LR Parsing

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Objectives

You should be able to ...

- Explain the difference between an LL and LR parser.
- Generate the finite state machine from an LR grammar.
- Use the state machine to detect ambiguity.

Further reading: See Dragon Book §4.x.

What Is LR Parsing?

- What is an LR parser?
  - An LR parser uses a **L**eft-to-right scan and produces a **R**ightmost derivation.
  - A.k.a. bottom-up parsing
  - Uses a *push-down automata* to do the work.
- There are four actions.

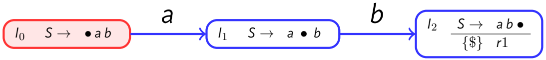
Shift	Consume a token from the input.
Reduce	Build a tree from components.
Goto	Jump to a different state, after a reduce.
Accept	Signal that we're done.

Shifting

Shifting involves three steps.

1. Consume a token from the input.
2. Push the token and the current state to the stack.
3. Go to the next state.

Example:



Grammar	$S \to a\, b$
Input	$\bullet$ a b \$
Stack	(empty)
Current State	0

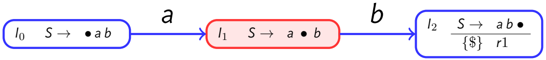
We will shift the a and then we go to state 1.

Shifting

Shifting involves three steps.

1. Consume a token from the input.
2. Push the token and the current state to the stack.
3. Go to the next state.

Example:



Grammar $S \rightarrow a\, b$

Input a • b $

Stack 0, a

Current State 1

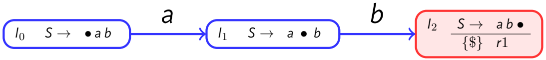
We will shift the b and then we go to state 2.

Shifting

Shifting involves three steps.

1. Consume a token from the input.
2. Push the token and the current state to the stack.
3. Go to the next state.

Example:



Grammar $S \to a\, b$

Input a b ● \$

Stack 0, a, 1, b

Current State 2

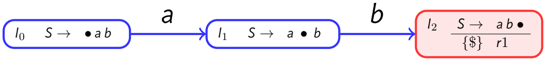
What should happen now?

Reducing

Reducing involves three steps.

1. Pop the tokens and states from the stack. (How many?)
2. Return to the last state popped.
3. Construct a new tree from the popped tokens.

Example:



Grammar $S \to a\, b$

Input `a b` $\bullet$ `$`

Stack 0, `a`, 1, `b`

Current State 2

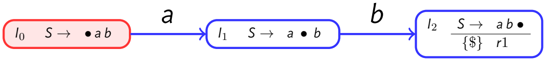
We are ready to reduce.

Reducing

Reducing involves three steps.

1. Pop the tokens and states from the stack. (How many?)
2. Return to the last state popped.
3. Construct a new tree from the popped tokens.

Example:



Grammar $S \to a\,b$

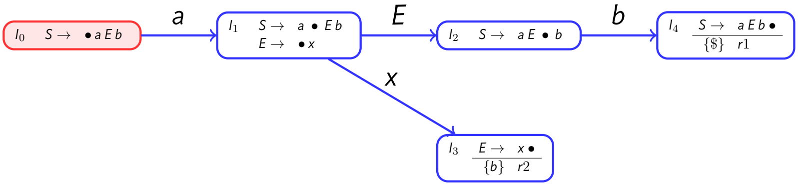
Input `a b` $\bullet$ `$`

Stack

Current State 0

Now we have an S tree. Go To or Accept could happen here.

A More Complex Example



Grammar

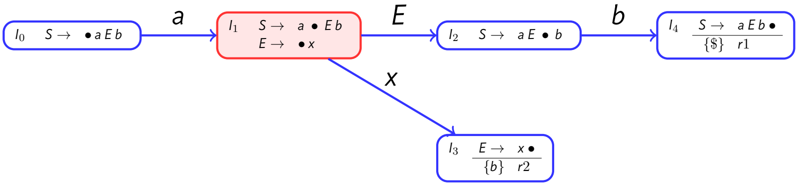
$$S \to a\, E\, b$$
$$E \to x$$

Input $\bullet$ `a x b $`

Stack (Empty)

