Causal Spillover Effects Using Instrumental Variables: Supplemental Appendix

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April 23, 2021

Abstract

This supplemental appendix provides the proofs of the results in the paper and additional results and discussions not included in the paper to conserve space.

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A1 Additional Identification Results

A1.1 Indirect ITT Effects

The following result characterizes the indirect ITT effect.

Lemma A1 *Under Assumptions 1-3,*

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=1]-\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]= \\
&\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|\{SC_{ig}\}\times\{GC_{jg},NT_{jg}\}] \\
&\quad\times\mathbb{P}[\{SC_{ig}\}\times\{GC_{jg},NT_{jg}\}] \\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,0)|\{SC_{ig}\}\times\{SC_{jg},C_{jg}\}] \\
&\quad\times\mathbb{P}[\{SC_{ig}\}\times\{SC_{jg},C_{jg}\}] \\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,1)|SC_{ig},AT_{jg}] \\
&\quad\times\mathbb{P}[SC_{ig},AT_{jg}] \\
&+\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|\{C_{ig},GC_{ig},NT_{ig}\}\times\{SC_{jg},C_{jg}\}] \\
&\quad\times\mathbb{P}[\{C_{ig},GC_{ig},NT_{ig}\}\times\{SC_{jg},C_{jg}\}] \\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)|\{AT_{ig}\}\times\{SC_{jg},C_{jg}\}] \\
&\quad\times\mathbb{P}[\{AT_{ig}\}\times\{SC_{jg},C_{jg}\}].
\end{aligned}
$$

A1.2 Total ITT Effects

The following result characterizes the total ITT effect.

Lemma A2 *Under Assumptions 1-3*

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=1]-\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]= \\
&\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|\{SC_{ig},C_{ig},GC_{ig}\}\times\{NT_{jg}\}] \\
&\quad\times\mathbb{P}[\{SC_{ig},C_{ig},GC_{ig}\}\times\{NT_{jg}\}] \\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)|\{AT_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}] \\
&\quad\times\mathbb{P}[\{AT_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}] \\
&+\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|\{NT_{ig}\},\{SC_{ig},C_{jg},GC_{jg}\}] \\
&\quad\times\mathbb{P}[\{NT_{ig}\},\{SC_{ig},C_{jg},GC_{jg}\}] \\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,1)|\{SC_{ig},C_{ig},GC_{ig}\}\times\{AT_{jg}\}] \\
&\quad\times\mathbb{P}[\{SC_{ig},C_{ig},GC_{ig}\}\times\{AT_{jg}\}] \\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,0)|\{SC_{ig},C_{ig},GC_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}] \\
&\quad\times\mathbb{P}[\{SC_{ig},C_{ig},GC_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}].
\end{aligned}
$$

Table A1: System of equations

D_{ig}	D_{jg}	Z_{ig}	Z_{jg}	$Y_{ig}(d,d')$	ξ_{ig}	ξ_{jg}
1	1	0	0	$Y_{ig}(1,1)$	AT	AT
1	1	0	1	$Y_{ig}(1,1)$	AT,SC	AT,SC,C
1	1	1	0	$Y_{ig}(1,1)$	AT,SC,C	AT,SC
1	1	1	1	$Y_{ig}(1,1)$	AT,SC,C,GC	AT,SC,C,GC
0	0	1	1	$Y_{ig}(0,0)$	NT	NT
0	0	1	0	$Y_{ig}(0,0)$	GC,NT	C,GC,NT
0	0	0	1	$Y_{ig}(0,0)$	C,GC,NT	GC,NT
0	0	0	0	$Y_{ig}(0,0)$	SC,C,GC,NT	SC,C,GC,NT
1	0	0	0	$Y_{ig}(1,0)$	AT	SC,C,GC,NT
1	0	0	1	$Y_{ig}(1,0)$	AT,SC	GC,NT
1	0	1	0	$Y_{ig}(1,0)$	AT,SC,C	C,GC,NT
1	0	1	1	$Y_{ig}(1,0)$	AT,SC,C,GC	NT
0	1	1	1	$Y_{ig}(0,1)$	NT	AT,SC,C,GC
0	1	1	0	$Y_{ig}(0,1)$	GC,NT	AT,SC
0	1	0	1	$Y_{ig}(0,1)$	C,GC,NT	AT,SC,C
0	1	0	0	$Y_{ig}(0,1)$	SC,C,GC,NT	AT

A1.3 Identification Under Monotonicity

In the absence of spillovers, Imbens and Rubin (1997) show that different combinations of D_{ig} and Z_{ig} can be exploited to identify average potential outcomes for compliers. The intuition behind this approach is that a unit with $D_{ig} = 1$ and $Z_{ig} = 0$ is necessarily an always-taker, whereas a unit with $D_{ig} = 1$ and $Z_{ig} = 1$ can be an always-taker or a complier, and hence the combination of these two cases identifies $\mathbb{E}[Y_{ig(1)}|C_{ig}]$ (and an analogous argument implies identification of $\mathbb{E}[Y_{ig}(0)|C_{ig}]$). To see why does approach does not work in the presence of spillovers, notice that a unit with $D_{ig} = 1, D_{jg} = 1, Z_{ig} = 0, Z_{jg} = 0$ is an always-taker with an always-taker peer. However, a unit with $D_{ig} = 1, D_{jg} = 1, Z_{ig} = 1, Z_{jg} = 0$ could be an always-taker, a social complier or a complier, and her peer could be an always-taker or a social complier, and it is not possible to disentangle each unit's compliance type.

Table A1 shows what can be identified under monotonicity without further restrictions by exploiting the variation in $(D_{ig}, D_{jg}, Z_{ig}, Z_{jg})$. For instance, the first row in the table indicates that:

$$\mathbb{E}[Y_{ig}|D_{ig} = 1, D_{jg} = 1, Z_{ig} = 0, Z_{jg} = 0] = \mathbb{E}[Y_{ig}(1,1)|AT_{ig}, AT_{jg}].$$

The table shows that, without further assumptions, is it not possible to point identify average potential outcomes for specific compliance types, with the exception of $\mathbb{E}[Y_{ig}(1,1)|AT_{ig}, AT_{jg}]$ and $\mathbb{E}[Y_{ig}(0,0)|NT_{ig}, NT_{jg}]$.

A1.4 Multiple Treatment Levels

The identification results in the paper can be extended to the case of multi-level treatments. To adapt the notation to this case, suppose that $D_{ig} \in \mathcal{D} = \{0, 1, \ldots, K\}$. The potential outcome is $Y_{ig}(k, k')$ (which implicitly imposes the exclusion restriction) where $k, k' \in \{0, 1, \ldots, K\}$ indicate different treatment levels. The observed outcome is:

$$Y_{ig} = \sum_{k \in \mathcal{D}} \sum_{k' \in \mathcal{D}} Y_{ig}(k, k') \mathbb{1}(D_{ig} = k) \mathbb{1}(D_{jg} = k').$$

Suppose that the instrument Z_{ig} takes as many values as the treatment, that is, $Z_{ig} \in \mathcal{Z}$ where $\mathcal{Z} = \mathcal{D}$. For example, Z_{ig} could indicate random assignment into different treatment levels, and D_{ig} indicates whether unit i actually receives the assigned treatment level. The potential treatment status is $D_{ig}(k, k')$ where $k, k' \in \mathcal{Z}$. Each unit i can have up to $(K+1)^2$ different potential treatment statuses given by own and peer's treatment assignment.

I will assume that non-compliance is one-sided, and that units cannot switch between non-control treatment statuses. For example, in Foos and de Rooij (2017), the treatment assignment consists of three treatment levels: control, low-intensity treatment and high-intensity treatment. In this setting, units may refuse the treatment they are assigned, but units assigned to the low-intensity treatment cannot receive the high-intensity treatment and vice versa.

Proposition A1 *Suppose that, in addition to Assumption 2, $\mathcal{D} = \mathcal{Z} = \{0, 1, 2, \ldots, K\}$, and:*

(a) $D_{ig}(0, k') = 0$ for all $k' \in \mathcal{Z}$,

(b) $D_{ig}(k, k') \in \{0, k\}$ for all $k, k' \in \mathcal{Z}$.

Then, for any k, $k' \in \{0, 1, \ldots, K\}$ such that $\mathbb{E}[\mathbb{1}(D_{ig}(k, 0) = k)] > 0$ and $\mathbb{E}[\mathbb{1}(D_{jg}(0, k') = k')] > 0$,

$$\mathbb{E}[\mathbb{1}(D_{ig} = k)|Z_{ig} = k, Z_{jg} = 0] = \mathbb{E}[\mathbb{1}(D_{ig}(k, 0) = k)]$$
$$\frac{\mathbb{E}[Y_{ig}|Z_{ig} = k, Z_{jg} = 0] - \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0]}{\mathbb{E}[\mathbb{1}(D_{ig} = k)|Z_{ig} = k, Z_{jg} = 0]} = \mathbb{E}[Y_{ig}(k, 0) - Y_{ig}(0, 0)|D_{ig}(k, 0) = k]$$
$$\frac{\mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = k'] - \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0]}{\mathbb{E}[\mathbb{1}(D_{jg} = k')|Z_{ig} = 0, Z_{jg} = k']} = \mathbb{E}[Y_{ig}(0, k') - Y_{ig}(0, 0)|D_{jg}(k', 0) = k'].$$

Condition (a) in Proposition A1 implies that units who are assigned to the control condition remain untreated, and Condition (b) states that a unit who is offered treatment level k can either receive that treatment level or remain untreated. This result shows how to identify the proportion of units who comply with treatment level k, $\mathbb{E}[\mathbb{1}(D_{ig}(k, 0) = k)]$, the average direct effect on units who comply with treatment level k, $\mathbb{E}[Y_{ig}(k, 0) - Y_{ig}(0, 0)|D_{ig}(k, 0) = k]$, and the average spillover effect on units whose peer complies with treatment level k'.

