Data Assimilation for Glacier Modeling

Mid-Term Presentation

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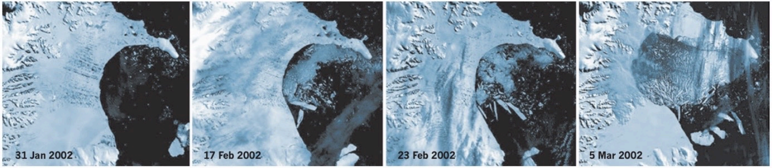
Supported by NSF grant DMS 2051019

30 June 2022

Introduction

- Dr. Mayo's work focuses on modeling storm surge inundation.
- Research has shown that climate change will likely impact storm surge inundation and make modeling this process more difficult. Sea-level rise caused by climate change plays a part in this impact. [Camelo et al., 2020].
- To better model sea-level rise, glaciers can be modeled.
- Our group is collaborating with Dr. Robel from Georgia Tech and working with his glacier model [Robel et al., 2018].



Ted Scambos, National Snow and Ice Data Centre [Robel, 2015]

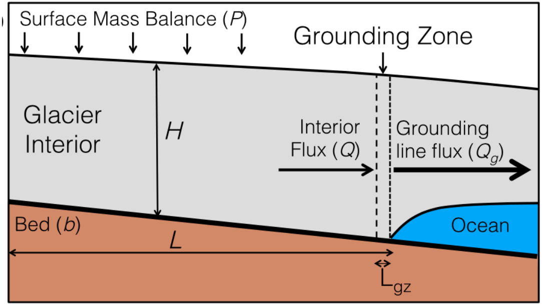
Glacier Background

- Marine-Terminating Glaciers have a natural flow towards the ocean, which contributes to sea level rise [Robel et al., 2019].
- By the year 2300, the Antarctic ice sheet is projected to cause up to 3 meters of sea level rise globally [Robel, 2015].
- Due to the severe impacts of glacial melting, modeling changes in ice sheets is an important task.
- There are challenges to modeling sea level rise, as ice sheet instability leads to significant sea-level rise uncertainty [Robel et al., 2019].

Michael Van Woert, National Oceanic and Atmospheric Association (NOAA) NESDIS, ORA
https://nsidc.org/cryosphere/quickfacts/iceshelves.html

Glacier Modeling

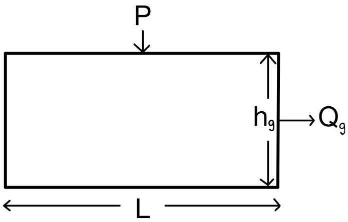
Ice sheet models aim to describe the changes in ice mass of marine-terminating glaciers, which may be impacted over time by climate change [Robel et al., 2018].



One-Stage Model

A glacier can be represented with a simplified box model.

This model is the best approximation for one variable and describes the dominant mode of the glacial system.



One-Stage Model Equations [Robel, 2022]

$$Q_g = \Omega h_g^{\beta}$$

$$\frac{dL}{dt} = \frac{1}{h_g}(PL - Q_g)$$

Two-Stage Model

The two-stage model incorporates a nested box into the system
This new box has a thickness, H, and an interior flux, Q

- This interior flux is typically less than the grounding line flux

Two-Stage Model Equations [Robel et al., 2018]

$$\frac{dH}{dt} = P - \frac{Q_g}{L} - \frac{H}{h_g L}(Q - Q_g)$$

$$\frac{dL}{dt} = \frac{1}{h_g}(Q - Q_g)$$

Diagram of the Model Code

Feed Initial Conditions and Parameters into Model *

↓

2-stage Model

↓

Forecast at time t

* H_{nd}, L_{nd}, b_x,
$sill_{min}$, $sill_{max}$, $sill_{slope}$,
smb_0, smb_1, smb_f
etc

Analysing the Model

- **How can we better understand the model?**

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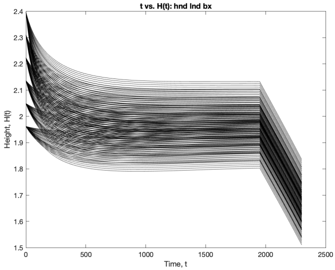
- Why?

