Causal Spillover Effects Using Instrumental Variables: Supplemental Appendix

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Abstract

This supplemental appendix provides the proofs of the results in the paper and additional results and discussions not included in the paper to conserve space.

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A1 Additional Identification Results

A1.1 Indirect ITT Effects

The following result characterizes the indirect ITT effect.

Lemma A1 *Under Assumptions 1-3,*

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=1]-\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]=&\\
&\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|\{SC_{ig}\}\times\{GC_{jg},NT_{jg}\}]\\
&\quad\times\mathbb{P}[\{SC_{ig}\}\times\{GC_{jg},NT_{jg}\}]\\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,0)|\{SC_{ig}\}\times\{SC_{jg},C_{jg}\}]\\
&\quad\times\mathbb{P}[\{SC_{ig}\}\times\{SC_{jg},C_{jg}\}]\\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,1)|SC_{ig},AT_{jg}]\\
&\quad\times\mathbb{P}[SC_{ig},AT_{jg}]\\
&+\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|\{C_{ig},GC_{ig},NT_{ig}\}\times\{SC_{jg},C_{jg}\}]\\
&\quad\times\mathbb{P}[\{C_{ig},GC_{ig},NT_{ig}\}\times\{SC_{jg},C_{jg}\}]\\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)|\{AT_{ig}\}\times\{SC_{jg},C_{jg}\}]\\
&\quad\times\mathbb{P}[\{AT_{ig}\}\times\{SC_{jg},C_{jg}\}].
\end{aligned}
$$

A1.2 Total ITT Effects

The following result characterizes the indirect ITT effect.

Lemma A2 *Under Assumptions 1-3,*

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=1]-\mathbb{E}[Y_{ig}|Z_{ig}=0,Z_{jg}=0]=&\\
&\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|\{SC_{ig},C_{ig},GC_{ig}\}\times\{NT_{jg}\}]\\
&\quad\times\mathbb{P}[\{SC_{ig},C_{ig},GC_{ig}\}\times\{NT_{jg}\}]\\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)|\{AT_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}]\\
&\quad\times\mathbb{P}[\{AT_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}]\\
&+\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|\{NT_{ig}\},\{SC_{ig},C_{jg},GC_{jg}\}]\\
&\quad\times\mathbb{P}[\{NT_{ig}\},\{SC_{ig},C_{jg},GC_{jg}\}]\\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,1)|\{SC_{ig},C_{ig},GC_{ig}\}\times\{AT_{jg}\}]\\
&\quad\times\mathbb{P}[\{SC_{ig},C_{ig},GC_{ig}\}\times\{AT_{jg}\}]\\
&+\mathbb{E}[Y_{ig}(1,1)-Y_{ig}(0,0)|\{SC_{ig},C_{ig},GC_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}]\\
&\quad\times\mathbb{P}[\{SC_{ig},C_{ig},GC_{ig}\}\times\{SC_{jg},C_{jg},GC_{jg}\}].
\end{aligned}
$$

A1.3 Identification with Multiple Units per Group

In this section I generalize the results to the case where each group g has $n_g + 1$ identically-distributed units, so that each unit in group g has n_g neighbors or peers. The vector of treatment statuses in each group is given by $\mathbf{D}_g = (D_{1g}, \ldots, D_{n_g+1,g})$. For each unit i, $D_{j,ig}$ is the treatment indicator corresponding to unit i's j-th neighbor, collected in the vector $\mathbf{D}_{(i)g} = (D_{1,ig}, D_{2,ig}, \ldots, D_{n_g,ig})$. This vector takes values $\mathbf{d}_g = (d_1, d_2, \ldots, d_{n_g}) \in \mathcal{D}_g \subseteq \{0,1\}^{n_g}$. For a given realization of the treatment status $(d, \mathbf{d}_g)$, the potential outcome for unit i in group g is $Y_{ig}(d, \mathbf{d}_g)$ with observed outcome $Y_{ig} = Y_{ig}(D_{ig}, \mathbf{D}_{(i)g})$. In what follows, $\mathbf{0}_g$ and $\mathbf{1}_g$ will denote n_g-dimensional vectors of zeros and ones, respectively. The observed outcome can be written as:

$$Y_{ig} = \sum_{d \in \{0,1\}} \sum_{\mathbf{d}_g \in \mathcal{D}_g} Y_{ig}(d, \mathbf{d}_g) \mathbb{1}(D_{ig} = d) \mathbb{1}(\mathbf{D}_{(i)g} = \mathbf{d}_g).$$

Let $\mathbf{Z}_{(i)g}$ be the vector of unit i's peers' instruments, taking values $\mathbf{z}_g \in \{0,1\}^{n_g}$. For simplicity, I will assume that potential statuses and outcomes satisfy an exchangeability condition under which the identities of the treated peers do not matter, and thus the variables depend on the vectors $\mathbf{z}_g$ and $\mathbf{d}_g$, respectively, only through the sum of its elements. See Vazquez-Bare (2017) for a detailed discussion on this assumption. Under this condition, we have that $D_{ig}(z, \mathbf{z}_g) = D_{ig}(z, w_g)$ where $w_g = \mathbf{1}_g' \mathbf{z}_g$ and $Y_{ig}(d, \mathbf{d}_g) = Y_{ig}(d, s_g)$ where $s_g = \mathbf{1}_g' \mathbf{d}_g$.

