Causal Spillover Effects Using Instrumental Variables: Supplemental Appendix

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Abstract

This supplemental appendix provides the proofs of the results in the paper and additional discussions not included in the paper to conserve space.

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Contents

A1 Spillovers on treatment status

When spillovers only occur on the treatment take-up stage, individual outcomes are not affected by neighbors' treatment status, so that

$$Y_{ig}(d, \mathbf{d}_g) = Y_{ig}(d).$$

Assuming exchangeability in the first stage, the (endogenous) treatment status will depend on the number of peers assigned to treatment,

$$D_{ig} = D_{ig}(Z_{ig}, W_{ig})$$

where $W_{ig} = \sum_{j \neq i} Z_{jg}$ is the number of unit i's peers assigned to treatment. The observed outcome is therefore:

$$Y_{ig} = Y_{ig}(0) + \tau_{ig} D_{ig}(Z_{ig}, W_{ig}) \tag{1}$$

where

$$\tau_{ig} = Y_{ig}(1) - Y_{ig}(0)$$

is the unit-level treatment effect. Under Assumption 1, we have that

$$\begin{aligned}
\Delta(z, w, z', w') &= \mathbb{E}[Y_{ig}|Z_{ig} = z, W_{ig} = w] - \mathbb{E}[Y_{ig}|Z_{ig} = z', W_{ig} = w'] \\
&= \mathbb{E}[\tau_{ig}(D_{ig}(z, w) - D_{ig}(z', w'))] \\
&= \mathbb{E}[\tau_{ig}|D_{ig}(z, w) > D_{ig}(z', w')]\mathbb{P}[D_{ig}(z, w) > D_{ig}(z', w')] \\
&\quad - \mathbb{E}[\tau_{ig}|D_{ig}(z, w) < D_{ig}(z', w')]\mathbb{P}[D_{ig}(z, w) < D_{ig}(z', w')]
\end{aligned}$$

A fact that stems from the above expression is that LATE-type estimands are identified only for the assignments such that $\mathbb{P}[D_{ig}(z, w) > D_{ig}(z', w')] > 0$ and $\mathbb{P}[D_{ig}(z, w) < D_{ig}(z', w')] = 0$ (or vice versa). The following result formalizes this idea. While the result is analogous to Theorem 1 in Imbens and Angrist (1994), I state it as a lemma for future reference.

Lemma A1 (Identification of LATE) *Under Assumption 1, for any pair of assignments* (z, w) *and* (z', w') *such that* $\mathbb{P}[D_{ig}(z, w) < D_{ig}(z', w')] = 0$ *and* $\mathbb{P}[D_{ig}(z, w) > D_{ig}(z', w')] > 0$,

$$\mathbb{E}[\tau_{ig}|D_{ig}(z, w) > D_{ig}(z', w')] = \frac{\mathbb{E}[Y_{ig}|Z_{ig} = z, W_{ig} = w] - \mathbb{E}[Y_{ig}|Z_{ig} = z', W_{ig} = w']}{\mathbb{E}[D_{ig}|Z_{ig} = z, W_{ig} = w] - \mathbb{E}[D_{ig}|Z_{ig} = z', W_{ig} = w']}$$

For instance, one may be willing to assume that $\mathbb{P}[D_{ig}(1, w) < D_{ig}(0, w)] = 0$ for all w so that given a number of neighbors assigned to treatment, being assigned to treatment can never reduce the likelihood of receiving it, which is a version of the monotonicity assumption. In particular, the following parameters have clear interpretations. First,

$$\mathbb{E}[\tau_{ig}|D_{ig}(1, w) > D_{ig}(0, w)]$$

is the average treatment effect for units who are compliers when w neighbors assigned to treatment. Observe that this is a function of w, so there are in fact $n+1$ of these parameters. On the other hand, for example,

$$\mathbb{E}[\tau_{ig}|D_{ig}(0,w+k) > D_{ig}(0,w)]$$

is the average treatment effect for units assigned to control but are pushed to get the treatment by having k additional neighbors being assigned to treatment.

In fact, Equation (1) can be seen as a standard IV setting with a multivalued instrument and a binary treatment, where the values of the instrument are given by the different possible combinations of own and neighbors' assignments. For this reason, Lemma A1 is very similar to the result in Imbens and Angrist (1994). The main difference is that Lemma A1 does not require monotonicity to hold equally for all treatment assignments. More precisely, it could be the case that for some values the instrument never decreases the probability of receiving treatment while for other values the instrument never increases it, and the LATEs for those assignments are identified (although for different subpopulations).

It is also interesting to analyze identification using the methods usually employed in empirical work. Under imperfect compliance, the most common estimand of interest is the Wald estimand:

$$\tau_{\mathsf{W}} = \frac{\mathbb{E}[Y_{ig}|Z_{ig}=1] - \mathbb{E}[Y_{ig}|Z_{ig}=0]}{\mathbb{E}[Z_{ig}|D_{ig}=1] - \mathbb{E}[D_{ig}|Z_{ig}=0]}$$

Assuming $\mathbb{P}[D_{ig}(1,w) < D_{ig}(0,w)] = 0$ for all w, this estimand becomes:

$$\tau_{\mathsf{W}} = \sum_{w=0}^{n} \mathbb{E}[\tau_{ig}|D_{ig}(1,w) > D_{ig}(0,w)]\rho(w)$$

where

$$\rho(w) = \frac{\mathbb{P}[D_{ig}(1,w) > D_{ig}(0,w)]\mathbb{P}[W_{ig}=w]}{\sum_{w=0}^{n}\mathbb{P}[D_{ig}(1,w) > D_{ig}(0,w)]\mathbb{P}[W_{ig}=w]}$$

so τ_{W} recovers an average of LATEs weighted by the proportion of each type of complier in the population and the probability of observing each possible number of neighbors assigned to treatment. This result is similar to the ones in Angrist and Imbens (1995) for the case of variable treatment intensity.