Analysing the Model

- **How can we better understand the model?**
- Why?
- This model is a simplification of a real-world scenario, so some uncertainty will always be present. Understanding the model allows us to know what uncertainty is most significant

What is Sensitivity Analysis?

Sensitivity analyses study how various sources of uncertainty in a mathematical model contribute to the model's overall uncertainty. [mod, 2005]



Computational method > analytical method

Define the groups of variables

The uncertain model parameters we considered are:

- Initial conditions

Define the groups of variables

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- Sill parameters

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The uncertain model parameters we considered are:

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- SMB values

Define the groups of variables

The uncertain model parameters we considered are:

- Initial conditions : Hnd, Lnd, b_x
- Sill parameters
- SMB values

Define the groups of variables

The uncertain model parameters we considered are:

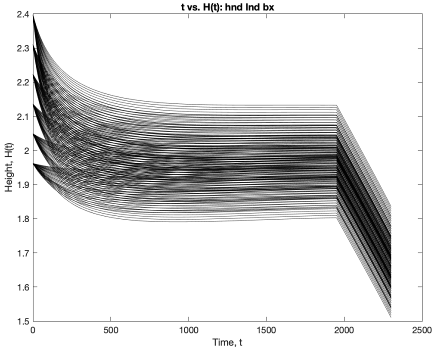
- Initial conditions
- Sill parameters : sillmin, sillmax, sillslope
- SMB values

Define the groups of variables

The uncertain model parameters we considered are:

- Initial conditions
- Sill parameters
- SMB values: smb0, smb1, smbf

Graphs with Parameter Variation



Input Values for simulation of H (height)		
Parameter Varied	Nominal Value	Range
initial height	2.18	1.962 - 2.398
initial length	4.44	3.966 - 4.854
slope	-0.001	-0.0011 to -0.0009

Graphs with Parameter Variation

t vs H(t)

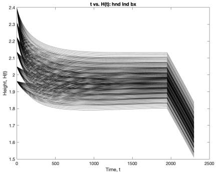


Figure: ±10% initial conditions

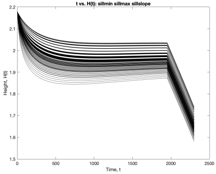


Figure: ±10% sill

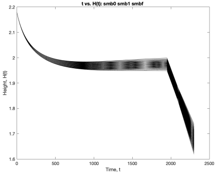
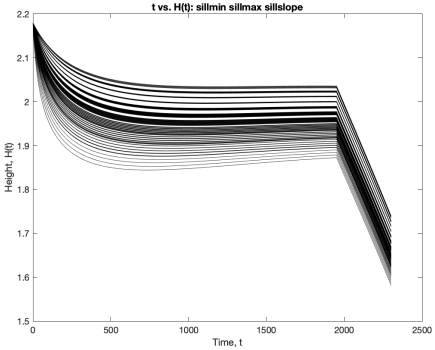


Figure: ±10% SMB

Graphs with Parameter Variation



Input Values for simulation of H (height)		
Parameter Varied	Nominal Value	Range
sill min	430e3	404.625 - 425.375
sill max	440e3	414.37 - 435.625
sill slope	0.01	0.009 - 0.011

Sill Parameter Variation

Figure: Ensembles H

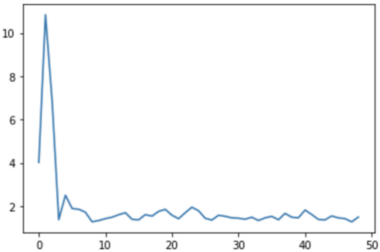
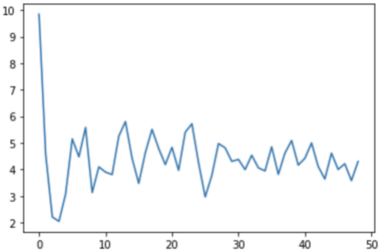
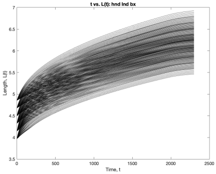
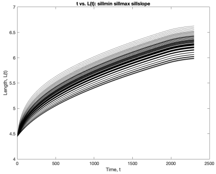