Current State 0

A More Complex Example



Grammar

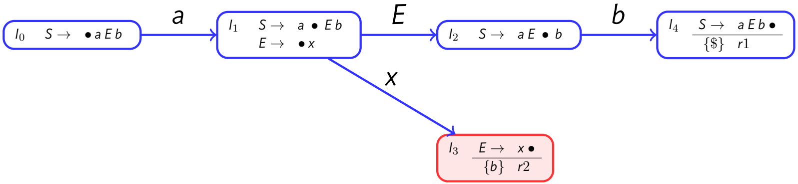
$$
\begin{aligned}
S &\rightarrow a\, E\, b \\
E &\rightarrow x
\end{aligned}
$$

Input `a ● x b $`

Stack 0,a

Current State 1

A More Complex Example



Grammar

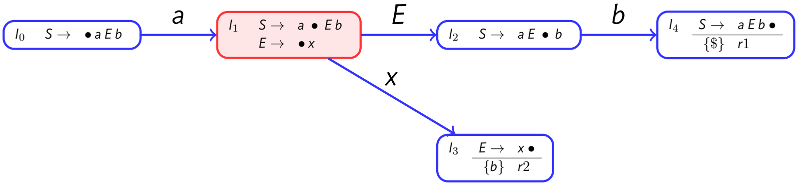
$$\begin{aligned} S &\rightarrow a\, E\, b \\ E &\rightarrow x \end{aligned}$$

Input a x ● b $

Current State 4

Stack 0,a,1,x

A More Complex Example



Grammar

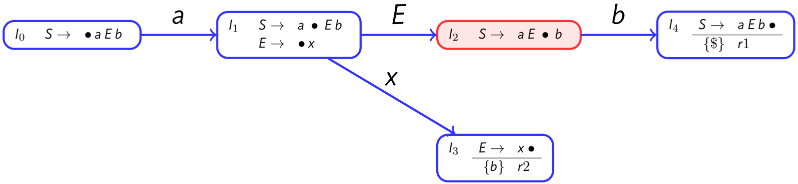
$$S \to a\, E\, b$$
$$E \to x$$

Input a x ● b $

Current State 1

Stack 0,a

A More Complex Example



Grammar

$$S \rightarrow a\,E\,b$$
$$E \rightarrow x$$

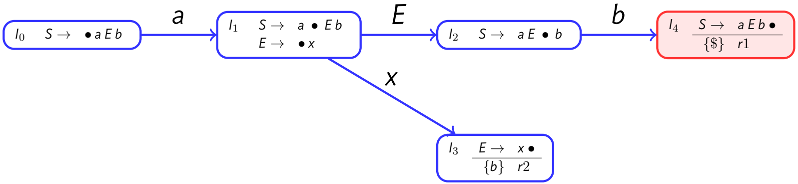
Input a x ● b $

Stack 0,a, 1, E

x

Current State 2

A More Complex Example



Grammar

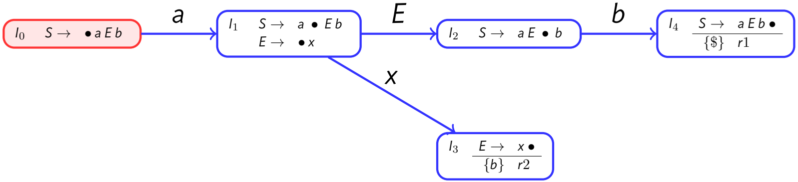
$$
\begin{aligned}
S &\rightarrow a\, E\, b \\
E &\rightarrow x
\end{aligned}
$$

Input a x b ● $

Current State 3

Stack 0,a, 1, $\overset{E}{\overset{|}{\mathtt{x}}}$, 2, b

A More Complex Example



Stack (Empty)

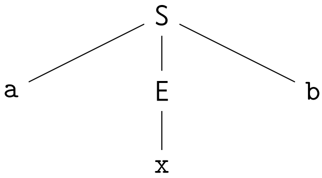
Grammar

$$
\begin{aligned}
S &\to a\, E\, b \\
E &\to x
\end{aligned}
$$

Input `a x b ● $`

Current State 0

Now we have the result:



Representing the Automata

We will represent the automata using two tables.