The presence of spillovers on treatment status is actually straightforward to test in practice by exploring the variation in D_{ig} induced by $(Z_{ig}, \mathbf{Z}_{(i)g})$. By the previous results above, assuming exchangeability for treatment take-up,

$$\mathbb{E}[D_{ig}|Z_{ig}=z, W_{ig}] = \mathbb{E}[D_{ig}(z)] + \sum_{w=1}^{n} \delta_w(z)\mathbb{1}(W_{ig}=w)$$

so the average spillover effects on treatment take-up, captured by the coefficients $\delta_w(z)$. Then, failure to reject the null that $\delta_w(z)=0$ for $w=1,\ldots,n$ would indicate the absence of average spillover effects in treatment status.

A2 Proofs

A2.1 Proof of Proposition 1

By assumption 1, $\mathbb{E}[D_{ig}|Z_{ig}=z, Z_{jg}=z'] = \mathbb{E}[D_{ig}(z,z')]$. Thus, under monotonicity (assumption 2),

$$\begin{aligned}
\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{ig}=0] &= \mathbb{E}[D_{ig}(0,0)] = \mathbb{P}[AT_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{ig}=1] &= \mathbb{E}[D_{ig}(0,1)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=0] &= \mathbb{E}[D_{ig}(1,0)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] + \mathbb{P}[C_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=1] &= \mathbb{E}[D_{ig}(1,1)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] + \mathbb{P}[C_{ig}] + \mathbb{P}[GC_{ig}]
\end{aligned}$$

and by simply solving the system it follows that

$$\begin{aligned}
\mathbb{P}[AT_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig}=0, Z_{ig}=0] \\
\mathbb{P}[SC_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig}=0, Z_{ig}=1] - \mathbb{E}[D_{ig}|Z_{ig}=0, Z_{ig}=0] \\
\mathbb{P}[C_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=0] - \mathbb{E}[D_{ig}|Z_{ig}=0, Z_{ig}=1] \\
\mathbb{P}[GC_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=1] - \mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=0]
\end{aligned}$$

and by monotonicity $\mathbb{P}[NT_{ig}] = 1 - \mathbb{P}[AT_{ig}] - \mathbb{P}[SC_{ig}] - \mathbb{P}[C_{ig}] - \mathbb{P}[GC_{ig}]$. Finally,

$$\begin{aligned}
\mathbb{E}[D_{ig}D_{jg}|Z_{ig}=0, Z_{ig}=0] &= \mathbb{E}[D_{ig}(0,0)D_{jg}(0,0)] = \mathbb{P}[AT_{ig}, AT_{jg}] \\
\mathbb{E}[(1-D_{ig})(1-D_{jg})|Z_{ig}=1, Z_{ig}=1] &= \mathbb{E}[(1-D_{ig}(1,1))(1-D_{jg}(1,1))] = \mathbb{P}[NT_{ig}, NT_{jg}].
\end{aligned}$$

See Tables 1 and 2 for the whole system of equations. □

A2.2 Proof of Lemma 1

Using that:

$$\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)] & \\
&+ \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))D_{ig}(z,z')(1-D_{jg}(z',z))] \\
&+ \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))(1-D_{ig}(z,z'))D_{jg}(z',z)] \\
&+ \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)],
\end{aligned}$$

Table 1: System of equations

D_{ig}	D_{jg}	Z_{ig}	Z_{jg}	Probabilities
1	1	0	0	p_{AA}
1	1	0	1	$p_{AA} + p_{AS} + p_{AC} + p_{SA} + p_{SS} + p_{SC}$
1	1	1	0	$p_{AA} + p_{AS} + p_{AC} + p_{SA} + p_{SS} + p_{SC}$
1	1	1	1	$1 - p_N - p_{NN}$
0	0	1	1	p_{NN}
0	0	1	0	$p_{GC} + p_{GG} + p_{GN} + p_{NC} + p_{NG} + p_{NN}$
0	0	0	1	$p_{GC} + p_{GG} + p_{GN} + p_{NC} + p_{NG} + p_{NN}$
0	0	0	0	$1 - p_A - p_{AA}$
1	0	0	0	$p_{AS} + p_{AC} + p_{AG} + p_{AN}$
1	0	1	1	$p_{NA} + p_{NS} + p_{NC} + p_{NG}$
1	0	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$
1	0	1	0	$p_{AC} + p_{AG} + p_{AN} + p_{SC} + p_{SG} + p_{SN} + p_{CC} + p_{CG} + p_{CN}$
0	1	0	0	$p_{AS} + p_{AC} + p_{AG} + p_{AN}$
0	1	1	1	$p_{NA} + p_{NS} + p_{NC} + p_{NG}$
0	1	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$
0	1	1	0	$p_{AC} + p_{AG} + p_{AN} + p_{SC} + p_{SG} + p_{SN} + p_{CC} + p_{CG} + p_{CN}$

