Hybrid algorithm for incompressible flow simulation on quantum hardware

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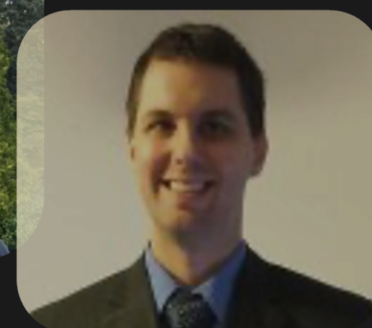

Bryan

CRNCH Summit

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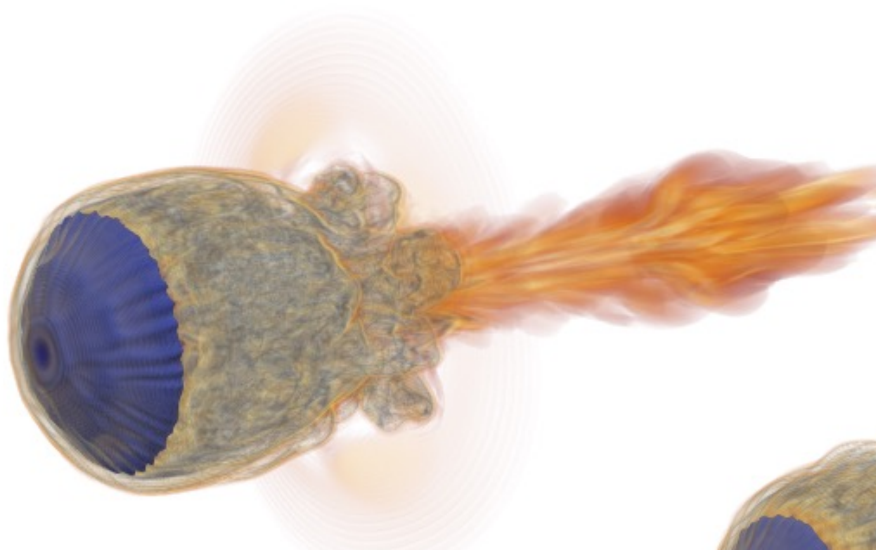
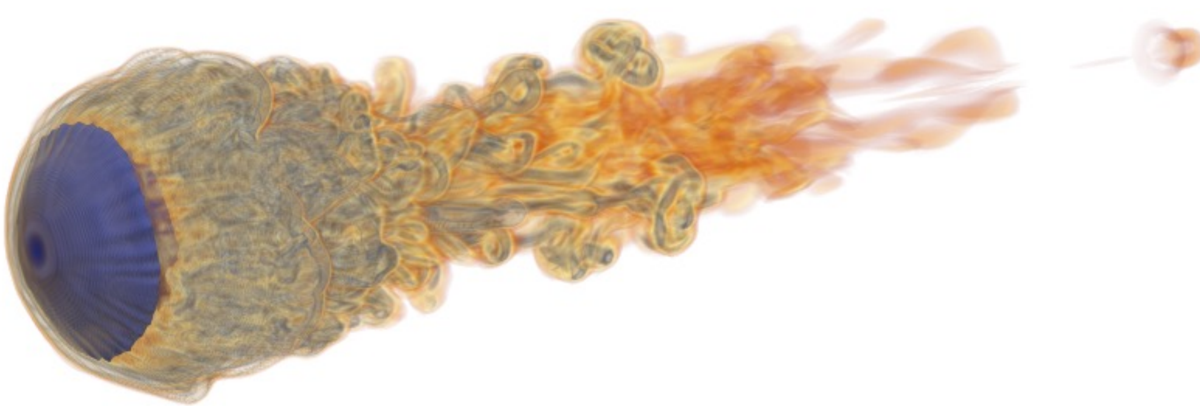
CPC Computational Physics Group

Introduction

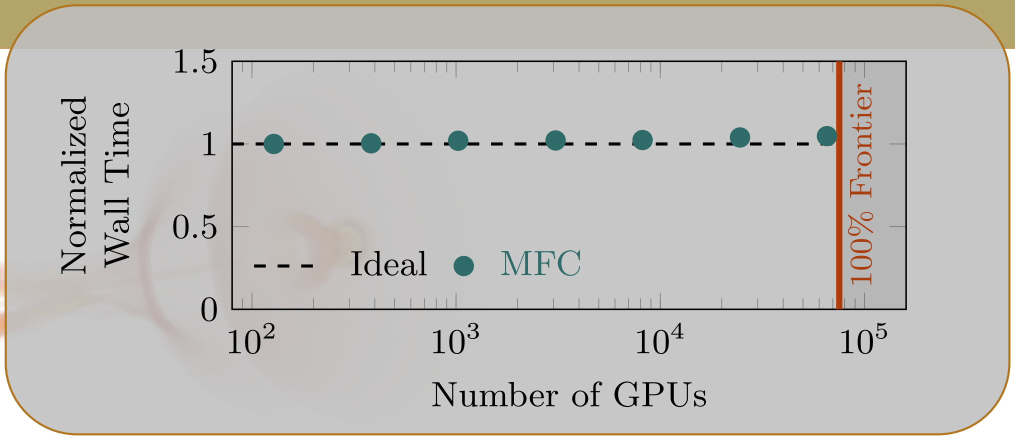
- CFD simulations computationally expensive/prohibitive
  - We're always eager for a new supercomputer
- Can new computer architectures avoid this reliance?