A2 Further Details on Estimation and Inference

All the parameters of interest in Section 4 can be recovered by estimating expectations using sample means. More precisely, let

$$\mathbb{1}^{\mathbf{z}}_{ig} = \begin{bmatrix} \mathbb{1}(Z_{ig}=0, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=1, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=0, Z_{jg}=1) \\ \mathbb{1}(Z_{ig}=1, Z_{jg}=1) \end{bmatrix}$$

and let $\mathbf{H}(\cdot)$ be a vector-valued function whose exact shape depends on the parameters to be estimated, as illustrated below. Then the goal is to estimate:

$$\boldsymbol{\mu} = \mathbb{E}\begin{bmatrix} \mathbb{1}^{\mathbf{z}}_{ig} \\ \mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) \otimes \mathbb{1}^{\mathbf{z}}_{ig} \end{bmatrix}$$

where the first four elements correspond to the assignment probabilities $\mathbb{P}[Z_{ig}=z, Z_{jg}=z']$ and the remaining elements corresponds to estimands of the form $\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig}=d, D_{jg}=d')\mathbb{1}(Z_{ig}=z, Z_{jg}=z')]$. The most general choice of $\mathbf{H}$ in this setup is the following:

$$\mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) = \begin{bmatrix} \mathbb{1}(D_{ig}=0, D_{jg}=0) \\ \vdots \\ \mathbb{1}(D_{ig}=1, D_{jg}=1) \\ Y_{ig}\mathbb{1}(D_{ig}=0, D_{jg}=0) \\ \vdots \\ Y_{ig}\mathbb{1}(D_{ig}=1, D_{jg}=1) \end{bmatrix}$$

which is a vector of dimension equal to eight that can be used to estimate all the first-stage estimands $\mathbb{E}[\mathbb{1}(D_{ig}=d, D_{jg}=d')|Z_{ig}=z, Z_{jg}=z']$ and average outcomes $\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig}=d, D_{jg}=d')|Z_{ig}=z, Z_{jg}=z']$. In this general case, the total number of equations to be estimated is 36: four probabilities $\mathbb{P}[Z_{ig}=z, Z_{jg}=z']$ plus the four indicators $\mathbb{1}(Z_{ig}=z, Z_{jg}=z')$ times each of the eight elements in $\mathbf{H}(\cdot)$. The dimension of $\mathbf{H}(\cdot)$ can be reduced, for example, by focusing on ITT parameters $\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z']$, which corresponds to:

$$\mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) = Y_{ig},$$

or by imposing the assumptions described in previous sections. For instance, under one-sided noncompliance, the parameters in Corollaries 1 and 2 can be estimated by defining:

$$\mathbb{1}^{\mathbf{z}}_{ig} = \begin{bmatrix} \mathbb{1}(Z_{ig}=0, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=1, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=0, Z_{jg}=1) \end{bmatrix}$$

and

$$\mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) = \begin{bmatrix} D_{ig} \\ Y_{ig} \\ Y_{ig}(1 - D_{ig}) \\ Y_{ig}(1 - D_{jg}) \end{bmatrix}.$$

Regardless of the choice of $\mathbb{1}^{\mathbf{z}}_{ig}$ and $\mathbf{H}(\cdot)$, the dimension of the vector of parameters to be estimated is fixed (and at most 36). Consider the following sample mean estimator:

$$\hat{\boldsymbol{\mu}} = \frac{1}{G}\sum_{g=1}^{G} W_g$$

where

$$W_g = \begin{bmatrix} (\mathbb{1}^{\mathbf{z}}_{1g} + \mathbb{1}^{\mathbf{z}}_{2g})/2 \\ (\mathbf{H}(Y_{1g}, D_{1g}, D_{2g}) \otimes \mathbb{1}^{\mathbf{z}}_{1g} + \mathbf{H}(Y_{2g}, D_{2g}, D_{1g}) \otimes \mathbb{1}^{\mathbf{z}}_{2g})/2 \end{bmatrix}.$$

It is straightforward to see that under assumption 5, $\hat{\boldsymbol{\mu}}$ is consistent for $\boldsymbol{\mu}$ and converges in distribution to a normal random variable after centering and rescaling as $G \to \infty$:

$$\sqrt{G}(\hat{\boldsymbol{\mu}} - \boldsymbol{\mu}) \xrightarrow{\mathcal{D}} \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$$

where $\boldsymbol{\Sigma} = \mathbb{E}[(W_g - \boldsymbol{\mu})(W_g - \boldsymbol{\mu})']$, and where the limiting variance can be consistently estimated by:

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{G}\sum_{g}(W_g - \hat{\boldsymbol{\mu}})(W_g - \hat{\boldsymbol{\mu}})'.$$

Finally, once $\hat{\boldsymbol{\mu}}$ has been estimated, the treatment effects of interest can be estimated as (possibly nonlinear) transformations of $\hat{\boldsymbol{\mu}}$, and their variance estimated using the delta method.

A3 Further Details on AR confidence intervals

In this section I outline the procedure to construct weak-instrument-robust confidence intervals for the direct effect on compliers (the procedure for the spillover effect is analogous). Given some hypothesized value β_1^* for this effect, consider the following regression on the subsample of units with untreated peers:

$$Y_{ig} - \beta_1^* D_{ig} = \theta_0 + \theta_1 Z_{ig} + \epsilon_{ig}.$$

Since Z_{ig} is binary, $\hat{\theta}_1 = \hat{\gamma}_1 - \beta_1^* \bar{D}_{10}$ where $\bar{D}_{10} = \sum_{g,i} D_{ig} Z_{ig}(1-Z_{jg})/\sum_{g,i} Z_{ig}(1-Z_{jg})$ is the first-stage estimate and $\hat{\gamma}_1$ is the reduced-form estimate $\sum_{g,i} Y_{ig} Z_{ig}(1 - Z_{jg})/\sum_{g,i} Z_{ig}(1 - Z_{jg}) - \sum_{g,i} Y_{ig}(1 - Z_{ig})(1 - Z_{jg})/\sum_{g,i}(1 - Z_{ig})(1 - Z_{jg})$. By previous results, as $G \to \infty$, $\hat{\theta}_1 \to_{\mathbb{P}} (\mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)|C_{ig}] - \beta_1^*)\,\mathbb{P}[C_{ig}]$ and thus under the null hypothesis that

$\mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)|C_{ig}] = \beta_1^*$, $\theta_1 = 0$ and:

$$\frac{\hat{\theta}_1^2}{V(\hat{\theta}_1)} \rightarrow_{\mathcal{D}} \chi_1^2$$

where χ_1^2 is the chi-squared distribution and $V(\hat{\theta}_1)$ is the variance of $\hat{\theta}_1$. Noting that both $\hat{\theta}_1$ and its variance depend on β_1^*, an AR confidence interval for $\mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)|C_{ig}]$ is given by:

$$\{\beta_1^* : \hat{\theta}_1^2 \leq V(\hat{\theta}_1)\chi^2_{1,1-\alpha}\}.$$

Finally, since $\hat{\theta}_1 = \hat{\gamma}_1 - \beta_1^* \bar{D}_{10}$, $V(\hat{\theta}_1) = V(\hat{\gamma}_1) + (\beta_1^*)^2 V(\bar{D}_{10}) - 2\beta_1^* Cov(\hat{\gamma}_1, \bar{D}_{10})$. Therefore, the AR confidence interval can be obtained by solving the inequality:

$$(\beta_1^*)^2 \left[\bar{D}_{10}^2 - \chi^2_{1,1-\alpha} V(\bar{D}_{10})\right] + 2\beta_1^* \left[\chi^2_{1,1-\alpha} Cov(\hat{\gamma}_1, \bar{D}_{10}) - \hat{\gamma}_1 \bar{D}_{10}\right] + \hat{\gamma}_1^2 - \chi^2_{1,1-\alpha} V(\hat{\gamma}_1) \leq 0$$

which depends only on the vector of estimated coefficients, their variance matrix and a quantile from the χ_1^2 distribution. In particular, notice that this quadratic function is strictly convex whenever $\bar{D}_{10}^2 / V(\bar{D}_{10}) > \chi^2_{1,1-\alpha}$, which holds when the null hypothesis the first-stage coefficient is zero is rejected at the α level. In this case, the AR confidence interval is bounded and convex.

A4 Additional Empirical Results

The experiment conducted by Foos and de Rooij (2017) included two different treatment levels. More precisely, the sample of 5,190 two-voter households with landline numbers were stratified into three blocks based on their last recorded party preference (Labor party supporter, rival party supported, unattached) and randomly assigned to one of three treatment arms:

- High-intensity treatment: the telephone message had a strong partisan tone, explicitly mentioning the Labour party and policies, taking an antagonistic stance toward the main rival party.
- Low-intensity treatment: the telephone message avoided statements about party competition and did not mention the candidate's affiliation nor the rival party.
- Control: did not receive any form of contact from the campaign.

Finally, within the households assigned to the low- or high-intensity treatment arms, only one household member was randomly selected to receive the telephone message.

In this section, I apply the results from Proposition A1 to analyze the effect of each treatment arm by separately comparing households exposed to each treatment intensity to the control households. The empirical results are shown in Table A2. The estimates suggest that both treatment arms had very similar effects.

