Multi Criteria Decision Making and Analytic Hierarchy Process

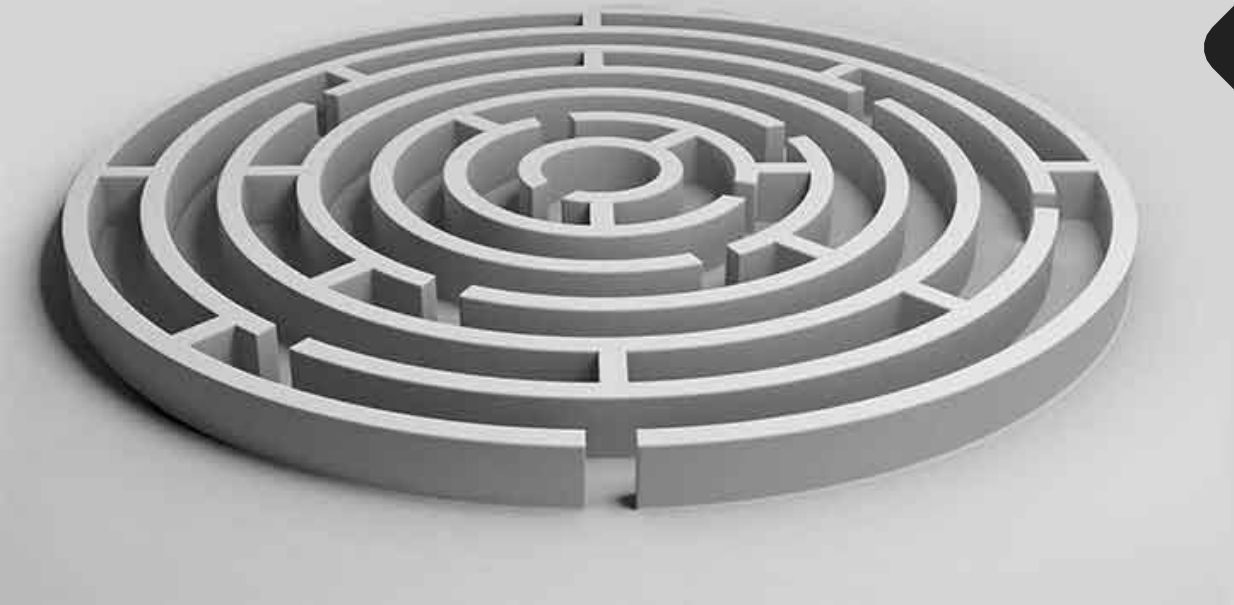
AT84.02 Business Intelligence and Analytics
January 2023 Semester

Discussion topics

01

Multi Criteria Decision Making (MCDM)

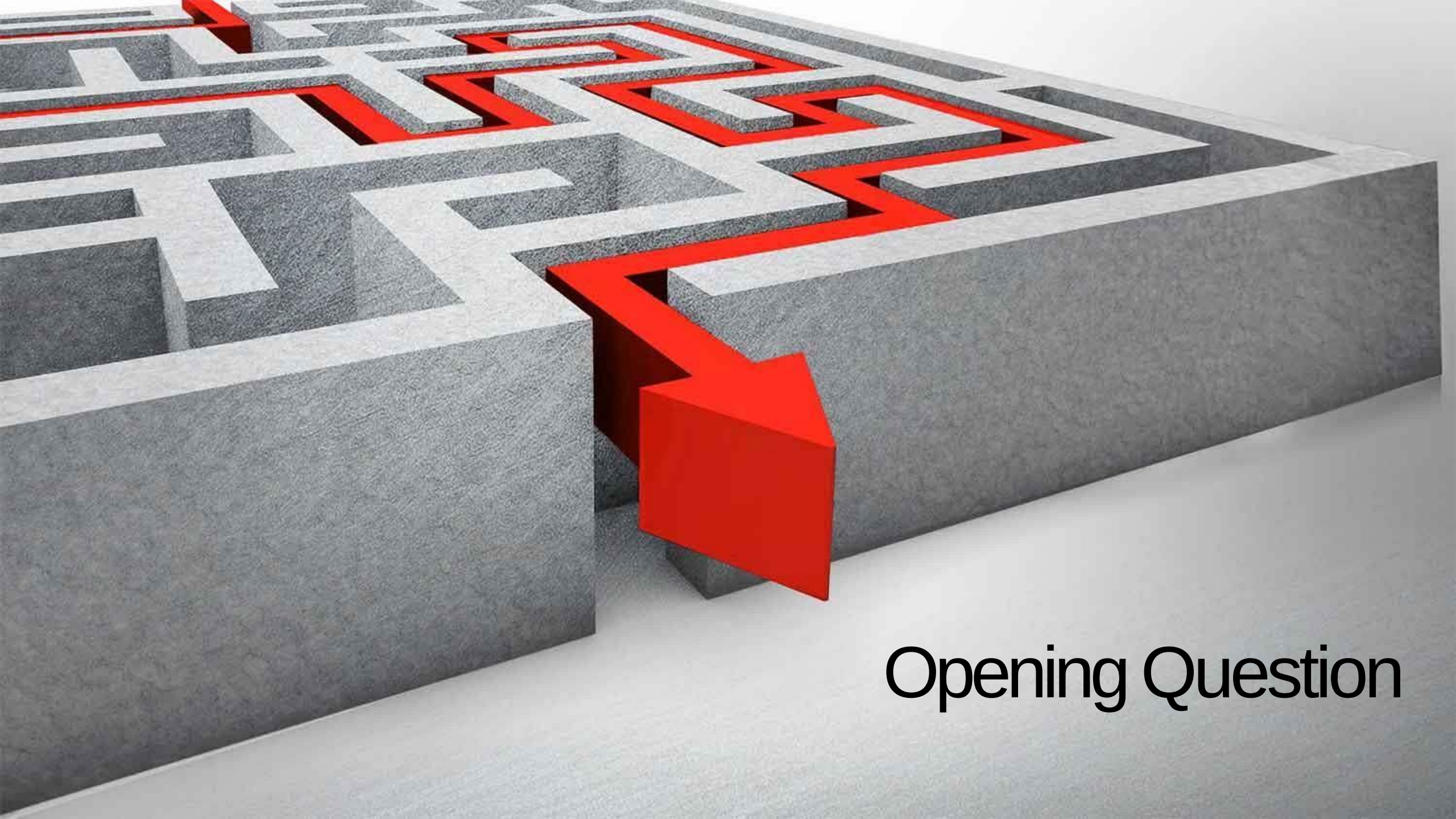
- Introduction
- Method
- Software



02

Analytic Hierarchy Process (AHP)

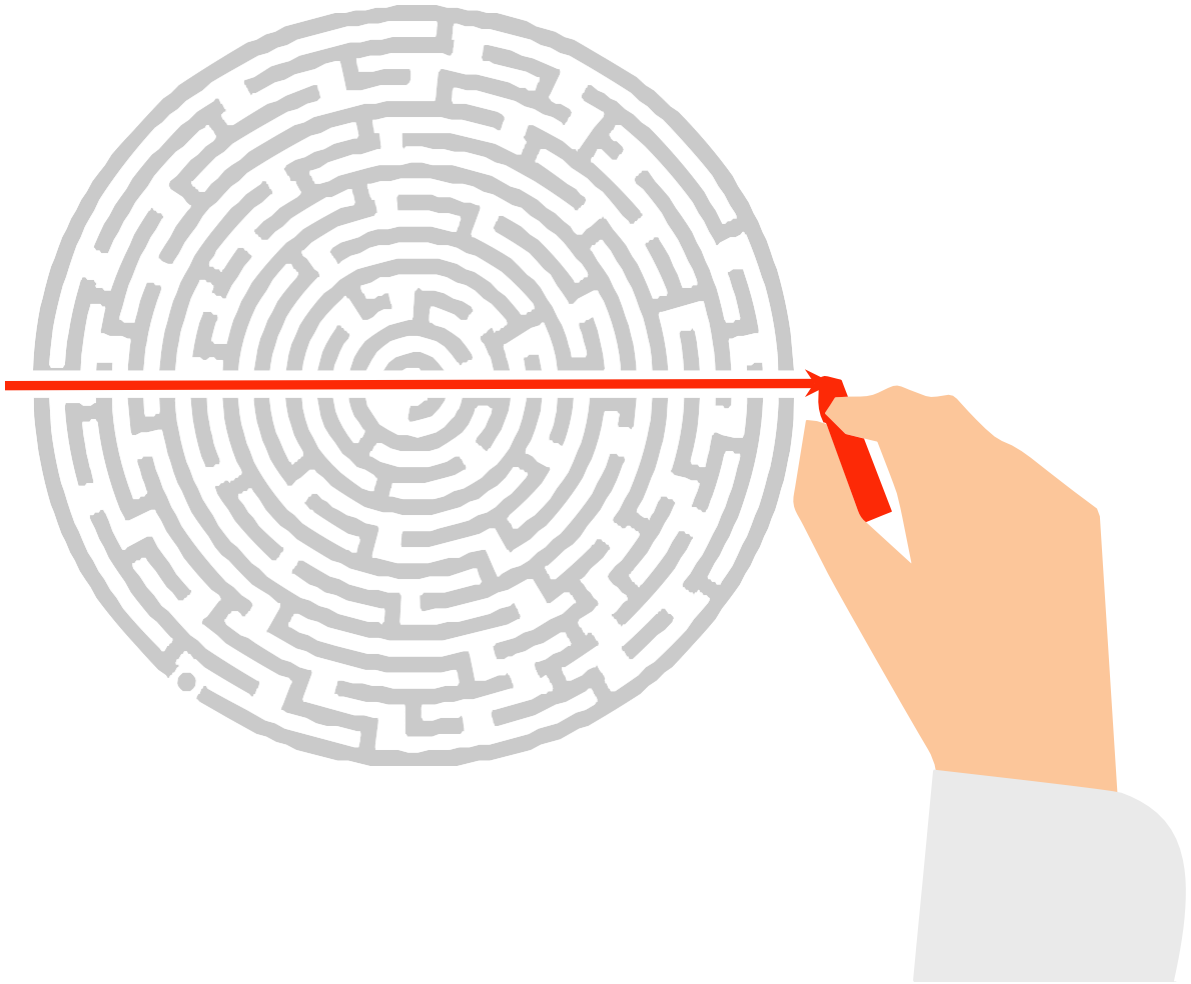
- AHP Concepts
- AHP Process
- AHP Software - SuperDecision



Opening Question

When you are making decision ?
How will you make this decision ?