Action Table Shift, Reduce *n*, Accept

Goto Table Destination State

The rows are the state numbers, the columns are the symbols.

The Algorithm

- To create the start state, add the *transitive closure* of the start symbol.

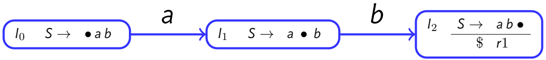
Example 1	Start	Example 2	Start
$S \rightarrow x\,S\,e$	$S \rightarrow \bullet\, x\,S\,e$	$S \rightarrow x\,S\,e$	
$\mid E\,x$	$\mid \bullet\, E\,x$	$\mid F\,x$	$S \rightarrow \bullet\, x\,S\,e$
$E \rightarrow a\,E$	$E \rightarrow \bullet\, a\,E$	$E \rightarrow a\,E$	$\mid \bullet\, F\,x$
$\mid F\,x$	$\mid \bullet\, F\,x$	$\mid F\,x$	$F \rightarrow \bullet\, q$
$F \rightarrow q$	$F \rightarrow \bullet\, q$	$F \rightarrow q$	

The Algorithm, ctd

- Let x be an arbitrary terminal, A be an aribtrary nonterminal, and α and β be arbitrary (possibly empty) strings of symbols.
- In an item set i, take every production of the form $E \rightarrow \alpha \bullet x\beta$ and produce a new state j containing the transitive closure of $E \rightarrow \alpha x \bullet \beta$. Add a shift in the action table for column x and state i, and destination state j in the goto table for column x and state i.
- In an item set i, take every production of the form $E \rightarrow \alpha \bullet A\beta$ and produce a new state j containing the transitive closure of $E \rightarrow \alpha A \bullet \beta$. Add j to the goto table in column A and state i.
- In an item set i, take every rule of the form $E \rightarrow \alpha\bullet$ and add a reduce actions for state i for each terminal in the follow set of E.
- If an item set is recreated, reuse the original; do not create a duplicate.

Automata Example 1

Automata



Tabular Representation

Grammar

$$S \rightarrow a\ b$$

Action

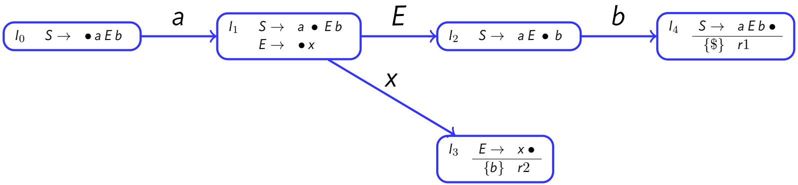
	a	b	$
0	s		
1		s	
2			r1

Goto

	a	b	$	S
0	1			
1		2		
2				

Automata Example 2

Automata



Grammar

$$
\begin{aligned}
S &\to a\, E\, b \\
E &\to x
\end{aligned}
$$

Action

	a	b	x	$
0	s			
1			s	
2		s		
3		r2		
4				r1

Goto

	a	b	x	$	S	E
0	1					
1			3			2
2		4				
3						
4						

Automata Example 3.1

- Let's build the table for this automata.

$$
\begin{array}{lcl}
S & \rightarrow & a\ E\ b \\
 & | & a\ b\ S \\
E & \rightarrow & E\ x \\
 & | & b
\end{array}
$$

Automata Example 3.2

I_0 $S \to$ $\bullet\, a\, E\, b$
$\bullet\, a\, b\, S$

Grammar

$$
\begin{aligned}
S &\to a\,E\,b \\
&\mid a\,b\,S \\
E &\to E\,x \\
&\mid b
\end{aligned}
$$

Action

	a	b	x	$
0				
1				
2				
3				
4				
5				
6				

Goto

	a	b	x	$	S	E
0						
1						
2						
3						
4						
5						
6						

Automata Example 3.2

$$I_0 \quad S \rightarrow \; \bullet\, a\, E\, b \Leftarrow$$
$$\bullet\, a\, b\, S \Leftarrow$$

