Introduction to MCDM and AHP

Junar Landicho

AT84.02: Business Intelligence and Analytics

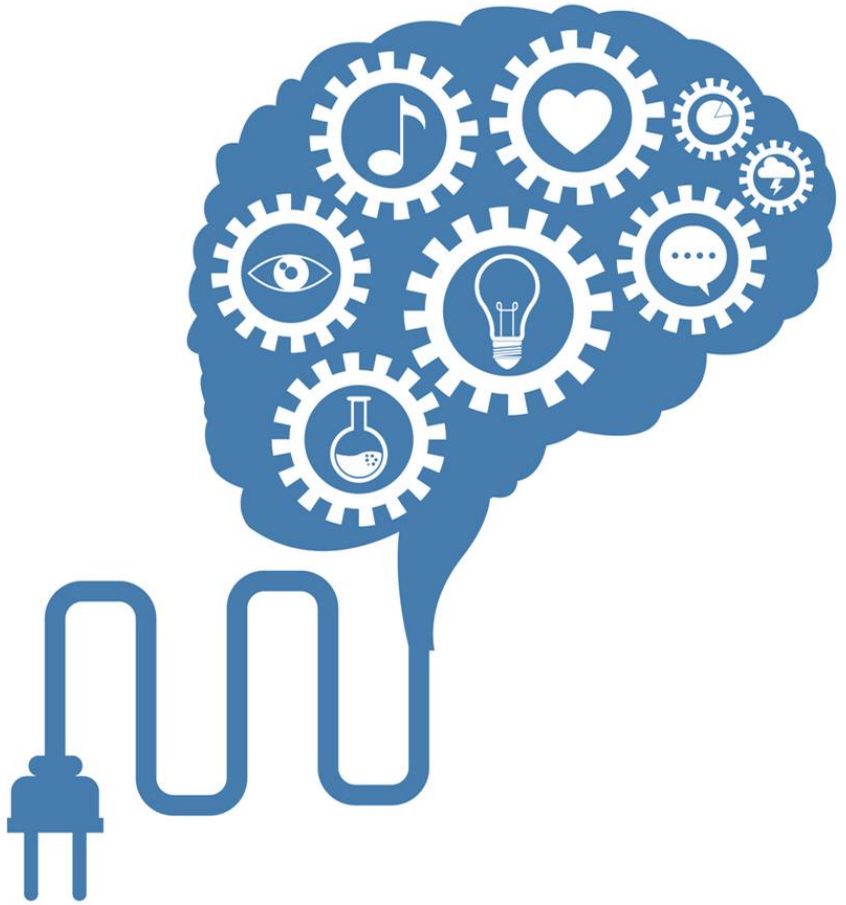
January 2021

Agenda for Today

- Multi-criteria decision making (MCDM)
  - Introduction to MCDM
  - MCDM methods
  - MCDM software
- Analytic Hierarchy Process (AHP)
  - Concepts of AHP
  - AHP Process
  - Build AHP using SuperDecisions Software

Introduction to MCDM

- **Multi-criteria decision making (MCDM)** is a method used to prioritize, rank or choose from a variety of different alternatives or options, based on multiple criteria.
- MCDM methods have been developed to **support the decision maker** in their unique and personal decision process.
- MCDM is useful for:
  - Dividing the decision into smaller, more understandable parts
  - Analyzing each part
  - Integrating the parts to produce a meaningful solution



Source: Multi-Criteria Decision Analysis: Methods and Software (2013)

Introduction to MCDM

- MCDA is a discipline that encompasses mathematics, management, informatics, psychology, social science and economics.
- Its application is even wider as it can be used to solve any problem where a significant decision needs to be made.

Types of Decision Making

Decision	Time perspective	Novelty	Degree of structure	Automation
Strategic	long term	new	low	low
Tactical	medium term	adaptive	semi-structured	middle
Operational	short term	every day	well defined	high

Source: Multi-Criteria Decision Analysis: Methods and Software (2013)

Introduction to MCDM

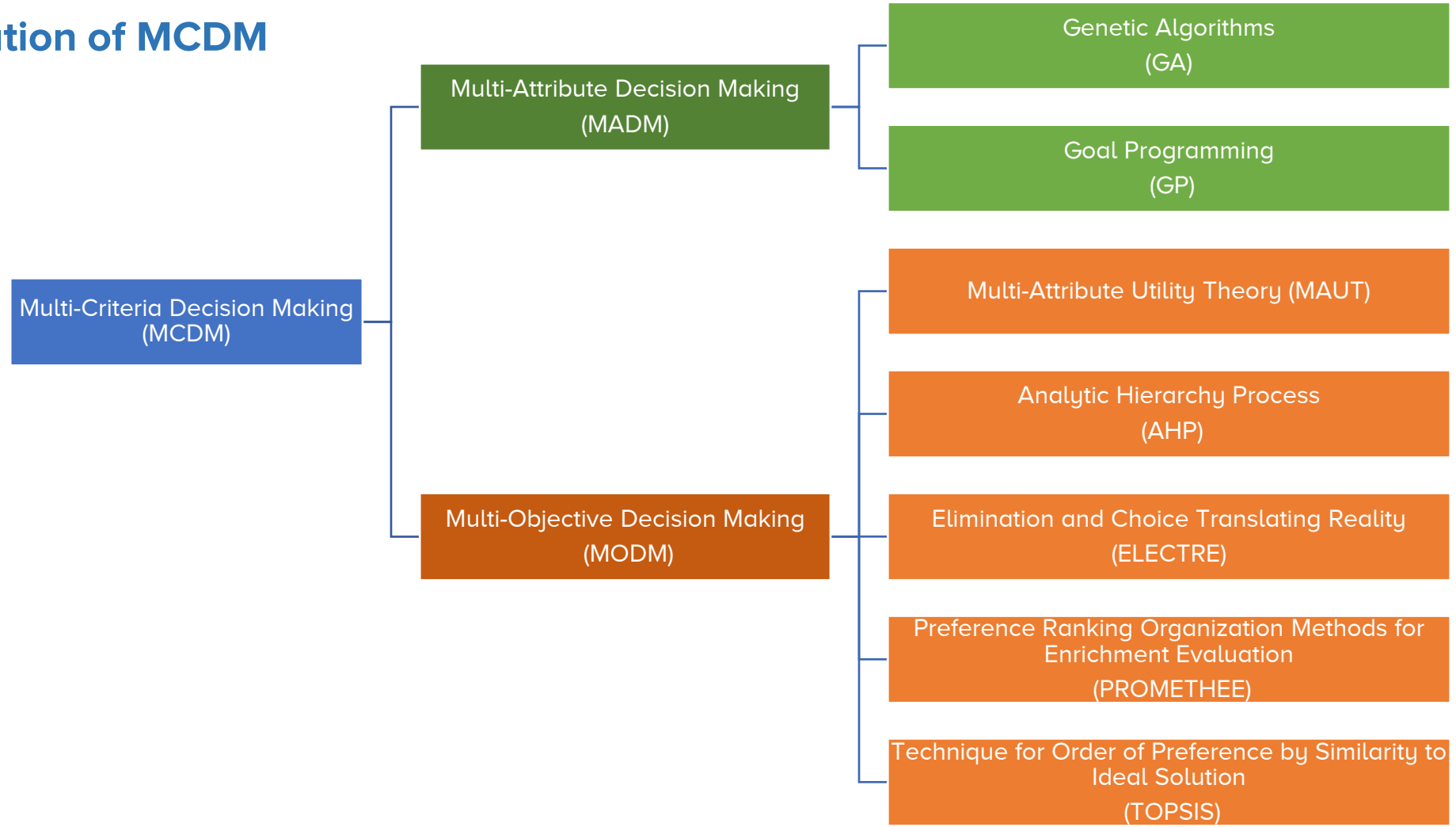
Types of Decision Problems

1. Choice Problem
2. Sorting Problem
3. Ranking Problem
4. Description Problem
5. Elimination Problem
6. Design Problem

Source: Roy (1981), Bana e Costa (1996) and Keeney (1992)

MCDM Methods

Classification of MCDM



MCDM Methods

General Stages of MCDM

- Identifying decision makers
- Defining the goal
- Analyzing the feasible alternatives

- Weighting the criteria
- Scoring each alternative
- Calculating the overall weighted scores of alternatives
- Ranking of all feasible alternatives

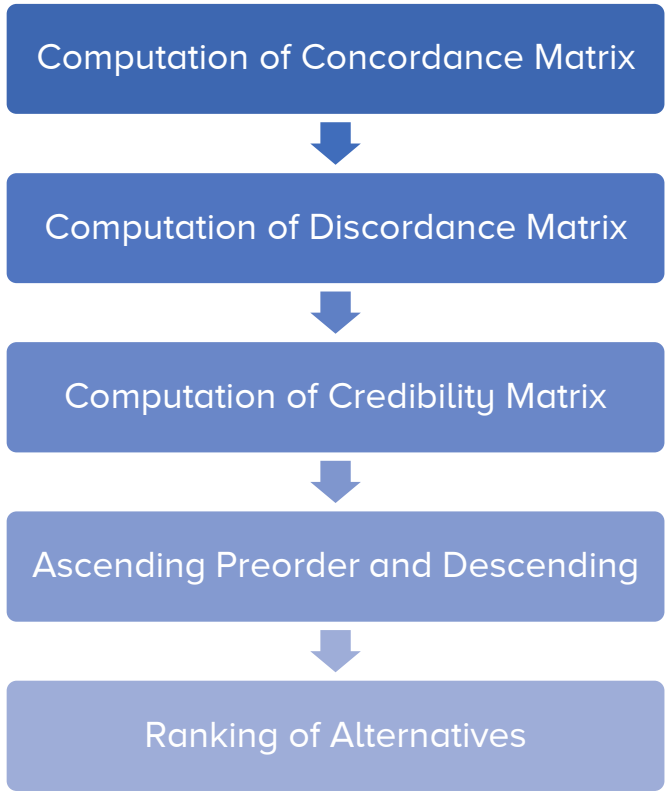
- The higher the overall weighted score is more preferable the alternative
- Obtained results should be examined further

MCDM Methods

Elimination and Choice Translating Reality (ELECTRE)

- First presented by **Benayoun et al. (1966)** as a response to limitations of existing decision-making methods for resolving the choice problem.
- Employed in a wide variety of fields including **risk evaluation, supplier selection** and **multiple criteria decision making**.
- Extended subsequently as **ELECTRE I, IS, Iv, II, III, IV, III-H** and **Tri** method.

