# Assignment #1

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**Question 1. Prove by induction that for $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$**

- Base case: For the base case we know that $n = 1$, then:
  $\sum_{i=1}^{1} \frac{1}{2^i} = \frac{1}{2}$; and also $1 - \frac{1}{2} = \frac{1}{2}$.
  Therefore the equation holds for the base case.

- Inductive hypothesis: Assume that the equation holds for $2 \leq n \leq k$ for some integer $k$

- Inductive step: For the case where $n = k + 1$, we have that:

$$\begin{aligned}\sum_{i=1}^{k+1} \frac{1}{2^i} &= \sum_{i=1}^{k} \frac{1}{2^i} + \frac{1}{2^{k+1}} \\ &= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} \text{ (by inductive hypothesis)} \\ &= 1 - (\frac{1}{2^k})(1 - \frac{1}{2}) \\ &= 1 - \frac{1}{2^{k+1}}\end{aligned}$$

  Then, if the equations holds for $k$ it also holds for $k + 1$. Now by induction, using the base case, inductive hypothesis and inductive step, we have that for all $n \geq 1$ the equation holds, as we wanted to proof.

**Question 2. Show that any integer postage greater than 7 cents can be formed by using only 3-cent and 5-cent stamps**

Basically, what we want to proof is that, for any $n > 7$, $n$ can be written as

$$n = 3a + 5b \tag{1}$$

where $a$ and $b$ non negative numbers. Let $k$ be an integer such that $k > 7$ and take $k \mod 3$:

- if $k \equiv 0 \mod 3$:
  Then, for some integer $q \geq 0$, $k = 3q$, so equation 1 holds.

- if $k \equiv 1 \mod 3$:
  We have that
  $$k - 10 \equiv 1 - 10 \mod 3$$
  $$k - 10 \equiv 0 \mod 3$$
  and, since $k > 7$ and $k \equiv 1 \mod 3$ we have that $k \geq 10$, hence $k - 10 \geq 0$ and there is $q \geq$ integer such that
  $$k - 10 = 3q$$
  $$k = 3(q) + 5(2)$$
  Therefore, equation 1 also holds for this case.

- if $k \equiv 2 \mod 3$:
  We have that
  $$k - 5 \equiv 2 - 5 \mod 3$$
  $$k - 5 \equiv 0 \mod 3$$
  and, since $k > 7$, there is $q \geq 0$ integer such that
  $$k - 5 = 3q$$
  $$k = 3(q) + 5(1)$$

**Question 3. Constructing DFA's**