Grammar

$$\begin{aligned} S &\rightarrow a\,E\,b \\ &\mid a\,b\,S \\ E &\rightarrow E\,x \\ &\mid b \end{aligned}$$

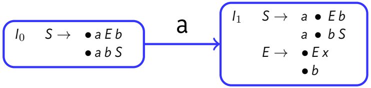
Action

	a	b	x	$
0				
1				
2				
3				
4				
5				
6				

Goto

	a	b	x	$	S	E
0						
1						
2						
3						
4						
5						
6						

Automata Example 3.3

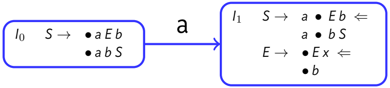


$$
\begin{aligned}
S &\rightarrow a\,E\,b \\
  &\mid\ a\,b\,S \\
E &\rightarrow E\,x \\
  &\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						
2						
3						
4						
5						
6						

Automata Example 3.3

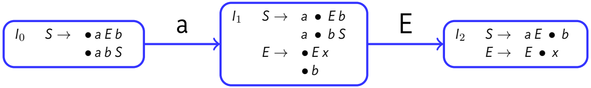


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
  & | & a\,b\,S \\
E & \to & E\,x \\
  & | & b
\end{array}
$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						
2						
3						
4						
5						
6						

Automata Example 3.4

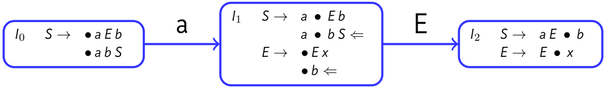


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
 & | & a\,b\,S \\
E & \to & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						2
2						
3						
4						
5						
6						

Automata Example 3.4

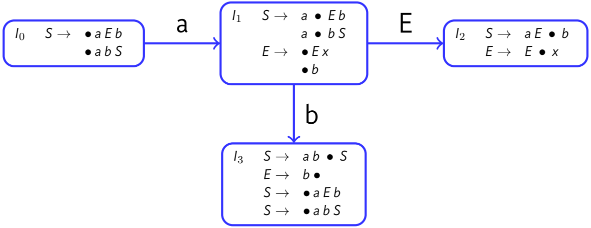


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
 & | & a\,b\,S \\
E & \to & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						2
2						
3						
4						
5						
6						

Automata Example 3.5

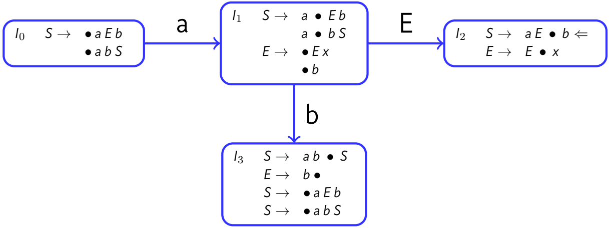


$$
\begin{array}{lcl}
S & \rightarrow & a\,E\,b \\
 & | & a\,b\,S \\
E & \rightarrow & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2						
3						
4						
5						
6						

Automata Example 3.5

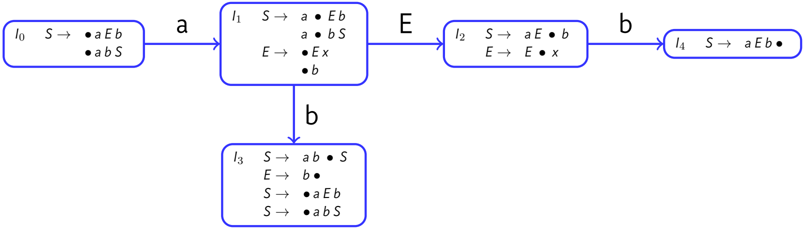


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
 & | & a\,b\,S \\
E & \to & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2						
3						
4						
5						
6						