ELECTRE Steps



Source: Mary and Suganya (2016)

MCDM Methods

Preference ranking organisation Method for Enrichment Evaluation (PROMETHEE)

- Initial proposed by **Brans (1982)** is another outranking method for a finite set of alternatives that is to be ranked and selected.
- PROMETHEE method was developed in order to assist decision makers when defining the preference relations in a particular level of the hierarchy.
- **Six methods** developed within the PROMETHEE (I-VI) family have been applied for solving MCDM problem.

Stepwise procedure for PROMETHEE II

Step 1. Determination of deviations based on pair-wise comparisons.

Step 2. Application of the preference function.

Step 3. Calculation of an overall or global preference index.

Step 4. Calculation of outranking flows.

Step 5. Calculation of net outranking flow.

Source: Brans (1982) and Corrente et al.(2013)

MCDM Methods

Technique for order of preference by similarity to ideal solution (TOPSIS)

- Developed by **Hwang and Yoon (1981)** , is another well-known MCDM method that evaluates the performance of alternatives based on the distance from the ideal solution.
- The preferred alternative must have the **shortest distance from the positive ideal solution** and the **longest distance from the negative ideal solution**.
- The TOPSIS methodology is applied widely in MCDM field, especially in the **fuzzy extension of linguistic variables.**

Stages for Implementing TOPSIS

1. Develop and normalize decision matrix.
2. Construction of weighted normalized decision matrix
3. Determination of positive and negative ideal solutions
4. Calculation of separation measures for each alternative.
5. Calculation of the relative closeness to the ideal solution.
6. Rank the alternatives according to the ideal solution

Source: Fei et al. (2016) and Sitorus et al.(2019)

MCDM Software

Problems	MCDM Methods	Software
• Ranking • Description (sorting) • Choice (sorting)	• PROMTHEE – GAIA • ELECTRE	• Decision Lab • D-Sight • Smart Picker Pro, • Visual Promethee • J-Electre

DecisionLab

D-SIGHT®

PROMETHEE METHODS

J-ELECTRE-v2.0

SMART PICKER

Source: Multi-Criteria Decision Analysis: Methods and Software (2013)

MCDM Software

Problems	MCDM Methods	Software
• Ranking • Choice	• PROMTHEE • UTA • AHP	• DECERNS • **UTA+** • MakeItRational • HIPRE 3+, • Criterium • EasyMind • Questfox • ChoiceResults • 123AHP

Decerns

Make It Rational

Web-HIPRE

CRITERIUM DECISION PLUS InfoHarvest Inc. 1998

123AHP my choice, my decision

easy mind

questfox

Source: Multi-Criteria Decision Analysis: Methods and Software (2013)

MCDM Software

Problems	MCDM Methods	Software
• Ranking • choice	• ANP • MACBETH • TOPSIS • DEA	• Super Decisions • Decision Lens • M-MACBETH • Win4DEAP • Dea Solver Pro • Frontier Analyst

SUPER DECISIONS CDF

DECISION LENS

MACBETH

Win4Deap 2

Professional version 15.1
DEA-Solver™

Frontier Analyst

Source: Multi-Criteria Decision Analysis: Methods and Software (2013)

MCDM Software

Required inputs for MCDM ranking or choice method

	Inputs	Effort input	MCDA method	Output
Ranking/choice problem	utility function	Very HIGH	**MAUT**	Complete ranking with scores
	pairwise comparisons on a ratio scale and interdependencies	↑	**ANP**	Complete ranking with scores
	pairwise comparisons on an interval scale	|	**MACBETH**	Complete ranking with scores
	pairwise comparisons on a ratio scale	|	**AHP**	Complete ranking with scores
	indifference, preference and veto thresholds	|	**ELECTRE**	Partial and complete ranking (pairwise outranking degrees)
	indifference and preference thresholds	|	**PROMETHEE**	Partial and complete ranking (pairwise preference degrees and scores)
	ideal option and constraints	|	**Goal programming**	Feasible solution with deviation score
	ideal and anti-ideal option	↓	**TOPSIS**	Complete ranking with closeness score
	no subjective inputs required	Very LOW	**DEA**	Partial ranking with effectiveness score

Source: Multi-Criteria Decision Analysis: Methods and Software (2013)

MCDM Software

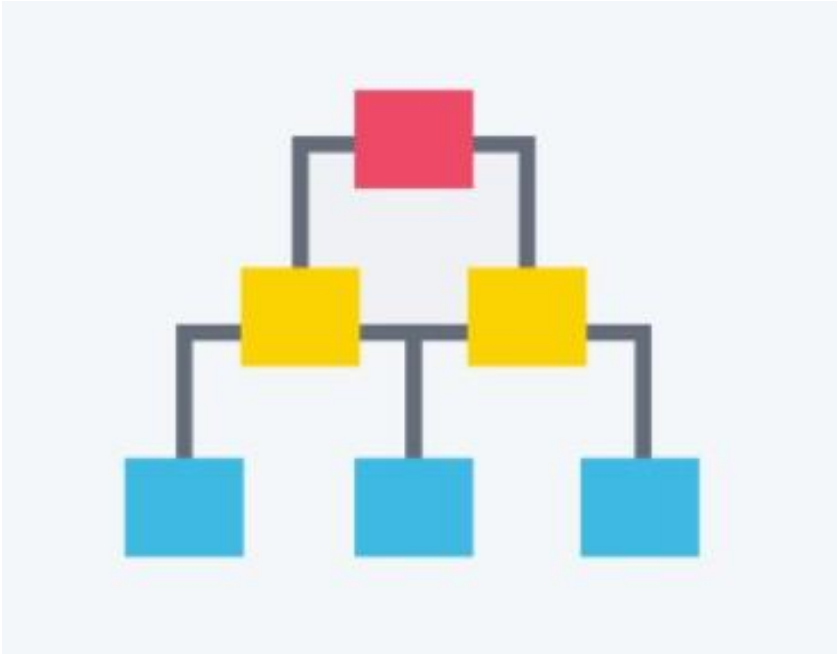
Required inputs for MCDM sorting method

	Inputs	Effort Input	MCDA method	Output
Sorting method	utility function	HIGH	UTADIS	Classification with scoring
	pairwise comparisons on a ratio scale	↕	AHPSort	Classification with scoring
	indifference, preference and veto thresholds	↕	ELECTRE-TRI	Classification with pairwise outranking degrees
	indifference and preference thresholds	LOW	FLOWSORT	Classification with pairwise outranking degrees and scores

Source: Multi-Criteria Decision Analysis: Methods and Software (2013)

Analytic hierarchy process (AHP)

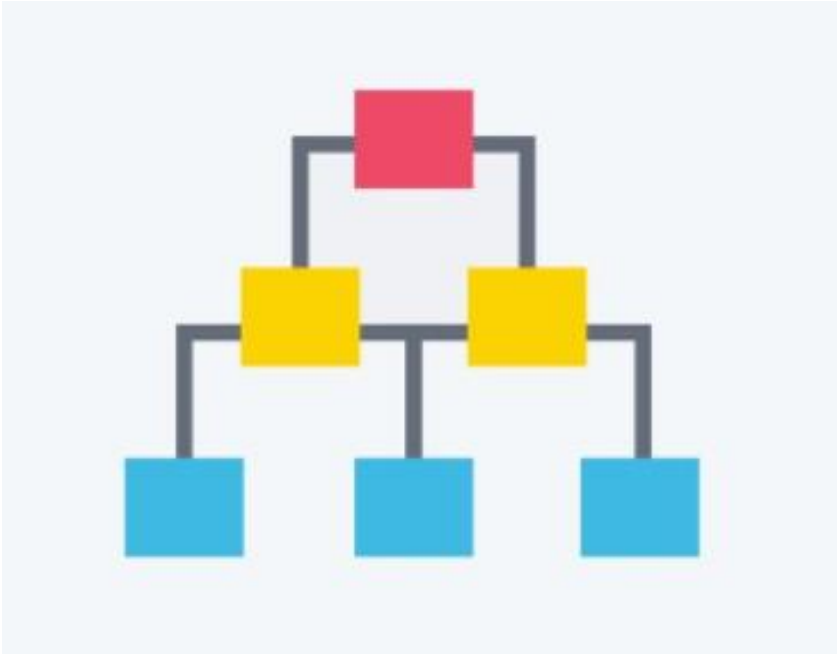
- Originally designed by **Saaty (1980)**, provides a systematic process to incorporate factors such as logic, experience or knowledge, emotion, and a sense of optimization into a decision making methodology
- It is based on the idea that the complexity inherent in a decision making problem with multiple criteria can be **solved by the hierarchy of the problems**.
- Decomposing a **decision into smaller parts**.



Source: Fei et al. (2016) and Sitorus et al.(2019)

Analytic hierarchy process (AHP)

- **Pairwise comparisons** were performed using **ratios of preference** (in comparison alternatives), and **ratios of importance** (when compared criteria) which are evaluated on a numerical scale by the method proposed.
- The method allows to **analyze the degree of inconsistency** of the judgements of the decision maker.