Introduction

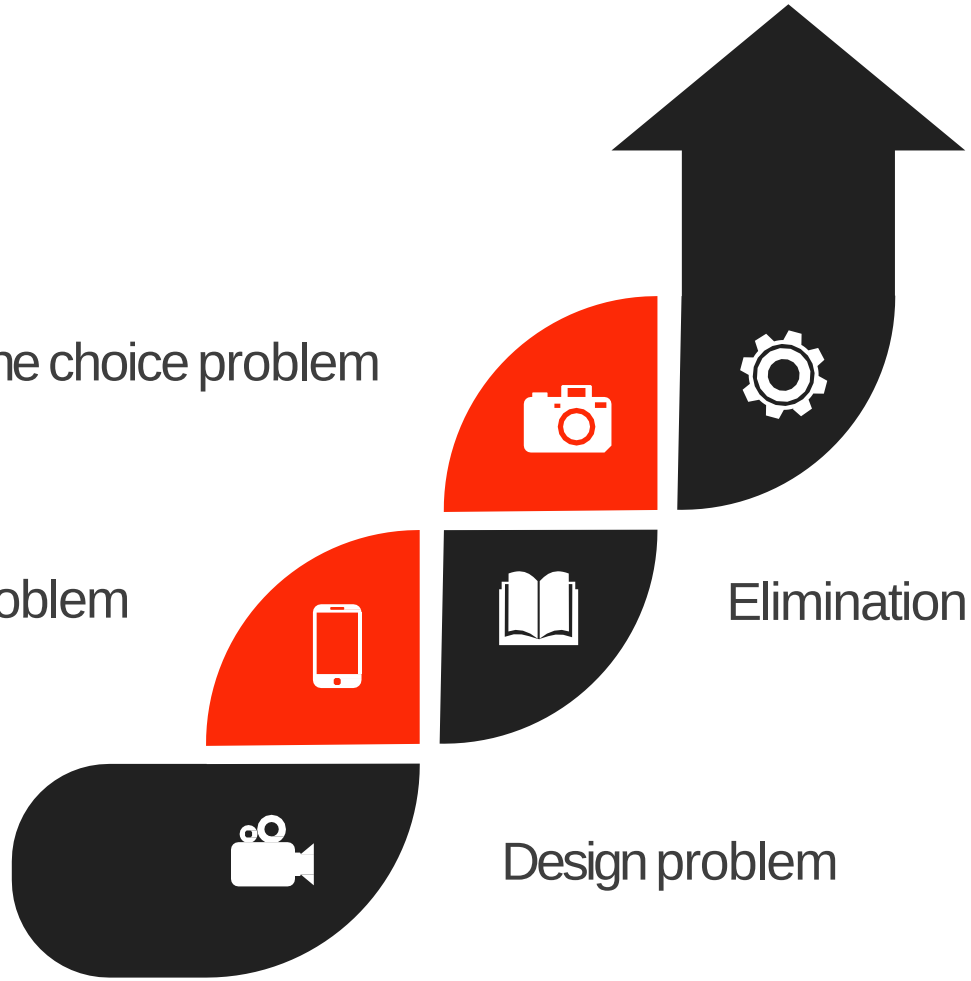


Decision making occur naturally in our daily life to figure our general problems.

1. Desire for immortality
2. Desire for pleasure

General decision problems usually involve with multiple criteria for decision process.

Decision problems



The choice problem

The description problem

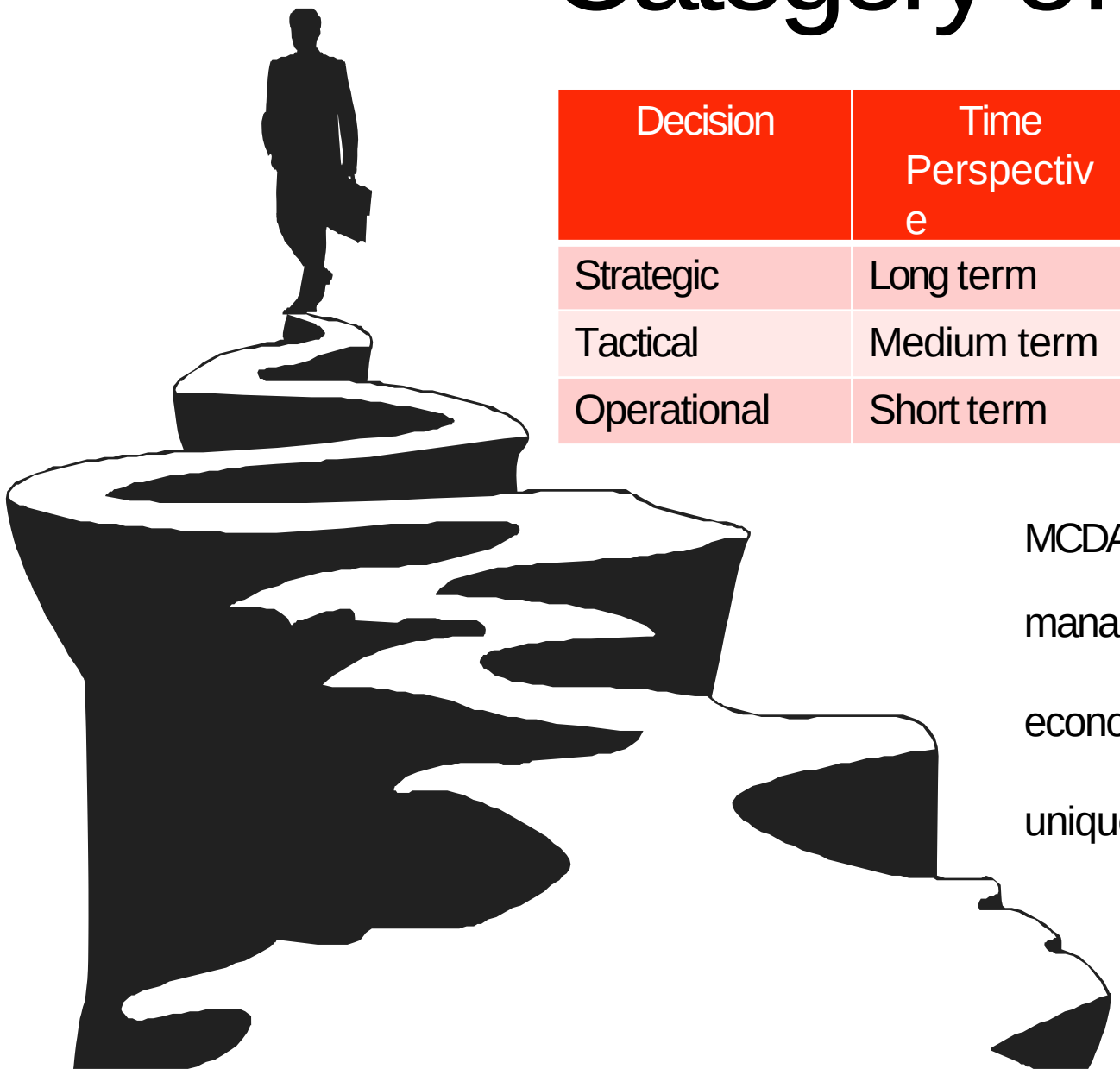
The sorting problem

Elimination problem

The ranking problem

Design problem

Category of Decision problem



Decision	Time Perspective	Novelty	Degree of structure	Automation
Strategic	Long term	New	Low	Low
Tactical	Medium term	Adaptive	Semi-structured	Medium
Operational	Short term	Every day	Well defined	High

MCDA(MCDM) is multidiscipline applying mathematics, management informatics, psychology, social science and economic, developed to support decision maker in their unique and personal decision process.