The monotonicity assumption can be adapted to the general case as:

$$D_{ig}(1, w_g) \geq D_{ig}(1, 0) \geq D_{ig}(0, n_g) \geq D_{ig}(0, w_g)$$

for all $w_g = 0, 1, \ldots, n_g$. Under this assumption, we can define five compliance classes. First, always-takers, AT, are units with $D_{ig}(0,0) = 1$ which implies $D_{ig}(z, w_g) = 1$ for all (z, w_g). Next, w^*-social compliers, SC(w^*), are units for whom $D_{ig}(1, w_g) = 1$ for all w_g, and for which there exists a $0 < w^* < n_g$ such that $D_{ig}(0, w_g) = 1$ for all $w_g \geq w^*$. Thus, w^*-social compliers start receiving treatment as soon as w^* of their peers are assigned to treatment. Compliers, C, are units with $D_{ig}(1, w_g) = 1$ and $D_{ig}(0, w_g) = 0$ for all w_g. Next, w^*-group compliers, GC(w^*) have $D_{ig}(0, w_g) = 0$ for all w_g and there exists a $0 < w^* < n_g$ such that $D_{ig}(1, w_g) = 1$ for all $w_g \geq w^*$. That is, w^*-group compliers need to be assigned to treatment and have at least w^* peers assigned to treatment to actually receive the treatment. Finally, never-takers, NT, are units with $D_{ig}(z, w_g) = 0$ for all (z, w_g).

Let ξ_{ig} be a random variable indicating unit i's compliance type, with

$$\xi_{ig} \in \Xi = \{\mathrm{NT}, \mathrm{GC}(w^*), \mathrm{C}, \mathrm{SC}(w^*), \mathrm{AT} | w^* = 1, \ldots, n_g - 1\},$$

and $\boldsymbol{\xi}_{(i)g}$ the vector collecting ξ_{jg} for $j \neq i$. As before, let the event $AT_{ig} = \{\xi_{ig} = \mathrm{AT}\}$, $C_{ig} = \{\xi_{ig} = \mathrm{C}\}$ and similarly for the other compliance types. I will also assume that peers' types are exchangeable, which means that average potential outcomes depend only on how many of their peers belong to each compliance class, regardless of their identities.

The following assumption collects the required conditions for the upcoming results.

Assumption A1 (Identification conditions for general n_g)

1. *Existence of instruments:*
   (a) $Y_{ig}(d, \mathbf{d}_g)$ *is not a function of* $(z, \mathbf{z}_g)$.
   (b) *For all* i, $j \neq i$ *and* g, $((Y_{ig}(d, \mathbf{d}_g))_{(d,\mathbf{d}_g)}, (D_{ig}(z, \mathbf{z}_g))_{z,\mathbf{z}_g}) \perp\!\!\!\perp (Z_{ig}, \mathbf{Z}_{(i)g})$.
2. *Exchangeability:*
   (a) $D_{ig}(z, \mathbf{z}_g) = D_{ig}(z, w_g)$ *where* $w_g = \mathbf{1}_g'\mathbf{z}_g$
   (b) $Y_{ig}(d, \mathbf{d}_g) = Y_{ig}(d, s_g)$ *where* $s_g = \mathbf{1}_g'\mathbf{d}_g$.
3. *Monotonicity: for all* $w_g = 0, 1, \ldots, n_g$, $D_{ig}(1, w_g) \geq D_{ig}(1, 0) \geq D_{ig}(0, n_g) \geq D_{ig}(0, w_g)$.
4. *Relevance:* $\mathbb{P}[AT_{ig}] + \mathbb{P}[NT_{ig}] < 1$.

Let $W_{ig} = \sum_{j \neq i} Z_{jg}$ be the observed number of unit i' peers assigned to treatment. The following result discusses identification of the distribution of compliance types.

Proposition A1 *Under Assumption A1,*

$$
\begin{aligned}
\mathbb{P}[AT_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 0, W_{ig} = 0] \\
\mathbb{P}[SC_{ig}(w^*)] &= \mathbb{E}[D_{ig}|Z_{ig} = 0, W_{ig} = w^*] - \mathbb{E}[D_{ig}|Z_{ig} = 0, W_{ig} = w^* - 1], \quad 1 < w^* < n_g \\
\mathbb{P}[C_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 1, W_{ig} = 0] - \mathbb{E}[D_{ig}|Z_{ig} = 0, W_{ig} = n_g] \\
\mathbb{P}[GC_{ig}(w^*)] &= \mathbb{E}[D_{ig}|Z_{ig} = 1, W_{ig} = w^*] - \mathbb{E}[D_{ig}|Z_{ig} = 1, W_{ig} = w^* - 1], \quad 1 < w^* < n_g \\
\mathbb{P}[NT_{ig}] &= \mathbb{E}[1 - D_{ig}|Z_{ig} = 1, W_{ig} = n_g].
\end{aligned}
$$