Table 2: System of equations - simplified

D_{ig}	D_{jg}	Z_{ig}	Z_{jg}	Probabilities	Independent?
1	1	0	0	p_{AA}	1
1	1	0	1	$p_A + p_S - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	2
1	1	1	0	$p_A + p_S - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-
1	1	1	1	$1 - p_N - p_{NN}$	3
0	0	1	1	p_{NN}	4
0	0	1	0	$p_G + p_N - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	5
0	0	0	1	$p_G + p_N - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-
0	0	0	0	$1 - p_A - p_{AA}$	6
1	0	0	0	$p_A - p_{AA}$	-
1	0	1	1	$p_N - p_{NN}$	-
1	0	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$	7
1	0	1	0	$p_C + (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-
0	1	0	0	$p_A - p_{AA}$	-
0	1	1	1	$p_N - p_{NN}$	-
0	1	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$	-
0	1	1	0	$p_C + (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-

Table 3: $D_{ig}(1,0)(1-D_{jg}(0,1))-D_{ig}(0,0)(1-D_{jg}(0,0))$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	-1	AT	SC
1	1	0	1	0		
1	1	0	0	0		
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	0		
1	0	0	0	1	C,SC	C,GC,NT
0	0	0	0	0		

Table 4: $(1-D_{ig}(1,0))D_{jg}(0,1)-(1-D_{ig}(0,0))D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	0		
1	1	0	1	-1	C,SC	AT
1	1	0	0	0		
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	1	GC,NT	SC
1	0	0	0	0		
0	0	0	0	0		

we have:

$$\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=0]-\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]= \\
&+\mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))(D_{ig}(1,0)(1-D_{jg}(0,1))-D_{ig}(0,0)(1-D_{jg}(0,0)))] \\
&+\mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))((1-D_{ig}(1,0))D_{jg}(0,1)-(1-D_{ig}(0,0))D_{jg}(0,0))] \\
&+\mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))(D_{ig}(1,0)D_{jg}(0,1)-D_{ig}(0,0)D_{jg}(0,0))].
\end{aligned}$$

Tables 3, 4 and 5 list the possible values that the terms that depend on the potential treatment statuses can take, which gives the desired result after simple algebra. □

Table 5: $D_{ig}(1,0))D_{jg}(0,1) - D_{ig}(0,0)D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	1	AT	SC
1	1	0	1	1	C,SC	AT
1	1	0	0	1	C,SC	SC
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	0		
1	0	0	0	0		
0	0	0	0	0		

A2.3 Proof of Proposition 2

Under personalized assignment,

$$\begin{aligned}\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig}=d)\mathbb{1}(D_{jg}=d')|Z_{ig}=z,Z_{jg}=z'] &= \mathbb{E}[Y_{ig}(d,d')\mathbb{1}(D_{ig}(z')=d)\mathbb{1}(D_{jg}(z')=d')] \\ &= \mathbb{E}[Y_{ig}(d,d')|D_{ig}(z)=d,D_{jg}(z')=d'] \\ &\quad \times \mathbb{P}[D_{ig}(z')=d,D_{jg}(z')=d'].\end{aligned}$$

The proof follows the same lines as the one for Proposition 5. The case in which $z = 1-d$ determines unit i's type, namely, always-taker if d=1 or never-taker if $d = 0$. If $z = d$, unit i can have two types, namely, never-taker or complier if $d = 0$ and always-taker or complier if $d = 1$. Combining this information with the assignment and treatment statuses of unit j gives the desired result. $\square$.

A2.4 Proof of Proposition 3

The result follows using that

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z,Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)] & \\ +\mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))D_{ig}(z,z')(1-D_{jg}(z',z))] & \\ +\mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))(1-D_{ig}(z,z'))D_{jg}(z',z)] & \\ +\mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)], &\end{aligned}$$

combined with the facts that under one-sided noncompliance, $D_{ig}(0,1) = D_{ig}(0,0) = 0$, for all i, $D_{ig}(1,0) = 1$ implies that i is a complier and $D_{ig}(1,1) = 0$ implies i is a never-taker. $\square$

A2.5 Proof of Corollary 1

Combine lines 2 and 5 from the display in Proposition 3 and the results in Proposition 1, noting that under one-sided noncompliance $\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{jg}=1]=0$. $\square$

A2.6 Proof of Corollary 2

We have that $\mathbb{E}[Y_{ig}(0,0)] = \mathbb{E}[Y_{ig}(0,0)|C_{ig}]\mathbb{P}[C_{ig}] + \mathbb{E}[Y_{ig}(0,0)|C_{ig}^c]\mathbb{P}[C_{ig}^c]$ and thus

$$\mathbb{E}[Y_{ig}(0,0)|C_{ig}^c] = \frac{\mathbb{E}[Y_{ig}(0,0)] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}]\mathbb{P}[C_{ig}]}{1-\mathbb{P}[C_{ig}]}$$

from which

$$\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}^c] = \frac{\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)]}{1-\mathbb{P}[C_{ig}]}.$$

Using Proposition 3, we obtain

$$\begin{aligned}&\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}^c] = \\ &\quad\left\{\frac{\mathbb{E}[Y_{ig}D_{ig}|Z_{ig}=1, Z_{jg}=0]}{\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{jg}=0]} - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0]\right\}\frac{1}{1-\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=0]}.\end{aligned}$$