Source: Fei et al. (2016) and Sitorus et al.(2019)

Analytic hierarchy process (AHP)

Stages for Implementing AHP

1 Set up the decision hierarchy

- Level 1: Main Objective
- Leve 2: Criteria
- Level 3: Alternatives

2 Collect the pairwise data

- Each criterion or alternative is compared to each criterion or alternative

3 Construction of decision matrix

- Weighting of Criteria
- Assessment of alternatives

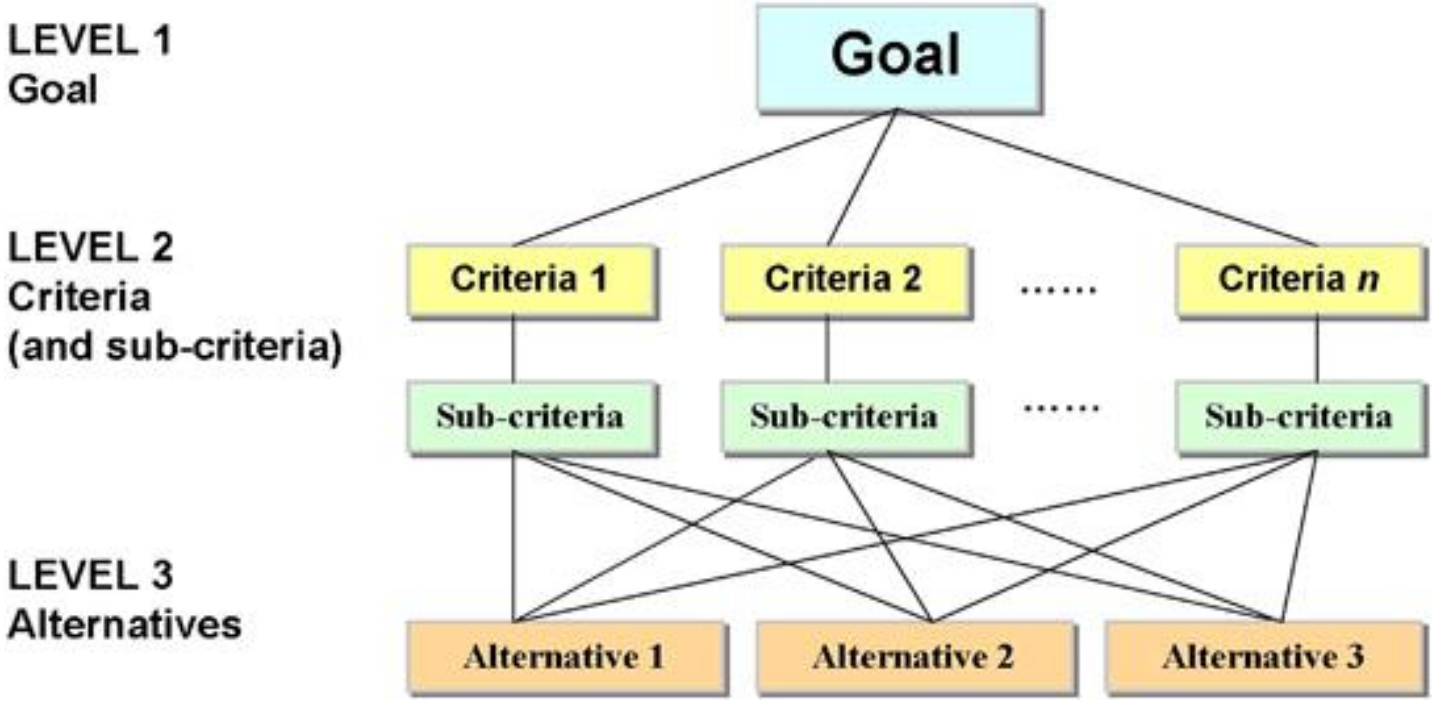
4 Calculations of overall priorities associated with each alternative

- Aggregation by weighted sum of priorities obtained in each level

Source: Fei et al. (2016) and Sitorus et al.(2019)

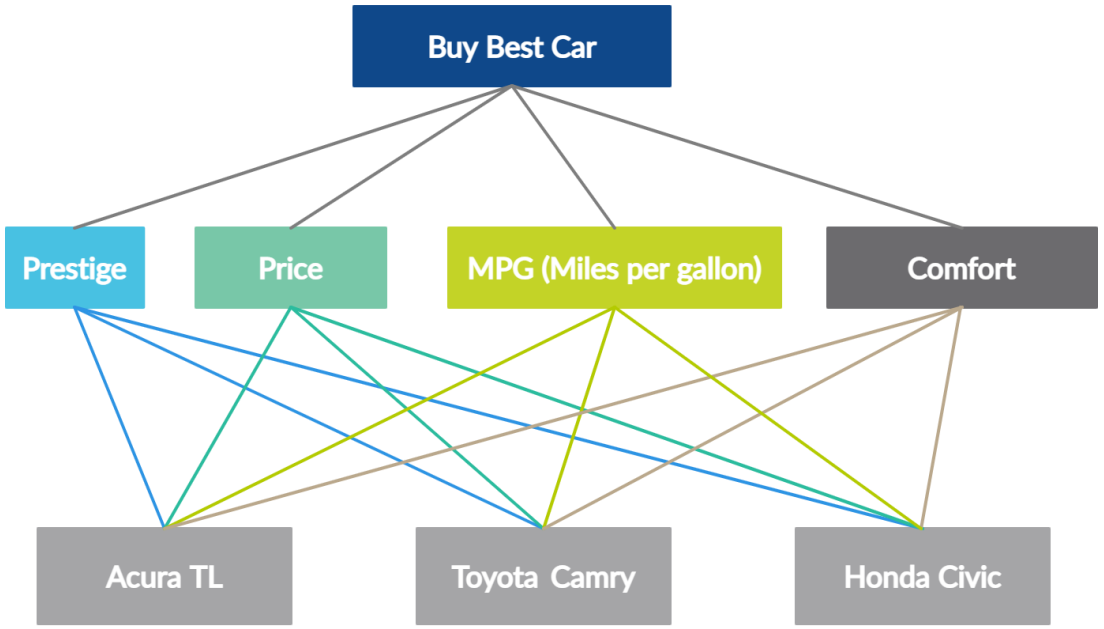
Analytic hierarchy process (AHP)

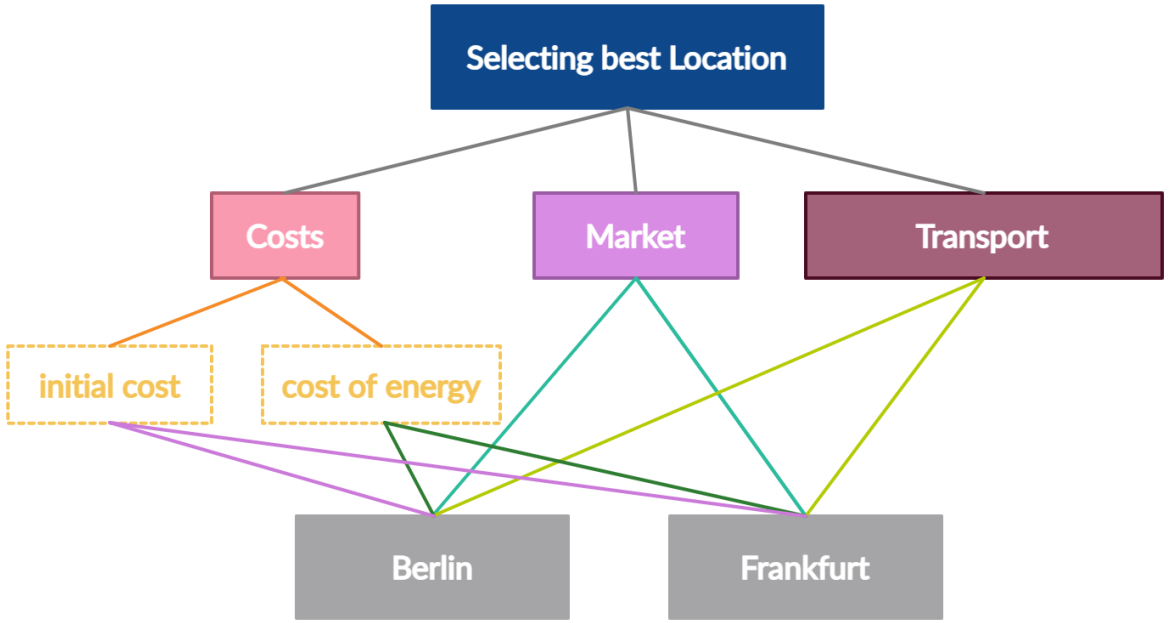
1. Set up the decision hierarchy



Analytic hierarchy process (AHP)

1. Set up the decision hierarchy





Analytic hierarchy process (AHP)

2. Collect the pairwise data

	to A1	A2	A3
Alternative 1 (A1)	a_{11}	a_{12}	a_{13}
Alternative 2 (A2)	a_{21}	a_{22}	a_{32}
Alternative 3 (A3)	a_{31}	a_{32}	a_{33}

→ Pairwise Comparison Matrix A = (a_{ij})

Properties of Pairwise Comparison Matrices

- Homogeneity : $a_{ij} = 1$
- Reciprocity : $a_{ij} * a_{ji} = 1$
- Transitivity : $a_{ij} * a_{jk} = a_{jk}$