Proposition A2 (Identification under OSN) *Suppose that* $D_{ig}(0, w_g) = 0$ *for all* i, g *and* w_g. *Under Assumption A1, for any* $j \neq i$,

$$
\begin{aligned}
\mathbb{E}[Y_{ig}(0, 0)] &= \mathbb{E}[Y_{ig}|Z_{ig} = 0, W_{ig} = 0] \\
\mathbb{E}[Y_{ig}(1, 0)|C_{ig}]\mathbb{P}[C_{ig}] &= \mathbb{E}[Y_{ig}D_{ig}|Z_{ig} = 1, W_{ig} = 0] \\
\mathbb{E}[Y_{ig}(0, 1)|C_{jg}]\mathbb{P}[C_{jg}] &= \mathbb{E}[Y_{ig}D_{jg}|Z_{ig} = 0, Z_{jg} = 1, W_{ig} = 1] \\
\mathbb{E}[Y_{ig}(0, 0)|C_{ig}]\mathbb{P}[C_{ig}] &= \mathbb{E}[Y_{ig}|Z_{ig} = 0, W_{ig} = 0] - \mathbb{E}[Y_{ig}(1 - D_{ig})|Z_{ig} = 1, W_{ig} = 0] \\
\mathbb{E}[Y_{ig}(0, 0)|C_{jg}]\mathbb{P}[C_{jg}] &= \mathbb{E}[Y_{ig}|Z_{ig} = 0, W_{ig} = 0] - \mathbb{E}[Y_{ig}(1 - D_{jg})|Z_{ig} = 0, Z_{jg} = 1, W_{ig} = 1] \\
\mathbb{E}[Y_{ig}(0, 0)|NT_{ig}, N_{ig}^{NT} = n_g]\mathbb{P}[NT_{ig}, N_{ig}^{NT} = n_g] &= \mathbb{E}\left[Y_{ig} \prod_{i=1}^{n_g+1} (1 - D_{ig}) \middle| Z_{ig} + W_{ig} = n_g + 1 \right]
\end{aligned}
$$

where

$$
\mathbb{P}[NT_{ig}, N_{ig}^{NT} = n_g] = \mathbb{E}\left[\prod_{i=1}^{n_g+1} (1 - D_{ig}) \middle| Z_{ig} + W_{ig} = n_g + 1 \right].
$$

Notice that this result only identifies the average direct and spillover effects when only one unit is treated. The reason is that, as soon as there is more than one unit assigned to treatment, it is not possible to distinguish between compliers and group compliers or between group compliers and never-takers. I provide this result to show that the conclusions from Proposition 2 still hold in the general case. I leave the question of what other parameters can be identified in this more general setting for future research.

A2 Estimation and Inference

The parameters of interest can be recovered by estimating expectations using sample means. More precisely, let

$$\mathbb{1}_{ig}^{\mathbf{z}} = \begin{bmatrix} \mathbb{1}(Z_{ig}=0, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=1, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=0, Z_{jg}=1) \\ \mathbb{1}(Z_{ig}=1, Z_{jg}=1) \end{bmatrix}$$

and let $\mathbf{H}(\cdot)$ be a vector-valued function whose exact shape depends on the parameters to be estimated, as illustrated below. Then the goal is to estimate:

$$\boldsymbol{\mu} = \mathbb{E}\begin{bmatrix} \mathbb{1}_{ig}^{\mathbf{z}} \\ \mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) \otimes \mathbb{1}_{ig}^{\mathbf{z}} \end{bmatrix}$$

where the first four elements correspond to the assignment probabilities $\mathbb{P}[Z_{ig}=z, Z_{jg}=z']$ and the remaining elements corresponds to estimands of the form $\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig}=d, D_{jg}=d')\mathbb{1}(Z_{ig}=z, Z_{jg}=z')]$. The most general choice of $\mathbf{H}$ in this setup is the following:

$$\mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) = \begin{bmatrix} \mathbb{1}(D_{ig}=0, D_{jg}=0) \\ \vdots \\ \mathbb{1}(D_{ig}=1, D_{jg}=1) \\ Y_{ig}\mathbb{1}(D_{ig}=0, D_{jg}=0) \\ \vdots \\ Y_{ig}\mathbb{1}(D_{ig}=1, D_{jg}=1) \end{bmatrix}$$

which is a vector of dimension equal to eight that can be used to estimate all the first-stage estimands $\mathbb{E}[\mathbb{1}(D_{ig}=d, D_{jg}=d')|Z_{ig}=z, Z_{jg}=z']$ and average outcomes $\mathbb{E}[Y_{ig}\mathbb{1}(D_{ig}=d, D_{jg}=d')|Z_{ig}=z, Z_{jg}=z']$. In this general case, the total number of equations to be estimated is 36: four probabilities $\mathbb{P}[Z_{ig}=z, Z_{jg}=z']$ plus the four indicators $\mathbb{1}(Z_{ig}=z, Z_{jg}=z')$ times each of the eight elements in $\mathbf{H}(\cdot)$. The dimension of $\mathbf{H}(\cdot)$ can be reduced, for example, by focusing on ITT parameters $\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z']$, which corresponds to:

$$\mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) = Y_{ig},$$

or by imposing the assumptions described in previous sections. For instance, under one-sided noncompliance, the parameters in Corollaries 1 and 2 can be estimated by defining:

$$\mathbb{1}_{ig}^{\mathbf{z}} = \begin{bmatrix} \mathbb{1}(Z_{ig}=0, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=1, Z_{jg}=0) \\ \mathbb{1}(Z_{ig}=0, Z_{jg}=1) \end{bmatrix}$$

and

$$\mathbf{H}(Y_{ig}, D_{ig}, D_{jg}) = \begin{bmatrix} D_{ig} \\ Y_{ig} \\ Y_{ig}(1-D_{ig}) \\ Y_{ig}(1-D_{jg}) \end{bmatrix}.$$

Regardless of the choice of $\mathbb{1}_{ig}^{\mathbf{z}}$ and $\mathbf{H}(\cdot)$, the dimension of the vector of parameters to be estimated is fixed (and at most 36). Consider the following sample mean estimator:

$$\hat{\boldsymbol{\mu}} = \frac{1}{G}\sum_{g=1}^{G} W_g$$

where

$$W_g = \begin{bmatrix} (\mathbb{1}_{1g}^{\mathbf{z}} + \mathbb{1}_{2g}^{\mathbf{z}})/2 \\ (\mathbf{H}(Y_{1g}, D_{1g}, D_{2g}) \otimes \mathbb{1}_{1g}^{\mathbf{z}} + \mathbf{H}(Y_{2g}, D_{2g}, D_{1g}) \otimes \mathbb{1}_{2g}^{\mathbf{z}})/2 \end{bmatrix}.$$

I will assume the following.

