

Assignmentsss #X

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CSCE 433: Formal Languages and Automata

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Question 1. Mathematical Symbols

This is an example of an answer to a homework question. In your answer, you may incorporate a variety of mathematical symbols:

- Fractions: $\frac{2}{3}$, $\frac{n}{10}$, $\frac{x^2+x+5}{x-10}$
- Binomial coefficients: $\binom{5}{2} = 10$
- Subscripts and superscripts: t_0 , t^2 , $t_0^{\frac{2}{3}}$,
- Greek letters: α , β , δ , γ , λ , π .
- Summations: $\sum_{i=1}^n i = \frac{n(n+1)}{2}$.

Question 2. Proving Gauss's Formula by Mathematical Induction

This is another example of a question. In this case, it's a multi-part question.

(a) Consider *Gauss's formula*:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \tag{1}$$

(b)

Proof. We will prove equation 1 by mathematical induction.

- Base case: If $n = 1$, then $1 = \frac{1(2)}{2} = 1$. Base case holds.

- Inductive hypothesis: Assume the equation holds for $2 \leq n \leq k$.
- Inductive step: For $n = k + 1$, we have

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= (k+1) + \sum_{i=1}^k i \\
&= (k+1) + \frac{k(k+1)}{2} \quad (\text{by inductive hypothesis}) \\
&= \frac{2k+2 + k(k+1)}{2} \\
&= \frac{k^2 + 3k + 2}{2} \\
&= \frac{(k+1)(k+2)}{2}
\end{aligned}$$

The last line shows that for $n = k + 1$, Gauss' formula still holds! We've shown that the formula holds for $n = 1$. And, we've shown that if it holds for $n = k$, it must hold for $n = k + 1$. Therefore, the equation must hold for all n . \square

Question 3. Constructing DFA's

- (a) Design a DFA to accept the language $L = \{w : w \text{ is a binary string containing the substring } 01\}$.

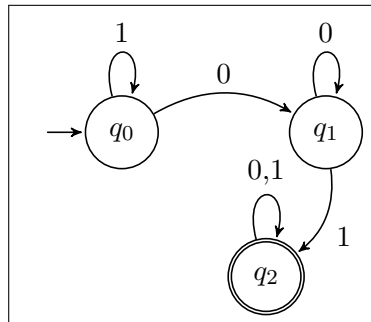


Figure 1: A DFA M that accepts the language L . The diagram was drawn using the `automata` library for the `tikz` package.

- (b) The formal specification of DFA $M = (Q, \Sigma, \delta, q, F)$, where

- $Q = \{q_0, q_1, q_2\}$,
- $\Sigma = \{0, 1\}$,

- $q = q_0$,
- $F = \{q_2\}$, and
- δ is defined as follows.

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_2	q_2

(c) You may hand-draw your figures as long as they are readable and in a format (e.g., pdf, jpg) that \LaTeX can handle.

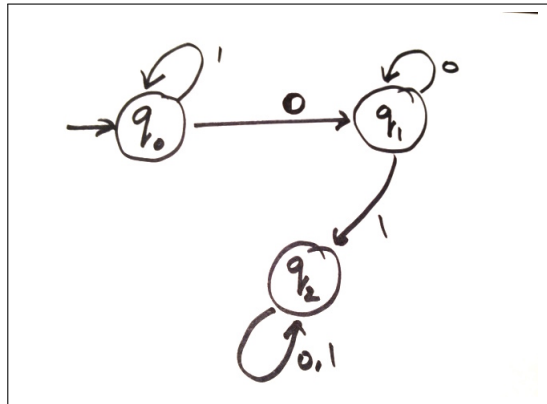


Figure 2: A hand-drawn DFA included into \LaTeX as a JPG image.