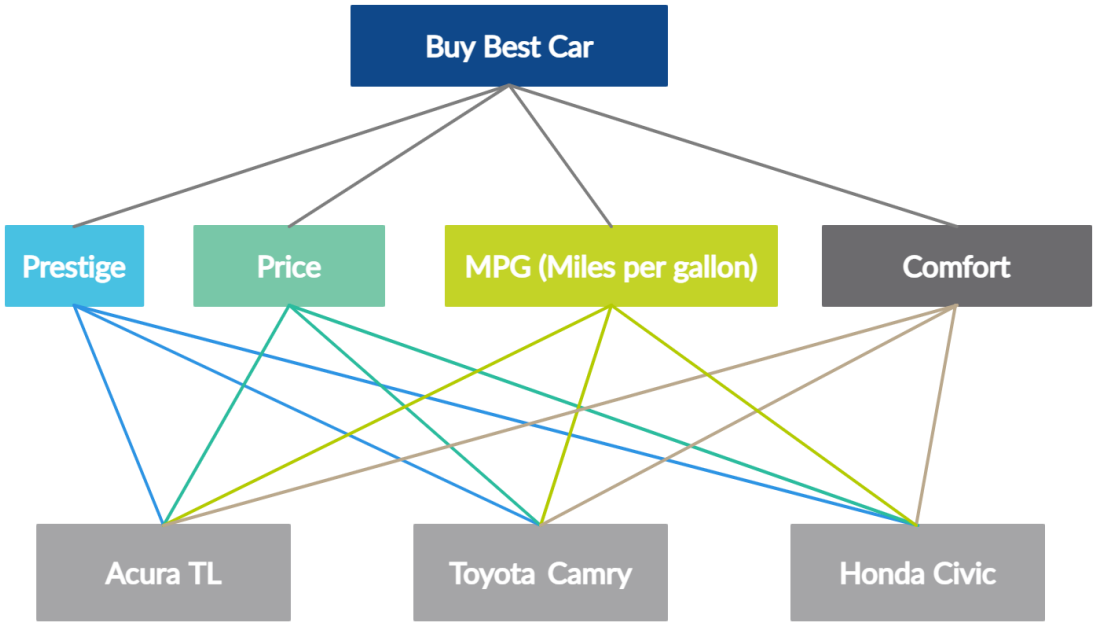
Scale of Relative Importance

Degree of importance	Definition
1	**Equal importance**
2	Weak
3	**Moderate importance**
4	Moderate plus
5	**Strong importance**
6	Strong plus
7	**Very strong or demon-strated importance**
8	Very, very strong
9	**Extreme importance**

Source: Fei et al. (2016) and Sitorus et al.(2019)

Analytic hierarchy process (AHP)

2. Collect the pairwise data

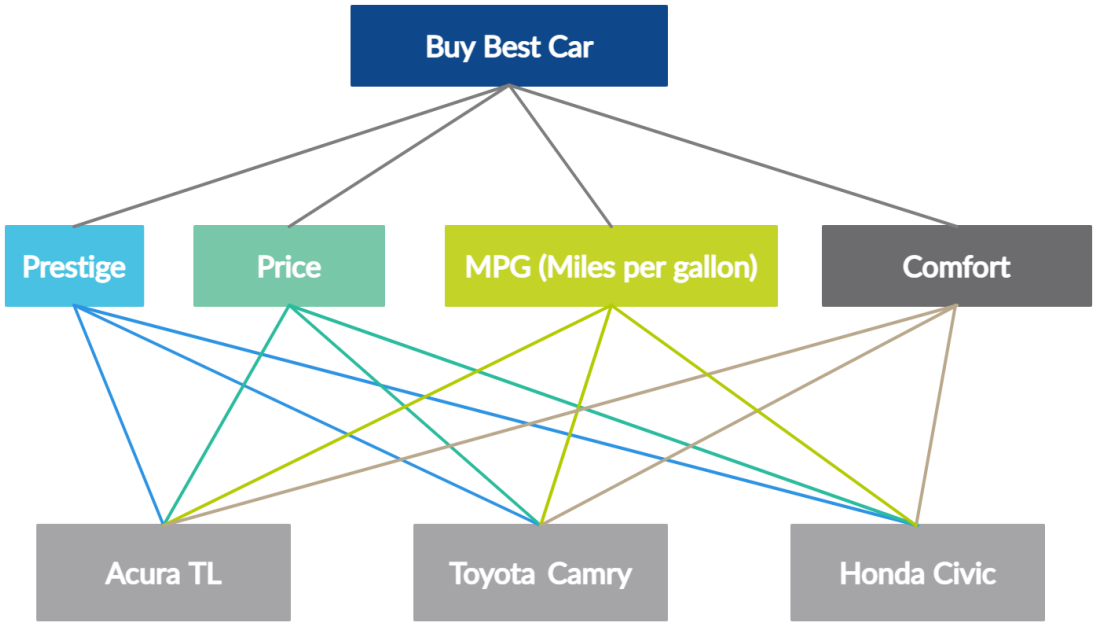


Pairwise Comparison Matrix

	Prestige	Price	MPG	Comfort
Prestige	1	5	3	6
Price	1/5	1	1/3	3
MPG	1/3	3	1	3
Comfort	1/6	1/3	1/3	1

Analytic hierarchy process (AHP)

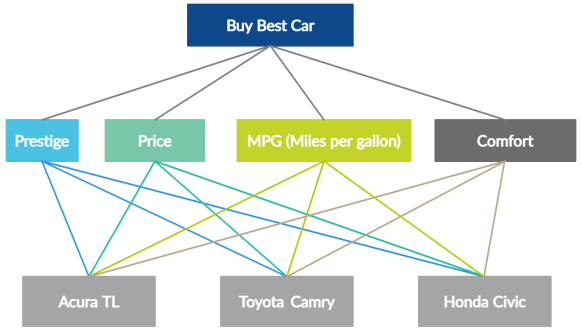
2. Collect the pairwise data



Pairwise Comparison Matrix

	Prestige	Price	MPG	Comfort
Prestige	1	5	3	6
Price	0.20	1	0.33	3
MPG	0.33	3	1	3
Comfort	0.17	0.33	0.33	1
Sum	1.70	9.33	4.67	13.00

Analytic hierarchy process (AHP)



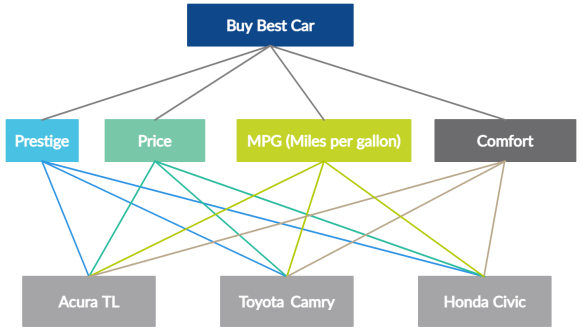
2. Collect the pairwise data

Normalised Pairwise Comparison Matrix

	Prestige	Price	MPG	Comfort	Criteria Weights
Prestige	0.5882	0.5357	0.6429	0.4615	0.5571
Price	0.1176	0.1071	0.0714	0.2308	0.1317
MPG	0.1961	0.3214	0.2143	0.2308	0.2406
Comfort	0.0980	0.0357	0.0714	0.0769	0.0705

$$\frac{0.5882 + 0.5357 + 0.6429 + 0.4615}{4} = \frac{2.2883}{4} = 0.5571$$

Analytic hierarchy process (AHP)

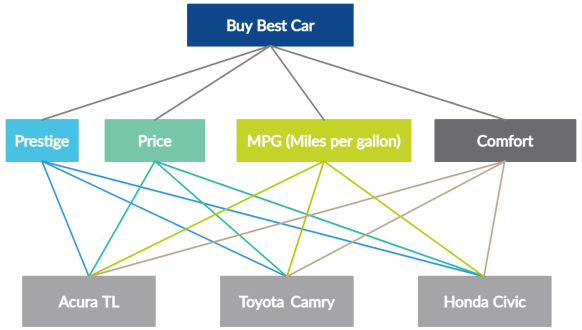


3. Construction of decision matrix

Calculating the Consistency

	Prestige	Price	MPG	Comfort
Prestige	1 * 0.5571	5 * 0.1317	3 * 0.2406	6 * 0.0705
Price	0.20 * 0.5571	1 * 0.1317	0.33 * 0.2406	3 * 0.0705
MPG	0.33 * 0.5571	3 * 0.1317	1 * 0.2406	3 * 0.0705
Comfort	0.17 * 0.5571	0.33 * 0.1317	0.33 * 0.2406	1 * 0.0705

Analytic hierarchy process (AHP)



4. Calculations of overall priorities associated with each alternative

Calculating tche Consistency

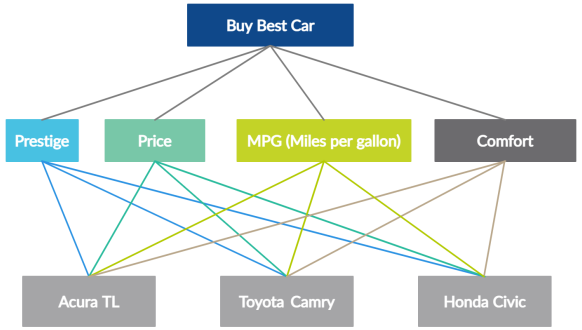
	Prestige	Price	MPG	Comfort	Weighted Sum Value	Criteria Weights	Weighted Sum Value / Criteria Weights
Prestige	0.5571	0.6585	0.7218	0.4230	2.3604	0.5571	4.2369
Price	0.1114	0.1317	0.0794	0.2115	0.5340	0.1317	4.0548
MPG	0.1838	0.3951	0.2406	0.2115	1.0310	0.2406	4.2853
Comfort	0.0947	0.0435	0.0794	0.0705	0.2881	0.0705	4.0860

$0.5571 + 0.6585 + 0.7218 + 0.4230 = 2.3604$

$$\lambda_{max} = \frac{4.2369 + 4.0548 + 4.2853 + 4.0860}{4} = 4.1658$$

Analytic hierarchy process (AHP)

4. Calculations of overall priorities associated with each alternative

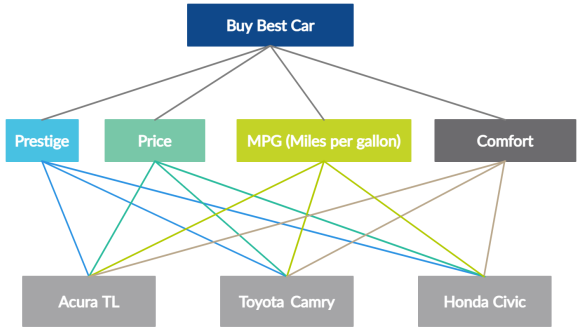


$$\boldsymbol{\lambda}_{max} = \frac{4.2369 + 4.0548 + 4.2853 + 4.0860}{4} = \mathbf{4.1658}$$

$$Consistency\ Index\ (CI) = \frac{\boldsymbol{\lambda}_{max} - n}{n - 1} = \frac{4.1658 - 4}{4 - 1} = \mathbf{0.0553}$$

Analytic hierarchy process (AHP)