Assumption A2 (Sampling and moments) *Let* $V_{ig} = (Y_{ig}, D_{ig}, Z_{ig})'$ *for* $i = 1, 2$, *and* $V_g = (V_{1g}', V_{2g}')'$.

1. $(V_g)_{g=1}^G$ *are independent and identically distributed.*
2. *For each* g, V_{1g} *and* V_{2g} *are identically distributed but not necessarily independent.*
3. $\mathbb{E}[Y_{ig}^4] < \infty$.

Assumption A2 requires the groups to be independent and identically distributed, but allows units within groups to be arbitrarily correlated.

It is straightforward to see that under assumption A2, $\hat{\boldsymbol{\mu}}$ is unbiased and consistent for $\boldsymbol{\mu}$, and it converges in distribution to a normal random variable after centering and rescaling as $G \to \infty$:

$$\sqrt{G}(\hat{\boldsymbol{\mu}} - \boldsymbol{\mu}) \xrightarrow{\mathcal{D}} \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$$

where $\boldsymbol{\Sigma} = \mathbb{E}[(W_g - \boldsymbol{\mu})(W_g - \boldsymbol{\mu})']$, and where the limiting variance can be consistently estimated by:

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{G}\sum_{g}(W_g - \hat{\boldsymbol{\mu}})(W_g - \hat{\boldsymbol{\mu}})'.$$

Finally, once $\hat{\boldsymbol{\mu}}$ has been estimated, the treatment effects of interest can be estimated as (possibly nonlinear) transformations of $\hat{\boldsymbol{\mu}}$, and their variance estimated using the delta method.

A3 Proofs of Main Results

A3.1 Proof of Proposition 1

By assumption 1, $\mathbb{E}[D_{ig}|Z_{ig} = z, Z_{jg} = z'] = \mathbb{E}[D_{ig}(z, z')]$. Thus, under monotonicity (assumption 2),

$$\begin{aligned}
\mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 0] &= \mathbb{E}[D_{ig}(0,0)] = \mathbb{P}[AT_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 1] &= \mathbb{E}[D_{ig}(0,1)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 0] &= \mathbb{E}[D_{ig}(1,0)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] + \mathbb{P}[C_{ig}] \\
\mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 1] &= \mathbb{E}[D_{ig}(1,1)] = \mathbb{P}[AT_{ig}] + \mathbb{P}[SC_{ig}] + \mathbb{P}[C_{ig}] + \mathbb{P}[GC_{ig}]
\end{aligned}$$

and by simply solving the system it follows that

$$\begin{aligned}
\mathbb{P}[AT_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 0] \\
\mathbb{P}[SC_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 1] - \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 0] \\
\mathbb{P}[C_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 0] - \mathbb{E}[D_{ig}|Z_{ig} = 0, Z_{ig} = 1] \\
\mathbb{P}[GC_{ig}] &= \mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 1] - \mathbb{E}[D_{ig}|Z_{ig} = 1, Z_{ig} = 0]
\end{aligned}$$

and by monotonicity $\mathbb{P}[NT_{ig}] = 1 - \mathbb{P}[AT_{ig}] - \mathbb{P}[SC_{ig}] - \mathbb{P}[C_{ig}] - \mathbb{P}[GC_{ig}]$. Finally,

$$\begin{aligned}
\mathbb{E}[D_{ig}D_{jg}|Z_{ig} = 0, Z_{ig} = 0] &= \mathbb{E}[D_{ig}(0,0)D_{jg}(0,0)] = \mathbb{P}[AT_{ig}, AT_{jg}] \\
\mathbb{E}[(1 - D_{ig})(1 - D_{jg})|Z_{ig} = 1, Z_{ig} = 1] &= \mathbb{E}[(1 - D_{ig}(1,1))(1 - D_{jg}(1,1))] = \mathbb{P}[NT_{ig}, NT_{jg}].
\end{aligned}$$

See Tables A1 and A2 for the whole system of equations. □

A3.2 Proof of Lemma 1

Using that:

$$\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig} = z, Z_{jg} = z'] = \mathbb{E}[Y_{ig}(0,0)]& \\
&+ \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))D_{ig}(z,z')(1 - D_{jg}(z',z))] \\
&+ \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))(1 - D_{ig}(z,z'))D_{jg}(z',z)] \\
&+ \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)],
\end{aligned}$$