Similarly,

$$\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)|C_{jg}^c] = \frac{\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)]}{1-\mathbb{P}[C_{jg}]}.$$

and thus

$$\begin{aligned}&\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)|C_{jg}^c] = \\ &\quad\left\{\frac{\mathbb{E}[Y_{ig}D_{jg}|Z_{ig}=0, Z_{jg}=1]}{\mathbb{E}[D_{jg}|Z_{ig}=0, Z_{jg}=1]} - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0]\right\}\frac{1}{1-\mathbb{E}[D_{jg}|Z_{ig}=0, Z_{ig}=1]}.\end{aligned}$$

A2.7 Proof of Proposition 4

Since $\mathbb{E}[u_{ig}|Z_{ig}=z, Z_{jg}=z']=0$ for all (z,z') by assumption,

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \beta_0 &+ \beta_1\mathbb{E}[D_{ig}|Z_{ig}=z, Z_{jg}=z'] \\ &+ \beta_2\mathbb{E}[D_{jg}|Z_{ig}=z, Z_{jg}=z'] \\ &+ \beta_3\mathbb{E}[D_{ig}D_{jg}|Z_{ig}=z, Z_{jg}=z']\end{aligned}$$

Under one-sided noncompliance, $\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{jg}=z'] = \mathbb{E}[D_{jg}|Z_{ig}=z, Z_{jg}=0]=0$

and thus:

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0] &= \beta_0 \\
\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=0] &= \beta_0+\beta_1\mathbb{E}[D_{ig}|Z_{ig}=1,Z_{jg}=0] \\
\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=1] &= \beta_0+\beta_2\mathbb{E}[D_{ig}|Z_{ig}=0,Z_{jg}=1]
\end{aligned}
$$

from which:

$$
\begin{aligned}
\beta_0 &= \mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]=\mathbb{E}[Y_{ig}(0,0)] \\
\beta_1 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=0]-\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]}{\mathbb{E}[D_{ig}|Z_{ig}=1,Z_{jg}=0]}=\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|C_{ig}] \\
\beta_2 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=1]-\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]}{\mathbb{E}[D_{jg}|Z_{ig}=0,Z_{jg}=1]}=\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|C_{jg}] \\
\beta_3 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=1]-\beta_0-\beta_1\mathbb{E}[D_{ig}(1,1)]-\beta_2\mathbb{E}[D_{jg}(1,1)]}{\mathbb{E}[D_{jg}(1,1)D_{jg}(1,1)]}
\end{aligned}
$$

as long as $\mathbb{E}[D_{jg}(1,1)D_{jg}(1,1)]>0$ (otherwise, β_3 is not identified). Finally, note that

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=1] = \mathbb{E}[Y_{ig}(0,0)]& \\
&+\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|D_{ig}(1,1)=1]\mathbb{E}[D_{ig}(1,1)] \\
&+\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|D_{jg}(1,1)=1]\mathbb{E}[D_{jg}(1,1)] \\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)-Y_{ig}(0,1)+Y_{ig}(0,0)|D_{ig}(1,1)=1,D_{jg}(1,1)=1] \\
&\quad\times\mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]
\end{aligned}
$$

and use the fact that $D_{ig}(1,1)=1$ if i is a complier or a group complier to get that:

$$
\begin{aligned}
\beta_3 &= (\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|GC_{ig}]-\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|C_{ig}])\frac{\mathbb{P}[GC_{ig}]}{\mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]} \\
&\quad+(\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|GC_{jg}]-\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|C_{jg}])\frac{\mathbb{P}[GC_{jg}]}{\mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]} \\
&\quad+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)-(Y_{ig}(0,1)-Y_{ig}(0,0))|D_{ig}(1,1)=1,D_{jg}(1,1)=1].
\end{aligned}
$$

which gives the result. □

A2.8 Proof of Proposition 5

Under IPT,

$$
\begin{aligned}
\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig}=d)|Z_{ig}=z,Z_{jg}=z',D_{jg}=d'] &= \mathbb{E}[Y_{ig}(d,d')\mathbb{1}(D_{ig}(z,z')=d)|D_{jg}(z',z)=d'] \\
&= \mathbb{E}[Y_{ig}(d,d')\mathbb{1}(D_{ig}(z,z')=d)] \\
&= \mathbb{E}[Y_{ig}(d,d')|D_{ig}(z,z')=d]\mathbb{P}[D_{ig}(z,z')=d].
\end{aligned}
$$

If $z = 1-d$ and $z' = 1-d$, the event $D_{ig}(1-d, 1-d) = d$ completely determines unit i's type, which is always-taker if $d = 1$ and never-taker if $d = 0.$, which identifies $\mathbb{E}[Y_{ig}(d, d')|\xi_{ig} = AT \cdot d + NT \cdot (1-d)]$. If $z = 1-d$ and $z = d$, the event $D_{ig}(1-d, d) = d$ implies two possible types for unit i, namely, always-taker or social complier if $d = 1$ and never-taker or group complier if $d = 0$. Since the case always-taker (never-taker) was already determined from $d = 1$ ($d = 0$), this identifies $\mathbb{E}[Y_{ig}(d, d')|\xi_{ig} = SC \cdot d + GC \cdot (1-d)]$. Similarly, when $z = d$ and $z' = 1-d$, the event $D_{ig}(d, 1-d) = d$ implies three possible types for unit i, namely, always-taker, social complier or complier if $d = 1$ and never-taker, group complier or complier if $d = 0$. Combined with the previous two cases, this identifies $\mathbb{E}[Y_{ig}(d, d')|\xi_{ig} = C]$. Finally, the case $z = d, z' = d$ implies four possible types for unit i, namely, always-taker, social complier, complier or group complier if $d = 1$ and never-taker, group complier, complier or social complier if $d = 0$. Combined with the previous three cases, this identifies $\mathbb{E}[Y_{ig}(d, d')|\xi_{ig} = GD \cdot d + SC \cdot (1-d)]$. $\square$