Automata Example 3.6

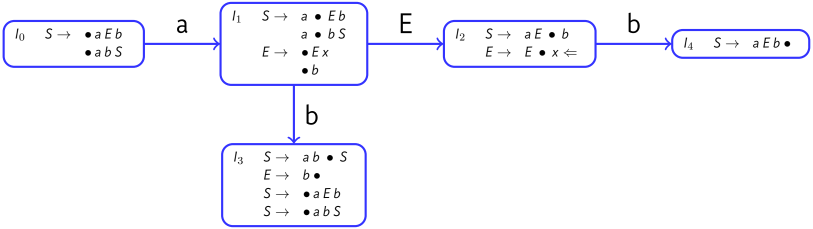


$$
\begin{aligned}
S &\to a\, E\, b \\
&\mid\ a\, b\, S \\
E &\to E\, x \\
&\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.6

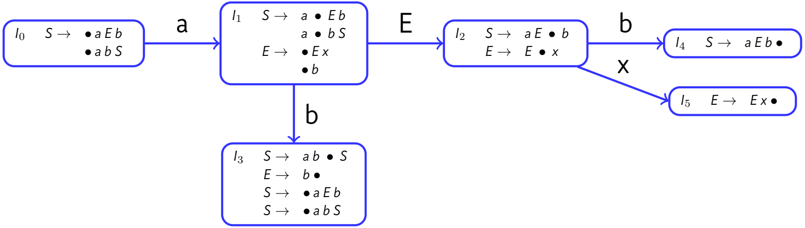


$$
\begin{aligned}
S &\rightarrow a\, E\, b \\
  &\mid\; a\, b\, S \\
E &\rightarrow E\, x \\
  &\mid\; b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.7

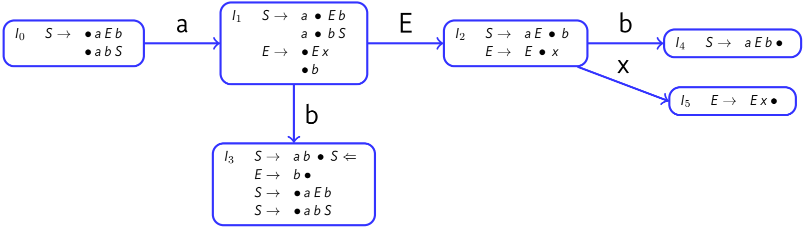


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
 & | & a\,b\,S \\
E & \to & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.7

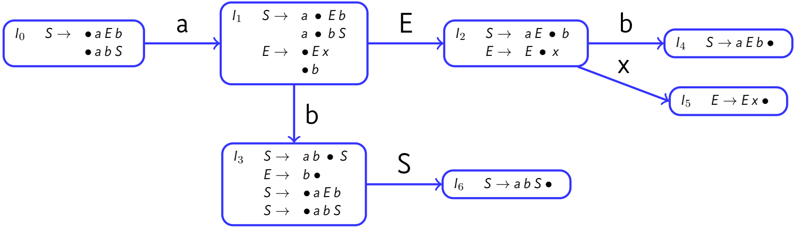


$$
\begin{array}{lcl}
S & \to & a\, E\, b \\
 & | & a\, b\, S \\
E & \to & E\, x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.8

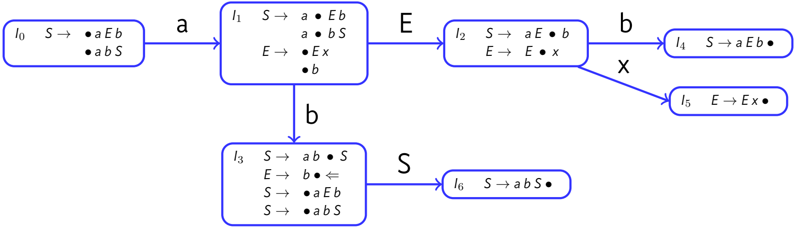


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
 & | & a\,b\,S \\
E & \to & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.8

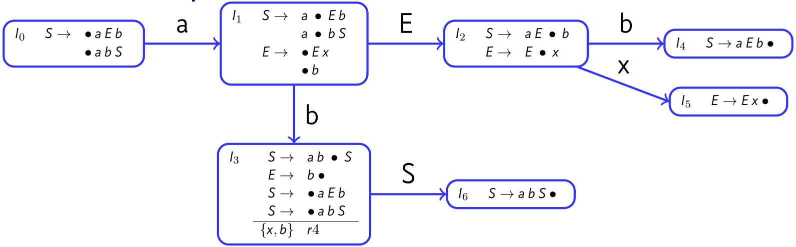


$$
\begin{array}{lcl}
S & \to & a\, E\, b \\
  & \mid & a\, b\, S \\
E & \to & E\, x \\
  & \mid & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.8