Introduction



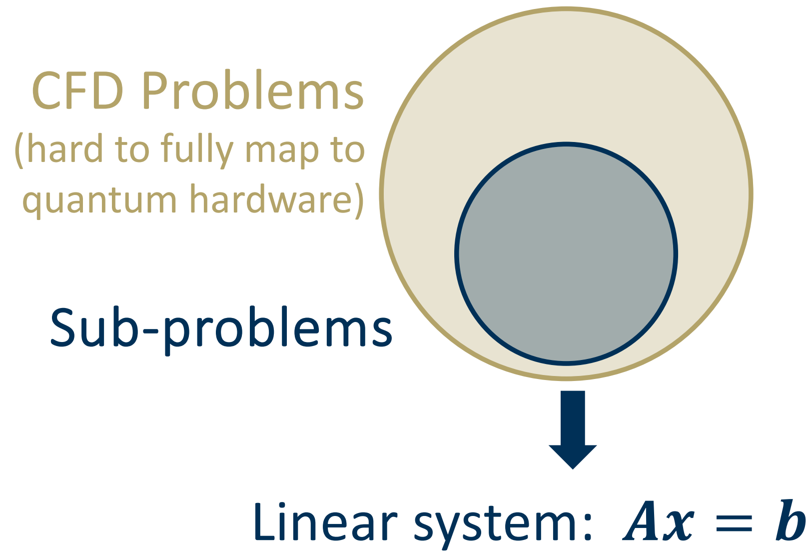


MFC Performance: >0.5 ExaFLOPs, 10T grid points



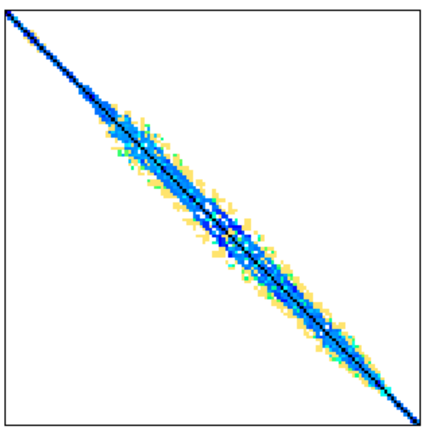
Introduction

- CFD simulations computationally expensive/prohibitive
  - We’re always eager for a new supercomputer
- Can new computer architectures avoid this reliance?

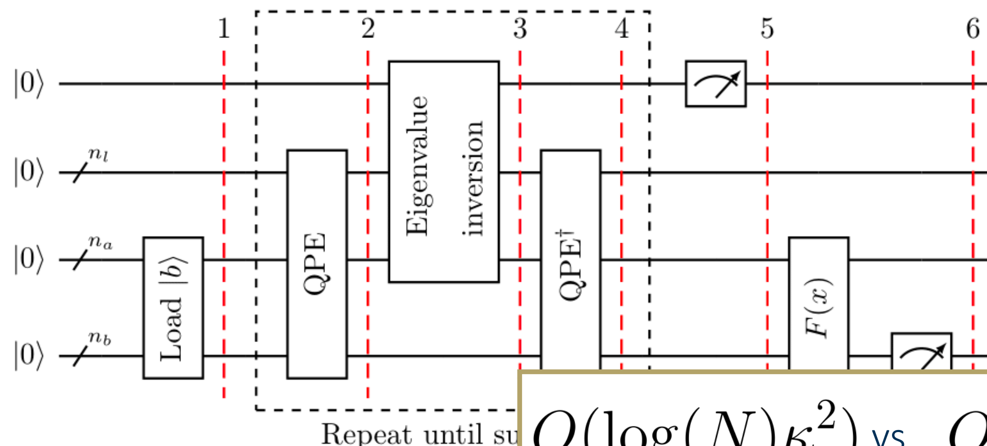


Motivation: Why Quantum?