4. Calculations of overall priorities associated with each alternative



$$Consistency\ Ratio\ (CR) = CI\ /\ Random\ Index$$

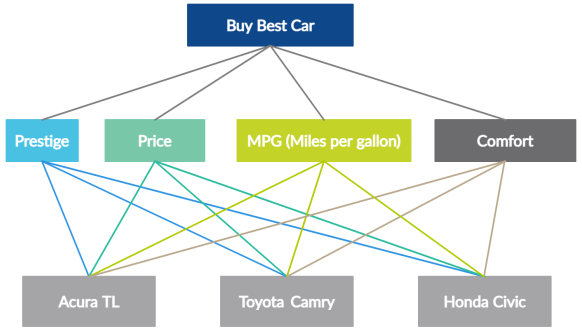
$$Consistency\ Ratio\ (CR) = \frac{0.0553}{0.90} = 0.614$$

Matrix size	Random consistency index (RI)
1	0.00
2	0.00
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.49

Source: Saaty 1981)

Analytic hierarchy process (AHP)

4. Calculations of overall priorities associated with each alternative



	Prestige	Price	MPG	Comfort	Criteria Weights
Prestige	0.5571	0.6585	0.7218	0.4230	0.5571
Price	0.1114	0.1317	0.0794	0.2115	0.1317
MPG	0.1838	0.3951	0.2406	0.2115	0.2406
Comfort	0.0947	0.0435	0.0794	0.0705	0.0705

$$Consistency\ Ratio\ (CR) = 0.614 < 0.10$$

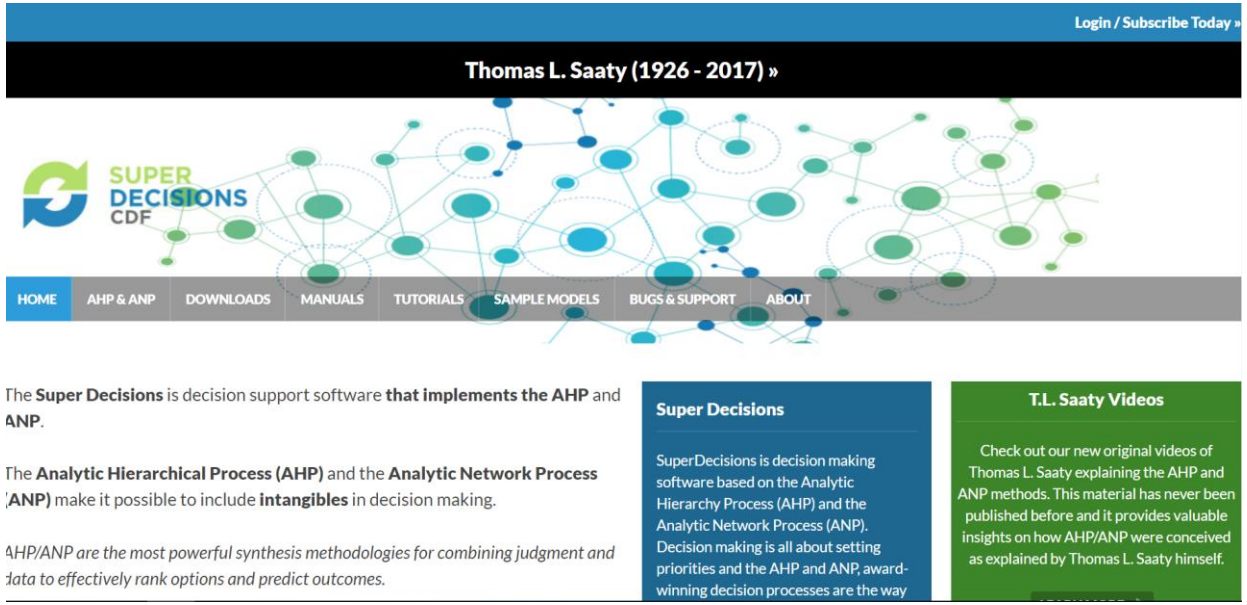
Build AHP using SuperDecisions Software

- The **Super Decisions** is decision support software that implements the **Analytic Hierarchical Process (AHP) and the Analytic Network Process (ANP).**
- A software provides tools to create and manage AHP and ANP models, enter your judgments, get results and perform sensitivity analysis on the results. It also provides support for complex, multilevel BOCR models (Benefits - Opportunities - Costs - Risks).

Source: http://www.superdecisions.com/

Build AHP using SuperDecisions Software

- Visit the website **http://www.superdecisions.com/**
- User need to log-in or register before can download the software
- On the download page, user can choose the system and version

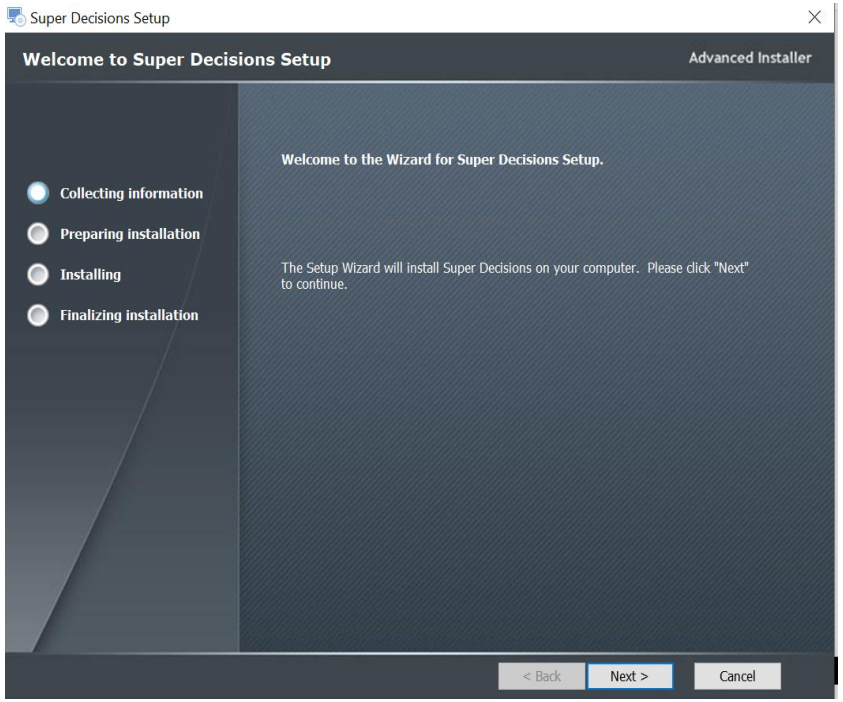


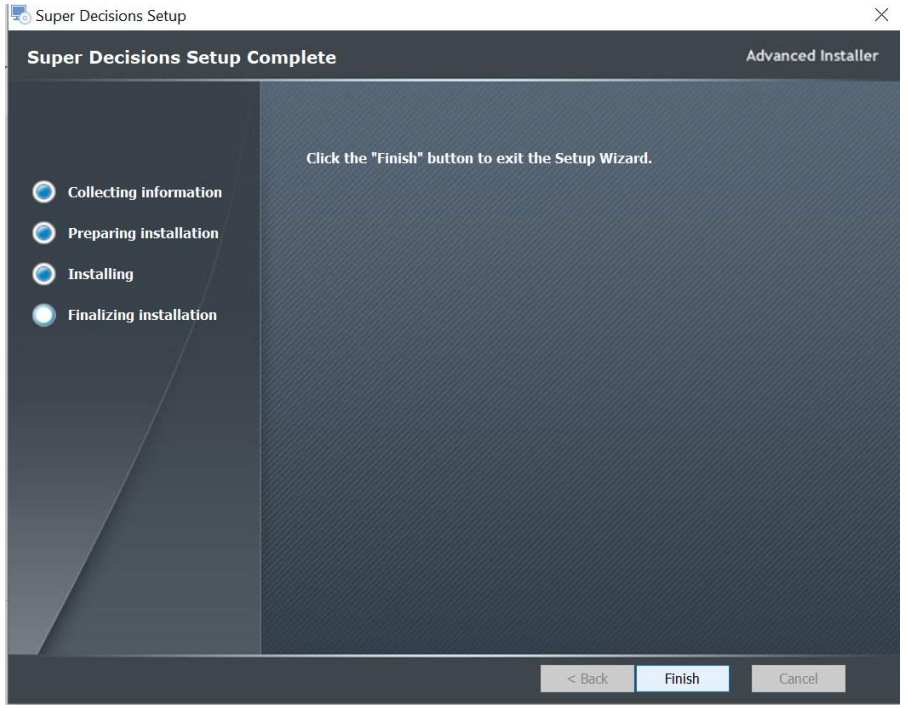
Quick Start Guide

- log-in or register to this website, if you have an account for the creativedecisions.net website you can use that information to login
- choose your operating system and the version you want from the left side menu
- click the download link in the version's page to start the download
- don't leave the page before copying your serial number
- if you forget to copy the serial number, you can always log-in to this website and go to your user page (the link with your user name on the top menu bar) and retrieve it or renew it if it has expired