Table A1: System of equations

D_{ig}	D_{jg}	Z_{ig}	Z_{jg}	Probabilities
1	1	0	0	p_{AA}
1	1	0	1	$p_{AA} + p_{AS} + p_{AC} + p_{SA} + p_{SS} + p_{SC}$
1	1	1	0	$p_{AA} + p_{AS} + p_{AC} + p_{SA} + p_{SS} + p_{SC}$
1	1	1	1	$1 - p_N - p_{NN}$
0	0	1	1	p_{NN}
0	0	1	0	$p_{GC} + p_{GG} + p_{GN} + p_{NC} + p_{NG} + p_{NN}$
0	0	0	1	$p_{GC} + p_{GG} + p_{GN} + p_{NC} + p_{NG} + p_{NN}$
0	0	0	0	$1 - p_A - p_{AA}$
1	0	0	0	$p_{AS} + p_{AC} + p_{AG} + p_{AN}$
1	0	1	1	$p_{NA} + p_{NS} + p_{NC} + p_{NG}$
1	0	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$
1	0	1	0	$p_{AC} + p_{AG} + p_{AN} + p_{SC} + p_{SG} + p_{SN} + p_{CC} + p_{CG} + p_{CN}$
0	1	0	0	$p_{AS} + p_{AC} + p_{AG} + p_{AN}$
0	1	1	1	$p_{NA} + p_{NS} + p_{NC} + p_{NG}$
0	1	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$
0	1	1	0	$p_{AC} + p_{AG} + p_{AN} + p_{SC} + p_{SG} + p_{SN} + p_{CC} + p_{CG} + p_{CN}$

Table A2: System of equations - simplified

D_{ig}	D_{jg}	Z_{ig}	Z_{jg}	Probabilities	Independent?
1	1	0	0	p_{AA}	1
1	1	0	1	$p_A + p_S - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	2
1	1	1	0	$p_A + p_S - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-
1	1	1	1	$1 - p_N - p_{NN}$	3
0	0	1	1	p_{NN}	4
0	0	1	0	$p_G + p_N - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	5
0	0	0	1	$p_G + p_N - (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-
0	0	0	0	$1 - p_A - p_{AA}$	6
1	0	0	0	$p_A - p_{AA}$	-
1	0	1	1	$p_N - p_{NN}$	-
1	0	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$	7
1	0	1	0	$p_C + (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-
0	1	0	0	$p_A - p_{AA}$	-
0	1	1	1	$p_N - p_{NN}$	-
0	1	0	1	$p_{AG} + p_{AN} + p_{SG} + p_{SN}$	-
0	1	1	0	$p_C + (p_{AG} + p_{AN} + p_{SG} + p_{SN})$	-

Table A3: $D_{ig}(1,0)(1-D_{jg}(0,1)) - D_{ig}(0,0)(1-D_{jg}(0,0))$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	-1	AT	SC
1	1	0	1	0		
1	1	0	0	0		
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	0		
1	0	0	0	1	C,SC	C,GC,NT
0	0	0	0	0		

Table A4: $(1-D_{ig}(1,0))D_{jg}(0,1) - (1-D_{ig}(0,0))D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	0		
1	1	0	1	-1	C,SC	AT
1	1	0	0	0		
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	1	GC,NT	SC
1	0	0	0	0		
0	0	0	0	0		

we have:

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|Z_{ig}=1, Z_{jg}=0] - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] = \\
&+ \mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))(D_{ig}(1,0)(1-D_{jg}(0,1)) - D_{ig}(0,0)(1-D_{jg}(0,0)))] \\
&+ \mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))((1-D_{ig}(1,0))D_{jg}(0,1) - (1-D_{ig}(0,0))D_{jg}(0,0))] \\
&+ \mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))(D_{ig}(1,0)D_{jg}(0,1) - D_{ig}(0,0)D_{jg}(0,0))].
\end{aligned}
$$

Tables A3, A4 and A5 list the possible values that the terms that depend on the potential treatment statuses can take, which gives the desired result after some algebra. □

Table A5: $D_{ig}(1,0))D_{jg}(0,1) - D_{ig}(0,0)D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	1	AT	SC
1	1	0	1	1	C,SC	AT
1	1	0	0	1	C,SC	SC
1	0	1	0	0		
0	1	0	1	0		
0	1	0	0	0		
1	0	0	0	0		
0	0	0	0	0		

A3.3 Proof of Proposition 2

The result follows using that

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)] & \\ + \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))D_{ig}(z,z')(1 - D_{jg}(z',z))] & \\ + \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))(1 - D_{ig}(z,z'))D_{jg}(z',z)] & \\ + \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)], &\end{aligned}$$

combined with the facts that under one-sided noncompliance, $D_{ig}(0,1) = D_{ig}(0,0) = 0$, for all i, $D_{ig}(1,0) = 1$ implies that i is a complier and $D_{ig}(1,1) = 0$ implies i is a never-taker. □

A3.4 Proof of Corollary 1

Combine lines 2 and 5 from the display in Proposition 2 and the results in Proposition 1, noting that under one-sided noncompliance $\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{jg}=1] = 0$. □

A3.5 Proof of Corollary 2

We have that $\mathbb{E}[Y_{ig}(0,0)] = \mathbb{E}[Y_{ig}(0,0)|C_{ig}]\mathbb{P}[C_{ig}] + \mathbb{E}[Y_{ig}(0,0)|C^c_{ig}]\mathbb{P}[C^c_{ig}]$ and thus

$$\mathbb{E}[Y_{ig}(0,0)|C^c_{ig}] = \frac{\mathbb{E}[Y_{ig}(0,0)] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}]\mathbb{P}[C_{ig}]}{1 - \mathbb{P}[C_{ig}]}$$

from which

$$\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)|C^c_{ig}] = \frac{\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)]}{1 - \mathbb{P}[C_{ig}]}.$$

