Assignment #2

Gustavo Estrela de Matos
CSCE 433: Formal Languages and Automata

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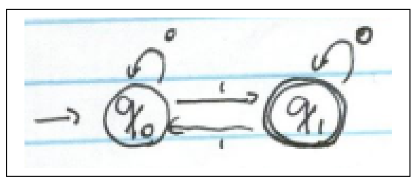
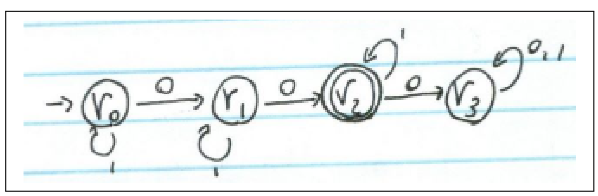
Question 1. Construct the NFA that accepts the language $\{w|w$ contains an odd number of 1's and exactly two 0's$\}$ with exatcly six states.

First let's build a machine that accepts strings with odd number of 1's:

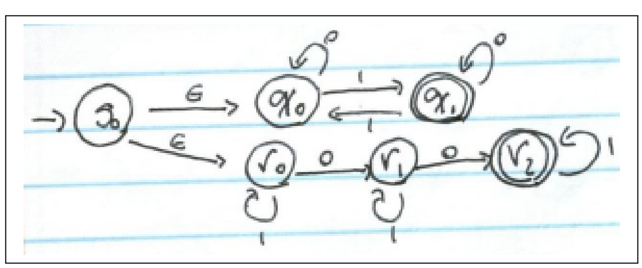


Now we build a machine that accepts strings with exactly 2 zeros:



Notice that we used 6 states already. Then, since we don't need to keep track of strings that go to state r_3, we could simply remove this state, then all the strings with more than 2 zeros would halt on this machine.

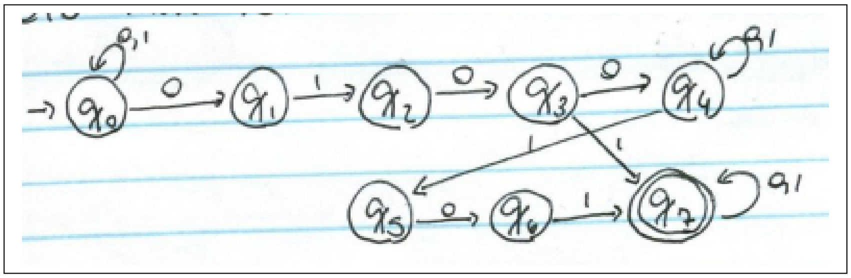
Now we can create a new state that goes to both machines without consuming characters of the string:



Question 2. Construct an NFA that accepts the set of binary strings that contain both substrings 010 and 101.

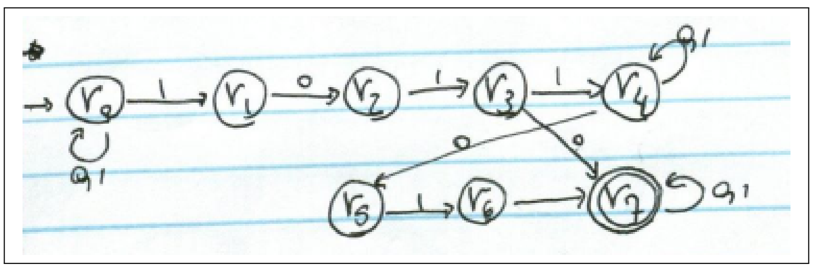
A machine that accepts strings with both substrings 010 and 101 has either the substring 010 or 101 first, then we could build different machines for both cases:

- 010 and then 101

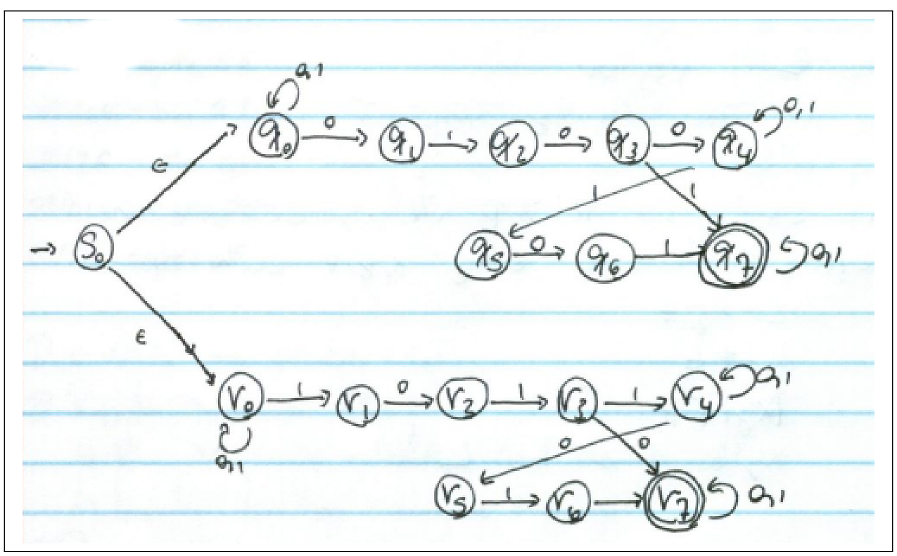


Notice that the machine consider the case in which 010 and 101 overlaps.

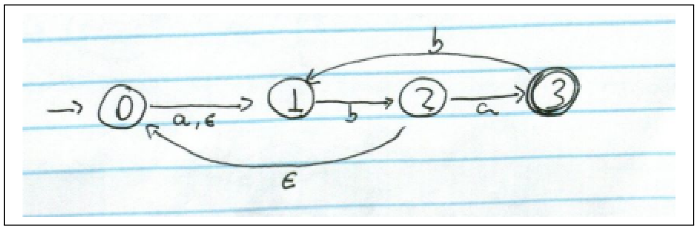
- 101 and then 010



- Which brings us to the final machine:

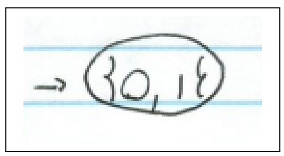


Question 3. Convert the NFA below to a DFA



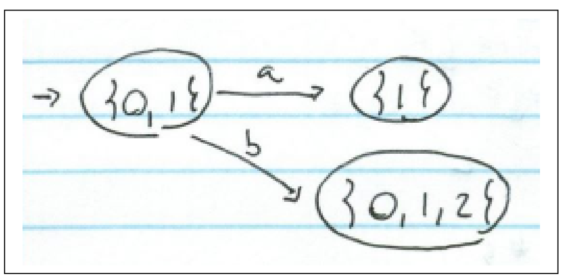
To solve this question we are going to use the same algorithm used to proove that ϵ-NFAs are equivalent to DFAs. We are going to call this NFA $N = (Q, \Sigma, \delta, q, F)$ and build an equivalent DFA $M = (Q', \Sigma, \delta', q', F')$ such that $q' = C_\epsilon(0) = \{0, 1\}$; $\delta' : \mathcal{P}(Q) \times \Sigma \to \mathcal{P}(Q)$ where $\delta'(R, a) = \bigcup_{r \in R} C_\epsilon(\delta(r, a))$ as it follows:

- Start with the initial state



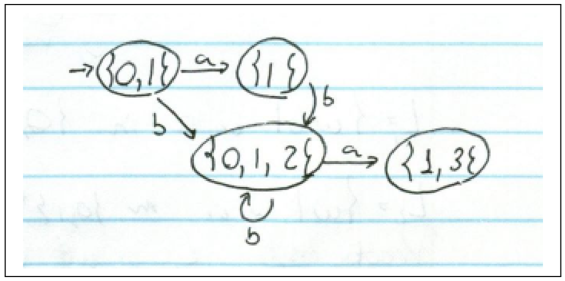
- Calculate the transitions of $\{0, 1\}$