- Computational “space” doubles with each added qubit
  - Billions of grid points → 10’s of qubits
- Some efficient quantum algorithms already exist! [HHL09]



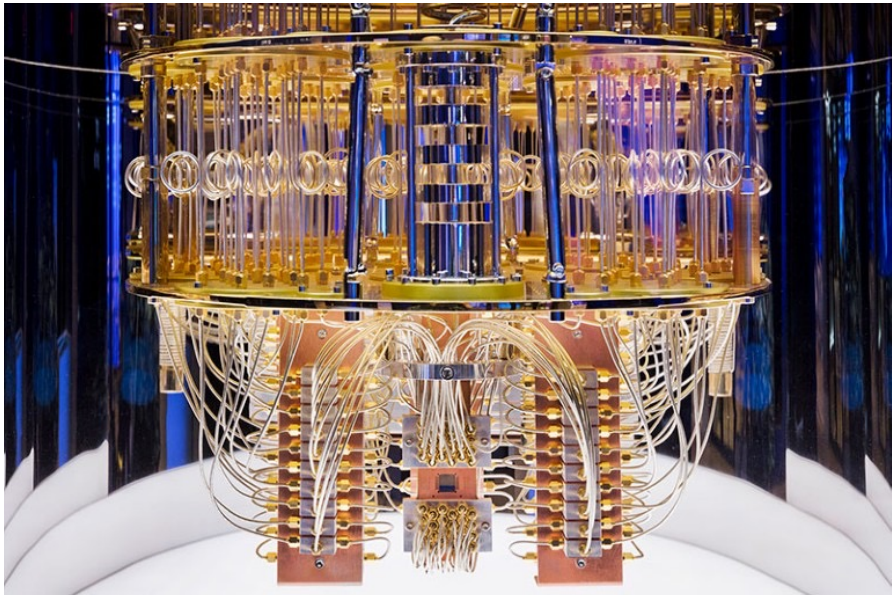
Example sparse A matrix in CFD simulation



$O(\log(N)\kappa^2)$ vs. $O(N\kappa)$

Harrow, Hassidim, Lloyd PRL (2009)

Limitation: Hardware & Nonlinearity



IBM quantum computer; interior

HHL-type QLSA*

- Only possible for tiny problems on current hardware

Variational QLSA*

- More suitable to near-term hardware
- Complexity advantage unclear

State Tomography

- Full state tomography → $O(3^N)$ circuits
- HTree tomography → $O(N+1)$ circuits