MCDM Classification

- Multi-Criteria Decision Making (MCDM)
  - Multi-Attribute Decision Making (MADM)
    - Genetic Algorithms (GA)
    - Goal Programming (GP)
  - Multi-Objective Decision Making (MODM)
    - Multi-Attribute Utility Theory (MAUT)
    - **Analytic Hierarchy Process (AHP)**
    - Elimination and Choice Translating Reality (ELECTRE)
    - Preference Ranking Organization Methods for Enrichment Evaluation (PROMETHEE)
    - Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

MCDM General Stage

Identifying decision makers
Define the goal
Feasible alternatives

Structuring a decision problem

Weighting
Scoring
Ranking

Determining and applying an MCDM method

Highest score selection
Examine selected option

Determining the final recommendation

MCDM Software

Problem:
1. Ranking
2. Description
3. Choice

Methods:
1. PROMETHEE (GAIA)
2. ELECTRE

PROMETHEE METHODS

J-ELECTRE-v2.0

D-SIGHT

DecisionLab

SMART PICKER

Problem:
1. Ranking
2. Choice

Methods:
1. PROMETHEE
2. UTA
3. AHP

Decerns

CRITERIUM DECISION PLUS

123AHP
my choice, my decision

questfox

Make It Rational

Problem:
1. Ranking
2. Choice

Methods:
1. ANP
2. MACBETH
3. TOPSIS
4. DEA

SUPER DECISIONS CDF

DECISION LENS

Frontier Analyst

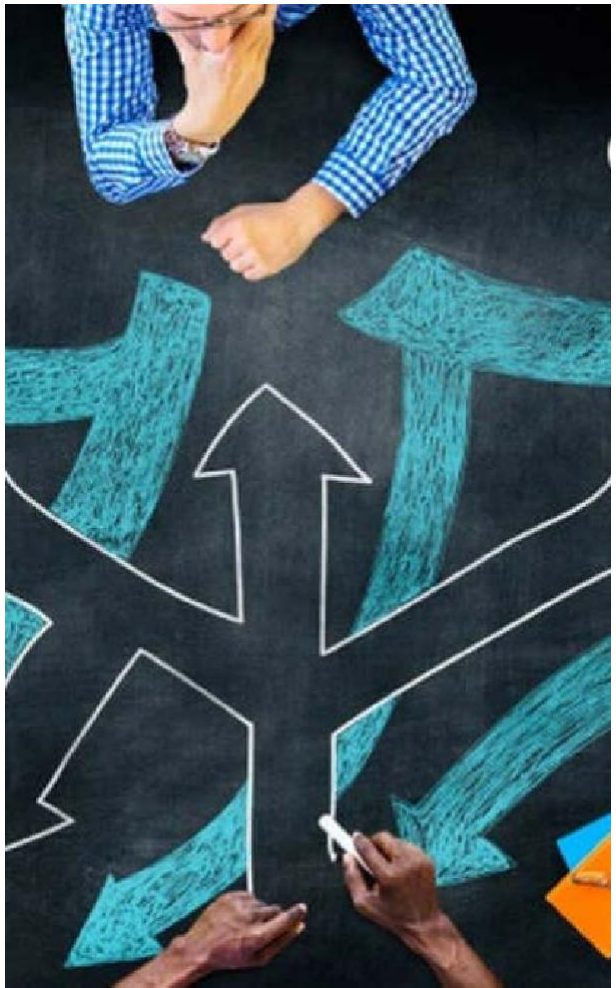
MACBETH

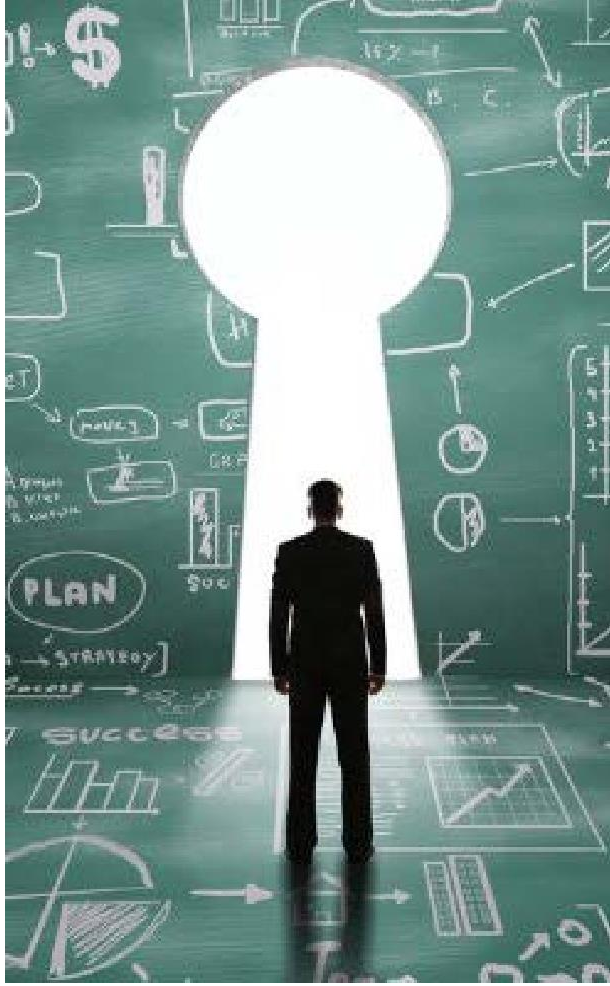
Win4Deap 2

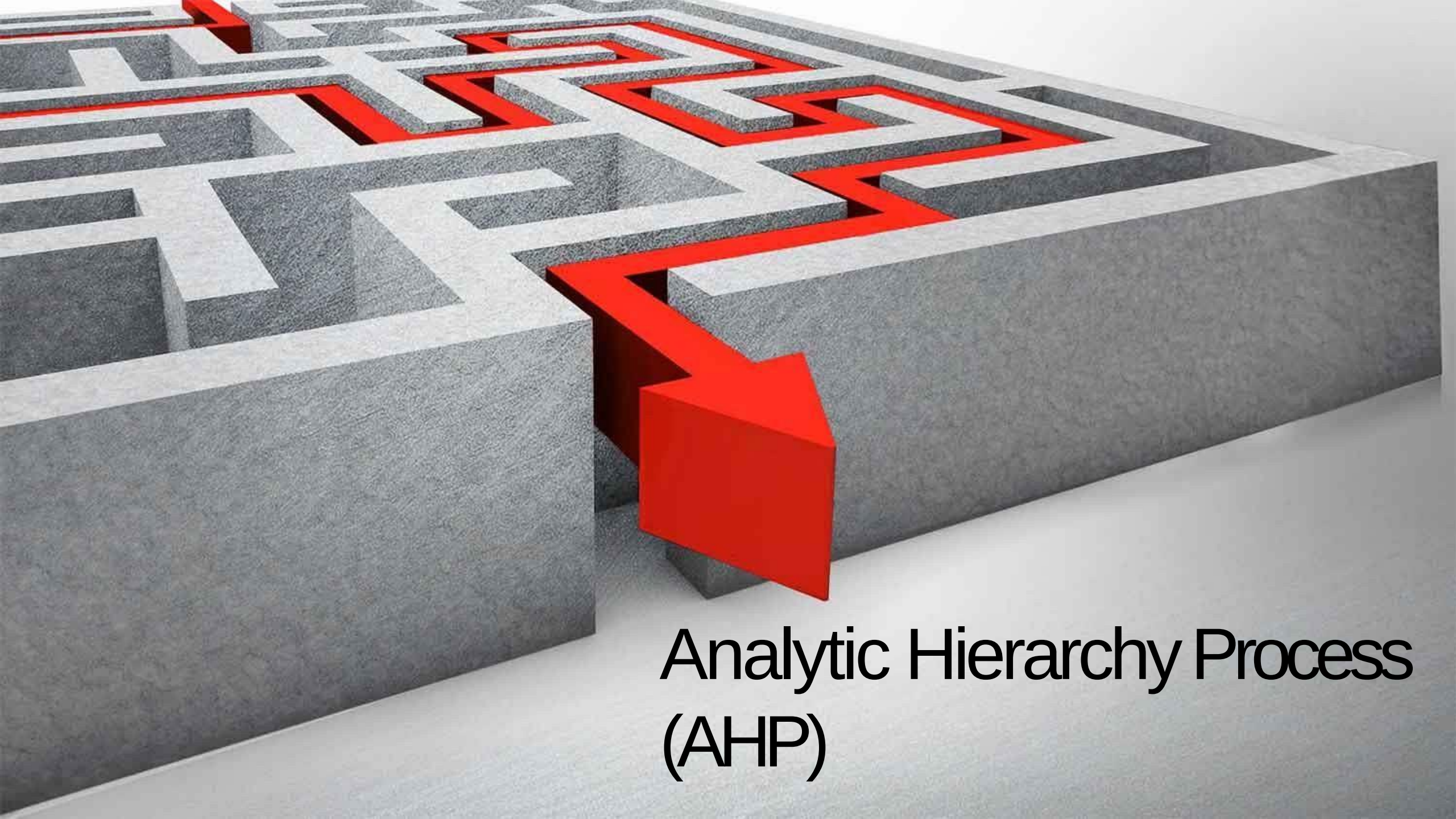
Key Takeaway

MCDM Concept

- The decision making process with multiple criteria
- Define weight on particular criteria base on priority
- Select the highest score or high ranking score if multiple options
- Evaluate the selected option