Table A2: Estimation Results with Multiple Treatments

	Low-intensity treatment	High-intensity treatment
ITT		
Z_{ig}	0.0300	0.0310
	(0.0145)	(0.0146)
	[0.0016 , 0.0584]	[0.0023 , 0.0597]
Z_{jg}	0.0485	0.0433
	(0.0148)	(0.0149)
	[0.0195 , 0.0775]	[0.0142 , 0.0724]
2SLS		
D_{ig}	0.0662	0.0688
	(0.0318)	(0.0323)
	[0.0039 , 0.1285]	[0.0055 , 0.1322]
D_{jg}	0.1070	0.0962
	(0.0324)	(0.0328)
	[0.0435 , 0.1706]	[0.0318 , 0.1606]
N	7,750	7,696
Clusters	3,875	3,848

Notes: estimated results from reduced-form regressions ("ITT") and 2SLS regressions ("2SLS") separately for each treatment arm against the control households. The first column shows the ITT and 2SLS results for the low-intensity treatment arm. The second column shows the ITT and 2SLS results for the high-intensity treatment arm. Standard errors in parentheses. 95%-confidence intervals are based on the large-sample normal approximation. Estimation accounts for clustering at the household level.

A5 Proofs of Main Results

A5.1 Proof of Proposition 1

By Assumption 2, $\mathbb{E}[D_{ig}|Z_{ig} = z, Z_{jg} = z'] = \mathbb{E}[D_{ig}(z, z')]$. Thus, under monotonicity (Assumption 3),

$$\begin{aligned}
\mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 0] &= \mathbb{E}[D_{ig}(0,0)] = \mathbb{P}[AT_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 1] &= \mathbb{E}[D_{ig}(0,1)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 0] &= \mathbb{E}[D_{ig}(1,0)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] + \mathbb{P}[C_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 1] &= \mathbb{E}[D_{ig}(1,1)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] + \mathbb{P}[C_{ig}] + \mathbb{P}[GC_{ig}]
\end{aligned}$$

and by solving the system it follows that:

$$\begin{aligned}
\mathbb{P}[AT_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 0] \\
\mathbb{P}[SC_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 1] - \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 0] \\
\mathbb{P}[C_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 0] - \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 1] \\
\mathbb{P}[GC_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 1] - \mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 0]
\end{aligned}$$

and by monotonicity $\mathbb{P}[NT_{ig}] = 1 - \mathbb{P}[AT_{ig}] - \mathbb{P}[SC_{ig}] - \mathbb{P}[C_{ig}] - \mathbb{P}[GC_{ig}]$. Finally,

$$\begin{aligned}
\mathbb{E}[D_{ig}D_{jg}|Z_{ig} = 0, Z_{ig} = 0] &= \mathbb{E}[D_{ig}(0,0)D_{jg}(0,0)] = \mathbb{P}[AT_{ig}, AT_{jg}] \\
\mathbb{E}[(1 - D_{ig})(1 - D_{jg})|Z_{ig} = 1, Z_{ig} = 1] &= \mathbb{E}[(1 - D_{ig}(1,1))(1 - D_{jg}(1,1))] = \mathbb{P}[NT_{ig}, NT_{jg}].
\end{aligned}$$

See Tables A3 and A4 for the whole system of equations. □

A5.2 Proof of Lemma 1

Using that:

$$\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig} = z, Z_{jg} = z'] = \mathbb{E}[Y_{ig}(0,0)] \\
&+ \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))D_{ig}(z,z')(1 - D_{jg}(z',z))] \\
&+ \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))(1 - D_{ig}(z,z'))D_{jg}(z',z)] \\
&+ \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)],
\end{aligned}$$

Table A3: System of equations

D_{ig}	D_{jg}	Z_{ig}	Z_{jg}	Probabilities
1	1	0	0	p_{AA}
1	1	0	1	$p_{AA}+p_{AS}+p_{AC}+p_{SA}+p_{SS}+p_{SC}$
1	1	1	0	$p_{AA}+p_{AS}+p_{AC}+p_{SA}+p_{SS}+p_{SC}$
1	1	1	1	$1-2p_N+p_{NN}$
0	0	1	1	p_{NN}
0	0	1	0	$p_{GC}+p_{GG}+p_{GN}+p_{NC}+p_{NG}+p_{NN}$
0	0	0	1	$p_{GC}+p_{GG}+p_{GN}+p_{NC}+p_{NG}+p_{NN}$
0	0	0	0	$1-2p_A+p_{AA}$
1	0	0	0	$p_{AS}+p_{AC}+p_{AG}+p_{AN}$
1	0	1	1	$p_{NA}+p_{NS}+p_{NC}+p_{NG}$
1	0	0	1	$p_{AG}+p_{AN}+p_{SG}+p_{SN}$
1	0	1	0	$p_{AC}+p_{AG}+p_{AN}+p_{SC}+p_{SG}+p_{SN}+p_{CC}+p_{CG}+p_{CN}$
0	1	0	0	$p_{AS}+p_{AC}+p_{AG}+p_{AN}$
0	1	1	1	$p_{NA}+p_{NS}+p_{NC}+p_{NG}$
0	1	1	0	$p_{AG}+p_{AN}+p_{SG}+p_{SN}$
0	1	0	1	$p_{AC}+p_{AG}+p_{AN}+p_{SC}+p_{SG}+p_{SN}+p_{CC}+p_{CG}+p_{CN}$

Table A4: System of equations - simplified

D_{ig}	D_{jg}	Z_{ig}	Z_{jg}	Probabilities	Independent?
1	1	0	0	p_{AA}	1
1	1	0	1	$p_A+p_S-(p_{AG}+p_{AN}+p_{SG}+p_{SN})$	2
1	1	1	0	$p_A+p_S-(p_{AG}+p_{AN}+p_{SG}+p_{SN})$	-
1	1	1	1	$1-2p_N+p_{NN}$	3
0	0	1	1	p_{NN}	4
0	0	1	0	$p_G+p_N-(p_{AG}+p_{AN}+p_{SG}+p_{SN})$	5
0	0	0	1	$p_G+p_N-(p_{AG}+p_{AN}+p_{SG}+p_{SN})$	-
0	0	0	0	$1-2p_A+p_{AA}$	6
1	0	0	0	p_A-p_{AA}	-
1	0	1	1	p_N-p_{NN}	-
1	0	0	1	$p_{AG}+p_{AN}+p_{SG}+p_{SN}$	7
1	0	1	0	$p_C+(p_{AG}+p_{AN}+p_{SG}+p_{SN})$	-
0	1	0	0	p_A-p_{AA}	-
0	1	1	1	p_N-p_{NN}	-
0	1	1	0	$p_{AG}+p_{AN}+p_{SG}+p_{SN}$	-
0	1	0	1	$p_C+(p_{AG}+p_{AN}+p_{SG}+p_{SN})$	-

Table A5: $D_{ig}(1,0)(1-D_{jg}(0,1)) - D_{ig}(0,0)(1-D_{jg}(0,0))$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	-1	AT	SC
1	1	0	1	0		
1	1	0	0	0		
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	0		
1	0	0	0	1	C,SC	C,GC,NT
0	0	0	0	0		

Table A6: $(1-D_{ig}(1,0))D_{jg}(0,1) - (1-D_{ig}(0,0))D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	0		
1	1	0	1	-1	C,SC	AT
1	1	0	0	0		
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	1	GC,NT	SC
1	0	0	0	0		
0	0	0	0	0		

we have:

$$
\begin{aligned}
&\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=0] - \mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0] = \\
&\qquad + \mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))(D_{ig}(1,0)(1-D_{jg}(0,1)) - D_{ig}(0,0)(1-D_{jg}(0,0)))] \\
&\qquad + \mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))((1-D_{ig}(1,0))D_{jg}(0,1) - (1-D_{ig}(0,0))D_{jg}(0,0))] \\
&\qquad + \mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))(D_{ig}(1,0)D_{jg}(0,1) - D_{ig}(0,0)D_{jg}(0,0))].
\end{aligned}
$$

Tables A5, A6 and A7 list the possible values that the terms that depend on the potential treatment statuses can take, which gives the desired result after some algebra. □

Table A7: $D_{ig}(1,0))D_{jg}(0,1) - D_{ig}(0,0)D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	1	AT	SC
1	1	0	1	1	C,SC	AT
1	1	0	0	1	C,SC	SC
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	0		
1	0	0	0	0		
0	0	0	0	0		

A5.3 Proof of Proposition 2

The result follows using that

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)] & \\ &+ \mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))D_{ig}(z,z')(1-D_{jg}(z',z))] \\ &+ \mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))(1-D_{ig}(z,z'))D_{jg}(z',z)] \\ &+ \mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)],\end{aligned}$$

combined with the facts that under one-sided noncompliance, $D_{ig}(0,1) = D_{ig}(0,0) = 0$, for all i, $D_{ig}(1,0) = 1$ implies that i is a complier and $D_{ig}(1,1) = 0$ implies i is a never-taker. □

A5.4 Proof of Corollary 1

Combine lines 2 and 5 from the display in Proposition 2 and the results in Proposition 1, noting that under one-sided noncompliance $\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{jg}=1] = 0$. □

