Regular Languages

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Objectives

You should be able to ...

- Use the syntax of regular expressions to model a given set of strings.
- Give examples of the limitations of regular expressions.

Motivation

- *Regular languages* were developed by Noam Chomsky in his quest to describe human languages.
- Computer scientists like them because they are able to describe “words” or “tokens” very easily.

Examples:

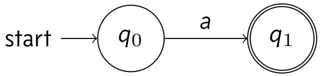
Integers	a bunch of digits
Reals	an integer, a dot, and an integer
Past Tense English Verbs	a bunch of letters ending with “ed”
Proper Nouns	a bunch of letters, the first of which must be capitalized

A Bunch of Digits?!

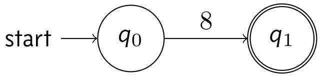
- We need something a bit more formal if we want to communicate properly.
- We will use a *pattern* (or a *regular expression*) to represent the kinds of words we want to describe.
- These expressions will correspond to NFAs.
- Kinds of patterns we will use:
  - Single letters
  - Repetition
  - Grouping
  - Choices

Single Letters

- To match a single character, just write the character.
- To match the letter “a” …
  - Regular expression: `a`
  - State machine:

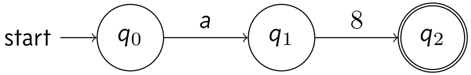


- To match the character “8” …
  - Regular expression: `8`
  - State machine:

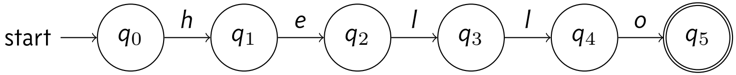


Juxtaposition

- To match longer things, just put two regular expressions together.
- To match the character “a” followed by the character “8” …
  - Regular expression: `a8`
  - State machine:

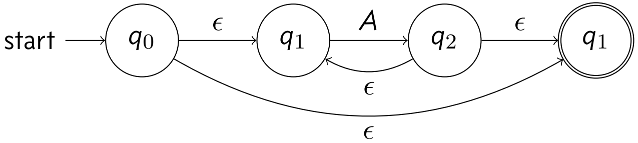


- To match the string “hello” …
  - Regular expression: `hello`
  - State machine:

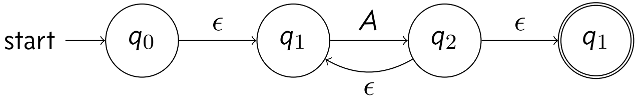


Repetition

- Zero or more copies of `A`, add `*`
  - Regular expression `A*`
  - State machine:

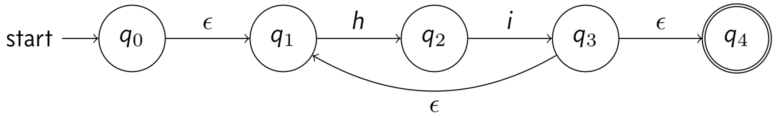


- One or more copies of `A`, add `+`
  - Regular expression `A+`
  - State machine:



Grouping

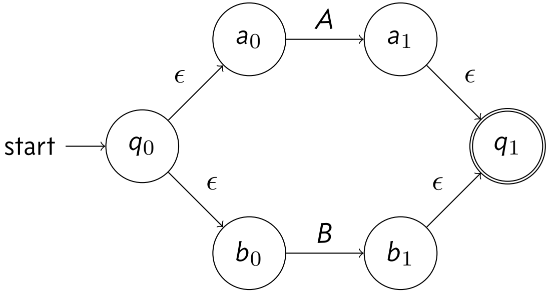
- To groups things together, use parenthesis.
- To match one or more copies of the word "hi" ...
  - Regular expression: `(hi)+`
  - State machine:



- We use Thompson's construction to build the state machine. The extra ϵ transitions are important!

Choice

- To make a choice, use the vertical bar (also called “pipe”).
- To match A or B …
    - Regular expression: `A|B`
    - State machine:



Examples

Expression	(Some) Matches	(Some) Rejects
`ab*a`	`aa`, `aba`, `abbba`	`ba`, `aaba`, `abaa`
`(0|1)*`	any binary number, ϵ	
`(0|1)+`	any binary number	empty string
`(0|1)*0`	even binary numbers	
`(aa)*a`	odd number of `a`s	
`(aa)*a(aa)*`	odd number of `a`s	

`(aa|bb)*((ab|ba)(aa|bb)*(ab|ba)(aa|bb)*)*`
even number of `a`s and `b`

Some Notational Shortcuts

- A range of characters: `[Xa-z]` matches `X` and between `a` and `z` (inclusively).
- Any character at all: `.`
- Escape: `\`

Expression	**(Some) Matches**
`[0-9]+`	integers
`X.*Y`	anything at all between an `X` and a `Y`
`[0-9]*\.[0-9]*`	floating point numbers (positive, without exponents)

Things to Know ...

- They are *greedy*.
  `X.*Y` will match `XabaaYaababY` entirely, not just `XabaaY`.
- They *cannot count* very well.
  - They can only count as high as you have states in the machine.
  - This regular expression matches some primes:

    aa | aaa | aaaaa | aaaaaaa
  - You cannot match an infinite number of primes.
  - You cannot match “nested comments.” `(\*.*\*)`