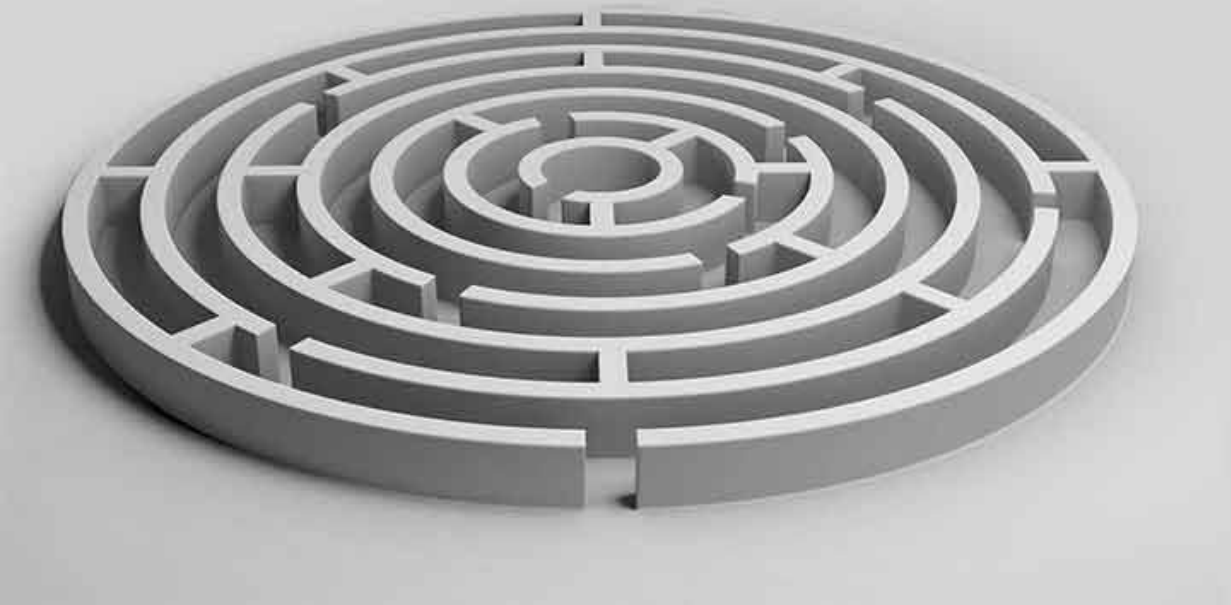
Analytic Hierarchy Process (AHP)

AHP - Introduction

The analytic hierarchy process (AHP), developed by Thomas Saaty (1995 - 1996), is an excellent modeling structure for representing multi-criteria (multiple goals, multiple objectives) problems- with sets of criteria and alternatives (choices)- commonly found in business environments.

AHP will decompose a decision making problem

into relevant criteria and alternatives.



The AHP separates the analysis of the criteria from the alternatives, which helps the decision maker to focus on small, manageable portions of the problem.

AHP Process

Here is the step-by-step approach for Analytic Hierarchy Process.

Step 1: Define the problem and Criteria (also sub criteria if available)

Step 2: Define Alternatives

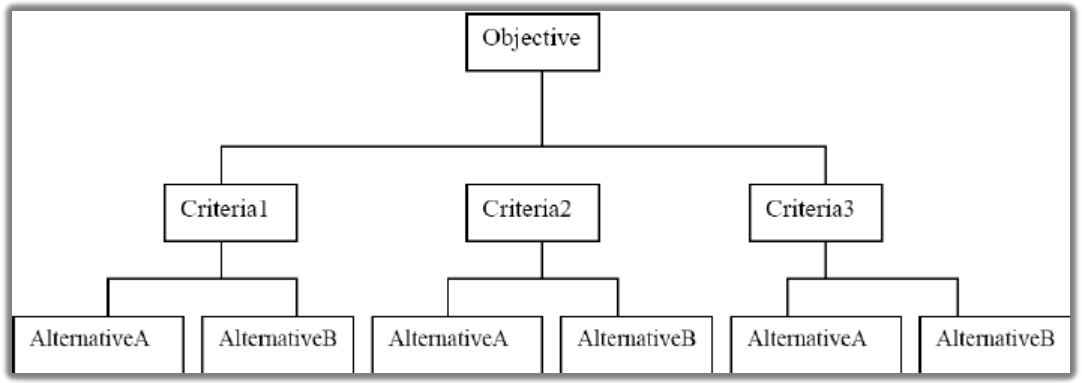
Step 3: Establish priority amongst criteria and alternatives using pairwise comparison.

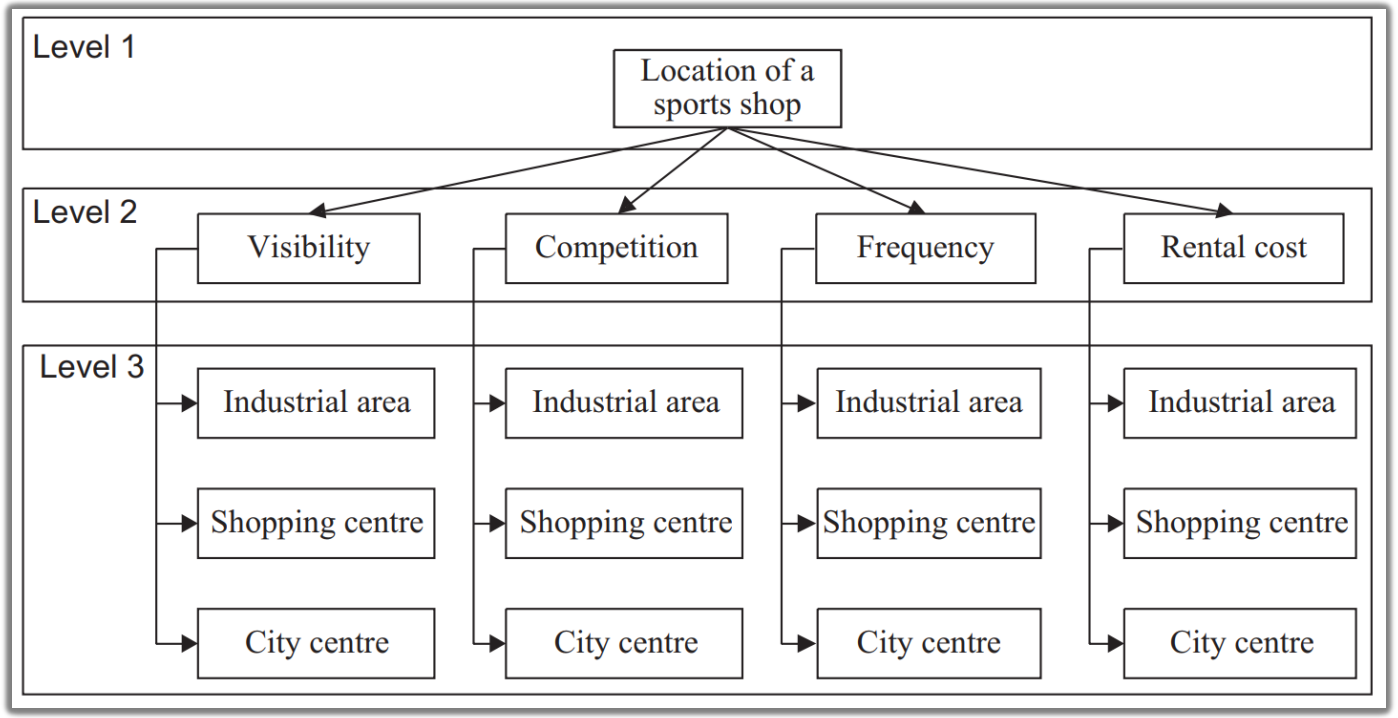
Step 4: Check consistency amongst the pairwise comparison.

Step 5: Evaluate relative weights from the pairwise comparisons and get the calculated overall priorities for the alternatives.

Step 6: Perform Sensitivity Analysis

Decision Hierarchy





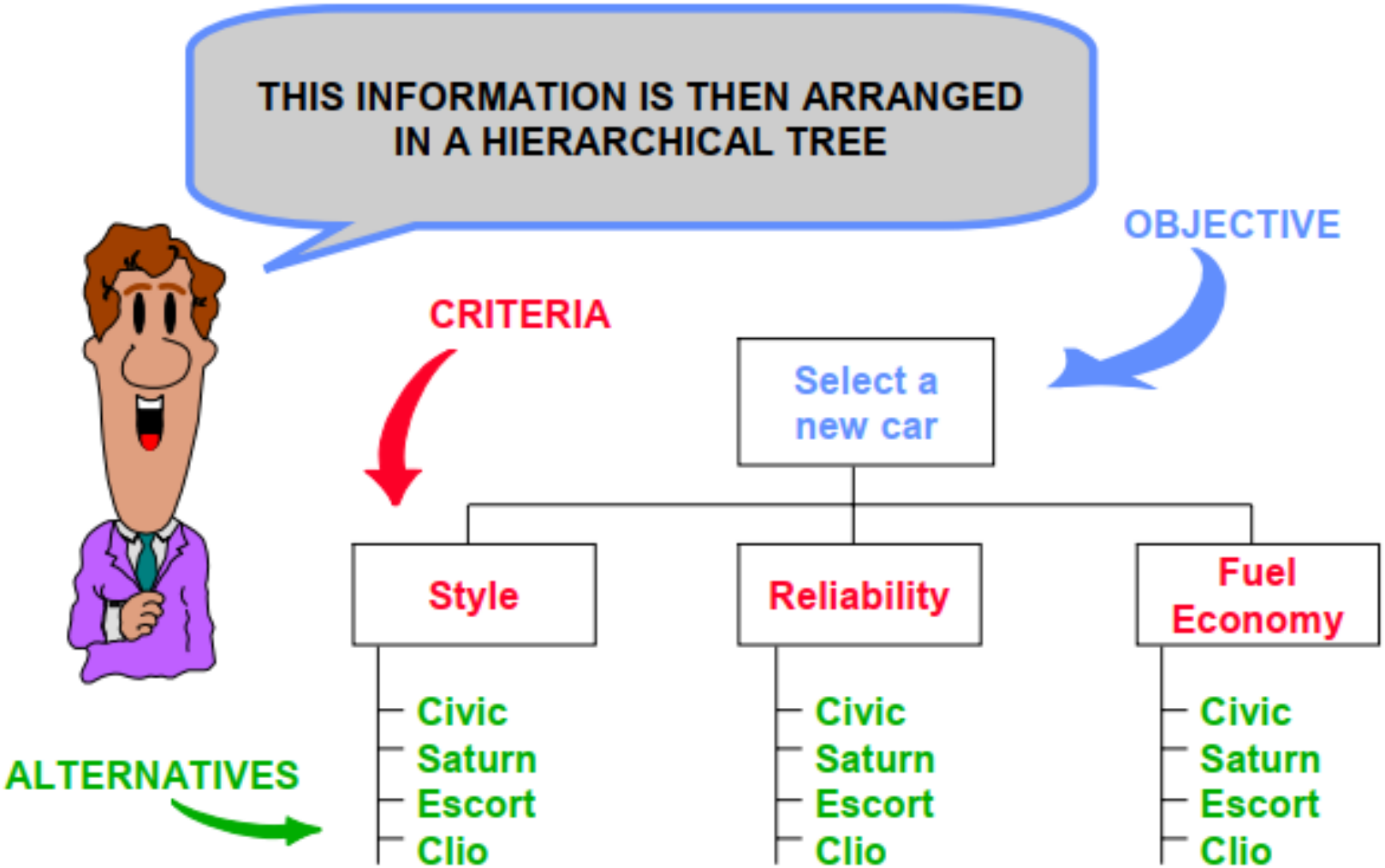
AHP Process (Example)