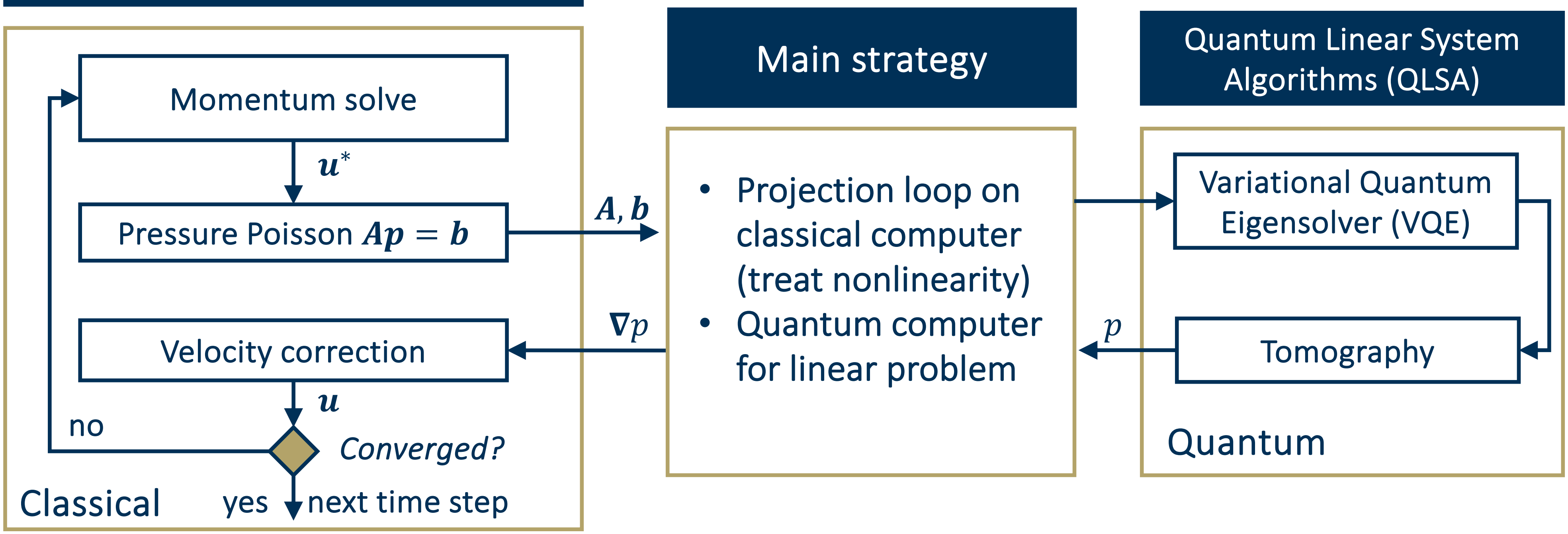
*QLSA: Quantum Linear System Algorithm

Approach: Hybrid Classical–Quantum

Incompressible Navier–Stokes

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{u}$$

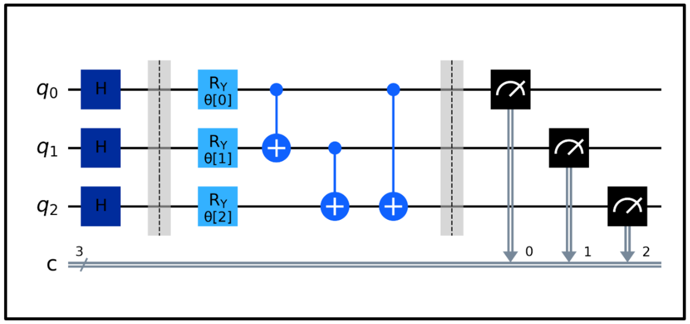
$$\nabla \cdot \mathbf{u} = 0$$



Variational Algorithms

Ground state of a Hamiltonian ↔ Solution of a linear system

Variational Quantum Circuit



Measurement

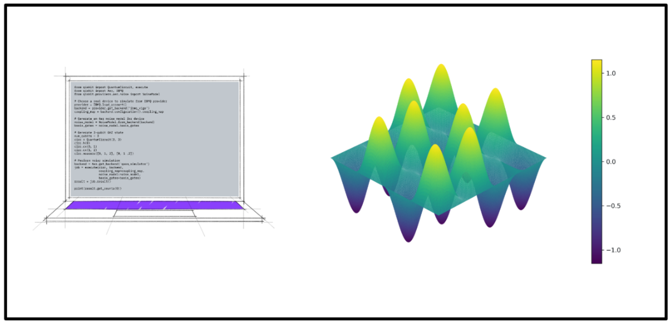
$$E(\theta) = \langle\psi(\theta)|H|\psi(\theta)\rangle$$

$$H = A^{\dagger}(I - |b\rangle\langle b|)A$$

Parameter update

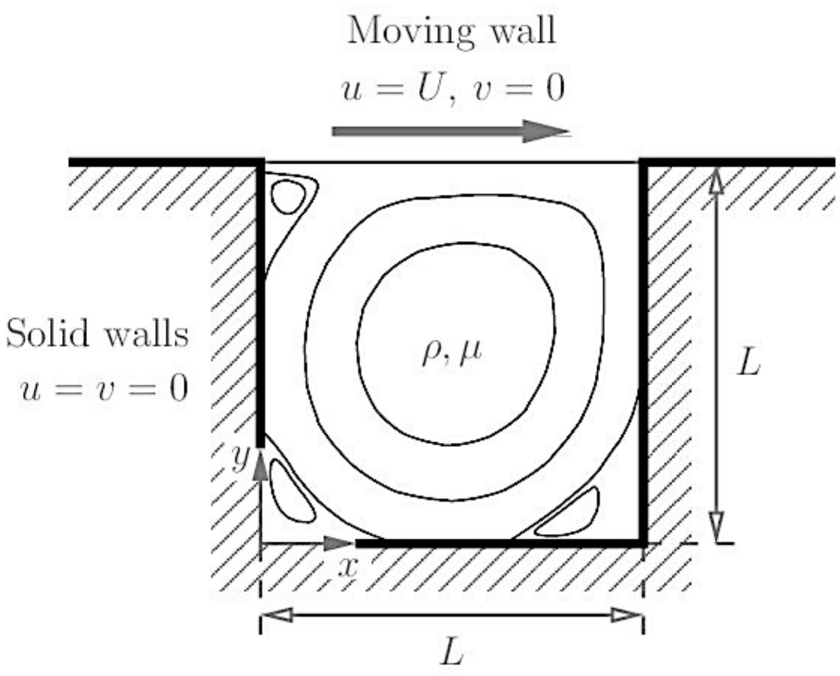
$$\theta_i \to \theta_{i+1}$$

Classical Optimizer

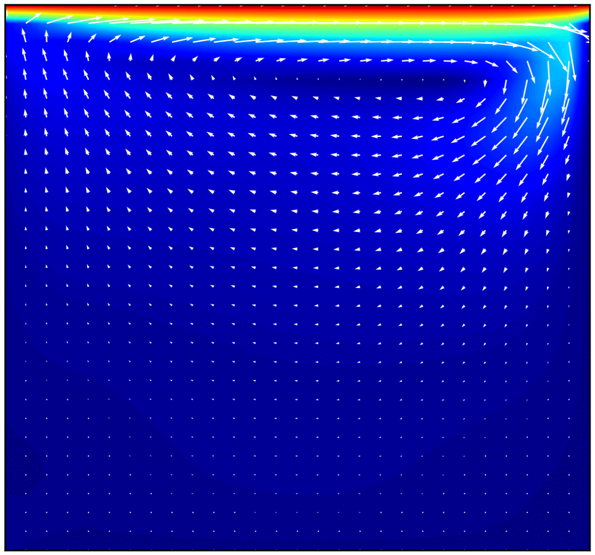


Cerezo et al. Nature Rev. Phys. (2021)