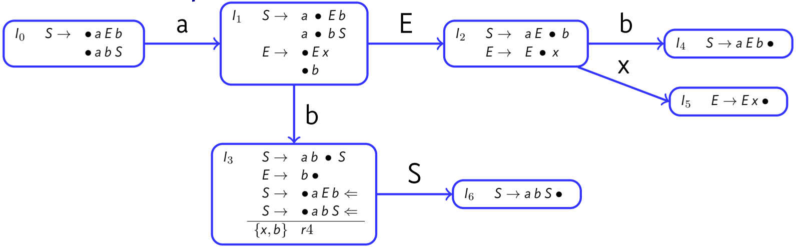


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
  & | & a\,b\,S \\
E & \to & E\,x \\
  & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3		r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.8

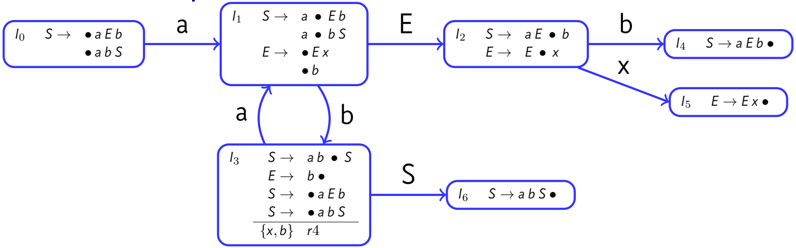


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
 & | & a\,b\,S \\
E & \to & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3		r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.9

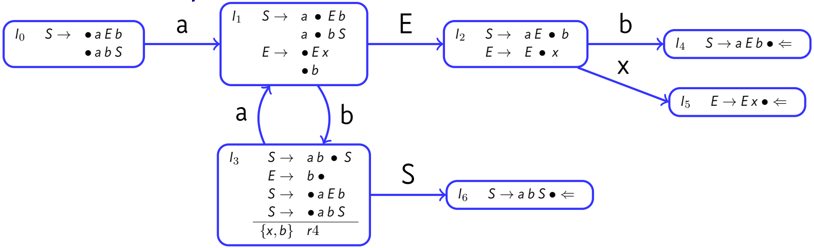


$$
\begin{array}{lcl}
S & \rightarrow & a\,E\,b \\
 & | & a\,b\,S \\
E & \rightarrow & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3	s	r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3	1				6	
4						
5						
6						

Automata Example 3.9

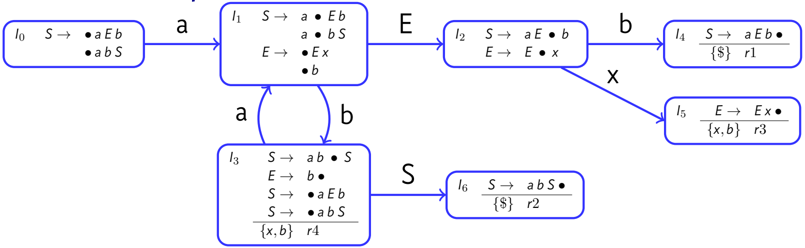


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
 & | & a\,b\,S \\
E & \to & E\,x \\
 & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3	s	r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3	1				6	
4						
5						
6						

Automata Example 3.10



$$
\begin{aligned}
S &\to a\,E\,b \\
&\mid\ a\,b\,S \\
E &\to E\,x \\
&\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3	s	r4	r4	
4				r1
5		r3	r3	
6				r2

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3	1				6	
4						
5						
6						

Activity

Your turn. Try to build the automata for this grammar. There's a surprise waiting for you!

$$
\begin{array}{lcl}
S & \rightarrow & a\,E\,b \\
 & | & x \\
E & \rightarrow & E\,x\,E \\
 & | & b
\end{array}
$$