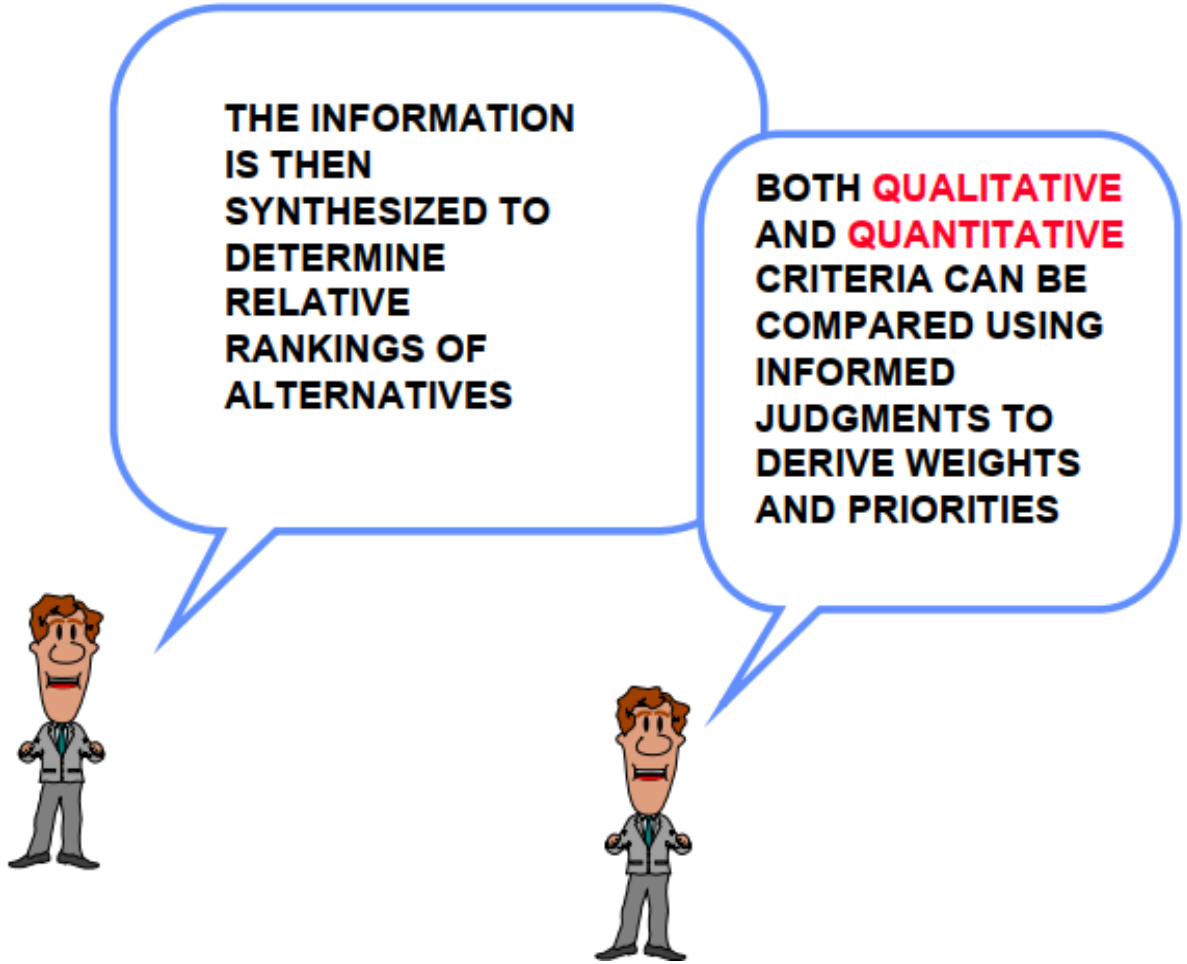
Page 18 – 44 are collected from the lecture of Dr. Rainer Haas and Dr. Oliver Meixner
Institute of Marketing & Innovation
University of Natural Resources and Applied Life Sciences, Vienna

I SEE A NEW CAR
IN YOUR FUTURE

AN IMPORTANT PART OF THE PROCESS IS TO ACCOMPLISH THESE THREE STEPS

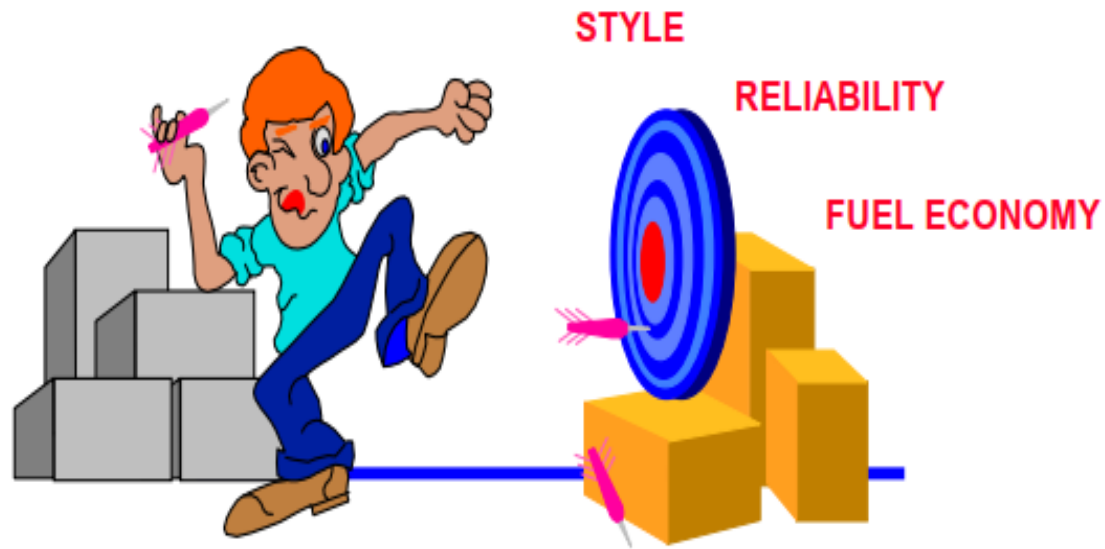
- **STATE THE OBJECTIVE:**
  - SELECT A NEW CAR
- **DEFINE THE CRITERIA:**
  - STYLE, RELIABILITY, FUEL ECONOMY
- **PICK THE ALTERNATIVES:**
  - CIVIC COUPE, SATURN COUPE, FORD ESCORT, RENAULT CLIO


THIS INFORMATION IS THEN ARRANGED IN A HIERARCHICAL TREE
OBJECTIVE
CRITERIA
Select a new car
Style
Reliability
Fuel Economy
Civic
Saturn
Escort
Clio
Civic
Saturn
Escort
Clio
Civic
Saturn
Escort
Clio
ALTERNATIVES


THE INFORMATION IS THEN SYNTHESIZED TO DETERMINE RELATIVE RANKINGS OF ALTERNATIVES
BOTH QUALITATIVE AND QUANTITATIVE CRITERIA CAN BE COMPARED USING INFORMED JUDGMENTS TO DERIVE WEIGHTS AND PRIORITIES

HOW DO YOU DETERMINE THE RELATIVE IMPORTANCE OF THE CRITERIA?

Here's one way !



HERE'S ANOTHER WAY

Hmm, I think reliability is the most important followed by style and fuel economy is least importeant so I will make the following judgements ....

USING JUDGMENTS TO DETERMINE THE RANKING OF THE CRITERIA

1. RELIABILITY IS 2 TIMES AS IMPORTANT AS STYLE
2. STYLE IS 3 TIMES AS IMPORTANT AS FUEL ECONOMY
3. RELIABILITY IS 4 TIMES AS IMPORTANT AS FUEL ECONOMY

he's not very consistent here ... that's o.k.