Using Proposition 2, we obtain

$$\mathbb{E}[Y_{ig}(0,0)|C_{ig}] - \mathbb{E}[Y_{ig}(0,0)|C_{ig}^c] = \\ \left\{ \frac{\mathbb{E}[Y_{ig}D_{ig}|Z_{ig}=1, Z_{jg}=0]}{\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{jg}=0]} - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] \right\} \frac{1}{1-\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{ig}=0]}.$$

Similarly,

$$\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)|C_{jg}^c] = \frac{\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)]}{1-\mathbb{P}[C_{jg}]}.$$

and thus

$$\mathbb{E}[Y_{ig}(0,0)|C_{jg}] - \mathbb{E}[Y_{ig}(0,0)|C_{jg}^c] = \\ \left\{ \frac{\mathbb{E}[Y_{ig}D_{jg}|Z_{ig}=0, Z_{jg}=1]}{\mathbb{E}[D_{jg}|Z_{ig}=0, Z_{jg}=1]} - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] \right\} \frac{1}{1-\mathbb{E}[D_{jg}|Z_{ig}=0, Z_{ig}=1]}.$$

A3.6 Proof of Proposition 3

Since $\mathbb{E}[u_{ig}|Z_{ig}=z, Z_{jg}=z']=0$ for all (z,z') by assumption,

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \beta_0 &+ \beta_1\mathbb{E}[D_{ig}|Z_{ig}=z, Z_{jg}=z'] \\ &+ \beta_2\mathbb{E}[D_{jg}|Z_{ig}=z, Z_{jg}=z'] \\ &+ \beta_3\mathbb{E}[D_{ig}D_{jg}|Z_{ig}=z, Z_{jg}=z']\end{aligned}$$

Under one-sided noncompliance, $\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{jg}=z'] = \mathbb{E}[D_{jg}|Z_{ig}=z, Z_{jg}=0]=0$ and thus:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] &= \beta_0 \\ \mathbb{E}[Y_{ig}|Z_{ig}=1, Z_{jg}=0] &= \beta_0 + \beta_1\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{jg}=0] \\ \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=1] &= \beta_0 + \beta_2\mathbb{E}[D_{ig}|Z_{ig}=0, Z_{jg}=1]\end{aligned}$$

from which:

$$\begin{aligned}\beta_0 &= \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] = \mathbb{E}[Y_{ig}(0,0)] \\ \beta_1 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig}=1, Z_{jg}=0] - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0]}{\mathbb{E}[D_{ig}|Z_{ig}=1, Z_{jg}=0]} = \mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|C_{ig}] \\ \beta_2 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=1] - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0]}{\mathbb{E}[D_{jg}|Z_{ig}=0, Z_{jg}=1]} = \mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|C_{jg}] \\ \beta_3 &= \frac{\mathbb{E}[Y_{ig}|Z_{ig}=1, Z_{jg}=1] - \beta_0 - \beta_1\mathbb{E}[D_{ig}(1,1)] - \beta_2\mathbb{E}[D_{jg}(1,1)]}{\mathbb{E}[D_{jg}(1,1)D_{jg}(1,1)]}\end{aligned}$$

as long as $\mathbb{E}[D_{jg}(1,1)D_{jg}(1,1)] > 0$ (otherwise, β_3 is not identified). Finally, note that

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=1,Z_{jg}=1] = \mathbb{E}[Y_{ig}(0,0)] & \\ &+ \mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|D_{ig}(1,1)=1]\mathbb{E}[D_{ig}(1,1)] \\ &+ \mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|D_{jg}(1,1)=1]\mathbb{E}[D_{jg}(1,1)] \\ &+ \mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)-Y_{ig}(0,1)+Y_{ig}(0,0)|D_{ig}(1,1)=1,D_{jg}(1,1)=1] \\ &\quad \times \mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]\end{aligned}$$

and use the fact that $D_{ig}(1,1) = 1$ if i is a complier or a group complier to get that:

$$\begin{aligned}\beta_3 &= (\mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|GC_{ig}] - \mathbb{E}[Y_{ig}(1,0)-Y_{ig}(0,0)|C_{ig}])\frac{\mathbb{P}[GC_{ig}]}{\mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]} \\ &+ (\mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|GC_{jg}] - \mathbb{E}[Y_{ig}(0,1)-Y_{ig}(0,0)|C_{jg}])\frac{\mathbb{P}[GC_{jg}]}{\mathbb{E}[D_{ig}(1,1)D_{jg}(1,1)]} \\ &+ \mathbb{E}[Y_{ig}(1,1)-Y_{ig}(1,0)-(Y_{ig}(0,1)-Y_{ig}(0,0))|D_{ig}(1,1)=1,D_{jg}(1,1)=1].\end{aligned}$$

which gives the result. □

A3.7 Proof of Proposition 4

First,

$$\begin{aligned}\mathbb{E}[D_{ig}|Z_{ig}=1,Z_{jg}=0,X_g] &= \mathbb{E}[D_{ig}(1,0)|Z_{ig}=1,Z_{jg}=0,X_g] \\ &= \mathbb{E}[D_{ig}(1,0)|X_g] = \mathbb{P}[C_{ig}|X_g]\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}[D_{ig}|Z_{ig}=1,Z_{jg}=1,X_g] &= \mathbb{E}[D_{ig}(1,1)|Z_{ig}=1,Z_{jg}=1,X_g] \\ &= \mathbb{E}[D_{ig}(1,1)|X_g] = \mathbb{P}[C_{ig}|X_g] + \mathbb{P}[GC_{ig}|X_g].\end{aligned}$$