Benchmark: 2D Lid-driven Cavity



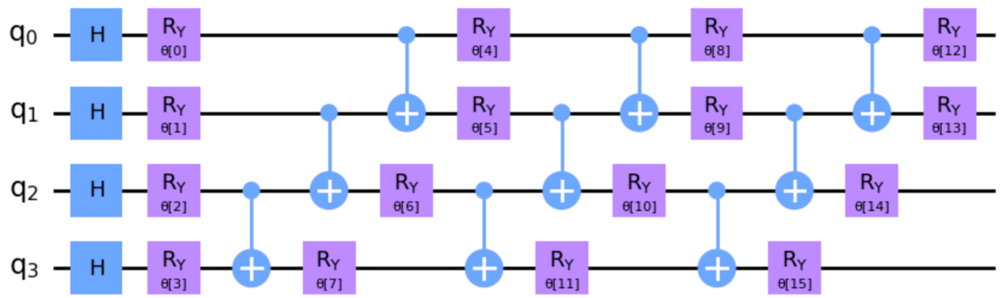
Classical simulation, $\mathrm{Re} = 400$



Ghia, K. (1977)

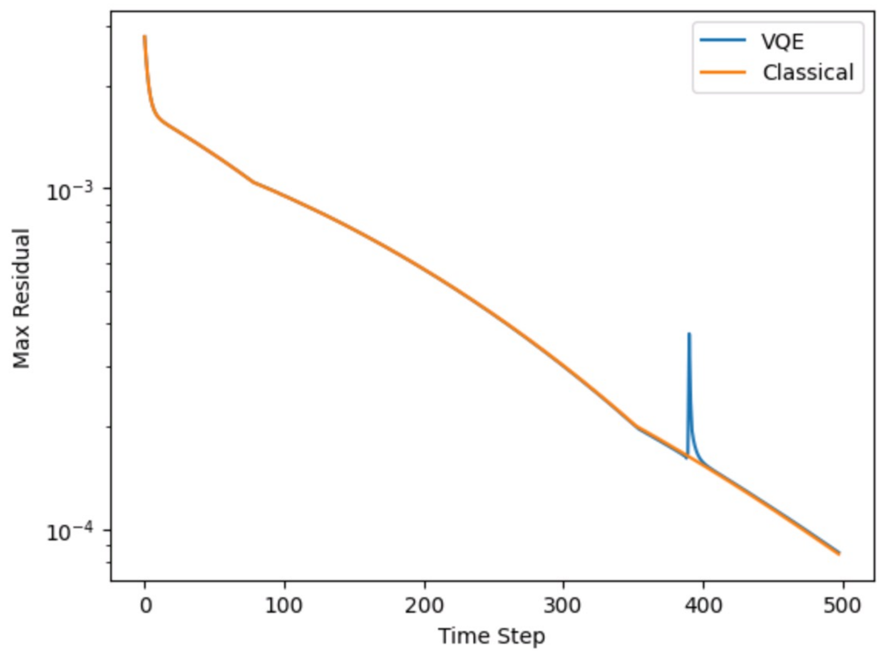
Benchmark: 2D Lid-driven Cavity

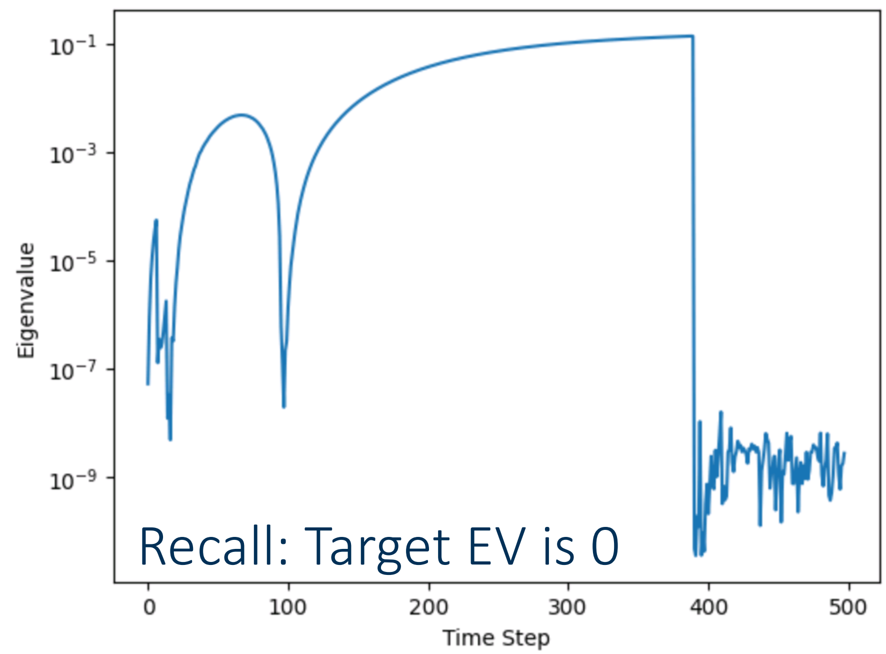
- Small 2D test case:
  - Grid: $16 \times 16 \rightarrow \#\text{qubits} = 2n = 4$
- VQE algorithm
  - Hardware efficient ansatz w/ R_Y and CNOT gates, $\textbf{depth} = 3$