Pairwise Comparisons

USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

Pairwise Comparisons

USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY		1/1	4/1
FUEL ECONOMY			1/1

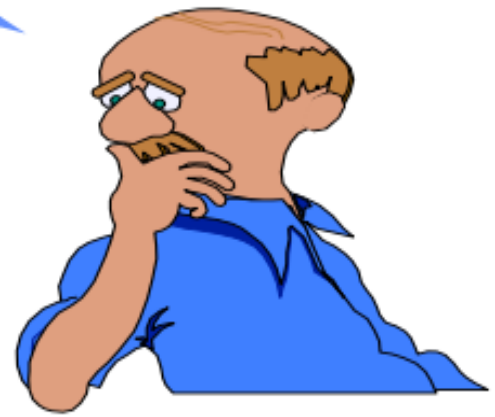
Pairwise Comparisons

USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

How do you turn this MATRIX into ranking of criteria?



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

HOW DO YOU GET A RANKING OF PRIORITIES FROM A PAIRWISE MATRIX?

ACTUALLY...

DR THOMAS L. SAATY, CURRENTLY WITH THE UNIVERSITY OF PITTSBURGH, DEMONSTRATED MATHEMATICALLY THAT THE EIGENVECTOR SOLUTION WAS THE BEST APPROACH.

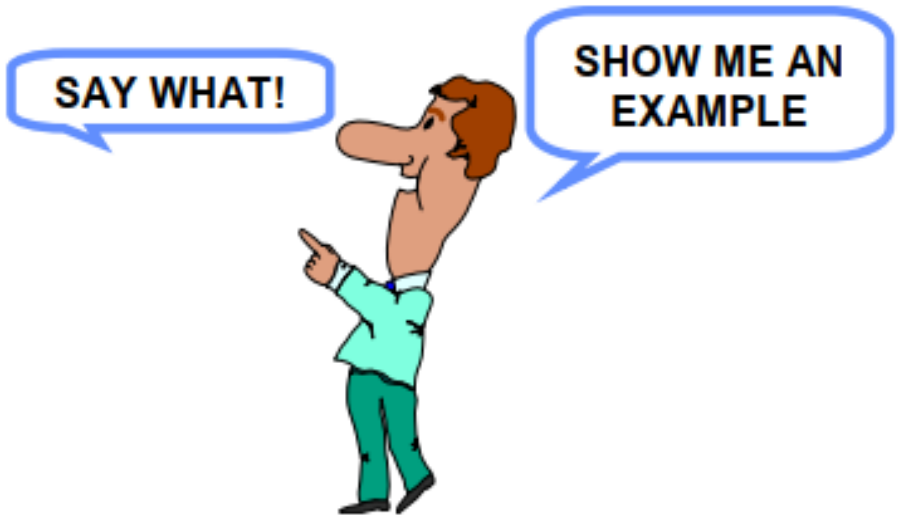
REFERENCE : THE ANALYTIC HIERARCHY PROCESS, 1990, THOMAS L. SAATY

HERE'S HOW TO SOLVE FOR THE EIGENVECTOR:

1. A SHORT COMPUTATIONAL WAY TO OBTAIN THIS RANKING IS TO RAISE THE PAIRWISE MATRIX TO POWERS THAT ARE SUCCESSIVELY SQUARED EACH TIME.

2. THE ROW SUMS ARE THEN CALCULATED AND NORMALIZED.

3. THE COMPUTER IS INSTRUCTED TO STOP WHEN THE DIFFERENCE BETWEEN THESE SUMS IN TWO CONSECUTIVE CALCULATIONS IS SMALLER THAN A PRESCRIBED VALUE.



IT'S MATRIX ALGEBRA TIME !!!

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

FOR NOW, LET'S REMOVE THE NAMES AND CONVERT THE FRACTIONS TO DECIMALS :

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

STEP 1: SQUARING THE MATRIX

THIS TIMES

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

THIS

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

I.E. (1.0000 * 1.0000) + (0.5000 * 2.0000) +(3.0000 * 0.3333) = 3.0000

RESULTS IN THIS

3.0000	1.7500	8.0000
5.3332	3.0000	14.0000
1.1666	0.6667	3.0000

STEP 2 : NOW, LET'S COMPUTE OUR FIRST EIGENVECTOR (TO FOUR DECIMAL PLACES)

FIRST, WE SUM THE ROWS

3.0000	+	1.7500	+	8.0000	=	12.7500	0.3194
5.3332	+	3.0000	+	14.0000	=	22.3332	0.5595
1.1666	+	0.6667	+	3.0000	=	4.8333	0.1211
						39.9165	1.0000

SECOND, WE SUM THE ROW TOTALS

FINALLY, WE NORMALIZE BY DIVIDING THE ROW SUM BY THE ROW TOTALS (I.E. 12.7500 DIVIDED BY 39.9165 EQUALS 0.3194)

THE RESULT IS OUR EIGENVECTOR (A LATER SLIDE WILL EXPLAIN THE MEANING IN TERMS OF OUR EXAMPLE)

$$\begin{bmatrix} 0.3194 \\ 0.5595 \\ 0.1211 \end{bmatrix}$$

THIS PROCESS MUST BE ITERATED UNTIL THE EIGENVECTOR SOLUTION DOES NOT CHANGE FROM THE PREVIOUS ITERATION (REMEMBER TO FOUR DECIMAL PLACES IN OUR EXAMPLE)

CONTINUING OUR EXAMPLE,
AGAIN, STEP 1: WE SQUARE THIS MATRIX

$$\begin{bmatrix} 3.0000 & 1.7500 & 8.0000 \\ 5.3332 & 3.0000 & 14.0000 \\ 1.1666 & 0.6667 & 3.0000 \end{bmatrix}$$

WITH THIS RESULT →

$$\begin{bmatrix} 27.6653 & 15.8330 & 72.4984 \\ 48.3311 & 27.6662 & 126.6642 \\ 10.5547 & 6.0414 & 27.6653 \end{bmatrix}$$

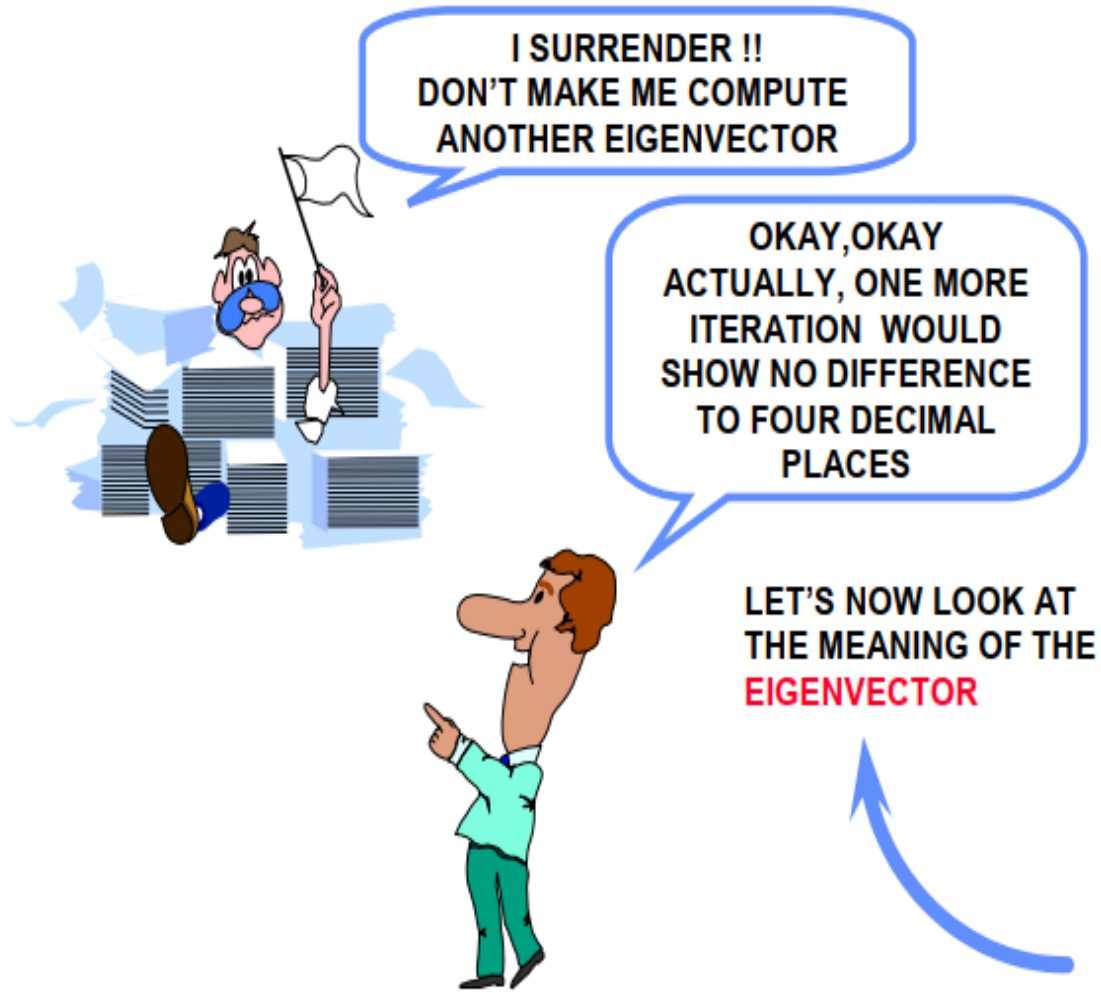
AGAIN STEP 2 : COMPUTE THE EIGENVECTOR (TO FOUR DECIMAL PLACES)

27.6653	+	15.8330	+	72.4984	=	115.9967	0.3196
48.3311	+	27.6662	+	126.6642	=	202.6615	0.5584
10.5547	+	6.0414	+	27.6653	=	44.2614	0.1220
						TOTALS 362.9196	1.0000

COMPUTE THE DIFFERENCE OF THE PREVIOUS COMPUTED EIGENVECTOR TO THIS ONE:

0.3194	−	0.3196	=	- 0.0002
0.5595	−	0.5584	=	0.0011
0.1211	−	0.1220	=	- 0.0009

TO FOUR DECIMAL PLACES THERE'S NOT MUCH DIFFERENCE
HOW ABOUT ONE MORE ITERATION?