A2.9 Proof of Proposition 6

First,

$$
\begin{aligned}
\mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{jg} = 0, X_g] &= \mathbb{E}[D_{ig}(1,0)|Z_{ig} = 1, Z_{jg} = 0, X_g] \\
&= \mathbb{E}[D_{ig}(1,0)|X_g] = \mathbb{P}[C_{ig}|X_g]
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{jg} = 1, X_g] &= \mathbb{E}[D_{ig}(1,1)|Z_{ig} = 1, Z_{jg} = 1, X_g] \\
&= \mathbb{E}[D_{ig}(1,1)|X_g] = \mathbb{P}[C_{ig}|X_g] + \mathbb{P}[GC_{ig}|X_g].
\end{aligned}
$$

For the second part, we have that for the first term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig}, X_g)\frac{(1-Z_{ig})(1-Z_{jg})}{p_{00}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}, X_g)\frac{(1-Z_{ig})(1-Z_{jg})}{p_{00}(X_g)}\middle| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}, X_g)\middle| Z_{ig} = 0, Z_{jg} = 0, X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0), X_g)\middle| Z_{ig} = 0, Z_{jg} = 0, X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0), X_g)\middle| X_g\right]\right\} \\
&= \mathbb{E}[g(Y_{ig}(0,0), X_g)].
\end{aligned}
$$

For the second term,

$$\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\frac{Z_{ig}(1-Z_{ig})}{p_{10}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\frac{Z_{ig}(1-Z_{ig})}{p_{10}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right| X_g]\right\} \\
&= \mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right] \\
&= \mathbb{E}\left[g(Y_{ig}(1,0),X_g)|C_{ig}\right]\mathbb{P}[C_{ig}].
\end{aligned}$$

For the third term,

$$\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\frac{(1-Z_{ig})Z_{ig}}{p_{01}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\frac{(1-Z_{ig})Z_{ig}}{p_{01}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right]\right\} \\
&= \mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right] \\
&= \mathbb{E}[g(Y_{ig}(0,1),X_g)|C_{jg}]\mathbb{P}[C_{jg}].
\end{aligned}$$

For the fourth term,

$$\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\frac{Z_{ig}(1-Z_{jg})}{p_{10}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\frac{Z_{ig}(1-Z_{jg})}{p_{10}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))\right| X_g]\right\} \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))] \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}^c]\mathbb{P}[C_{ig}^c]
\end{aligned}$$

and the result follows from $\mathbb{E}[g(Y_{ig}(0,0),X_g)] = \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}]\mathbb{P}[C_{ig}]+\mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}^c]\mathbb{P}[C_{ig}^c]$. Similarly for the fifth term,

$$\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\frac{(1-Z_{ig})Z_{jg}}{p_{01}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\frac{(1-Z_{ig})Z_{jg}}{p_{01}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))\right| X_g]\right\} \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))] \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{jg}^c]\mathbb{P}[C_{jg}^c]
\end{aligned}$$

and it can be seen that all these equalities also hold conditional on X_g. $\square$

A2.10 Proof of Proposition 7

The proof is the same as the one for Proposition 1, replacing $\{Z_{jg}=0\}$ by $\{W_{ig}=0\}$ and $\{Z_{jg}=1\}$ by $\{Z_{jg}=1, W_{ig}=w^*\}$. $\square$

A2.11 Proof of Lemma 2

$$
\begin{aligned}
S_{ig}(z,\mathbf{z}_g) &= \sum_{j\neq i} D_{jg}(z_j, w_g+z-z_j) \\
&= \mathbb{1}(w_g=0)\sum_{j\neq i} D_{jg}(0,0) + z\mathbb{1}(w_g=0)\sum_{j\neq i}[D_{jg}(0,1)-D_{jg}(0,0)] \\
&+ \mathbb{1}(w_g>0)\sum_{j\neq i} D_{jg}(0,w_g) + \mathbb{1}(w_g>0)\sum_{j\neq i}[D_{jg}(1,w_g-1)-D_{jg}(0,w_g)]z_j \\
&+ z\mathbb{1}(w_g>0)\sum_{j\neq i}[D_{jg}(0,w_g+1)-D_{jg}(0,w_g)] \\
&+ z\mathbb{1}(w_g>0)\sum_{j\neq i}[D_{jg}(1,w_g)-D_{jg}(1,w_g-1)-(D_{jg}(0,w_g+1)-D_{jg}(0,w_g))]z_j.
\end{aligned}
$$

