Sum Types

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Objectives

- Describe the syntax for declaring disjoint data types in HASKELL.
- Show how to use disjoint types to represent lists, expressions, and exceptions.
- Explain the operation and implementation of the list, `Maybe` and `Either` data types.
- Use a disjoint datatype to represent an arithmetic calculation.

Simple Type Definitions

Disjoint Type Syntax

$$\texttt{data}\ \mathit{TName} = \mathit{CName}\ [\mathit{type} \cdots]\ [\mid \mathit{CName}\ [\mathit{type} \cdots] \cdots]$$

A *sum type* has three components: a *name*, a set of `constructors`, and possible *arguments*.

```
1 data Contest = Rock | Scissors | Paper
2 data Velocity = MetersPerSecond Float
3               | FeetPerSecond Float
4 data List a = Cons a (List a)
5             | Nil
6 data Tree a = Node a (Tree a) (Tree a)
7             | Empty
```

Example of Contest and Velocity

```
1 winner Rock Scissors = "Player 1"
2 winner Scissors Paper = "Player 1"
3 winner Paper Rock = "Player 1"
4 winner Scissors Rock = "Player 2"
5 winner Paper Scissors = "Player 2"
6 winner Rock Paper = "Player 2"
7 winner _ _ = "Tie"
8
9 thrust (FeetPerSecond x) = x / 3.28
10 thrust (MetersPerSecond x)  = x
```

The Most Fun Datatypes Are Recursive

Our Own List Construct

```
1 data List = Cons Int List
2           | Nil
3   deriving Show
4 insertSorted a Nil = Cons a Nil
5 insertSorted a (Cons b bs)
6       | a < b      = Cons a (Cons b bs)
7       | otherwise = Cons b (insertSorted a bs)
```

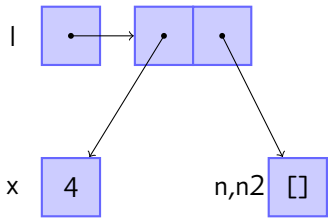
We can run it like this:

```
*Main> let l1 = insertSorted 3 (Cons 2 (Cons 4 Nil))
*Main> l1
Cons 2 (Cons 3 (Cons 4 Nil))
```

Type Constructors and Memory

- When a type constructor is invoked, it causes memory to be allocated.
  - Writing an integer
  - Writing `[]` or `Nil`
  - Using `:` or `Cons`
- Writing down a variable does not cause memory to be allocated.

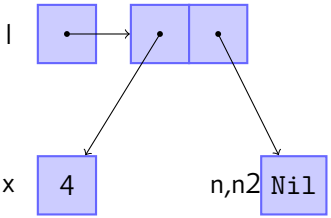
```
1 x = 4   -- allocates 4
2 n = []  -- allocates empty list
3 n2 = n  -- does NOT allocate memory
4 l = x:n -- A cons cell is allocated, but not the 4 or the empty list
```



Similarly ...

```
1 x = 4
2 n = Nil
3 n2 = n
4 l = Cons x n
```

- Our own types do the same thing.



Parameters

HASKELL supports *parametric polymorphism*, like templates in C++ or generics in JAVA.

Parametric Polymorphism

```
data List a = Cons a (List a)
            | Nil
  deriving Show
```

```
x1 = Cons 1 (Cons 2 (Cons 4 Nil)) -- List Int
x2 = Cons "hi" (Cons "there" Nil) -- List String
x3 = Cons Nil (Cons (Cons 5 Nil) Nil) -- List (List Int)
```

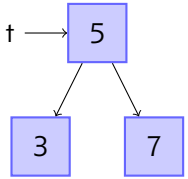
BST Add

- Here is some code for BST Add!
- Note the dual use of a constructor: both for building and for pattern matching.

```
1 data Tree a = Node a (Tree a) (Tree a)
2             | Empty
3 add_bst :: Integer -> Tree Integer -> Tree Integer
4 add_bst i Empty = Node i Empty Empty
5 add_bst i (Node x left right)
6    | i <= x    = Node x (add_bst i left) right
7    | otherwise = Node x left (add_bst i right)
```

Functional Updating

- It is important to understand functional updating.
- We don't update in place. We make copies, and share whatever we can.
  - Example: add 5,3,7 to a tree `t`
  - `let u = add t 6`
  - `let v = add u 1`



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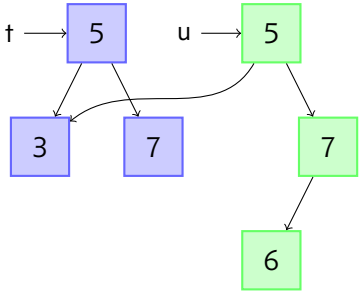

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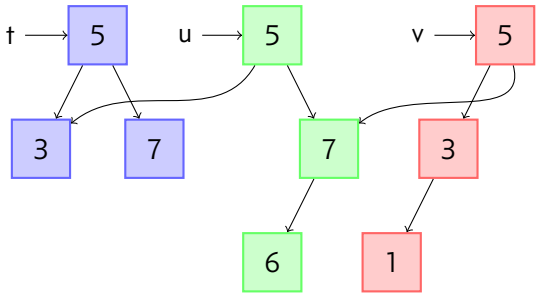

The Maybe Type

The Maybe Type

```
1 data Maybe a = Just a | Nothing
```

Remember the lookup function that didn't know what to do if the item wasn't in the list?

```
1 getItem key [] = Nothing
2 getItem key ((k,v):xs) =
3       if key == k then Just v
4                   else getItem key xs
```

Example:

```
*Main> getItem 3 [(2,"french hens"), (3,"turtle doves")]
Just "turtle doves"
*Main> getItem 5 [(2,"french hens"), (3,"turtle doves")]
Nothing
```

The Either Type

The Either Type

```
1 data Either a b = Left a | Right b
```

We can use it in places where we want to return something, or else an error message.

```
1 getItem key [] = Left "Key not found"
2 getItem key ((k,v):xs) =
3       if key == k then Right v
4                   else getItem key xs
```

Example:

```
*Main> getItem 3 [(2,"french hens"), (3,"turtle doves")]
Right "turtle doves"
*Main> getItem 5 [(2,"french hens"), (3,"turtle doves")]
Left "Key not found"
```

You try!

```haskell
data Tree a = Branch a (Tree a) (Tree a)
            | Empty
  deriving Show
```

1. Write `add :: Tree a -> a -> Tree a`
2. Write `find :: Tree a -> a -> Bool`
3. Write `lookup :: Tree (k,v) -> k -> Maybe v`
4. Write `delete :: Tree a -> a -> Tree a`