I SURRENDER !!
DON'T MAKE ME COMPUTE
ANOTHER EIGENVECTOR
OKAY,OKAY
ACTUALLY, ONE MORE
ITERATION WOULD
SHOW NO DIFFERENCE
TO FOUR DECIMAL
PLACES
LET'S NOW LOOK AT
THE MEANING OF THE
EIGENVECTOR

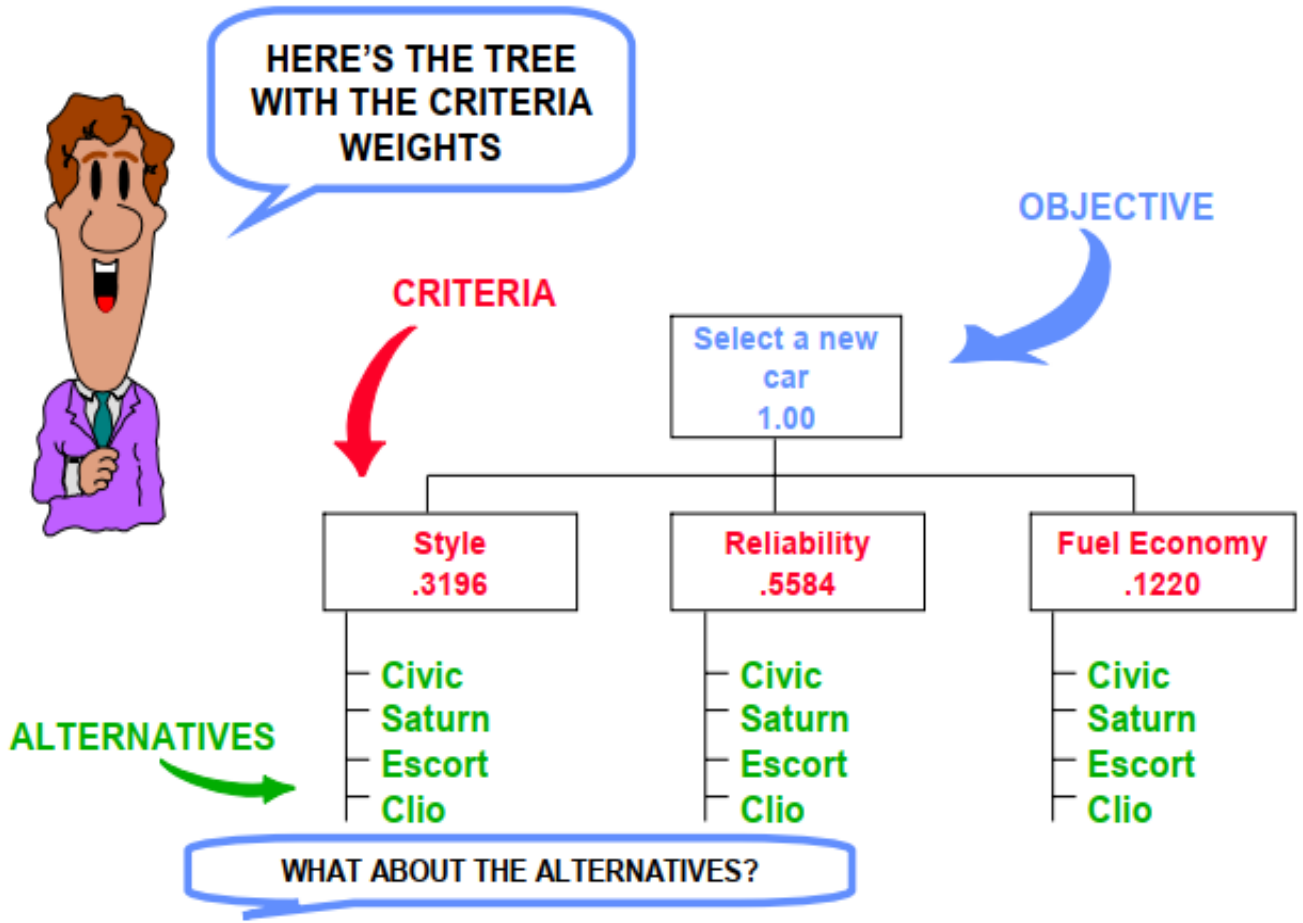
HERE'S OUR PAIRWISE MATRIX WITH THE NAMES

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

AND THE COMPUTED EIGENVECTOR GIVES US THE RELATIVE RANKING OF OUR CRITERIA

STYLE	0.3196	← THE SECOND MOST IMPORTANT CRITERION
RELIABILITY	0.5584	← THE MOST IMPORTANT CRITERION
FUEL ECONOMY	0.1220	← THE LEAST IMPORTANT CRITERION

NOW BACK TO THE HIEARCHICAL TREE...


HERE'S THE TREE WITH THE CRITERIA WEIGHTS
OBJECTIVE
CRITERIA
Select a new car 1.00
Style .3196
Reliability .5584
Fuel Economy .1220
ALTERNATIVES
Civic
Saturn
Escort
Clio
Civic
Saturn
Escort
Clio
Civic
Saturn
Escort
Clio
WHAT ABOUT THE ALTERNATIVES?

IN TERMS OF STYLE, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

STYLE

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	1/4	4/1	1/6
SATURN	4/1	1/1	4/1	1/4
ESCORT	1/4	1/4	1/1	1/5
CLIO	6/1	4/1	5/1	1/1

AND...

IN TERMS OF RELIABILITY, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

RELIABILITY

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	2/1	5/1	1/1
SATURN	1/2	1/1	3/1	2/1
ESCORT	1/5	1/3	1/1	1/4
CLIO	1/1	1/2	4/1	1/1

ITS MATRIX ALGEBRA TIME!!!

COMPUTING THE EIGENVECTOR DETERMINES THE RELATIVE RANKING OF ATERNATIVES UNDER EACH CRITERION

RANKING		STYLE
3	CIVIC	.1160
2	SATURN	.2470
4	ESCORT	.0600
1	CLIO	.5770

RANKING		RELIABILITY
1	CIVIC	.3790
2	SATURN	.2900
4	ESCORT	.0740
3	CLIO	.2570

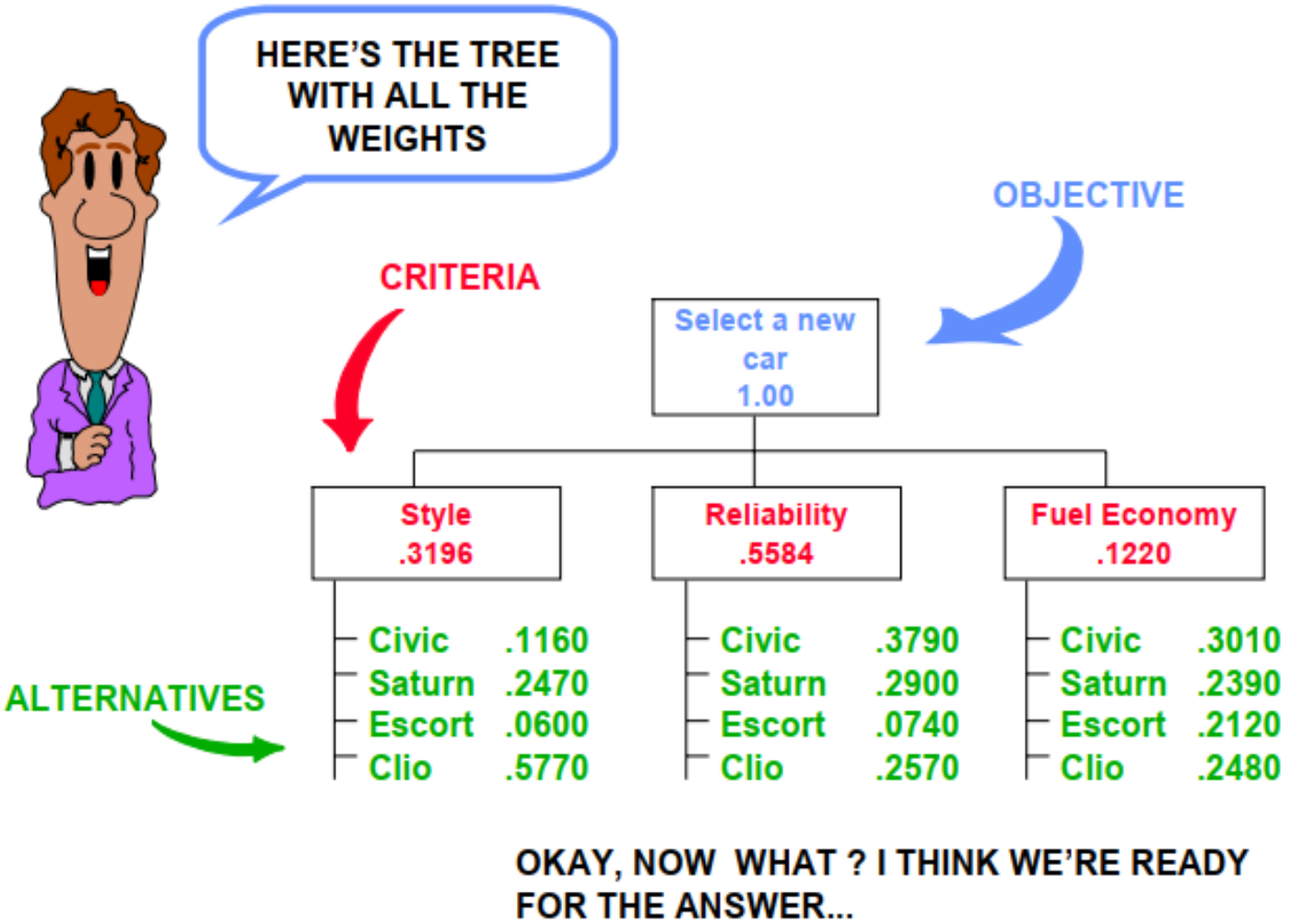
WHAT ABOUT FUEL ECONOMY?