A5.5 Proof of Corollary 2

We have that $\mathbb{E}[Y_{ig}(0,0)] = \mathbb{E}[Y_{ig}(0,0)|C_{ig}]\mathbb{P}[C_{ig}] + \mathbb{E}[Y_{ig}(0,0)|C_{ig}^c]\mathbb{P}[C_{ig}^c]$ and thus

$$\mathbb{E}[Y_{ig}(0,0)|C_{ig}^c] = \frac{\mathbb{E}[Y_{ig}(0,0)] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}]\mathbb{P}[C_{ig}]}{1-\mathbb{P}[C_{ig}]}$$

from which

$$\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}^c] = \frac{\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)]}{1-\mathbb{P}[C_{ig}]}.$$

Using Proposition 2, we obtain

$$\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}^c] = \left\{\frac{\mathbb{E}[Y_{ig}D_{ig}|Z_{ig}=1, Z_{jg}=0]}{\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{jg}=0]} - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0]\right\} \frac{1}{1-\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=0]}.$$

Similarly,

$$\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)|C_{jg}^c] = \frac{\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)]}{1-\mathbb{P}[C_{jg}]}.$$

and thus

$$\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)|C_{jg}^c] = \left\{\frac{\mathbb{E}[Y_{ig}D_{jg}|Z_{ig}=0, Z_{jg}=1]}{\mathbb{E}[D_{jg}|Z_{ig}=0, Z_{jg}=1]} - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0]\right\} \frac{1}{1-\mathbb{E}[D_{jg}|Z_{ig}=0, Z_{ig}=1]}. \square$$

A5.6 Proof of Proposition 3

Under one-sided noncompliance, for any Borel set $\mathcal{Y}$,

$$\begin{aligned}
\mathbb{P}[Y_{ig} \in \mathcal{Y}, D_{ig}=0|Z_{ig}=1, Z_{jg}=0] &= \mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, D_{ig}(1,0)=0|Z_{ig}=1, Z_{jg}=0] \\
&= \mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, D_{ig}(1,0)=0] \\
&= \mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, C_{ig}^c]
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{P}[Y_{ig} \in \mathcal{Y}|Z_{ig}=0, Z_{jg}=0] &= \mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}|Z_{ig}=0, Z_{jg}=0] \\
&= \mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}] \\
&= \mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, C_{ig}^c] + \mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, C_{ig}]
\end{aligned}$$

from which

$$\mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, C_{ig}] = \mathbb{P}[Y_{ig} \in \mathcal{Y}|Z_{ig}=0, Z_{jg}=0] - \mathbb{P}[Y_{ig} \in \mathcal{Y}, D_{ig}=0|Z_{ig}=1, Z_{jg}=0].$$

Since $\mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, C_{ig}] \geq 0$, a testable implication of this model is:

$$\mathbb{P}[Y_{ig} \in \mathcal{Y}|Z_{ig}=0, Z_{jg}=0] - \mathbb{P}[Y_{ig} \in \mathcal{Y}, D_{ig}=0|Z_{ig}=1, Z_{jg}=0] \geq 0.$$

By the same reasoning,

$$\mathbb{P}[Y_{ig}(0,0) \in \mathcal{Y}, C_{jg}] = \mathbb{P}[Y_{ig} \in \mathcal{Y}|Z_{ig}=0, Z_{jg}=0] - \mathbb{P}[Y_{ig} \in \mathcal{Y}, D_{jg}=0|Z_{ig}=0, Z_{jg}=1]$$

and the testable implication is;

$$\mathbb{P}[Y_{ig} \in \mathcal{Y} | Z_{ig} = 0, Z_{jg} = 0] - \mathbb{P}[Y_{ig} \in \mathcal{Y}, D_{jg} = 0 | Z_{ig} = 0, Z_{jg} = 1] \geq 0$$

as required. $\square$

A5.7 Proof of Theorem 1

First, consider the 2SLS regression of Y_{ig} on $1 - D_{ig}$ and D_{ig} (without an intercept) using Z_{ig} and $1 - Z_{ig}$ as instruments, on the subsample of units with $Z_{jg} = 0$. The 2SLS estimator is given by:

$$\hat{\boldsymbol{\alpha}} = (\tilde{\mathbf{Z}}'\tilde{\mathbf{D}})^{-1}\tilde{\mathbf{Z}}\mathbf{Y} = \left(\sum_{g,i} \begin{bmatrix} 1 - Z_{ig} \\ Z_{ig} \end{bmatrix} \begin{bmatrix} 1 - D_{ig} & D_{ig} \end{bmatrix} (1 - Z_{jg})\right)^{-1} \sum_{g,i} \begin{bmatrix} 1 - Z_{ig} \\ Z_{ig} \end{bmatrix} Y_{ig}(1 - Z_{jg}) = \begin{bmatrix} \bar{Y}_{00} \\ \frac{\bar{Y}_{10} - \bar{Y}_{00}}{\bar{D}_{10}} + \bar{Y}_{00} \end{bmatrix}$$

The cluster-robust variance estimator is:

$$\hat{V}_{\mathsf{cr}}(\hat{\boldsymbol{\alpha}}) = \frac{1}{4G^2}\left(\frac{1}{2G}\tilde{\mathbf{Z}}'\tilde{\mathbf{D}}\right)^{-1} \sum_g \tilde{\mathbf{z}}_g'\hat{\mathbf{u}}_g\hat{\mathbf{u}}_g'\tilde{\mathbf{z}}_g \left(\frac{1}{2G}\tilde{\mathbf{D}}'\tilde{\mathbf{Z}}\right)^{-1}$$

where $\hat{u}_{ig} = (Y_{ig} - \hat{\alpha}_0(1 - D_{ig}) - \hat{\alpha}_1 D_{ig})(1 - Z_{jg})$ and $\hat{\mathbf{u}}_g' = [\hat{u}_{1g}\ \hat{u}_{2g}]$. Next,

$$\tilde{\mathbf{z}}_g'\hat{\mathbf{u}}_g\hat{\mathbf{u}}_g'\tilde{\mathbf{z}}_g = \begin{bmatrix} (1 - Z_{1g})(1 - Z_{2g})(\hat{u}_{1g}^2 + \hat{u}_{2g}^2 + 2\hat{u}_{1g}\hat{u}_{2g}) & 0 \\ 0 & Z_{1g}(1 - Z_{2g})\hat{u}_{1g}^2 + Z_{2g}(1 - Z_{1g})\hat{u}_{2g}^2 \end{bmatrix}$$

Then,

$$\hat{V}_{\mathsf{cr},11}(\hat{\boldsymbol{\alpha}}) = \frac{1}{4G^2\hat{p}_{00}^2}\sum_{g,i}(1 - Z_{ig})(1 - Z_{jg})(\hat{u}_{ig}^2 + \hat{u}_{ig}\hat{u}_{jg})$$

and

$$\hat{V}_{\mathsf{cr},22}(\hat{\boldsymbol{\alpha}}) = \frac{1}{4G^2\bar{D}_{10}^2\hat{p}_{10}^2}\sum_{g,i} Z_{ig}(1 - Z_{jg})\hat{u}_{ig}^2 + \frac{(1 - \bar{D}_{10})^2}{4G^2\bar{D}_{10}^2\hat{p}_{00}^2}\sum_{g,i}(1 - Z_{ig})(1 - Z_{jg})(\hat{u}_{ig}^2 + \hat{u}_{ig}\hat{u}_{jg}).$$

Now, for any invertible transformation $\tilde{\mathbf{A}}$, consider the transformed variables $\tilde{\mathbf{Z}}\tilde{\mathbf{A}}$ and $\tilde{\mathbf{D}}\tilde{\mathbf{A}}$. Let: $\hat{\boldsymbol{\delta}}(\tilde{\mathbf{A}}) = ((\tilde{\mathbf{Z}}\tilde{\mathbf{A}})'(\tilde{\mathbf{D}}\tilde{\mathbf{A}}))^{-1}(\tilde{\mathbf{Z}}\tilde{\mathbf{A}})'\mathbf{Y}$. The 2SLS estimator using this transformed variables is:

$$\begin{aligned}
\hat{\boldsymbol{\delta}}(\tilde{\mathbf{A}}) &= ((\tilde{\mathbf{Z}}\tilde{\mathbf{A}})'(\tilde{\mathbf{D}}\tilde{\mathbf{A}}))^{-1}(\tilde{\mathbf{Z}}\tilde{\mathbf{A}})'\mathbf{Y} \\
&= (\tilde{\mathbf{A}}'\tilde{\mathbf{Z}}'\tilde{\mathbf{D}}\tilde{\mathbf{A}})^{-1}\tilde{\mathbf{A}}'\tilde{\mathbf{Z}}'\mathbf{Y} \\
&= \tilde{\mathbf{A}}^{-1}(\tilde{\mathbf{Z}}'\tilde{\mathbf{D}})^{-1}(\tilde{\mathbf{A}}')^{-1}\tilde{\mathbf{A}}'\tilde{\mathbf{Z}}'\mathbf{Y} \\
&= \tilde{\mathbf{A}}^{-1}(\tilde{\mathbf{Z}}'\tilde{\mathbf{D}})^{-1}\tilde{\mathbf{Z}}'\mathbf{Y} \\
&= \tilde{\mathbf{A}}^{-1}\hat{\boldsymbol{\alpha}}.
\end{aligned}$$

and

$$\hat{V}_{\mathsf{cr}}(\hat{\boldsymbol{\delta}}(\tilde{\mathbf{A}})) = \tilde{\mathbf{A}}^{-1}\hat{V}_{\mathsf{cr}}(\hat{\boldsymbol{\alpha}})(\tilde{\mathbf{A}}^{-1})'$$

Now, setting:

$$\tilde{\mathbf{A}} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

gives the 2SLS estimator from the regression of Y_{ig} on D_{ig}, and a constant using Z_{ig} as an instrument, conditional on $Z_{jg} = 0$, which is the parameterization considered in the paper. It follows that the conditional Wald estimator of the direct effect on compliers is:

$$\hat{\delta}_1 = \frac{\bar{Y}_{10} - \bar{Y}_{00}}{\bar{D}_{10}}$$

and

$$\hat{V}_{\mathsf{cr}}(\hat{\delta}_1) = \hat{V}_{\mathsf{cr},11}(\hat{\boldsymbol{\alpha}}) + \hat{V}_{\mathsf{cr},22}(\hat{\boldsymbol{\alpha}}).$$

The case for the spillover effect estimator based on the regression of Y_{ig} on D_{jg} using Z_{jg} as an instrument on the subsample of units with $Z_{ig} = 0$ follows analogously.

