

Assignment #4

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Question 1.

Question 2.

$$(a) \begin{cases} q_0 = \epsilon q_1 + bq_2 \\ q_1 = aq_1 + \epsilon q_2 + \epsilon \\ q_2 = \epsilon \end{cases}$$

- Substitute q_2 into the second equation

$$\begin{aligned} q_1 &= aq_1 + \epsilon + \epsilon \\ &= a^* \text{ (by Arden's Lemma)} \end{aligned}$$

Question 3.

(a) $G = (V, \Sigma, R, S)$ where:

- $V = \{S\}$
- $\Sigma = \{a, b\}$

Note that $\epsilon \in L_1$, therefore our first rule is $S \rightarrow \epsilon$. To build other strings we have to add 1's into the end and twice this number of 0's at the beginning of the string, then the second rule is $S \rightarrow 00S1$.

- $R = \{S \rightarrow \epsilon, S \rightarrow 00S1\}$

(b) $G = (V, \Sigma, R, S)$ where:

- $V = \{S\}$
- $\Sigma = \{a, b\}$

Note that $\epsilon \in L_1$, therefore our first rule is $S \rightarrow \epsilon$. To build other strings of this language we have to guarantee that $m \geq n$ and keep m and n both odd or even. Which means that:

- 1 $S \rightarrow aSb$; whenever adding an a you have to add a new b to keep $m \geq n$ and $m - n$ even.
- 2 $S \rightarrow Sbb$; whenever adding a b we need to add two of them to keep $m - n$ even.

Therefore the rules for this language are:

- $R = \{S \rightarrow \epsilon, S \rightarrow Sbb, S \rightarrow aSb\}$

(c) $G = (V, \Sigma, R, S)$ where:

- $V = \{S, S_1, R_1, T_1, S_2, R_2, T_2\}$
- $\Sigma = \{a, b, c, d\}$

Note that the equation $m + n = p + q$ means that for every a or b added we need to add a new c or d . To do that we are going to build the string from the outside to inside adding for every a or b a new d or a new c . You should also note that it's hard to have the pair of rules "for every a add a new c " and "for every b add a new d " because they intersect each other; to deal with this we are going to create rules for two different situations: $m \geq q$ and $m \leq q$.

In the case where $m \geq q$ we are looking for the language $L_1 = \{a^m b^n c^p d^q \mid m + n = p + q, m \geq q\}$ generate the strings with the following rules:

- 1 $S_1 \rightarrow aS_1d \mid R_1$
- 2 $R_1 \rightarrow aR_1c \mid T_1$
- 3 $T_1 \rightarrow bT_1c \mid \epsilon$

We do not need a rule that adds a new d when adding a b because we assumed that $m \geq q$ therefore every d of a string would already have been added by rule 1.

In the case where $m \leq q$ we are looking for the language $L_2 = \{a^m b^n c^p d^q \mid m + n = p + q, m \leq q\}$ generate the strings with the following rules:

- 4 $S_2 \rightarrow aS_2d \mid R_2$
- 5 $R_2 \rightarrow bR_2d \mid T_2$
- 6 $T_2 \rightarrow bT_2c \mid \epsilon$

Similarly we do not need a rule that adds a new c when adding a a because we assumed that $m \leq q$ therefore every a of a string would already have been added by rule 4. Now the last rule we need is the one that makes the union of L_1 and L_2 :

- 7 $S \rightarrow S_1 \mid S_2$

- $R = \{S \rightarrow S_1 \mid S_2, S_1 \rightarrow aS_1d \mid R_1, R_1 \rightarrow aR_1c \mid T_1, T_1 \rightarrow bT_1c \mid \epsilon, S_2 \rightarrow aS_2d \mid R_2, R_2 \rightarrow bR_2d \mid T_2, T_2 \rightarrow bT_2c \mid \epsilon\}$

Question 4.

$G = (V, \Sigma, R, S)$ where:

- $V = \{S, R\}$
- $\Sigma = \{a, b, *, (,), +, \varepsilon\}$; to avoid ambiguity we are going to use the symbol ε to represent the empty string in the regular expressions we want to build.

To create all the regular expressions we are going to use two different variables: R and S . The variable S represents any non-empty regular expression while R can be either S or the empty string ϵ .

- 1 $S \rightarrow aR$
- 2 $S \rightarrow bR$
- 3 $S \rightarrow \varepsilon R$
- 4 $S \rightarrow S^*R$
- 5 $S \rightarrow (S)R$
- 6 $S \rightarrow S + S$
- 7 $R \rightarrow S$
- 8 $R \rightarrow \epsilon$

- $R = \{S \rightarrow aR, S \rightarrow bR, S \rightarrow \varepsilon R, S \rightarrow S^*R, S \rightarrow (S)R, S \rightarrow S + S, R \rightarrow S, R \rightarrow \epsilon\}$