AS STATED EARLIER,
AHP CAN COMBINE BOTH QUALITATIVE
AND QUANITATIVE INFORMATION

FUEL ECONOMY INFORMATION IS OBTAINED FOR EACH ALTERNATIVE:

	FUEL ECONOMY (MILES/GALLON)		
CIVIC	34	34 / 113 =	.3010
SATURN	27	27 / 113 =	.2390
ESCORT	24	24 / 113 =	.2120
CLIO	28	28 / 113 =	.2480
	113		1.0000

NORMALIZING THE FUEL ECONOMY INFO
ALLOWS US TO USE IT WITH OTHER RANKINGS


HERE'S THE TREE WITH ALL THE WEIGHTS
OBJECTIVE
CRITERIA
Select a new car 1.00
Style .3196
Reliability .5584
Fuel Economy .1220
ALTERNATIVES
Civic .1160
Saturn .2470
Escort .0600
Clio .5770
Civic .3790
Saturn .2900
Escort .0740
Clio .2570
Civic .3010
Saturn .2390
Escort .2120
Clio .2480
OKAY, NOW WHAT ? I THINK WE'RE READY FOR THE ANSWER...

A LITTLE MORE MATRIX ALGEBRA GIVES US THE SOLUTION:

	STYLE	RELI-ABILITY	FUEL ECONOMY		CRITERIA RANKING	
CIVIC	.1160	.3790	.3010		0.3196	STYLE
SATURN	.2470	.2900	.2390	*	0.5584	RELIABILITY
ESCORT	.0600	.0740	.2120			
CLIO	.5770	.2570	.2480		0.1220	FUEL ECONOMY

I.E. FOR THE CIVIC (.1160 * .3196) + (.3790 * .5584) + (.3010 * .1220) = .3060

	Civic	.3060	AND THE WINNER IS !!!
=	Saturn	.2720	
	Escort	.0940	
	Clio	.3280	THE CLIO IS THE HIGHEST RANKED CAR

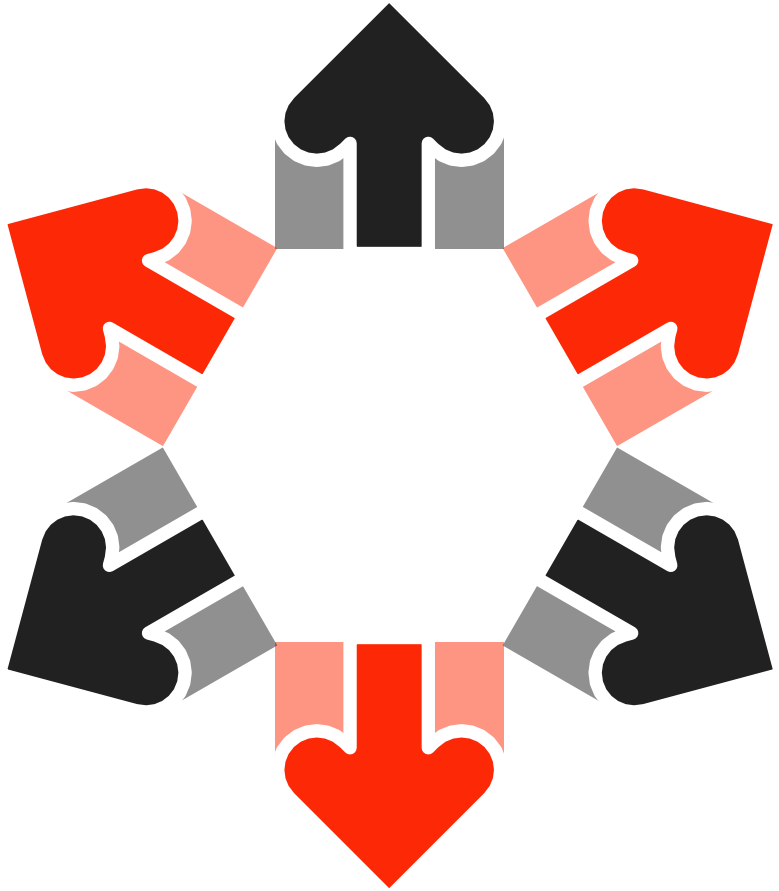
IN SUMMARY, THE ANALYTIC HIERARCHY PROCESS PROVIDES A LOGICAL FRAMEWORK TO DETERMINE THE BENEFITS OF EACH ALTERNATIVE

1. Clio	.3280
2. Civic	.3060
3. Saturn	.2720
4. Escort	.0940

AHP-Based Software

Make It Rational

transparent choice



SpiceLogic™ Inc.

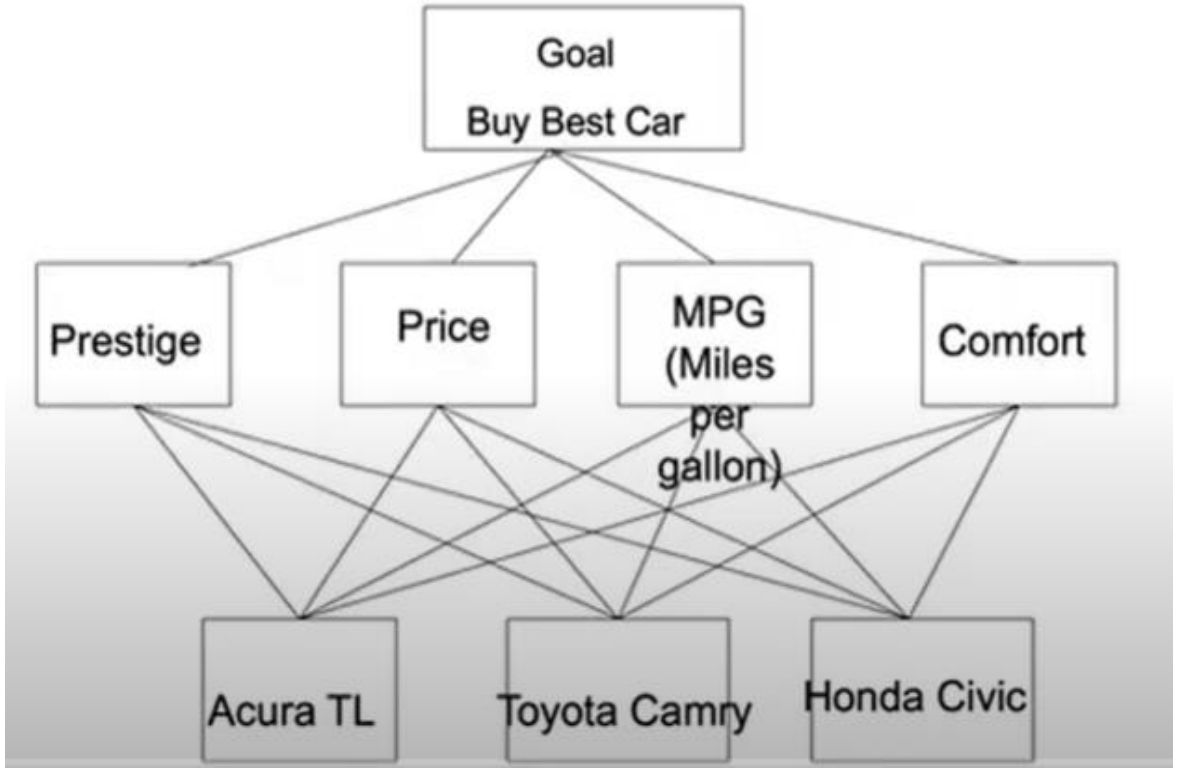
SUPER DECISIONS CDF

expertchoice®
For Collaborative Decision Making

XLSTAT by Addinsoft

Building an AHP model in SuperDecisions

Case study: Choose the best car



Reference: https://superdecisions.com/

Building an AHP model in SuperDecisions

Case study: Choose the best car.

We will follow official tutorial for this part. The links are given below.
-Building AHP Model

- I will also show Sensitivity Analysis in the class.

THANK YOU