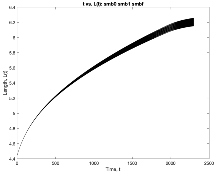
Figure: Ensembles L



t vs L(t)

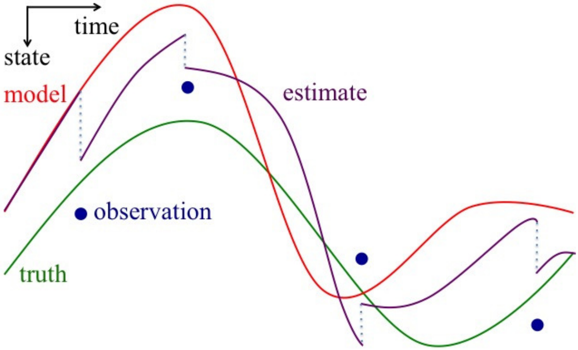






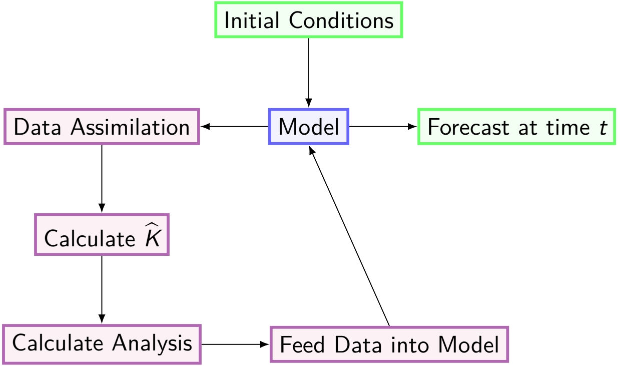
Data Assimilation

Data assimilation is a method to move models closer to reality using real world observations by readjusting the model state at specified times. [dat, 2022]



Data Assimilation

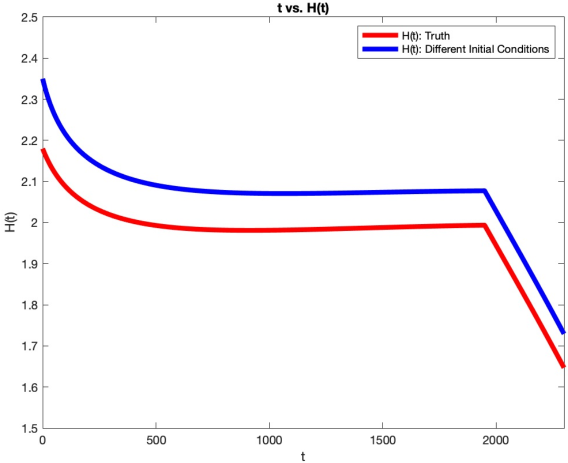
The Ensemble Kalman Filter process we use appears as follows:



Note: The forecast is at some steps in fact the output from the Data Assimilation.

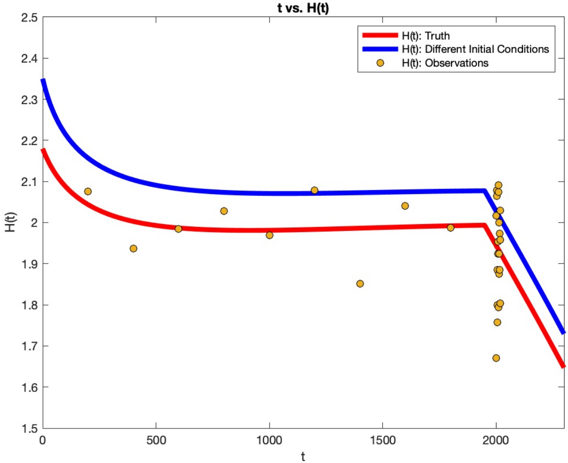
Data Assimilation: An Example

Height Truth Simulation:



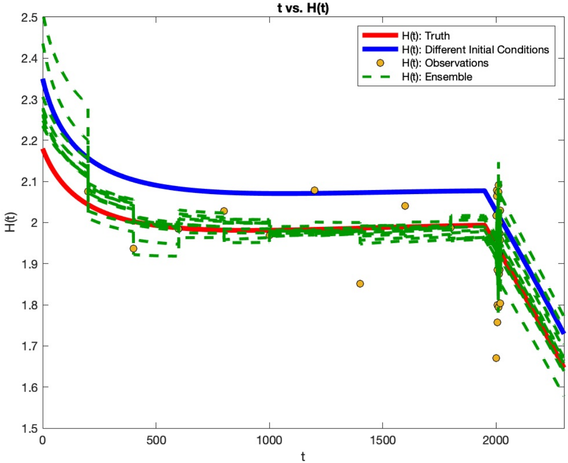
Data Assimilation: An Example

Height Truth Simulation with observations:



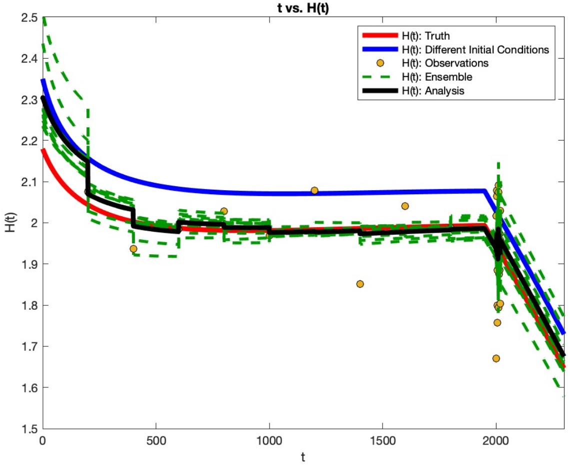
Data Assimilation: An Example

Height Truth Simulation and Ensemble Analysis Simulations:



Data Assimilation: An Example

Add the Mean of Ensemble Analysis Simulations:



Preliminary Results

- We have found that in our model the sill variables caused a great degree of variation.
- Data assimilation seems to improve the quality of our forecasts of Glacier average height and length.

Next Steps

Going forward, we will focus on data assimilation when making changes to the model.

We will explore these areas, focusing on recovering the truth

- The frequency at which data needs to be assimilated
- The smallest amount of data needed
- The essential time period of data
- The acceptable error bound on parameters
- A realistic range of values for the parameters

Overall Goals

- We will improve the data assimilation process of the glacier model
- We will integrate the output of the glacier model into the ADCIRC hurricane storm surge model

Final Remarks

Thank you for your time!

Feel free to contact us with any questions:

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EnKF Data Assimilation Algorithm

Let $\mathcal{M}$ be our model, $\widehat{x}_t^{(0)}, \widehat{x}_t^{(1)}, \ldots, \widehat{x}_t^{(N)}$, be our ensemble at time t, y_t an observation at time t, $w_t^{(i)} \sim \mathcal{N}_n(0, R_t)$, $v_t^{(i)} \sim \mathcal{N}_{m_t}(0, Q_t)$, H_t is the observation operator and $C_t = \tilde{S}_t$ where $\tilde{S}_t$ is the sample covariance of the current ensemble.

The following algorithm outlines the assimilation:

Generate $\widehat{x}_0^{(0)}, \widehat{x}_0^{(1)}, \ldots, \widehat{x}_0^{(N)}$
for $t = 0, 1, \ldots, T$ **do**
  Calculate $\widehat{K}_t = C_t H_t'(H_t C_t H_t' + R_t)^{-1}$ and
  for $i = 0, 1, \ldots, N$ **do**
    $\tilde{x}_t^{(i)} = \mathcal{M} x_{t-1}^{(i)} + w_t^{(i)}$
    $\widehat{x}_t^{(i)} = \tilde{x}_t^{(i)} + \widehat{K}_t(y_t + v_t^{(i)} - H_t \tilde{x}_t^{(i)})$
  end for
  where $\widehat{x}_t^{(i)}$ is the analysis output.
  Calculate analysis output: $x_t^a = \frac{1}{N}\sum_{i=0}^{N} \widehat{x}_t^{(i)}$.
end for