Rearranging the terms,

$$
\begin{aligned}
&= \sum_{j\neq i} D_{jg}(0,0) \\
&+ z\sum_{j\neq i}[D_{jg}(0,1)-D_{jg}(0,0)] \\
&+ \mathbb{1}(w_g>0)\sum_{j\neq i}[D_{jg}(0,w_g)-D_{jg}(0,0)] \\
&+ z\mathbb{1}(w_g>0)\sum_{j\neq i}[D_{jg}(0,w_g+1)-D_{jg}(0,w_g)-(D_{jg}(0,1)-D_{jg}(0,0))] \\
&+ \mathbb{1}(w_g>0)\sum_{j\neq i}[D_{jg}(1,w_g-1)-D_{jg}(0,w_g)]z_j \\
&+ z\mathbb{1}(w_g>0)\sum_{j\neq i}[D_{jg}(1,w_g)-D_{jg}(1,w_g-1)-(D_{jg}(0,w_g+1)-D_{jg}(0,w_g))]z_j \\
&= N_{ig}^{AT} + zN_{ig}^{SC(1)} + \mathbb{1}(w_g>0)\sum_{w=1}^{w_g} N_{ig}^{SC(w)} + z\mathbb{1}(w_g>0)\left[N_{ig}^{SC(w_g)} - N_{ig}^{SC(1)}\right] \\
&+ \mathbb{1}(w_g>0)\mathbf{z}_g'\left[\mathbf{C}_{(i)g} + \mathbb{1}(w_g>1)\left(\sum_{w=w_g+1}^{n_g}\mathbf{SC}_{ig}^{(w)} + \sum_{w=1}^{w_g-1}\mathbf{GC}_{ig}^{(w)}\right)\right] \\
&+ z\mathbb{1}(w_g>0)\mathbf{z}_g'\left[\mathbf{GC}_{(i)g}^{(w_g)} - \mathbf{SC}_{(i)g}^{(w_g)}\right]
\end{aligned}
$$

which gives the result. $\square$

A2.12 Proof of Lemma 3

Start by writing

$$\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=z,\mathbf{Z}_{(i)g}=\mathbf{0}_g] &= \mathbb{E}[Y_{ig}(D_{ig}(z,0),S_{ig}(z,\mathbf{0}_g))] \\
&= \mathbb{E}[Y_{ig}(0,S_{ig}(0,\mathbf{0}_g))] + \mathbb{E}[(Y_{ig}(1,S_{ig}(z,\mathbf{0}_g)) - Y_{ig}(0,S_{ig}(z,\mathbf{0}_g)))D_{ig}(z,0)] \\
&= \sum_{s=0}^{n_g}\mathbb{E}[Y_{ig}(0,s)\mathbb{1}(S_{ig}(z,\mathbf{0}_g)=s)] \\
&+ \sum_{s=0}^{n_g}\mathbb{E}[(Y_{ig}(1,s)-Y_{ig}(0,s))D_{ig}(z,0)\mathbb{1}(S_{ig}(z,\mathbf{0}_g)=s)]
\end{aligned}$$

and thus

$$\begin{aligned}
&\mathbb{E}[Y_{ig}|Z_{ig}=1,\mathbf{Z}_{(i)g}=\mathbf{0}_g] - \mathbb{E}[Y_{ig}|Z_{ig}=0,\mathbf{Z}_{(i)g}] \\
&\quad= \sum_{s=0}^{n_g}\mathbb{E}[Y_{ig}(0,s)\left(\mathbb{1}(S_{ig}(1,\mathbf{0}_g)=s) - \mathbb{1}(S_{ig}(0,\mathbf{0}_g)=s)\right)] \\
&\quad+ \sum_{s=0}^{n_g}\mathbb{E}[(Y_{ig}(1,s)-Y_{ig}(0,s))D_{ig}(1,0)\mathbb{1}(S_{ig}(1,\mathbf{0}_g)=s)] \\
&\quad- \sum_{s=0}^{n_g}\mathbb{E}[(Y_{ig}(1,s)-Y_{ig}(0,s))D_{ig}(0,0)\mathbb{1}(S_{ig}(0,\mathbf{0}_g)=s)].
\end{aligned}$$

The result follows by noting that $D_{ig}(0,0)=1$ implies that i is an always-taker and that by Lemma 2, $S_{ig}(0,\mathbf{0}_g)=N_{ig}^{AT}$ and $S_{ig}(1,\mathbf{0}_g)=N_{ig}^{AT}+N_{ig}^{SC(1)}$. $\square$

A2.13 Proof of Proposition 8

Take treatment status $(d,\mathbf{d}_g)$ and start by setting $(z,\mathbf{z}_g)=(1-d,\mathbf{1}_g-\mathbf{d}_g)$ under the condition that $\mathbb{P}[Z_{ig}=1-d,\mathbf{Z}_{(i)g}=\mathbf{1}_g-\mathbf{d}_g]>0$. Because for each unit $d_i\neq z_i$, this completely determines each unit's compliance type, $\xi_{ig}=\xi(d)$ where $\xi(d)=AT\cdot d+NT\cdot(1-d)$. Then,

$$\mathbb{P}[Z_{ig}=1-d,\mathbf{Z}_{(i)g}=\mathbf{1}_g-\mathbf{d}_g,D_{ig}=d_i,\mathbf{D}_{(i)g}=\mathbf{d}_g]=\mathbb{P}[\xi_{ig}=\xi(d_i),\forall\, i]$$

and

$$\mathbb{E}[Y_{ig}|Z_{ig}=1-d,\mathbf{Z}_{(i)g}=\mathbf{1}_g-\mathbf{d}_g,D_{ig}=d,\mathbf{D}_{(i)g}=\mathbf{d}_g]=\mathbb{E}[Y_{ig}(d,\mathbf{d}_g)|\xi_{ig}=\xi(d_i),\forall\, i].$$

