

Basic Recursion

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Objectives

- ▶ Diagram the stack frames that result from a series of function calls.
- ▶ Use `HASKELL` to write a recursive function on integers.
- ▶ Use `HASKELL` to write a recursive function on lists.

Function Calls

- ▶ Remember the syntax of a function definition in HASKELL.

Function Syntax

```
1 foo a =  
2   let aa = a * a  
3   in aa + a
```

- ▶ The above function has one parameter and one local.
- ▶ If we call it three times, what will happen in memory?

```
1 x = (foo 1) + (foo 2) + (foo 3)
```

Function Calls

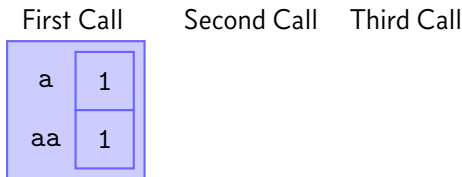
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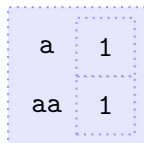
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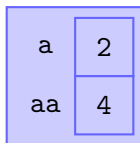
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Second Call



Third Call

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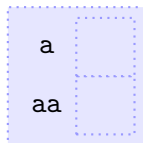
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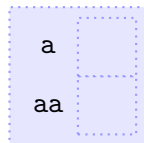
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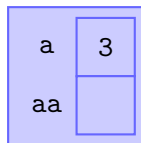
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Functions Calling Functions

- ▶ If one function calls another, *both* activation records exist simultaneously.

```
1 foo x = x + bar (x+1)
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2 bar y = y + baz (y+1)
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```
3 baz z = z * 10
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- ▶ What happens when we call `foo 1`?

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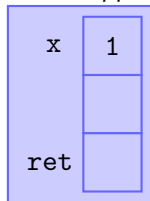
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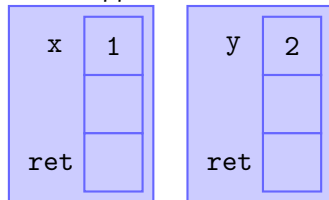
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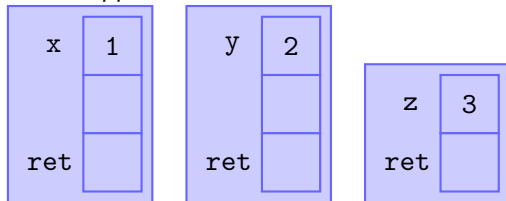
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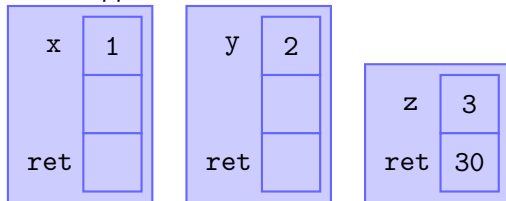
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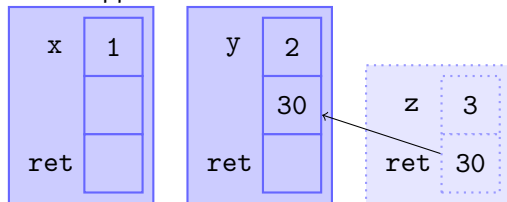


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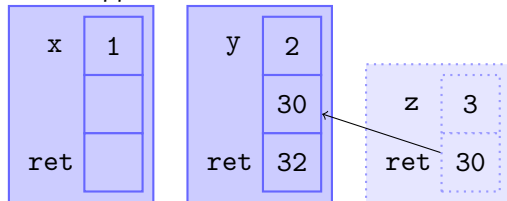
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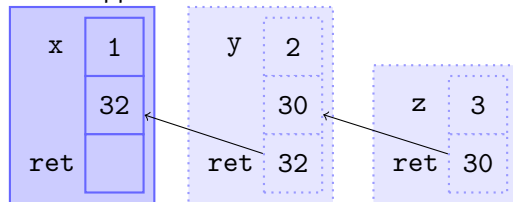


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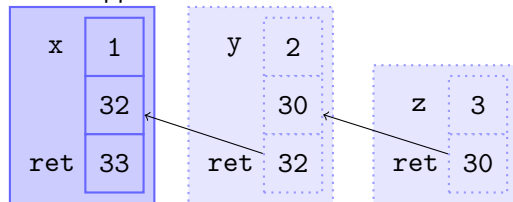


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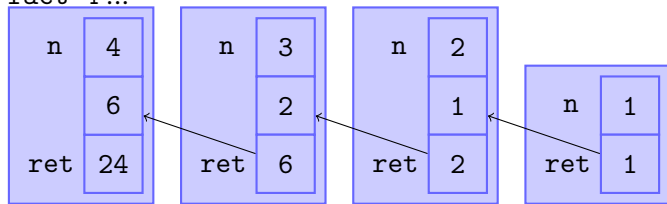
Factorial

- ▶ This works if the function calls itself.

Factorial

```
1 fact 0 = 1  
2 fact 1 = 1  
3 fact n = n * fact (n-1)
```

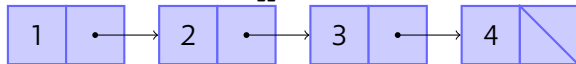
- ▶ `fact 4 ...`



Lists in HASKELL

- ▶ HASKELL has a built-in syntax for singly linked lists.
- ▶ The empty list is `[]`.
- ▶ You can use `:` to create a new list ...

`1 : 2 : 3 : 4 : []`



- ▶ You can also write `[1,2,3,4]`.

Lists

Because lists are recursive, functions that deal with lists tend to be recursive.

Length

```
1 mylength :: [a] -> Int
2 mylength [] = 0
3 mylength (x:xs) = 1 + mylength xs
4
5 mylength s -- would return 3
```

- ▶ The base case stops the computation.
- ▶ Your recursive case calls itself with a *smaller* argument than the original call.

Activity

- ▶ Write a function `fib` that computes the n th Fibonacci number F_n . Let $F_1 = 1$ and $F_2 = 1$.
- ▶ Write a function `sumList` that takes a list and sums its elements.
- ▶ Write a function `incList` that takes a list and increments its elements.

Solutions to fib and sumList

```
1 fib 1 = 1
2 fib 2 = 1
3 fib n = fib (n-1) + fib (n-2)
4
5 sumList [] = 0
6 sumList (x:xs) = x + sumList xs
```

Solution to incList

- ▶ Remember that you must create a new list!

```
1 incList [] = []  
2 incList (x:xs) = x+1 : incList xs
```

History

- ▶ The first programming language to implement recursion was LISP in 1958. [McC79]

References

- [McC79] John McCarthy. *History of Lisp*. Stanford University, 1979. URL: <http://www-formal.stanford.edu/jmc/history/lisp/lisp.html>.