For the second part, we have that for the first term,

$$\begin{aligned}\mathbb{E}\left[g(Y_{ig},X_g)\frac{(1-Z_{ig})(1-Z_{jg})}{p_{00}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)\frac{(1-Z_{ig})(1-Z_{jg})}{p_{00}(X_g)}\middle| X_g\right]\right\} \\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)\right| Z_{ig}=0,Z_{jg}=0,X_g]\right\} \\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)\right| Z_{ig}=0,Z_{jg}=0,X_g]\right\} \\ &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)\right| X_g]\right\} \\ &= \mathbb{E}[g(Y_{ig}(0,0),X_g)].\end{aligned}$$

For the second term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\frac{Z_{ig}(1-Z_{ig})}{p_{10}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\frac{Z_{ig}(1-Z_{ig})}{p_{10}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{ig}\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right| X_g]\right\} \\
&= \mathbb{E}\left[g(Y_{ig}(1,0),X_g)D_{ig}(1,0)\right] \\
&= \mathbb{E}\left[g(Y_{ig}(1,0),X_g)|C_{ig}\right]\mathbb{P}[C_{ig}].
\end{aligned}
$$

For the third term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\frac{(1-Z_{ig})Z_{ig}}{p_{01}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\frac{(1-Z_{ig})Z_{ig}}{p_{01}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)D_{jg}\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right]\right\} \\
&= \mathbb{E}\left[g(Y_{ig}(0,1),X_g)D_{jg}(0,1)\right] \\
&= \mathbb{E}[g(Y_{ig}(0,1),X_g)|C_{jg}]\mathbb{P}[C_{jg}].
\end{aligned}
$$

For the fourth term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\frac{Z_{ig}(1-Z_{jg})}{p_{10}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\frac{Z_{ig}(1-Z_{jg})}{p_{10}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{ig})\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))\right| Z_{ig}=1, Z_{jg}=0, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))\right| X_g]\right\} \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)(1-D_{ig}(1,0))] \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}^c]\mathbb{P}[C_{ig}^c]
\end{aligned}
$$

and the result follows from $\mathbb{E}[g(Y_{ig}(0,0),X_g)] = \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}]\mathbb{P}[C_{ig}]+\mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{ig}^c]\mathbb{P}[C_{ig}^c]$. Similarly for the fifth term,

$$
\begin{aligned}
\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\frac{(1-Z_{ig})Z_{jg}}{p_{01}(X_g)}\right] &= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\frac{(1-Z_{ig})Z_{jg}}{p_{01}(X_g)}\right| X_g\right]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig},X_g)(1-D_{jg})\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))\right| Z_{ig}=0, Z_{jg}=1, X_g]\right\} \\
&= \mathbb{E}_{X_g}\left\{\mathbb{E}\left[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))\right| X_g]\right\} \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)(1-D_{jg}(1,0))] \\
&= \mathbb{E}[g(Y_{ig}(0,0),X_g)|C_{jg}^c]\mathbb{P}[C_{jg}^c]
\end{aligned}
$$

Table A6: $D_{ig}(0,1)(1-D_{jg}(1,0)) - D_{ig}(0,0)(1-D_{jg}(0,0))$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	-1	AT	SC,C
1	0	1	0	0		
1	1	0	1	0		
1	1	0	0	0		
1	0	0	0	1	SC	GC,NT
0	1	0	1	0		
0	1	0	0	0		
0	0	0	0	0		

and it can be seen that all these equalities also hold conditional on X_g. □

A4 Proofs of Additional Results

A4.1 Proof of Lemma A1

Using that:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)] \\ &+ \mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))D_{ig}(z,z')(1-D_{jg}(z',z))] \\ &+ \mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))(1-D_{ig}(z,z'))D_{jg}(z',z)] \\ &+ \mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)],\end{aligned}$$

we have:

$$\begin{aligned}&\mathbb{E}[Y_{ig}|Z_{ig}=1, Z_{jg}=0] - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] = \\ &\quad + \mathbb{E}[(Y_{ig}(1,0)-Y_{ig}(0,0))(D_{ig}(0,1)(1-D_{jg}(1,0)) - D_{ig}(0,0)(1-D_{jg}(0,0)))] \\ &\quad + \mathbb{E}[(Y_{ig}(0,1)-Y_{ig}(0,0))((1-D_{ig}(0,1))D_{jg}(1,0) - (1-D_{ig}(0,0))D_{jg}(0,0))] \\ &\quad + \mathbb{E}[(Y_{ig}(1,1)-Y_{ig}(0,0))(D_{ig}(0,1)D_{jg}(1,0) - D_{ig}(0,0)D_{jg}(0,0))].\end{aligned}$$

Tables A6, A7 and A8 list the possible values that the terms that depend on the potential treatment statuses can take, which gives the desired result after some algebra. □

Table A7: $(1 - D_{ig}(0,1))D_{jg}(1,0) - (1 - D_{ig}(0,0))D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	0		
1	0	1	0	0		
1	1	0	1	-1	SC	AT
1	1	0	0	0		
1	0	0	0	0		
0	1	0	1	0		
0	1	0	0	1	C,CG,NT	SC,C
0	0	0	0	0		