Next, consider the 2SLS regression of Y_{ig} on $(1-D_{ig})(1-D_{jg})$, $D_{ig}(1-D_{jg})$, $(1-D_{ig})D_{jg}$ and $D_{ig}D_{jg}$ using $(1-Z_{ig})(1-Z_{jg})$, $Z_{ig}(1-Z_{jg})$, $(1-Z_{ig})Z_{jg}$ and $Z_{ig}Z_{jg}$ as instruments. The 2SLS estimator is:

$$\hat{\boldsymbol{\theta}} = \left(\frac{1}{2G}\mathbf{Z}'\mathbf{D}\right)^{-1}\frac{1}{2G}\mathbf{Z}'\mathbf{Y}.$$

Now,

$$\begin{aligned}
\frac{1}{2G}\mathbf{Z}'\mathbf{D} &= \frac{1}{2G}\sum_{g,i}\begin{bmatrix} (1-Z_{ig})(1-Z_{jg}) \\ Z_{ig}(1-Z_{jg}) \\ (1-Z_{ig})Z_{jg} \\ Z_{ig}Z_{jg} \end{bmatrix}\begin{bmatrix} (1-D_{ig})(1-D_{jg}) & D_{ig}(1-D_{jg}) & (1-D_{ig})D_{jg} & D_{ig}D_{jg} \end{bmatrix} \\
&= \begin{bmatrix} \hat{p}_{00} & 0 & 0 & 0 \\ (1-\bar{D}_{10})\hat{p}_{10} & \bar{D}_{10}\hat{p}_{10} & 0 & 0 \\ (1-\bar{D}_{10})\hat{p}_{10} & 0 & \bar{D}_{10}\hat{p}_{10} & 0 \\ \bar{D}_{11}^{00}\hat{p}_{11} & \bar{D}_{11}^{10}\hat{p}_{11} & \bar{D}_{11}^{01}\hat{p}_{11} & \bar{D}_{11}^{11}\hat{p}_{11} \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
\hat{p}_{zz'} &= \frac{1}{2G}\sum_{g,i}\mathbb{1}(Z_{ig}=z)\mathbb{1}(Z_{jg}=z') \\
\bar{D}_{z,z'} &= \frac{\sum_{g,i} D_{ig}\mathbb{1}(Z_{ig}=z)\mathbb{1}(Z_{jg}=z')}{\sum_{g,i}\mathbb{1}(Z_{ig}=z)\mathbb{1}(Z_{jg}=z')} \\
\bar{D}_{z,z'}^{d,d'} &= \frac{\sum_{g,i}\mathbb{1}(D_{ig}=d)\mathbb{1}(D_{jg}=d')\mathbb{1}(Z_{ig}=z)\mathbb{1}(Z_{jg}=z')}{\sum_{g,i}\mathbb{1}(Z_{ig}=z)\mathbb{1}(Z_{jg}=z')}.
\end{aligned}$$

The inverse can be found by direct calculation as the matrix $\hat{\mathbf{Q}}$ such that $\frac{1}{2G}\mathbf{Z}'\mathbf{D} \times \hat{\mathbf{Q}} = \mathbf{I}$. This gives:

$$\hat{\mathbf{Q}} = \left(\frac{1}{2G}\mathbf{Z}'\mathbf{D}\right)^{-1} = \begin{bmatrix} \frac{1}{\hat{p}_{00}} & 0 & 0 & 0 \\ -\frac{(1-\bar{D}_{10})}{\bar{D}_{10}\hat{p}_{00}} & \frac{1}{\bar{D}_{10}\hat{p}_{10}} & 0 & 0 \\ -\frac{(1-\bar{D}_{10})}{\bar{D}_{10}\hat{p}_{00}} & 0 & \frac{1}{\bar{D}_{10}\hat{p}_{10}} & 0 \\ \frac{2\bar{D}_{11}^{10}}{\hat{p}_{00}\bar{D}_{11}^{11}}\left(\frac{1-\bar{D}_{10}}{\bar{D}_{10}}\right) - \frac{\bar{D}_{11}^{00}}{\hat{p}_{00}\bar{D}_{11}^{11}} & -\frac{\bar{D}_{11}^{10}}{\bar{D}_{10}\bar{D}_{11}^{11}\hat{p}_{10}} & -\frac{\bar{D}_{11}^{10}}{\bar{D}_{10}\bar{D}_{11}^{11}\hat{p}_{10}} & \frac{1}{\bar{D}_{11}^{11}\hat{p}_{11}} \end{bmatrix}.$$

On the other hand,

$$\frac{1}{2G}\mathbf{Z}'\mathbf{Y} = \begin{bmatrix} \bar{Y}_{00}\hat{p}_{00} \\ \bar{Y}_{10}\hat{p}_{10} \\ \bar{Y}_{01}\hat{p}_{10} \\ \bar{Y}_{11}\hat{p}_{11} \end{bmatrix}$$

where

$$\bar{Y}_{zz'} = \frac{\sum_{g,i} Y_{ig}\mathbb{1}(Z_{ig} = z)\mathbb{1}(Z_{jg} = z')}{\sum_{g,i} \mathbb{1}(Z_{ig} = z)\mathbb{1}(Z_{jg} = z')}.$$

Thus:

$$\hat{\boldsymbol{\theta}} = \begin{bmatrix} \bar{Y}_{00} \\ \frac{\bar{Y}_{10}-\bar{Y}_{00}}{\bar{D}_{10}} + \bar{Y}_{00} \\ \frac{\bar{Y}_{01}-\bar{Y}_{00}}{\bar{D}_{10}} + \bar{Y}_{00} \\ \frac{\bar{Y}_{11}-\bar{Y}_{00}}{\bar{D}_{11}^{11}} - \frac{\bar{D}_{11}^{10}}{\bar{D}_{11}^{11}}\left(\frac{\bar{Y}_{10}+\bar{Y}_{01}-2\bar{Y}_{00}}{\bar{D}_{10}}\right) + \bar{Y}_{00} \end{bmatrix}.$$

Now, for any invertible transformation $\mathbf{A}$, consider the transformed variables $\mathbf{ZA}$ and $\mathbf{DA}$. Let: $\hat{\boldsymbol{\beta}}(\mathbf{A}) = ((\mathbf{ZA})'(\mathbf{DA}))^{-1}(\mathbf{ZA})'\mathbf{Y}$. The 2SLS estimator using this transformed variables is:

$$\begin{aligned} \hat{\boldsymbol{\beta}}(\mathbf{A}) &= ((\mathbf{ZA})'(\mathbf{DA}))^{-1}(\mathbf{ZA})'\mathbf{Y} \\ &= (\mathbf{A}'\mathbf{Z}'\mathbf{DA})^{-1}\mathbf{A}'\mathbf{Z}'\mathbf{Y} \\ &= \mathbf{A}^{-1}(\mathbf{Z}'\mathbf{D})^{-1}(\mathbf{A}')^{-1}\mathbf{A}'\mathbf{Z}'\mathbf{Y} \\ &= \mathbf{A}^{-1}(\mathbf{Z}'\mathbf{D})^{-1}\mathbf{Z}'\mathbf{Y} \\ &= \mathbf{A}^{-1}\hat{\boldsymbol{\theta}}. \end{aligned}$$

and

$$\hat{V}_{\mathsf{cr}}(\hat{\boldsymbol{\beta}}(\mathbf{A})) = \mathbf{A}^{-1}\hat{V}_{\mathsf{cr}}(\hat{\boldsymbol{\theta}})(\mathbf{A}^{-1})'$$

Now, setting:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

gives the 2SLS estimator from the regression of Y_{ig} on D_{ig}, D_{jg}, $D_{ig}D_{jg}$ and a constant using Z_{ig}, Z_{jg}, $Z_{ig}Z_{jg}$ as instruments, which is the parameterization considered in the paper. It follows that the 2SLS estimator, $\hat{\boldsymbol{\beta}}$, is:

$$\begin{aligned} \hat{\boldsymbol{\beta}} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 \end{bmatrix} \hat{\boldsymbol{\theta}} \\ &= \begin{bmatrix} \bar{Y}_{00} \\ \frac{\bar{Y}_{10}-\bar{Y}_{00}}{\bar{D}_{10}} \\ \frac{\bar{Y}_{01}-\bar{Y}_{00}}{\bar{D}_{10}} \\ \frac{\bar{Y}_{11}-\bar{Y}_{00}}{\bar{D}_{11}^{11}} - \left(\frac{\bar{D}_{11}^{11}+\bar{D}_{11}^{10}}{\bar{D}_{11}^{11}}\right)\left(\frac{\bar{Y}_{10}-\bar{Y}_{00}}{\bar{D}_{10}} + \frac{\bar{Y}_{01}-\bar{Y}_{00}}{\bar{D}_{10}}\right) \end{bmatrix} \end{aligned}$$

which implies that the treatment effect estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ are identical to the conditional Wald ratio estimators.