Next take an assignment $(z,\mathbf{z}_g)$ such that $z_j=d_j$ and $z_i=1-d_i$ for all $i\neq j$. That is, $(z,\mathbf{z}_g)$ and $(d,\mathbf{d}_g)$ differ in all but one coordinate. This combination of assignments and treatment statuses completely determines all compliance types $\xi_{ig}=AT\cdot d_i+NT\cdot(1-d_i)$ except for unit j, for whom we only know that $\xi_{jg}\in\{C,AT\}$ if $d_j=1$ and $\xi_{jg}\in\{NT,C\}$ if

$d_j = 0$. Hence,

$$\mathbb{P}[Z_{ig} = z, \mathbf{Z}_{(i)g} = \mathbf{z}_g, D_{ig} = d, \mathbf{D}_{(i)g} = \mathbf{d}_g] = \\ \mathbb{P}[\xi_{jg} = AT \cdot d_j + NT \cdot d_j, \xi_{jg} = \xi(d_j), \forall i \neq j] + \mathbb{P}[\xi_{jg} = C, \xi_{jg} = \xi(d_j), \forall i \neq j]$$

and

$$\mathbb{E}[Y_{ig}|Z_{ig} = z, \mathbf{Z}_{(i)g} = \mathbf{z}_g, D_{ig} = d, \mathbf{D}_{(i)g} = \mathbf{d}_g] = \\ \mathbb{E}[Y_{ig}(d, \mathbf{d}_g)|\xi_{jg} = AT \cdot d_j + NT \cdot d_j, \xi_{jg} = \xi(d_j), \forall i \neq j]p \\ + \mathbb{E}[Y_{ig}(d, \mathbf{d}_g)|\xi_{jg} = C, \xi_{jg} = \xi(d_j), \forall i \neq j](1-p)$$

where

$$p = \frac{\mathbb{P}[\xi_{jg} = AT \cdot d_j + NT \cdot d_j | \xi_{jg} = \xi(d_j), \forall i \neq j]}{\mathbb{P}[\xi_{jg} = AT \cdot d_j + NT \cdot d_j | \xi_{jg} = \xi(d_j), \forall i \neq j] + \mathbb{P}[\xi_{jg} = C | \xi_{jg} = \xi(d_j), \forall i \neq j]}.$$

Now, one of the two unknown terms on the right-hand side is identified from the previous assignment, and hence this gives identified of the remaining probability and expectation. Using the same logic, we can consider all assignments in which $(z, \mathbf{z}_g)$ and $(d, \mathbf{d}_g)$ differ in only one coordinate, then all assignments in which they differ in all but two, and so forth until $(z, \mathbf{z}_g) = (d, \mathbf{d}_g)$.

When the exchangeability condition in Assumption 9 holds, identification of average potential outcomes follows immediately, noting that for $N_{ig}^{AT} = n^{AT}, N_{ig}^{C} = n^{C}, N_{ig}^{NT} = n^{NT}$

$$\begin{aligned} 0 &\leq n^C \leq n_g \\ 0 &\leq n^{AT} \leq s_g \\ 0 &\leq n^{NT} \leq n_g - s_g \\ n_g &= n^C + n^{AT} + n^{NT}. \end{aligned}$$

From the third and fourth lines we get $0 \leq n_g - n^C - n^{AT} \leq n_g - s_g$ which implies that $s_g - n^C \leq n^{AT} \leq n_g - n^C$ and combining this with the second line we obtain:

$$\begin{gathered} 0 \leq n^C \leq n_g \\ \max\{0, s_g - n^C\} \leq n^{AT} \leq \min\{s_g, n_g - n^C\} \\ n^{NT} = n_g - n^C - n^{AT}. \end{gathered}$$

A2.14 Proof of Proposition 9

The proof is the same as the one for Proposition 3, replacing $\{Z_{jg} = 0\}$ by $\{W_{ig} = 0\}$ and $\{Z_{jg} = 1\}$ by $\{Z_{jg} = 1, W_{ig} = 1\}$. For the last two terms, note that

$$\begin{aligned}\mathbb{E}\left[\prod_{i=1}^{n_g+1}(1-D_{ig})\middle| Z_{ig}+W_{ig}=n_g+1\right] &= \mathbb{E}\left[\prod_{i=1}^{n_g+1}(1-D_{ig}(1,n_g))\right]\\ &= \mathbb{P}\left[\cap_{i=1}^{n_g+1} NT_{ig}\right]\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}\left[Y_{ig}\prod_{i=1}^{n_g+1}(1-D_{ig})\middle| Z_{ig}+W_{ig}=n_g+1\right] &= \mathbb{E}\left[Y_{ig}(0,0)\prod_{i=1}^{n_g+1}(1-D_{ig}(1,n_g))\right]\\ &= \mathbb{E}\left[Y_{ig}(0,0)|\cap_{i=1}^{n_g+1} NT_{ig}\right]\mathbb{P}\left[\cap_{i=1}^{n_g+1} NT_{ig}\right]\end{aligned}$$