Table A8: $D_{ig}(0,1))D_{jg}(1,0) - D_{ig}(0,0)D_{jg}(0,0)$

$D_{ig}(1,0)$	$D_{jg}(0,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	1	AT	SC,C
1	0	1	0	0		
1	1	0	1	1	SC	AT
1	1	0	0	1	SC	SC,C
1	0	0	0	0		
0	1	0	1	0		
0	1	0	0	0		
0	0	0	0	0		

Table A9: $D_{ig}(1,1)(1-D_{jg}(1,1)) - D_{ig}(0,0)(1-D_{jg}(0,0))$

$D_{ig}(1,1)$	$D_{jg}(1,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	0	1	0		
0	1	0	1	0		
1	1	1	0	-1	AT	SC,C,GC
1	1	0	0	0		
0	1	0	0	0		
1	0	1	0	0		
1	0	0	0	-1	SC,C,GC	NT
0	0	0	0	0		

A4.2 Proof of Lemma A2

Using that:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=z, Z_{jg}=z'] = \mathbb{E}[Y_{ig}(0,0)] & \\ + \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))D_{ig}(z,z')(1-D_{jg}(z',z))] & \\ + \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))(1-D_{ig}(z,z'))D_{jg}(z',z)] & \\ + \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))D_{ig}(z,z')D_{jg}(z',z)], &\end{aligned}$$

we have:

$$\begin{aligned}\mathbb{E}[Y_{ig}|Z_{ig}=1, Z_{jg}=1] - \mathbb{E}[Y_{ig}|Z_{ig}=0, Z_{jg}=0] = & \\ + \mathbb{E}[(Y_{ig}(1,0) - Y_{ig}(0,0))(D_{ig}(1,1)(1-D_{jg}(1,1)) - D_{ig}(0,0)(1-D_{jg}(0,0)))] & \\ + \mathbb{E}[(Y_{ig}(0,1) - Y_{ig}(0,0))((1-D_{ig}(1,1))D_{jg}(1,1) - (1-D_{ig}(0,0))D_{jg}(0,0))] & \\ + \mathbb{E}[(Y_{ig}(1,1) - Y_{ig}(0,0))(D_{ig}(1,1)D_{jg}(1,1) - D_{ig}(0,0)D_{jg}(0,0))]. &\end{aligned}$$

Tables A9, A10 and A11 list the possible values that the terms that depend on the potential treatment statuses can take, which gives the desired result after some algebra. □

A4.3 Proof of Proposition A1

The proof is the same as the one for Proposition 1, replacing $\{Z_{jg}=0\}$ by $\{W_{ig}=0\}$ and $\{Z_{jg}=1\}$ by $\{Z_{jg}=1, W_{ig}=w^*\}$. □

Table A10: $(1 - D_{ig}(1,1))D_{jg}(1,1) - (1 - D_{ig}(0,0))D_{jg}(0,0)$

$D_{ig}(1,1)$	$D_{jg}(1,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	0	1	-1	SC,C,GC	AT
0	1	0	1	0		
1	1	1	0	0		
1	1	0	0	0		
0	1	0	0	1	NT	SC,C,GC
1	0	1	0	0		
1	0	0	0	0		
0	0	0	0	0		

Table A11: $D_{ig}(1,1))D_{jg}(1,1) - D_{ig}(0,0)D_{jg}(0,0)$

$D_{ig}(1,1)$	$D_{jg}(1,1)$	$D_{ig}(0,0)$	$D_{jg}(0,0)$	Difference	ξ_{ig}	ξ_{jg}
1	1	1	1	0		
1	1	1	0	1	SC,C,GC	AT
1	0	1	0	0		
1	1	0	1	1	AT	SC,C,GC
1	1	0	0	1	SC,C,GC	SC,C,GC
1	0	0	0	0		
0	1	0	1	0		
0	1	0	0	0		
0	0	0	0	0		

A4.4 Proof of Proposition A2

The proof is the same as the one for Proposition 2, replacing $\{Z_{jg} = 0\}$ by $\{W_{ig} = 0\}$ and $\{Z_{jg} = 1\}$ by $\{Z_{jg} = 1, W_{ig} = 1\}$. For the last two terms, note that

$$\begin{aligned}
\mathbb{E}\left[\prod_{i=1}^{n_g+1}(1-D_{ig})\middle| Z_{ig}+W_{ig}=n_g+1\right] &= \mathbb{E}\left[\prod_{i=1}^{n_g+1}(1-D_{ig}(1,n_g))\right] \\
&= \mathbb{P}\left[\cap_{i=1}^{n_g+1} NT_{ig}\right]
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}\left[Y_{ig}\prod_{i=1}^{n_g+1}(1-D_{ig})\middle| Z_{ig}+W_{ig}=n_g+1\right] &= \mathbb{E}\left[Y_{ig}(0,0)\prod_{i=1}^{n_g+1}(1-D_{ig}(1,n_g))\right] \\
&= \mathbb{E}\left[Y_{ig}(0,0)|\cap_{i=1}^{n_g+1} NT_{ig}\right]\mathbb{P}\left[\cap_{i=1}^{n_g+1} NT_{ig}\right]
\end{aligned}$$

and the result follows. $\square$

References

Vazquez-Bare, Gonzalo, "Identification and Estimation of Spillover Effects in Randomized Experiments," *working paper*, 2017.