The cluster-robust variance estimator for $\hat{\boldsymbol{\theta}}$ is:

$$\hat{V}_{\mathsf{cr}}(\hat{\boldsymbol{\theta}}) = \frac{1}{4G^2}\left(\frac{1}{2G}\mathbf{Z}'\mathbf{D}\right)^{-1} \sum_g \mathbf{z}_g'\hat{\boldsymbol{\varepsilon}}_g\hat{\boldsymbol{\varepsilon}}_g'\mathbf{z}_g \left(\frac{1}{2G}\mathbf{D}'\mathbf{Z}\right)^{-1}$$

where $\hat{\varepsilon}_{ig} = Y_{ig} - \mathbf{D}_{ig}'\hat{\boldsymbol{\theta}}$. We have that:

$$\mathbf{z}_g'\hat{\boldsymbol{\varepsilon}}_g\hat{\boldsymbol{\varepsilon}}_g' = \begin{bmatrix} (1-Z_{1g})(1-Z_{2g})(\hat{\varepsilon}_{1g}^2 + \hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g}) & (1-Z_{2g})(1-Z_{1g})(\hat{\varepsilon}_{2g}^2 + \hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g}) \\ Z_{1g}(1-Z_{2g})\hat{\varepsilon}_{1g}^2 + Z_{2g}(1-Z_{1g})\hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g} & Z_{2g}(1-Z_{1g})\hat{\varepsilon}_{2g}^2 + Z_{1g}(1-Z_{2g})\hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g} \\ (1-Z_{1g})Z_{2g}\hat{\varepsilon}_{1g}^2 + (1-Z_{2g})Z_{1g}\hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g} & (1-Z_{2g})Z_{1g}\hat{\varepsilon}_{2g}^2 + (1-Z_{1g})Z_{2g}\hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g} \\ Z_{1g}Z_{2g}(\hat{\varepsilon}_{1g}^2 + \hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g}) & Z_{1g}Z_{2g}(\hat{\varepsilon}_{1g}^2 + \hat{\varepsilon}_{1g}\hat{\varepsilon}_{2g}) \end{bmatrix}$$

and

$$\begin{aligned} \hat{\boldsymbol{\Omega}} &= \sum_g \mathbf{z}_g'\hat{\boldsymbol{\varepsilon}}_g\hat{\boldsymbol{\varepsilon}}_g'\mathbf{z}_g \\ &= \begin{bmatrix} \omega_{11} & 0 & 0 & 0 \\ 0 & \omega_{22} & \omega_{23} & 0 \\ 0 & \omega_{23} & \omega_{33} & 0 \\ 0 & 0 & 0 & \omega_{44} \end{bmatrix} \end{aligned}$$

where

$$\begin{aligned}
\omega_{11} &= \sum_{g,i}(1-Z_{ig})(1-Z_{jg})(\hat{\varepsilon}_{ig}^2+\hat{\varepsilon}_{ig}\hat{\varepsilon}_{jg}) \\
\omega_{22} &= \sum_{g,i} Z_{ig}(1-Z_{jg})\hat{\varepsilon}_{ig}^2 \\
\omega_{23} &= \sum_{g,i} Z_{ig}(1-Z_{jg})\hat{\varepsilon}_{ig}\hat{\varepsilon}_{jg} \\
\omega_{33} &= \sum_{g,i}(1-Z_{ig})Z_{jg}\hat{\varepsilon}_{ig}^2 \\
\omega_{44} &= \sum_{g,i} Z_{ig}Z_{jg}(\hat{\varepsilon}_{ig}^2+\hat{\varepsilon}_{ig}\hat{\varepsilon}_{jg}).
\end{aligned}$$

Let

$$\hat{\mathbf{Q}} = \left(\frac{1}{2G}\mathbf{Z}'\mathbf{D}\right)^{-1} = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ q_{21} & q_{22} & 0 & 0 \\ q_{21} & 0 & q_{22} & 0 \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix}$$

where the exact values of the matrix were given above. The cluster-robust variance estimator for $\hat{\boldsymbol{\theta}}$ can be rewritten as:

$$\hat{V}_{\mathsf{cr}}(\hat{\boldsymbol{\theta}}) = \frac{1}{4G^2}\hat{\mathbf{Q}}\hat{\boldsymbol{\Omega}}\hat{\mathbf{Q}}'.$$

Then,

$$\begin{aligned}
\hat{V}_{\mathsf{cr},11}(\hat{\boldsymbol{\theta}}) &= \frac{1}{4G^2\hat{p}_{00}^2}\sum_{g,i}(1-Z_{ig})(1-Z_{jg})(\hat{\varepsilon}_{ig}^2+\hat{\varepsilon}_{ig}\hat{\varepsilon}_{jg}) \\
\hat{V}_{\mathsf{cr},22}(\hat{\boldsymbol{\theta}}) &= \frac{1}{4G^2\bar{D}_{10}^2\hat{p}_{10}^2}\sum_{g,i} Z_{ig}(1-Z_{jg})\hat{\varepsilon}_{ig}^2 + \frac{(1-\bar{D}_{10})^2}{4G^2\bar{D}_{10}^2\hat{p}_{00}^2}\sum_{g,i}(1-Z_{ig})(1-Z_{jg})(\hat{\varepsilon}_{ig}^2+\hat{\varepsilon}_{ig}\hat{\varepsilon}_{jg}).
\end{aligned}$$

But notice that, if the residuals are the same, we have that $\hat{V}_{\mathsf{cr},11}(\hat{\boldsymbol{\theta}}) = \hat{V}_{\mathsf{cr},11}(\hat{\boldsymbol{\alpha}})$ and $\hat{V}_{\mathsf{cr},22}(\hat{\boldsymbol{\theta}}) = \hat{V}_{\mathsf{cr},22}(\hat{\boldsymbol{\alpha}})$ which in turns implies that the cluster-robust variance estimators for $\hat{\beta}_1$ and $\hat{\delta}_1$ are equal.

To see that the residuals are indeed equal, note that:

$$\begin{aligned}
(1-Z_{jg})\hat{u}_{ig}^2 &= (1-Z_{jg})(Y_{ig}-\hat{\alpha}_0(1-D_{ig})-\hat{\alpha}_1 D_{ig}) \\
&= (1-Z_{jg})(Y_{ig}-\hat{\theta}_0(1-D_{ig})-\hat{\theta}_1 D_{ig}) \\
&= (1-Z_{jg})\hat{\varepsilon}_{ig}^2
\end{aligned}$$

which implies that $\hat{\delta}_0 = \hat{\beta}_0$, $\hat{V}_{\mathsf{cr}}(\hat{\delta}_0) = \hat{V}_{\mathsf{cr}}(\hat{\beta}_0)$, $\hat{\delta}_1 = \hat{\beta}_1$ and $\hat{V}_{\mathsf{cr}}(\hat{\delta}_1) = \hat{V}_{\mathsf{cr}}(\hat{\beta}_1)$. The results for $\hat{\delta}_2$ and $\hat{\beta}_2$ follow by the same argument. □

A5.8 Proof of Lemma 2

This result is well-known and follows from standard 2SLS properties and using Theorem 1 as $G \to \infty$. To find the exact formula for β_3, note that

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig} = z, Z_{jg} = z'] = \beta_0 &+ \beta_1 \mathbb{E}[D_{ig}|Z_{ig} = z, Z_{jg} = z'] \\ &+ \beta_2 \mathbb{E}[D_{jg}|Z_{ig} = z, Z_{jg} = z'] \\ &+ \beta_3 \mathbb{E}[D_{ig}D_{jg}|Z_{ig} = z, Z_{jg} = z']\end{aligned}$$

Under one-sided noncompliance, $\mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{jg} = z'] = \mathbb{E}[D_{jg}|Z_{ig} = z, Z_{jg} = 0] = 0$ and thus:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0] &= \beta_0 \\ \mathbb{E}[Y_{ig}|Z_{ig} = 1, Z_{jg} = 0] &= \beta_0 + \beta_1 \mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{jg} = 0] \\ \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 1] &= \beta_0 + \beta_2 \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{jg} = 1]\end{aligned}$$

from which:

$$\begin{aligned}\beta_0 &= \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0] = \mathbb{E}[Y_{ig}(0,0)] \\ \beta_1 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig} = 1, Z_{jg} = 0] - \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0]}{\mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{jg} = 0]} = \mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)|C_{ig}] \\ \beta_2 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 1] - \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0]}{\mathbb{E}[D_{jg}|Z_{ig} = 0, Z_{jg} = 1]} = \mathbb{E}[Y_{ig}(0,1) - Y_{ig}(0,0)|C_{jg}] \\ \beta_3 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig} = 1, Z_{jg} = 1] - \beta_0 - \beta_1 \mathbb{E}[D_{ig}(1,1)] - \beta_2 \mathbb{E}[D_{jg}(1,1)]}{\mathbb{E}[D_{jg}(1,1)D_{jg}(1,1)]}\end{aligned}$$

as long as $\mathbb{E}[D_{jg}(1,1)D_{jg}(1,1)] > 0$ (otherwise, β_3 is not identified). Finally, note that