  $\delta'(\{0, 1\}, a) = C_\epsilon(1) \cup \emptyset = \{1\}$
  $\delta'(\{0, 1\}, b) = \emptyset \cup C_\epsilon(2) = \{0, 1, 2\}$

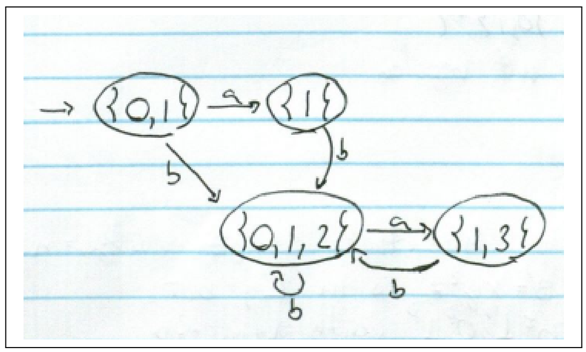


- Calculate the transitions of $\{1\}$ and $\{0, 1, 2\}$

  $\delta'(\{1\}, a) = \emptyset$
  $\delta'(\{1\}, b) = C_\epsilon(2) = \{0, 1, 2\}$
  $\delta'(\{0, 1, 2\}, a) = C_\epsilon(1) \cup \emptyset \cup C_\epsilon(3) = \{1, 3\}$
  $\delta'(\{0, 1, 2\}, b) = \emptyset \cup C_\epsilon(2) \cup \emptyset = \{0, 1, 2\}$



- Calculate the transitions of {1, 3}
  $\delta'(\{1,3\},a) = \emptyset \cup \emptyset = \emptyset$
  $\delta'(\{1,3\},b) = C_\epsilon(2) \cup C_\epsilon(1) = \{0,1,2\}$



Question 4. Prove that for every NFA with an arbitrary number of final states, there is an equivalent NFA with only one final state

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be an NFA with an arbitrary number of final states. We are going to build equivalent NFA $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ with only one final state that is reachable from N_1 final states by ϵ-transitions:

- First we determine F_2, which should composed by just one state, then $F_2 = q_f$.
- The inicial state stays the same $q_2 = q_1$.
- The final state q_f should be the only new state added to N_1, then $Q_2 = Q_1 \cup q_f$
- Now we only need to determine $\delta_2 : Q_2 \times \Sigma \to \mathcal{P}(Q_2)$ such that we keep the dynamics of N_1 and also add the ϵ-transitions from the final states of N_1 to the new final state f_2. Then we define:

$$\delta_2(q,a) = \begin{cases} \delta_1(q,a) \cup \{q_f\} & \text{if } q \in F_1 \text{ and } a = \epsilon \\ \delta_1(q,a) & \text{if } q \in Q_1 \text{ and } (q \notin F_1 \text{ or } a \neq \epsilon) \\ \emptyset & \text{otherwise} \end{cases}$$

Question 5. Give an inductive definition for the set S, of all strings in $\{0,1\}^\star$ with an equal number of 0's and 1's

- Base case: $\epsilon \in S$
- Induction: if $z \in S$, then $01z, 0z1, z01, 10z, 1z0$ and $z10$ are also in S

Question 6. What is wrong with the following proof?

Proposition: $6n = 0$ for all $n \in \mathbb{N}$

Proof. We will prove the above proposition by mathematical induction on $n \geq 0$.

- *Base case:* If $n = 0$, then $6n = 0$.
- *Induction hypothesis:* Suppose that $6n = 0$ for $0 \leq n \leq k$.
- *Induction step:* Let $n = k + 1 = a + b$, where a and b are natural numbers less than $k + 1$. By induction hypothesis, $6a = 0$ and $6b = 0$. Therefore,

$$6n = 6(k + 1) = 6(a + b) = 6a + 6b = 0 + 0 = 0$$

$\square$

The wrong statement in this proof is:

Let $n = k + 1 = a + b$, where a and b are natural numbers less than $k + 1$.

If we choose $n = 1$ we have that it is impossible to find a and b such that $a < 1$ and $b < 1$ and $a + b = 1$. Therefore everything that follows on the existance of a and b can't be guaranteed to be true.