Build AHP using SuperDecisions Software

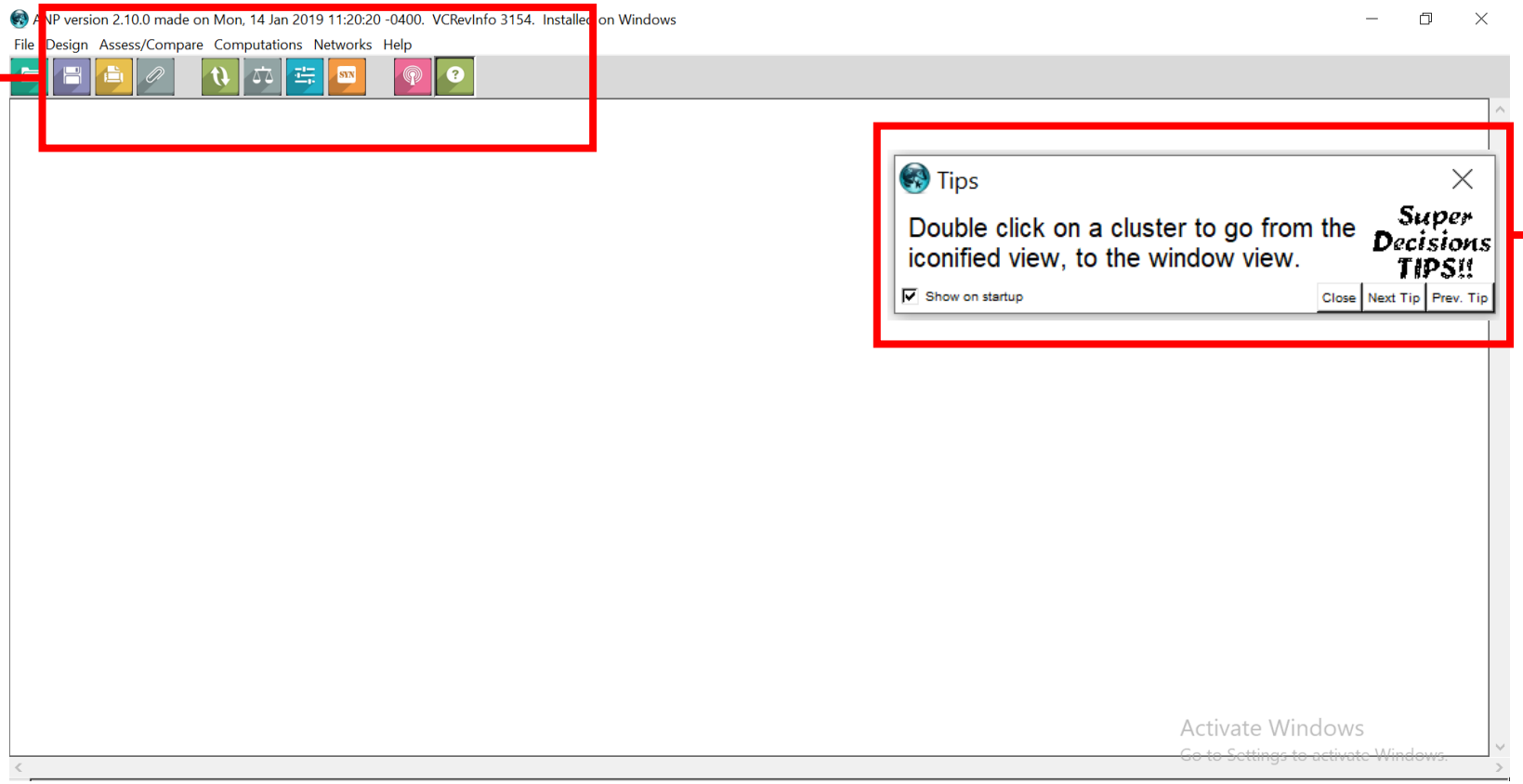
- Install the SuperDecision application in your computer.





Build AHP using SuperDecisions Software

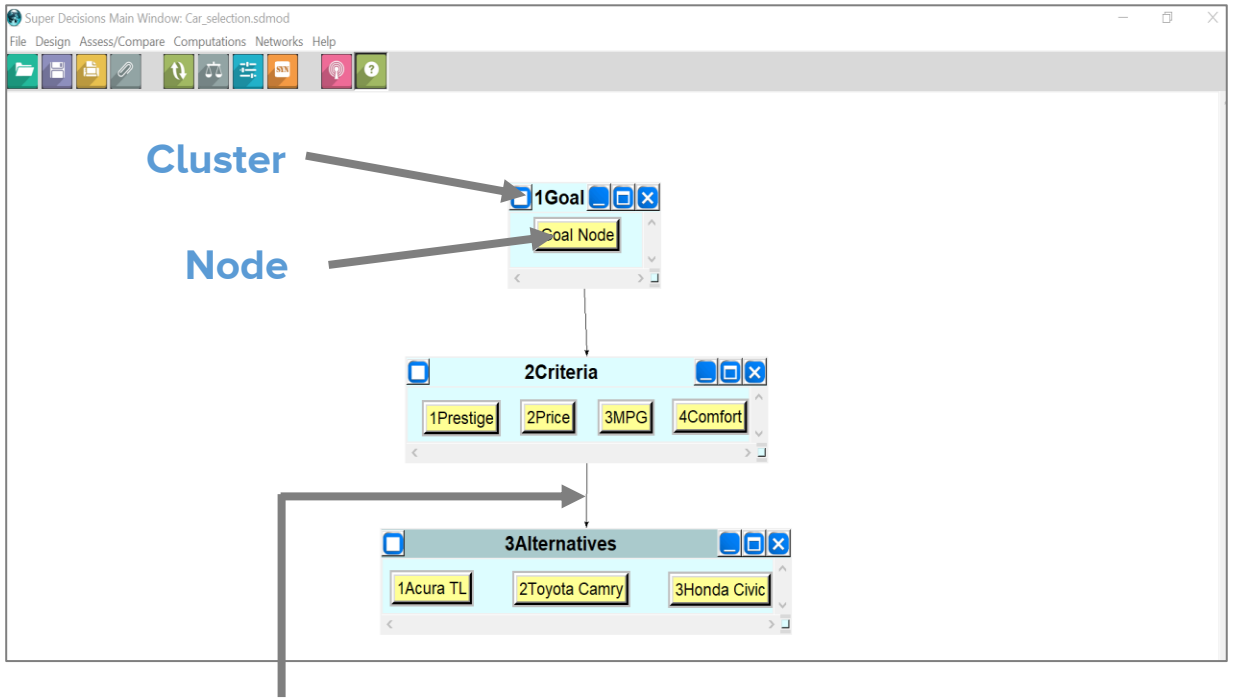
- SuperDecisions' Opening Screen



Main Menu Bar

SuperDecisions Tool Tips

Build AHP using SuperDecisions Software

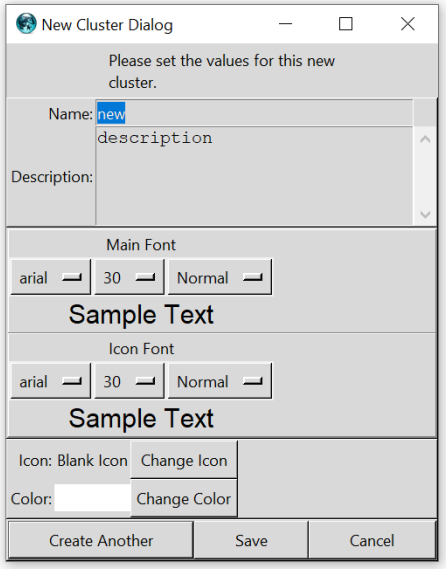


All links are among nodes: the cluster link is automatically created because some node(s) in the Criteria cluster are connected to some node(s) in the Alternatives Cluster

- To create the Goal cluster, select the **Design** command:

Design > Cluster > New

- A cluster editing window will appear where you can input the information and design needed.

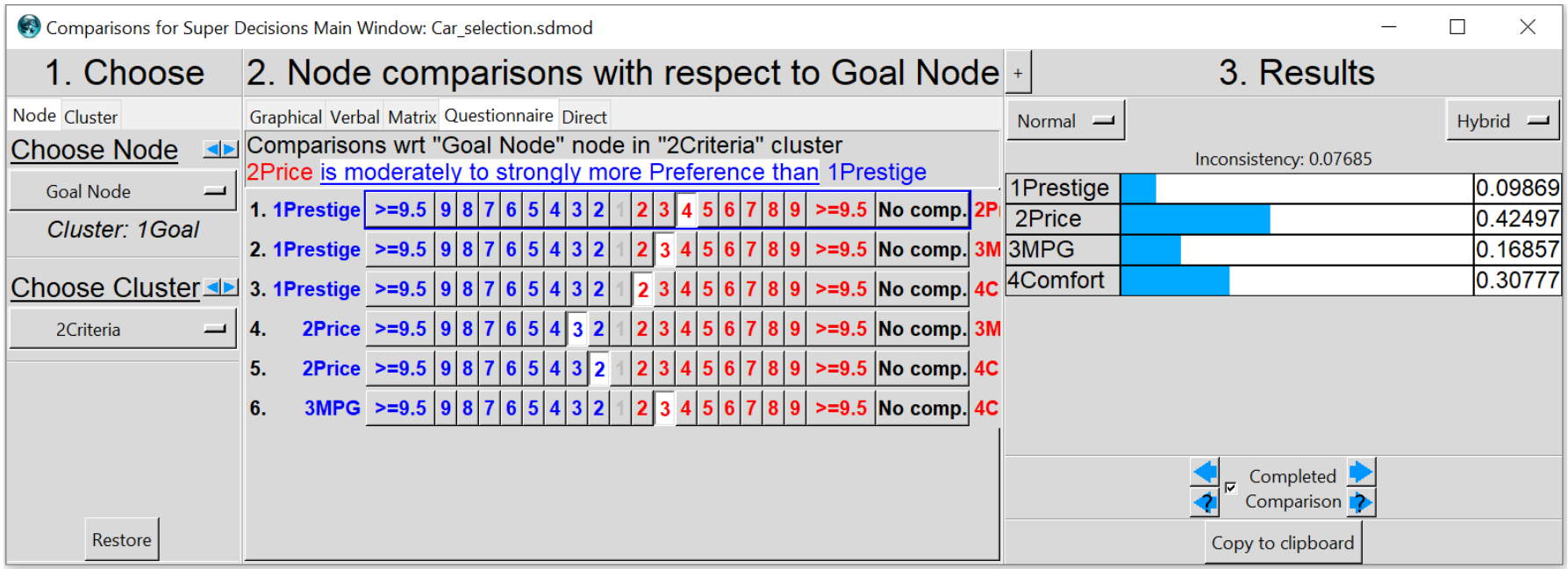


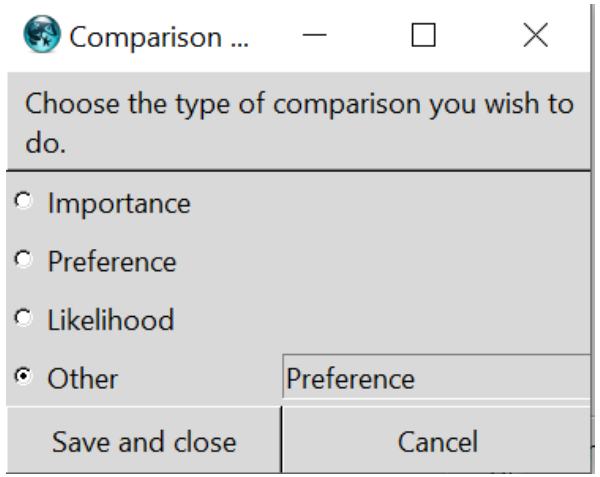
Build AHP using SuperDecisions Software

- To perform Pairwise Comparison:

  Assess/Compare > Pairwise Comparison

- There are 5 possible models for entering assessments; judgements entered in one mode will appear as the equivalent judgement in any other mode.