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig} = 1, Z_{jg} = 1] = \mathbb{E}[Y_{ig}(0,0)] & \\ &+ \mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)|D_{ig}(1,1) = 1]\mathbb{E}[D_{ig}(1,1)] \\ &+ \mathbb{E}[Y_{ig}(0,1) - Y_{ig}(0,0)|D_{jg}(1,1) = 1]\mathbb{E}[D_{jg}(1,1)] \\ &+ \mathbb{E}[Y_{ig}(1,1) - Y_{ig}(1,0) - Y_{ig}(0,1) + Y_{ig}(0,0)|D_{ig}(1,1) = 1, D_{jg}(1,1) = 1] \\ &\quad \times \mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]\end{aligned}$$

and use the fact that $D_{ig}(1,1) = 1$ if i is a complier or a group complier to get that:

$$\begin{aligned}\beta_3 &= (\mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)|GC_{ig}] - \mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)|C_{ig}]) \frac{\mathbb{P}[GC_{ig}]}{\mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]} \\ &+ (\mathbb{E}[Y_{ig}(0,1) - Y_{ig}(0,0)|GC_{jg}] - \mathbb{E}[Y_{ig}(0,1) - Y_{ig}(0,0)|C_{jg}]) \frac{\mathbb{P}[GC_{jg}]}{\mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]} \\ &+ \mathbb{E}[Y_{ig}(1,1) - Y_{ig}(1,0) - (Y_{ig}(0,1) - Y_{ig}(0,0))|D_{ig}(1,1) = 1, D_{jg}(1,1) = 1].\end{aligned}$$

which gives the desired result. $\square$

A5.9 Proof of Proposition 4

First,

$$\begin{aligned}\mathbb{E}[D_{ig}|Z_{ig}=1,Z_{jg}=0,X_g] &= \mathbb{E}[D_{ig}(1,0)|Z_{ig}=1,Z_{jg}=0,X_g]\\ &= \mathbb{E}[D_{ig}(1,0)|X_g] = \mathbb{P}[C_{ig}|X_g]\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}[D_{ig}|Z_{ig}=1,Z_{jg}=1,X_g] &= \mathbb{E}[D_{ig}(1,1)|Z_{ig}=1,Z_{jg}=1,X_g]\\ &= \mathbb{E}[D_{ig}(1,1)|X_g] = \mathbb{P}[C_{ig}|X_g]+\mathbb{P}[GC_{ig}|X_g].\end{aligned}$$

For the second part, we have that for the first term,

$$\begin{aligned}\mathbb{E}\left[g(Y_{ig},X_g)\frac{(1-Z_{ig})(1-Z_{jg})}{p_{00}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)\frac{(1-Z_{ig})(1-Z_{jg})}{p_{00}(X_g)}\middle| X_g\right]\right\}\\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)\right| Z_{ig}=0,Z_{jg}=0,X_g]\right\}\\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)\right| Z_{ig}=0,Z_{jg}=0,X_g]\right\}\\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)\right| X_g]\right\}\\ &= \mathbb{E}[g(Y_{ig}(0,0),X_g)].\end{aligned}$$

For the second term,

$$\begin{aligned}\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\frac{Z_{ig}(1-Z_{ig})}{p_{10}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\frac{Z_{ig}(1-Z_{ig})}{p_{10}(X_g)}\middle| X_g\right]\right\}\\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\right| Z_{ig}=1,Z_{jg}=0,X_g]\right\}\\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right| Z_{ig}=1,Z_{jg}=0,X_g]\right\}\\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right| X_g]\right\}\\ &= \mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right]\\ &= \mathbb{E}\left[g(Y_{ig}(1,0),X_g)|C_{ig}\right]\mathbb{P}[C_{ig}].\end{aligned}$$

For the third term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\frac{(1-Z_{ig})Z_{ig}}{p_{01}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\frac{(1-Z_{ig})Z_{ig}}{p_{01}(X_g)}\middle| X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\middle| Z_{ig}=0,Z_{jg}=1,X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\middle| Z_{ig}=0,Z_{jg}=1,X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right]\right\}\\
&= \mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right]\\
&= \mathbb{E}[g(Y_{ig}(0,1),X_g)|C_{jg}]\mathbb{P}[C_{jg}].
\end{aligned}
$$

For the fourth term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\frac{Z_{ig}(1-Z_{jg})}{p_{10}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\frac{Z_{ig}(1-Z_{jg})}{p_{10}(X_g)}\middle| X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\middle| Z_{ig}=1,Z_{jg}=0,X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))\middle| Z_{ig}=1,Z_{jg}=0,X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))\middle| X_g\right]\right\}\\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))]\\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}^c]\mathbb{P}[C_{ig}^c]
\end{aligned}
$$

and the result follows from $\mathbb{E}[g(Y_{ig}(0,0),X_g)] = \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}]\mathbb{P}[C_{ig}]+\mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}^c]\mathbb{P}[C_{ig}^c]$. Similarly for the fifth term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\frac{(1-Z_{ig})Z_{jg}}{p_{01}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\frac{(1-Z_{ig})Z_{jg}}{p_{01}(X_g)}\middle| X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\middle| Z_{ig}=0,Z_{jg}=1,X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))\middle| Z_{ig}=0,Z_{jg}=1,X_g\right]\right\}\\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))\middle| X_g\right]\right\}\\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))]\\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{jg}^c]\mathbb{P}[C_{jg}^c]
\end{aligned}
$$

and it can be seen that all these equalities also hold conditional on X_g. □

A5.10 Proof of Proposition 5

By independence, $\mathbb{E}[D_{ig}|Z_{ig}=z,W_{ig}=w]=\mathbb{E}[D_{ig}(z,w)]$. Then, under monotonicity, $\mathbb{E}[D_{ig}|Z_{ig}=0,W_{ig}=0]=\mathbb{E}[D_{ig}(0,0)]=\mathbb{P}[D_{ig}(0,0)=1]=\mathbb{P}[AT_{ig}]$. Next, $\mathbb{E}[D_{ig}|Z_{ig}=0,W_{ig}=w^*]-\mathbb{E}[D_{ig}|Z_{ig}=0,W_{ig}=w^*-1]=\mathbb{E}[D_{ig}(0,w^*)]-\mathbb{E}[D_{ig}(0,w^*-1)]=\mathbb{P}[D_{ig}(0,w^*)>D_{ig}(0,w^*-1)]=\mathbb{P}[SC_{ig}(w^*)]$. Similarly, $\mathbb{E}[D_{ig}|Z_{ig}=1,W_{ig}=0]-\mathbb{E}[D_{ig}|Z_{ig}=0,W_{ig}=n_g]=\mathbb{P}[D_{ig}(1,0)>D_{ig}(0,n_g)]=\mathbb{P}[C_{ig}]$ and $\mathbb{E}[D_{ig}|Z_{ig}=1,W_{ig}=w^*]-\mathbb{E}[D_{ig}|Z_{ig}=1,W_{ig}=$

$w^* - 1] = \mathbb{P}[D_{ig}(1, w^*) > D_{ig}(1, w^* - 1)] = \mathbb{P}[GC_{ig}(w^*)]$. Finally, $\mathbb{E}[1 - D_{ig}|Z_{ig} = 1, W_{ig} = n_g] = \mathbb{P}[D_{ig}(1, n_g) = 0] = \mathbb{P}[NT_{ig}]$. □

A5.11 Proof of Proposition 6

Under the assumptions in the proposition,

$$\begin{aligned}
\mathbb{E}[Y_{ig}|D_{ig} = d, S_{ig} = s, Z_{ig} = z, W_{ig} = w] &= \sum_{\mathbf{z}_g} \mathbb{E}[Y_{ig}|D_{ig} = d, S_{ig} = s, Z_{ig} = z, W_{ig} = w, \mathbf{Z}_{(i)g} = \mathbf{z}_g] \\
&\quad \times \mathbb{P}[\mathbf{Z}_{(i)g} = \mathbf{z}_g|D_{ig} = d, S_{ig} = s, Z_{ig} = z, W_{ig} = w] \\
&= \sum_{\mathbf{z}_g} \mathbb{E}[Y_{ig}(d, s)|D_{ig}(z, w) = d, S_{ig}(z, \mathbf{z}_g) = s, Z_{ig} = z, \mathbf{Z}_{(i)g} = \mathbf{z}_g] \\
&\quad \times \mathbb{P}[\mathbf{Z}_{(i)g} = \mathbf{z}_g|D_{ig} = d, S_{ig} = s, Z_{ig} = z, W_{ig} = w] \\
&= \sum_{\mathbf{z}_g} \mathbb{E}[Y_{ig}(d, s)|D_{ig}(z, w) = d, S_{ig}(z, \mathbf{z}_g) = s] \\
&\quad \times \mathbb{P}[\mathbf{Z}_{(i)g} = \mathbf{z}_g|D_{ig} = d, S_{ig} = s, Z_{ig} = z, W_{ig} = w] \\
&= \mathbb{E}[Y_{ig}(d, s)|D_{ig}(z, w) = d]
\end{aligned}$$

where the second equality uses the fact that S_{ig} depends on the whole vector of instruments, the third equality follows by independence and the fourth equality uses independence of peers' types.