Results: “Perfect” simulator

- Initial VQE parameters come from previous time step
- Converged solution tends to steady state

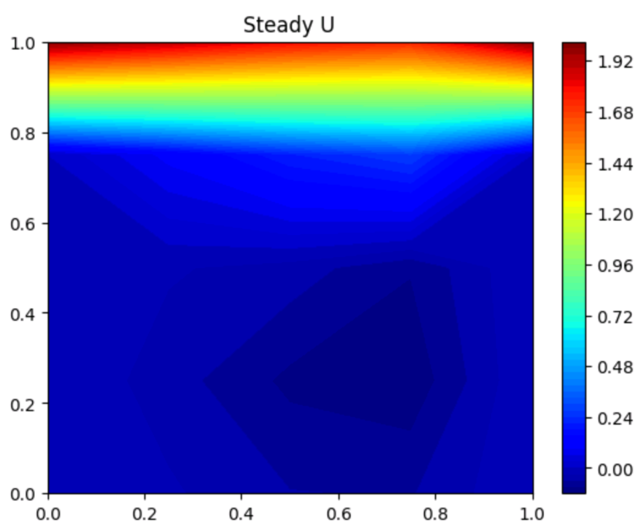




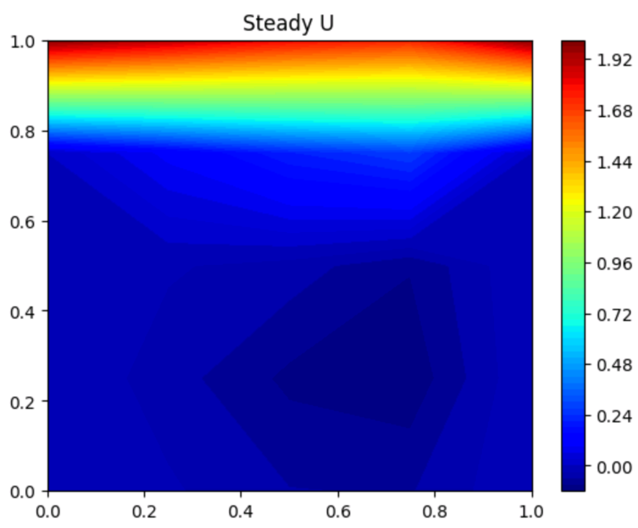
Results: Solution viz.

Comparing quantum to classical solution (statevec. sim.)