and the result follows. □

A2.15 Proof of Proposition 10

The proof follows the same argument as that of Proposition 5. Start by considering The assignment $(D_{ig}, Z_{ig}, S_{ig}, W_{ig}) = (1, n, 0, 0)$. As long as the probability of this assignment is nonzero, this identifies $\mathbb{P}[AT_{ig}]$ and $\mathbb{E}[Y_{ig}(1, n)|AT_{ig}]$. Next, switch W_{ig} to one leaving the remaining assignments constant. In this case we know that unit i can be an always-taker or a 1-social complier, so combined with the previous result, this identifies $\mathbb{P}[SC_{ig}(1)]$ and $\mathbb{E}[Y_{ig}(1, n)|SC_{ig}(1)]$. With a similar reasoning, switching the value of W_{ig} to $2, 3 \ldots, n$ we identify $\mathbb{E}[Y_{ig}(1, n)|SC_{ig}(w)]$ for $w = 1, \ldots, n$. Next, switch W_{ig} back to zero and change Z_{ig} to 1. From the fact that $D_{ig}(1, 0) = 1$ together with the results so far we identify $\mathbb{P}[C_{ig}]$ and $\mathbb{E}[Y_{ig}(1, n)|C_{ig}]$. Similarly, the assignment $(Z_{ig}, W_{ig}) = (1, w)$ identifies $\mathbb{P}[GC_{ig}(w)]$ and $\mathbb{E}[Y_{ig}(1, n)|GC_{ig}(w)]$. The same reasoning can be applied to any other treatment status $(D_{ig}), S_{ig}) = (d, s)$. □

A2.16 Proof of Proposition 11

First,

$$\mathbb{E}[\mathbb{1}(D_{ig} = d)|Z_{ig} = z] = \mathbb{P}[D_{ig}(z) = d]$$

from which:

$$\begin{aligned}
\mathbb{P}[AT_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 0] \\
\mathbb{P}[NT_{ig}] &= \mathbb{E}[1 - D_{ig}|Z_{ig} = 1] \\
\mathbb{P}[C_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 1] - \mathbb{E}[D_{ig}|Z_{ig} = 0]
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig} = d)|Z_{ig} = 1, S_{ig} = s] &= \sum_{\mathbf{z}_g} \mathbb{E}[Y_{ig}\mathbb{1}(D_{ig} = d)|Z_{ig} = 1, S_{ig} = s, Z_{(i)g} = \mathbf{z}_g] \\
&\quad \times \mathbb{P}[Z_{(i)g} = \mathbf{z}_g|Z_{ig} = 1, S_{ig} = s] \\
&= \sum_{\mathbf{z}_g} \mathbb{E}[Y_{ig}(d, s)\mathbb{1}(D_{ig}(z) = d)|S_{ig}(\mathbf{z}_g) = s]\mathbb{P}[Z_{(i)g} = \mathbf{z}_g|Z_{ig} = 1, S_{ig} = s] \\
&= \sum_{\mathbf{z}_g} \mathbb{E}[Y_{ig}(d, s)\mathbb{1}(D_{ig}(z) = d)]\mathbb{P}[Z_{(i)g} = \mathbf{z}_g|Z_{ig} = 1, S_{ig} = s] \\
&= \mathbb{E}[Y_{ig}(d, s)\mathbb{1}(D_{ig}(z) = d)] \sum_{\mathbf{z}_g} \mathbb{P}[Z_{(i)g} = \mathbf{z}_g|Z_{ig} = 1, S_{ig} = s] \\
&= \mathbb{E}[Y_{ig}(d, s)\mathbb{1}(D_{ig}(z) = d)] \\
&= \mathbb{E}[Y_{ig}(d, s)|D_{ig}z = d]\mathbb{P}[D_{ig}(z) = d]
\end{aligned}$$

It follows that:

$$\begin{aligned}
\mathbb{E}[Y_{ig}D_{ig}|Z_{ig} = 0, S_{ig} = s] &= \mathbb{E}[Y_{ig}(1, s)|AT_{ig}]\mathbb{P}[AT_{ig}] \\
\mathbb{E}[Y_{ig}(1 - D_{ig})|Z_{ig} = 1, S_{ig} = s] &= \mathbb{E}[Y_{ig}(0, s)|NT_{ig}]\mathbb{P}[NT_{ig}] \\
\mathbb{E}[Y_{ig}D_{ig}|Z_{ig} = 1, S_{ig} = s] &= \mathbb{E}[Y_{ig}(1, s)|AT_{ig}]\mathbb{P}[AT_{ig}] + \mathbb{E}[Y_{ig}(1, s)|C_{ig}]\mathbb{P}[C_{ig}] \\
\mathbb{E}[Y_{ig}(1 - D_{ig})|Z_{ig} = 0, S_{ig} = s] &= \mathbb{E}[Y_{ig}(0, s)|NT_{ig}]\mathbb{P}[NT_{ig}] + \mathbb{E}[Y_{ig}(0, s)|C_{ig}]\mathbb{P}[C_{ig}]
\end{aligned}$$

and the result follows. □

A2.17 Proof of Lemma A1

We have that

$$\mathbb{E}[Y_{ig}|Z_{ig} = z, W_{ig} = w] = \mathbb{E}[Y_{ig}(0)] + \mathbb{E}[\tau_{ig}D_{ig}(z, w)]$$

from which

$$\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig} = z, W_{ig} = w] - \mathbb{E}[Y_{ig}|Z_{ig} = z', W_{ig} = w'] &= \mathbb{E}[\tau_{ig}(D_{ig}(z, w) - D_{ig}(z', w'))] \\
&= \mathbb{E}[\tau_{ig}|D_{ig}(z, w) > D_{ig}(z', w')] \\
&\quad \times \mathbb{P}[D_{ig}(z, w) > D_{ig}(z', w')]
\end{aligned}$$

But

$$\mathbb{E}[D_{ig}|Z_{ig} = z, W_{ig} = w] - \mathbb{E}[D_{ig}|Z_{ig} = z', W_{ig} = w'] = \mathbb{E}[D_{ig}(z, w) - D_{ig}(z', w')]$$

which gives the desired result. $\square$

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