Now, note that for any s, $\mathbb{E}[Y_{ig}|D_{ig} = 1, S_{ig} = s, Z_{ig} = 0, W_{ig} = 0] = \mathbb{E}[Y_{ig}(1, s)|D_{ig}(0, 0) = 1] = \mathbb{E}[Y_{ig}(1, s)|AT_{ig}]$ which shows that $\mathbb{E}[Y_{ig}(1, s)|AT_{ig}]$ is identified. Then,

$$\begin{aligned}
\mathbb{E}[Y_{ig}|D_{ig} = 1, S_{ig} = s, Z_{ig} = 0, W_{ig} = 1] &= \mathbb{E}[Y_{ig}(1, s)|D_{ig}(0, 1) = 1] \\
&= \mathbb{E}[Y_{ig}(1, s)|AT_{ig}] \frac{\mathbb{P}[AT_{ig}]}{\mathbb{P}[AT_{ig}] + \mathbb{P}[SC(1)_{ig}]} \\
&\quad + \mathbb{E}[Y_{ig}(1, s)|SC_{ig}(1)] \frac{\mathbb{P}[SC_{ig}(1)]}{\mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}(1)]}
\end{aligned}$$

and hence $\mathbb{E}[Y_{ig}(1, s)|SC_{ig}(1)]$ is identified by the results above and Proposition 5. By the same logic,

$$\begin{aligned}
\mathbb{E}[Y_{ig}|D_{ig} = 1, S_{ig} = s, Z_{ig} = 0, W_{ig} = 2] &= \mathbb{E}[Y_{ig}(1, s)|D_{ig}(0, 2) = 1] \\
&= \mathbb{E}[Y_{ig}(1, s)|AT_{ig}] \frac{\mathbb{P}[AT_{ig}]}{\mathbb{P}[AT_{ig}] + \mathbb{P}[SC(1)_{ig}] + \mathbb{P}[SC(2)_{ig}]} \\
&\quad + \mathbb{E}[Y_{ig}(1, s)|SC_{ig}(1)] \frac{\mathbb{P}[SC_{ig}(1)]}{\mathbb{P}[AT_{ig}] + \mathbb{P}[SC(1)_{ig}] + \mathbb{P}[SC(2)_{ig}]} \\
&\quad + \mathbb{E}[Y_{ig}(1, s)|SC_{ig}(2)] \frac{\mathbb{P}[SC_{ig}(2)]}{\mathbb{P}[AT_{ig}] + \mathbb{P}[SC(1)_{ig}] + \mathbb{P}[SC(2)_{ig}]}
\end{aligned}$$

and thus $\mathbb{E}[Y_{ig}(1, s)|SC_{ig}(2)]$ is identified. The same reasoning shows that as long as all required probabilities are non-zero, $\mathbb{E}[Y_{ig}(1, s)|\xi_{ig}]$ is identified for all values of ξ_{ig} except for

Table A8: $D_{ig}(0,1)(1-D_{jg}(1,0)) - D_{ig}(0,0)(1-D_{jg}(0,0))$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	-1	AT	SC,C
1	0	1	0	0		
1	1	0	1	0		
1	1	0	0	0		
1	0	0	0	1	SC	GC,NT
0	1	0	1	0		
0	1	0	0	0		
0	0	0	0	0		

$\xi_{ig} = NT$, since the assignment $(1, s)$ is never observed for never-takers. Identification of $\mathbb{E}[Y_{ig}(0,s)|\xi_{ig}]$ for all values of $\xi_{ig} \neq AT$ follows similarly. □

A6 Proofs of Additional Results

A6.1 Proof of Lemma A1

Using that:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)]&\\ + \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))D_{ig}(z,z')(1-D_{jg}(z',z))]&\\ + \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))(1-D_{ig}(z,z'))D_{jg}(z',z)]&\\ + \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)],&\end{aligned}$$

we have:

$$\begin{aligned}&\mathbb{E}[Y_{ig}|Z_{ig}=1, Z_{jg}=0] - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] =\\ &\quad + \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))(D_{ig}(0,1)(1-D_{jg}(1,0)) - D_{ig}(0,0)(1-D_{jg}(0,0)))]\\ &\quad + \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))((1-D_{ig}(0,1))D_{jg}(1,0) - (1-D_{ig}(0,0))D_{jg}(0,0))]\\ &\quad + \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))(D_{ig}(0,1)D_{jg}(1,0) - D_{ig}(0,0)D_{jg}(0,0))].\end{aligned}$$

Tables A8, A9 and A10 list the possible values that the terms that depend on the potential treatment statuses can take, which gives the desired result after some algebra. □

Table A9: $(1 - D_{ig}(0,1))D_{jg}(1,0) - (1 - D_{ig}(0,0))D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	0		
1	0	1	0	0		
1	1	0	1	-1	SC	AT
1	1	0	0	0		
1	0	0	0	0		
0	1	0	1	0		
0	1	0	0	1	C,CG,NT	SC,C
0	0	0	0	0		

Table A10: $D_{ig}(0,1)D_{jg}(1,0) - D_{ig}(0,0)D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	1	AT	SC,C
1	0	1	0	0		
1	1	0	1	1	SC	AT
1	1	0	0	1	SC	SC,C
1	0	0	0	0		
0	1	0	1	0		
0	1	0	0	0		
0	0	0	0	0		

Table A11: $D_{ig}(1,1)(1-D_{jg}(1,1)) - D_{ig}(0,0)(1-D_{jg}(0,0))$

$D_{ig}(1,1)$	$D_{jg}(1,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	0	1	0		
0	1	0	1	0		
1	1	1	0	-1	AT	SC,C,GC
1	1	0	0	0		
0	1	0	0	0		
1	0	1	0	0		
1	0	0	0	-1	SC,C,GC	NT
0	0	0	0	0		

A6.2 Proof of Lemma A2

Using that:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z,Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)] & \\ + \mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))D_{ig}(z,z')(1-D_{jg}(z',z))] & \\ + \mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))(1-D_{ig}(z,z'))D_{jg}(z',z)] & \\ + \mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)], &\end{aligned}$$

we have:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=1] - \mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0] = & \\ + \mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))(D_{ig}(1,1)(1-D_{jg}(1,1)) - D_{ig}(0,0)(1-D_{jg}(0,0)))] & \\ + \mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))((1-D_{ig}(1,1))D_{jg}(1,1) - (1-D_{ig}(0,0))D_{jg}(0,0))] & \\ + \mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))(D_{ig}(1,1)D_{jg}(1,1) - D_{ig}(0,0)D_{jg}(0,0))]. &\end{aligned}$$

Tables A11, A12 and A13 list the possible values that the terms that depend on the potential treatment statuses can take, which gives the desired result after some algebra. □

A6.3 Proof of Proposition A1

Under the conditions of the proposition,

$$\mathbb{E}[\mathbb{1}(D_{ig}=k)|Z_{ig}=k,Z_{jg}=0] = \mathbb{E}[\mathbb{1}(D_{ig}(k,0)=d)|Z_{ig}=k,Z_{jg}=0] = \mathbb{E}[\mathbb{1}(D_{ig}(k,0)=k)]$$

Table A12: $(1 - D_{ig}(1,1))D_{jg}(1,1) - (1 - D_{ig}(0,0))D_{jg}(0,0)$

$D_{ig}(1,1)$	$D_{jg}(1,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	0	1	-1	SC,C,GC	AT
0	1	0	1	0		
1	1	1	0	0		
1	1	0	0	0		
0	1	0	0	1	NT	SC,C,GC
1	0	1	0	0		
1	0	0	0	0		
0	0	0	0	0		

Table A13: $D_{ig}(1,1)D_{jg}(1,1) - D_{ig}(0,0)D_{jg}(0,0)$

$D_{ig}(1,1)$	$D_{jg}(1,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	1	SC,C,GC	AT
1	0	1	0	0		
1	1	0	1	1	AT	SC,C,GC
1	1	0	0	1	SC,C,GC	SC,C,GC
1	0	0	0	0		
0	1	0	1	0		
0	1	0	0	0		
0	0	0	0	0		

On the other hand, for any $k \in \{0, 1, \ldots, K\}$,

$$\begin{aligned}
\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig} = k)|Z_{ig} = k, Z_{jg} = 0] &= \mathbb{E}[Y_{ig}(k, 0)\mathbb{1}(D_{ig}(k, 0) = k)] \\
\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig} = 0)|Z_{ig} = k, Z_{jg} = 0] &= \mathbb{E}[Y_{ig}(0, 0)\mathbb{1}(D_{ig}(k, 0) = 0)] \\
&= \mathbb{E}[Y_{ig}(0, 0)|D_{ig}(k, 0) = 0]\mathbb{E}[\mathbb{1}(D_{ig}(k, 0) = 0)] \\
&= \mathbb{E}[Y_{ig}(0, 0)\mathbb{1}(D_{ig}(k, 0) = k)] + \mathbb{E}[Y_{ig}(0, 0)\mathbb{1}(D_{ig}(k, 0) = 0)]
\end{aligned}$$

and $\mathbb{E}[Y_{ig}|Z_{ig} = k, Z_{jg} = 0] = \mathbb{E}[Y_{ig}\mathbb{1}(D_{ig} = k)|Z_{ig} = k, Z_{jg} = 0] + \mathbb{E}[Y_{ig}\mathbb{1}(D_{ig} = 0)|Z_{ig} = k, Z_{jg} = 0]$, from which:

$$\frac{\mathbb{E}[Y_{ig}|Z_{ig} = k, Z_{jg} = 0] - \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0]}{\mathbb{E}[\mathbb{1}(D_{ig} = k)|Z_{ig} = k, Z_{jg} = 0]} = \mathbb{E}[Y_{ig}(k, 0)|D_{ig}(k, 0) = k]$$

Similarly,

$$\frac{\mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = k] - \mathbb{E}[Y_{ig}|Z_{ig} = 0, Z_{jg} = 0]}{\mathbb{E}[\mathbb{1}(D_{jg} = k)|Z_{ig} = 0, Z_{jg} = k]} = \mathbb{E}[Y_{ig}(0, k)|D_{jg}(k, 0) = k]$$

as required. $\square$