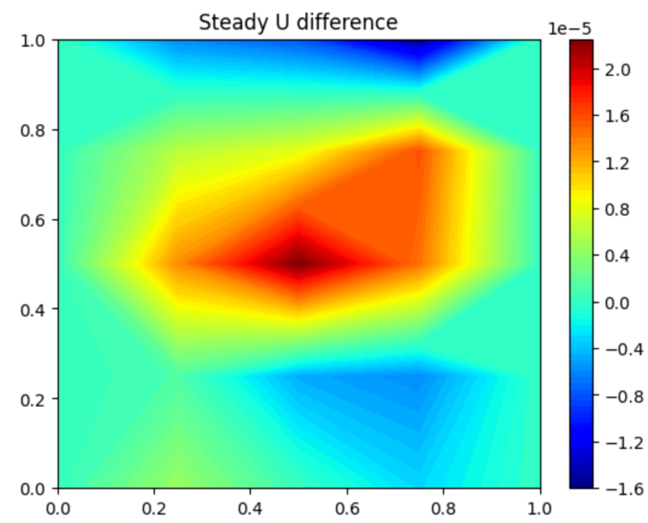
Classical



Quantum

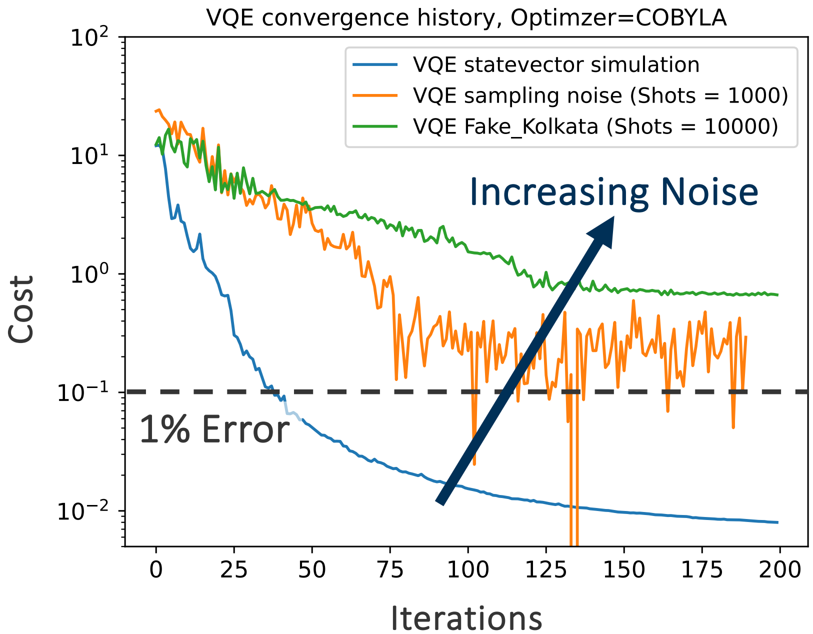


Difference

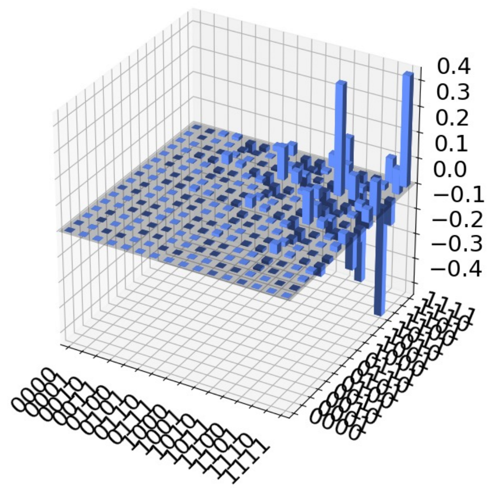


Results: Simulator + Artificial Noise

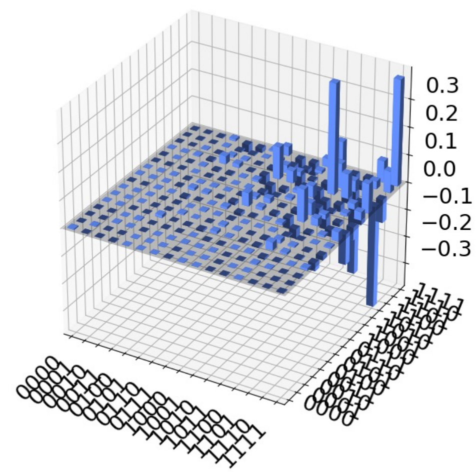
- Use VQE to solve pressure Poisson eq’n
- 4-qubit simulation, different noise levels



Truth



Fake_Kolkata +
State Tomography



Error (with artificial noise): 0.5%

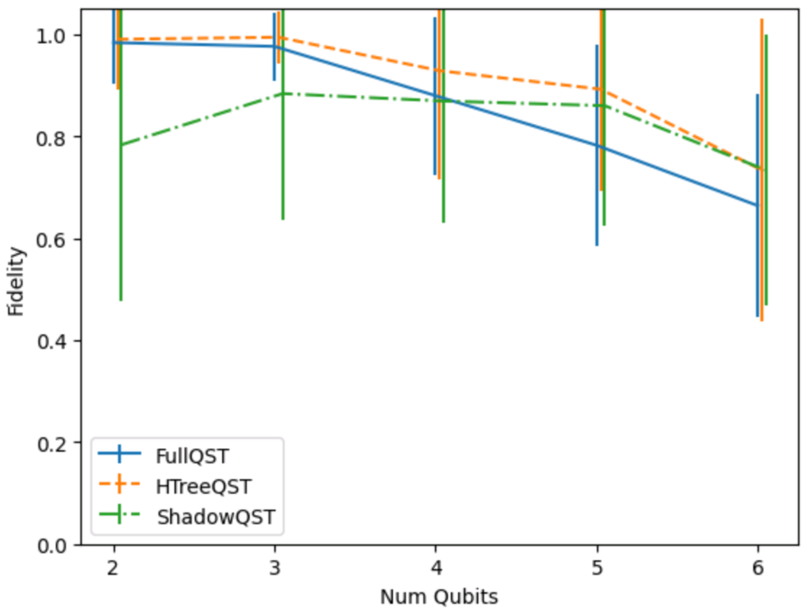
Mind your Measurements: Tomography

Standard quantum tomography

- 3^N circuits characterize output
  - *Computational bottleneck*
  - This is a **complex** valued state
- CFD output is **real** valued