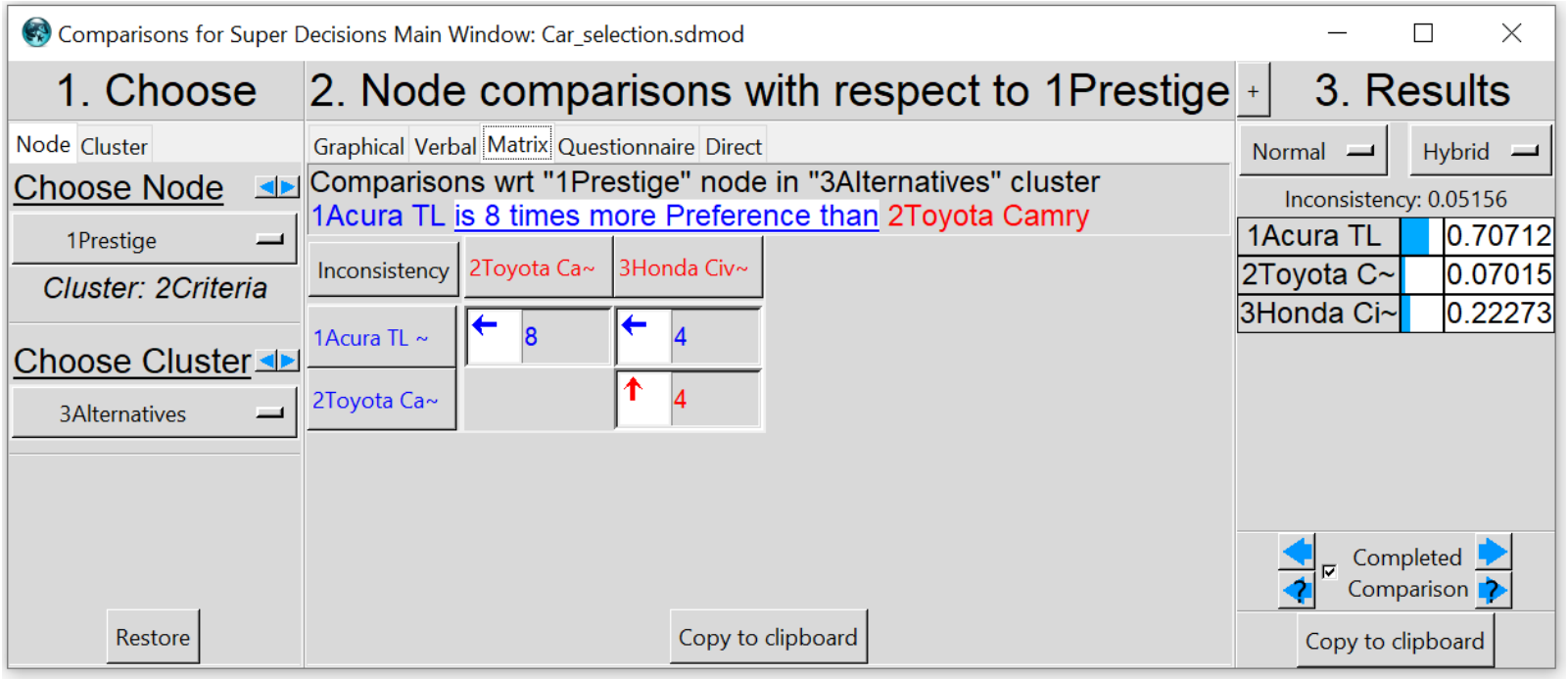




Left click on the dominance phrase itself to get the menu of possible phrases and select the most appropriate word.

Build AHP using SuperDecisions Software

- Only 3 judgements are necessary, AHP reciprocals are shown in **RED** in SuperDecisions



Build AHP using SuperDecisions Software

- Inconsistency button can click in the top left corner of matrix and select the Basic Consistency Report.
- Prestige versus MPG has a **red (3.00)** meaning that MPG are more important than Prestige but the Best Value of 1.05 (in blue) means Prestige is a little more important than MPG and the inconsistency would be improve down to 0.01 if that were the judgement.
- Return to the matrix and may input new value. Possible to use the suggested value or leave it as it is and proceed to the next number and may change the value, and so on.

Inconsistency Report

Rank	Row	Col	Current Val	Best Val	Old Inconsist.	New Inconsist.	% Improvement
1.	1Prestige	3MPG	3.000003	1.055480	0.076853	0.011444	85.11 %
2.	3MPG	4Comfort	3.000300	1.116959	0.076853	0.031811	58.61 %
3.	1Prestige	4Comfort	2.000000	4.763935	0.076853	0.036958	51.91 %
4.	1Prestige	2Price	4.000000	4.595083	0.076853	0.074284	3.34 %
5.	2Price	3MPG	3.000000	2.160580	0.076853	0.075626	1.60 %
6.	2Price	4Comfort	1.500000	1.281717	0.076853	0.077146	-0.38 %

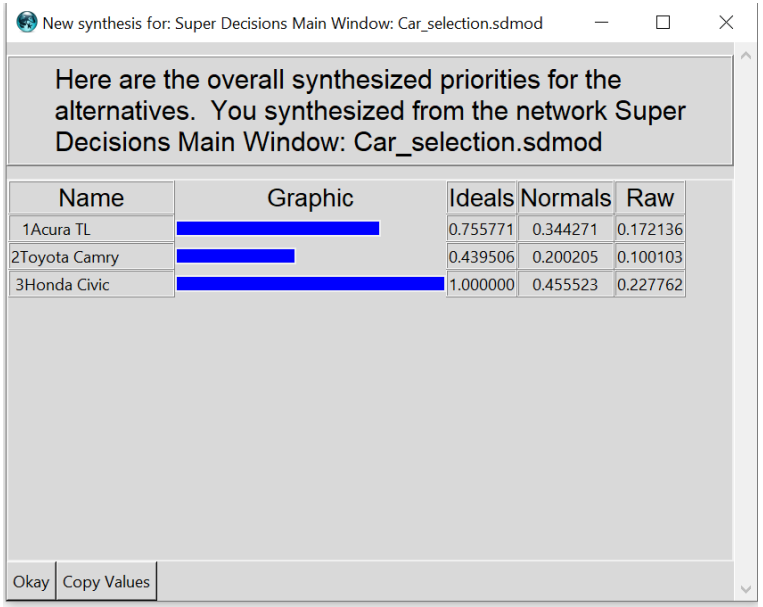
Build AHP using SuperDecisions Software

- The Unweighted Super Matrix after all Judgments completed.

Super Decisions Main Window: Car_selection.sdmod: Unweighted Super Matrix

Cluster Node Labels		1Goal	2Criteria				3Alternatives		
		Goal Node	1Prestige	2Price	3MPG	4Comfort	1Acura TL	2Toyota Camry	3Honda Civic
1Goal	Goal Node	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2Criteria	1Prestige	0.098689	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	2Price	0.424972	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	3MPG	0.168571	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	4Comfort	0.307768	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
3Alternatives	1Acura TL	0.000000	0.707117	0.063252	0.181821	0.704936	0.000000	0.000000	0.000000
	2Toyota Camry	0.000000	0.070155	0.193882	0.272723	0.210920	0.000000	0.000000	0.000000
	3Honda Civic	0.000000	0.222728	0.742867	0.545455	0.084144	0.000000	0.000000	0.000000

Done

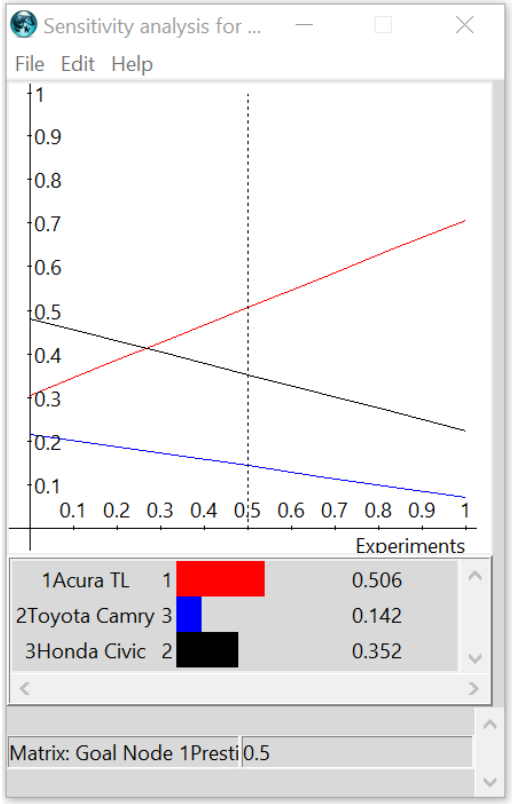


- Synthesize to get Overall Results
- The **RAW values** come from the Limit Super Matrix.
- The **NORMALIZED values** obtained form them by summing and dividing each by the sum.
- The **IDEALS values** are obtained by dividing the Raw values by the largest Raw value.

