If the scatterers are sufficiently slowly varying, the displacement becomes of the order of $(\Delta X)^2\sim\tau^2 l^2\langle(l\partial V/\partial y)^2\rangle/\hbar^2$, since the center coordinate moves according to $\dot{X}=l^2/\hbar\cdot\partial V/\partial y$, and an electron feels the potential during the order of the life-time τ (the duration time τ_d is about the same as τ). The level width Γ becomes of the order of $\Gamma^2\sim\langle(V(r)-\langle V(r)\rangle)^2\rangle$. Therefore, the peak value of the conductivity becomes

$$\sigma_{xx}\sim e^2\frac{1}{2\pi l^2\Gamma}\frac{\tau}{\hbar^2}l^2\left\langle\left(l\frac{\partial V}{\partial y}\right)^2\right\rangle$$
$$\sim\frac{e^2}{\hbar}\frac{\langle(l(\partial V/\partial y)^2\rangle}{\langle(V(r)-\langle V(r)\rangle)^2\rangle}.$$

The formulas obtained above coincide with the results eqs. (3.10) and (3.13) obtained in §3 within a multiplicative factor. Therefore, the transport phenomena in our system is well described by the simple diffusion process.

The situation under extremely strong magnetic fields can be realized experimentally in MOS inversion layers made on semiconductor surfaces. The inversion layers made on (100) surface of silicon has been studied extensively, and we will confine ourselves to it in the following. Main scatterers in this system are considered to be of short range,[7] and the characteristic of the peak value of the conductivity that it depends only on the Landau level index and the natural constants is expected to be observed. As a matter of fact, it has been confirmed by the experiment of Kobayashi and Komatsubara as has already been reported before.[10] An example of the results of their experiment is shown in Fig. 5. The axis of abscissa represents the gate voltage, which can be considered to be proportional to the total number of electrons in the inversion layer. Each of the first two Landau levels $N=0$, 1 splits into four peaks, which is

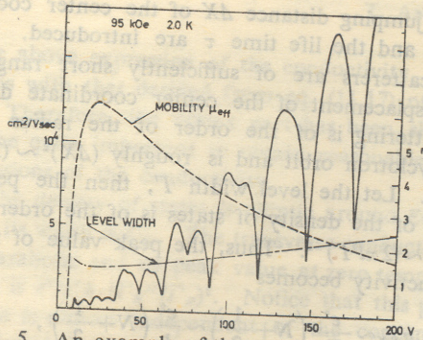

Fig. 5. An example of the oscillation of the conductivity at H=95 kOe, and the effective mobility obtained experimentally. The level width calculated theoretically by the use of the mobility is also shown.

thought of to be due to the removal of the spin and valley degeneracies.* In the next two levels N=2, 3, only the spin splitting is observed, and in higher levels, the splitting does not seem to be observed. If the intervalley scattering and the "inter-spin" scattering are neglected, the conductivity becomes the simple sum of that of each kind of electrons. The absolute peak values agree with the theoretical one quite well if the above mentioned degree of the degeneracy of each Landau level is taken into account.[10]

It is therefore expected that one can also explain such degree of the splitting of each Landau level, although our simple approximation results in the sharp cutoff at tails of the spectrum and is not adequate to explain the actual form of the oscillation. As has been shown in the previous section, the level width Γ is obtained from the mobility of the system under no magnetic field in case of short-ranged scatterers. The mobility obtained experimentally and the level width Γ calculated from it are shown in Fig. 5. The spin Zeeman energy $g\mu_B H/2$ is about 0.55 meV under this magnetic field (95 kOe), which is always much smaller than the level width. Thus, the spin splitting should not be observed, much less the valley splitting, if our theory on the level width is correct and the g factor is set equal to the value of the conduction band of silicon, i.e. 1.99. As will be discussed in subsequent papers, the effective level width becomes smaller in still higher approximations, but the most important cause of such discrepancy will be the enhancement of spin splitting from the free value in our system due to the Coulomb many-body effect. The valley-splitting is possibly enhanced by the same mechanism. As a matter of fact, Fang and Stiles[14] measured the effective g factor under tilted magnetic fields and concluded that it is much greater than the free value and decreases with the increase of the total number of electrons. Janak[15] attributed this g-shift to the Coulomb many-body effect. The present authors[16] investigated this under strong magnetic fields, and showed that it is substantially large and strongly dependent on the difference in number of electrons with different spins. Recently, Lakhani and Stiles[17] reported the considerably large value of the g factor ($\sim$6). Their experimental analysis neglected the dependence of the level width on the magnetic field and also on the gate voltage through the change of the mobility, and did not include the dependence of the conductivity on the Landau level. There remain some problems in the accuracy of their results. However, it can be said that the g factor is remarkably large. As is seen in Fig. 5, the spin and valley splitting of each Landau level changes rather drastically with the increase of the Landau level index N. This fact can not be explained if the level width is assumed to be proportional to $\mu^{-1/2}$ and supports the possibility of their enhancement due to many-body effects.

We have studied the characteristics of the level broadening and the transport of the two-dimensional system under strong magnetic fields in the simplest approximation. Various cases of the nature of scatterers have been considered. The results have been compared with experiments made on Si (100) inversion layers, where scatterers are of short range, and excellent agreement between the theory and experiments has been obtained for the peak value of the transverse conductivity. If experiments are performed on such inversion layers as charged centers play an important role as scatterers in them, it will be possible to observe the range dependence of the peak value obtained in this paper.

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* The ground electric sub-band is doubly degenerate within the effective mass approximation in the (100) surface.

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