More efficient tomography technique

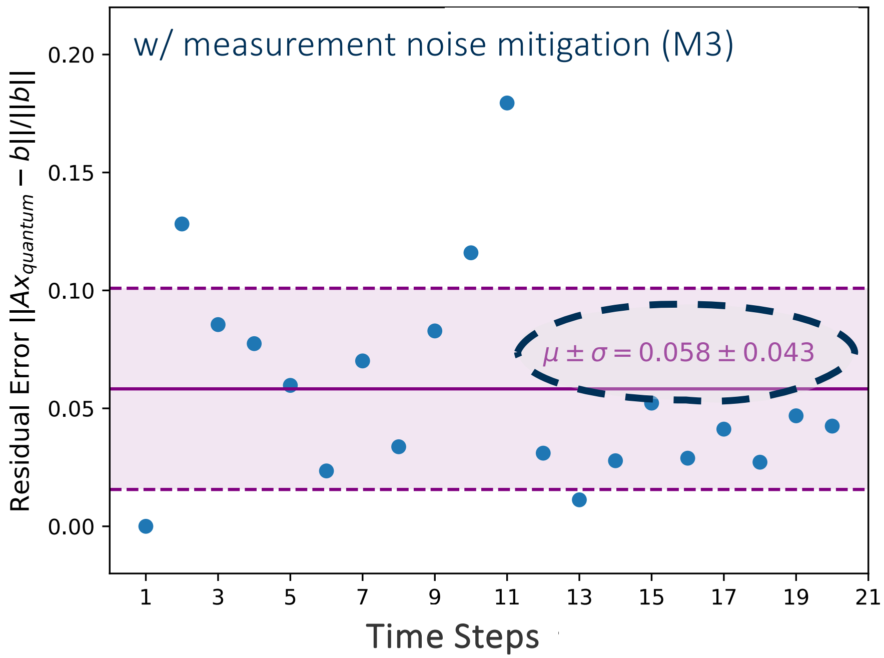
- Formulate **HTree**
- Requires $N + 1$ circuits (https://github.com/QuantumApplicationLab/vqls-prototype)

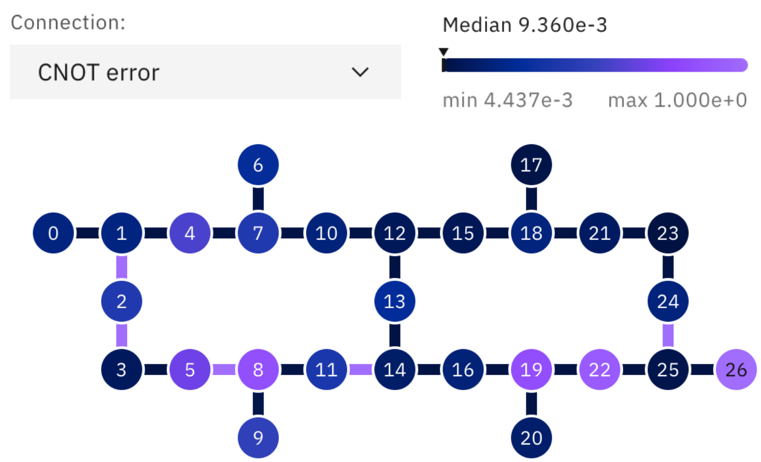


Compare Htree to standard full state and shadow tomography

Results: Quantum Hardware

- VQE solve on IBM quantum computer Kolkata
- With HTree, measurement cost and final errors small





27-qubit IBMQ_Kolkata

To come: Scaling!

- Current and upcoming quantum devices

Backend	# qubits	T_1 (µs)	T_2 (µs)	Error_{1q}	Error_{2q}	t_{2q} (ns)	t_{mea} (ns)
`ibmq_kolkata`	27	102.88	66.51	2.55×10^{-4}	8.89×10^{-3}	452	640
`ibm_sherbrooke`	127	257.56	167.28	2.17×10^{-4}	7.16×10^{-3}	533	1244
`ibm_torino`	133	167.79	132.44	3.08×10^{-4}	3.40×10^{-3}	124	1560

- Early results promising, low 2-qubit error `ibm_torino` critical

Summary

Hybrid classical-quantum CFD solver

- Acceptable performance on current quantum computers
- State tomography could bottleneck hybrid schemes
- Novel HTree method makes our method practical

Next: 3D Navier–Stokes on 133+ qubit devices

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Thank You!

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