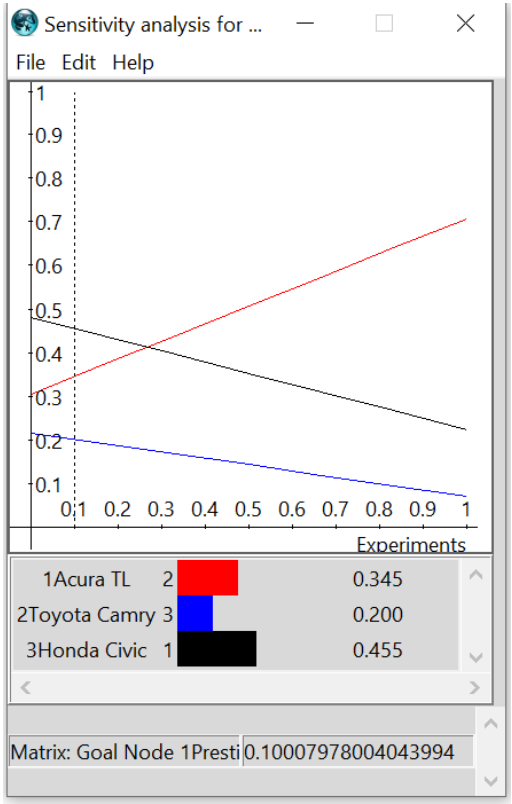
Build AHP using SuperDecisions Software

- To do graphical sensitivity select the Computations > Sensitivity
- The first graph that appears has the first node, alphabetically, selected as the with respect to node. **It is generally not the one you want.**
- You can edit in Select Edit > Independent Variable
- To get the Sensitivity input selector box and change the Independent Variable to the Goal

At Prestige = 50%, Acura is Best

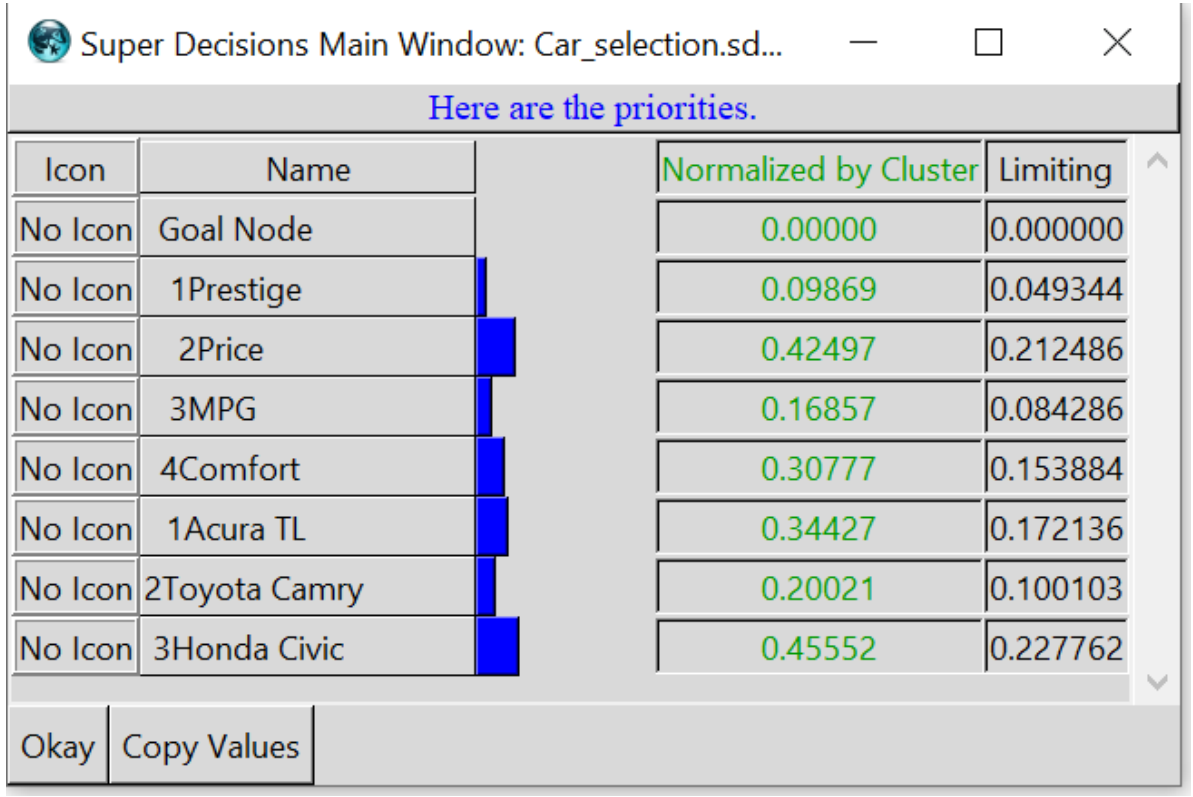


At Prestige = 10%, Civic is Best



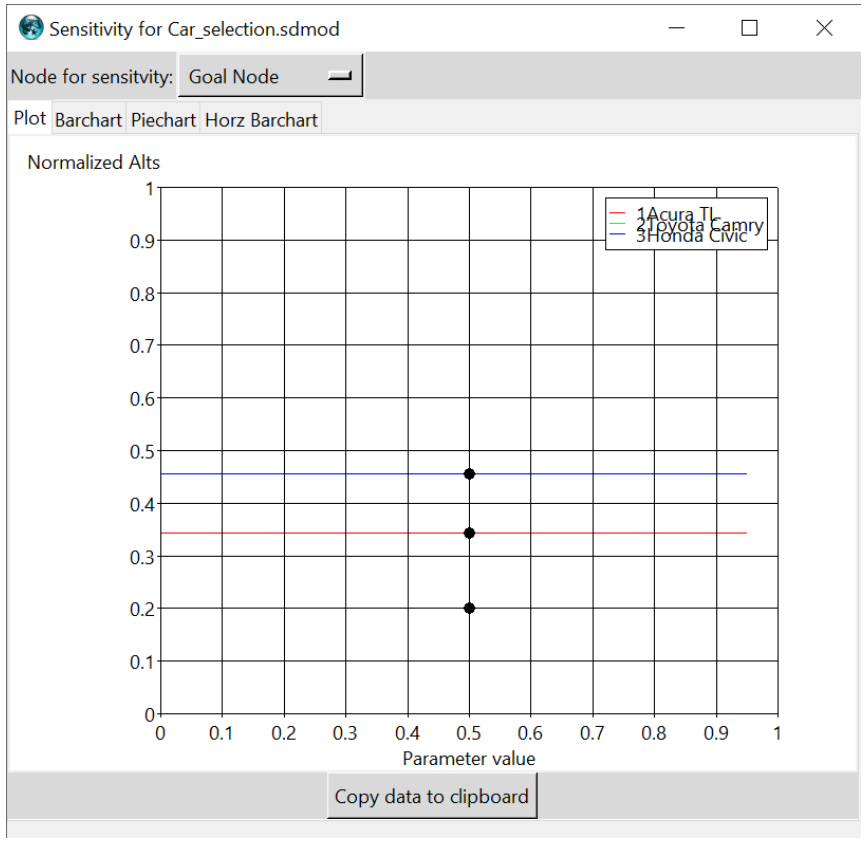
Build AHP using SuperDecisions Software

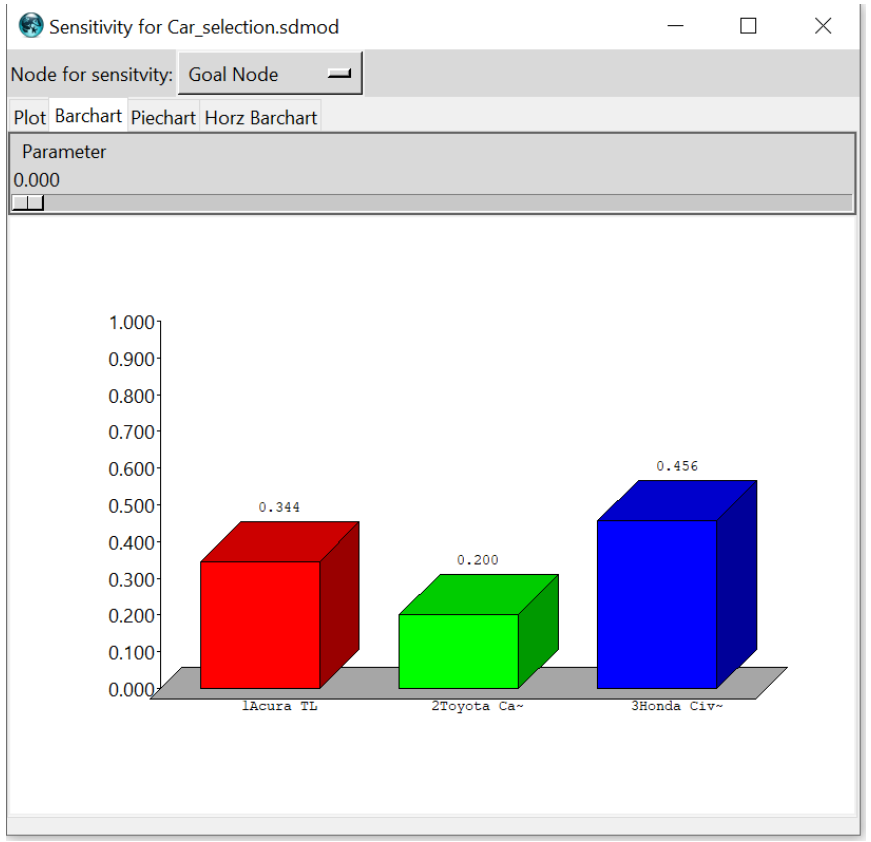
- Priorities of all nodes in model
- Select Computations > Priorities command to see the priorities of all nodes in model
- **Limiting Priority** column shows priority of Prestige compared to all the other nodes in the entire model.
- **Normalized by Cluster** column shows the priority of Prestige (0.098) compared to the other criteria in its cluster



Build AHP using SuperDecisions Software

- Dynamic Sensitivity
- Select Computations > Node Sensitivity





Thank You

AT84.02: Business Intelligence and Analytics | January 2021

Asian Institute of Technology

ENGINEERING AND TECHNOLOGY