Logistic regression

Predict binary outcomes (success/failure) from numerical or categorical predictors.

Linear vs. logistic regression

Linear regression:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

Linear vs. logistic regression

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$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

Logistic regression:

$$\Pr(success) = \frac{e^t}{1 + e^t}$$

$$t = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

Linear vs. logistic regression

Linear regression:

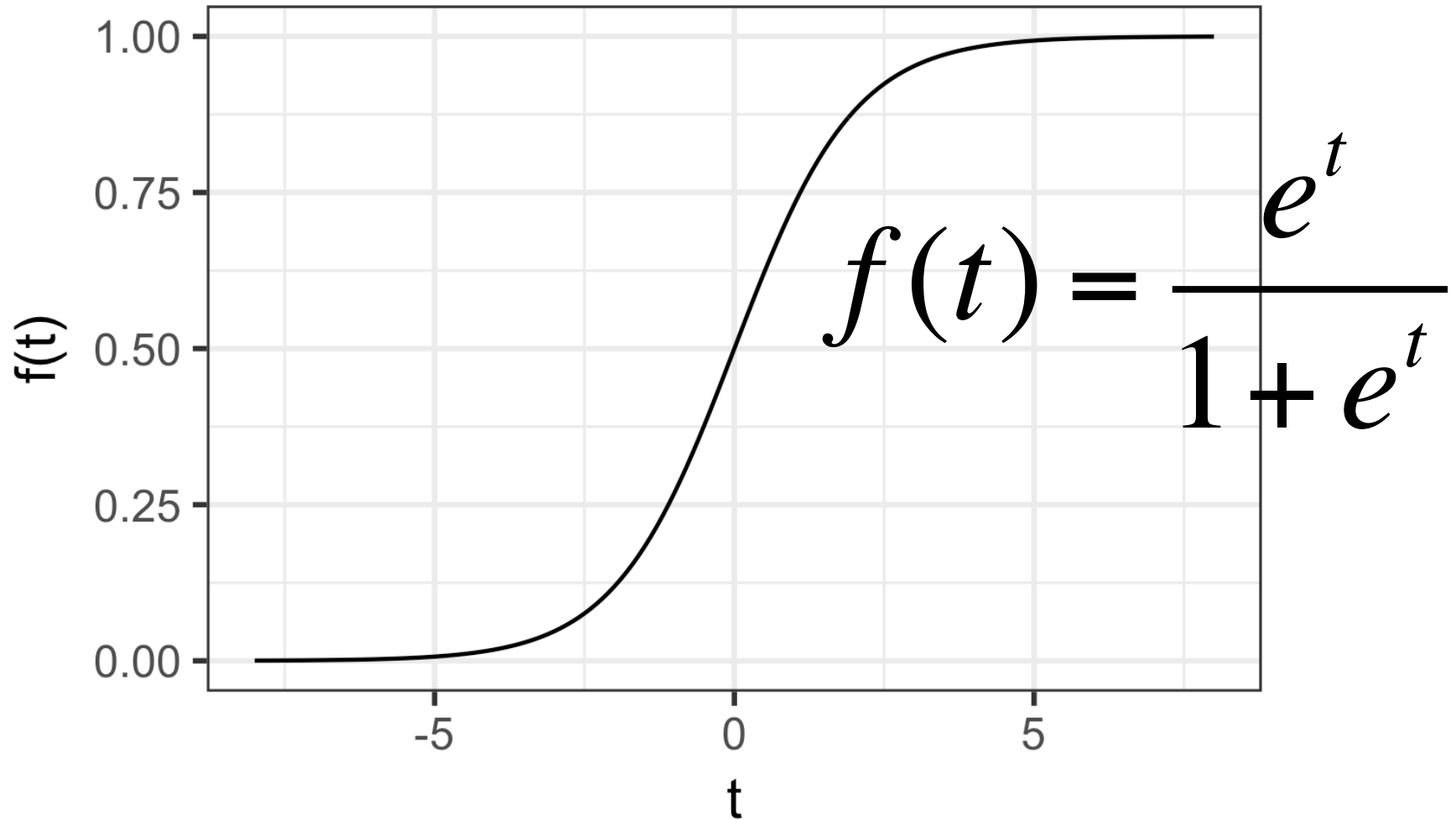$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

Logistic regression:

$$\Pr(success) = \frac{e^t}{1 + e^t}$$

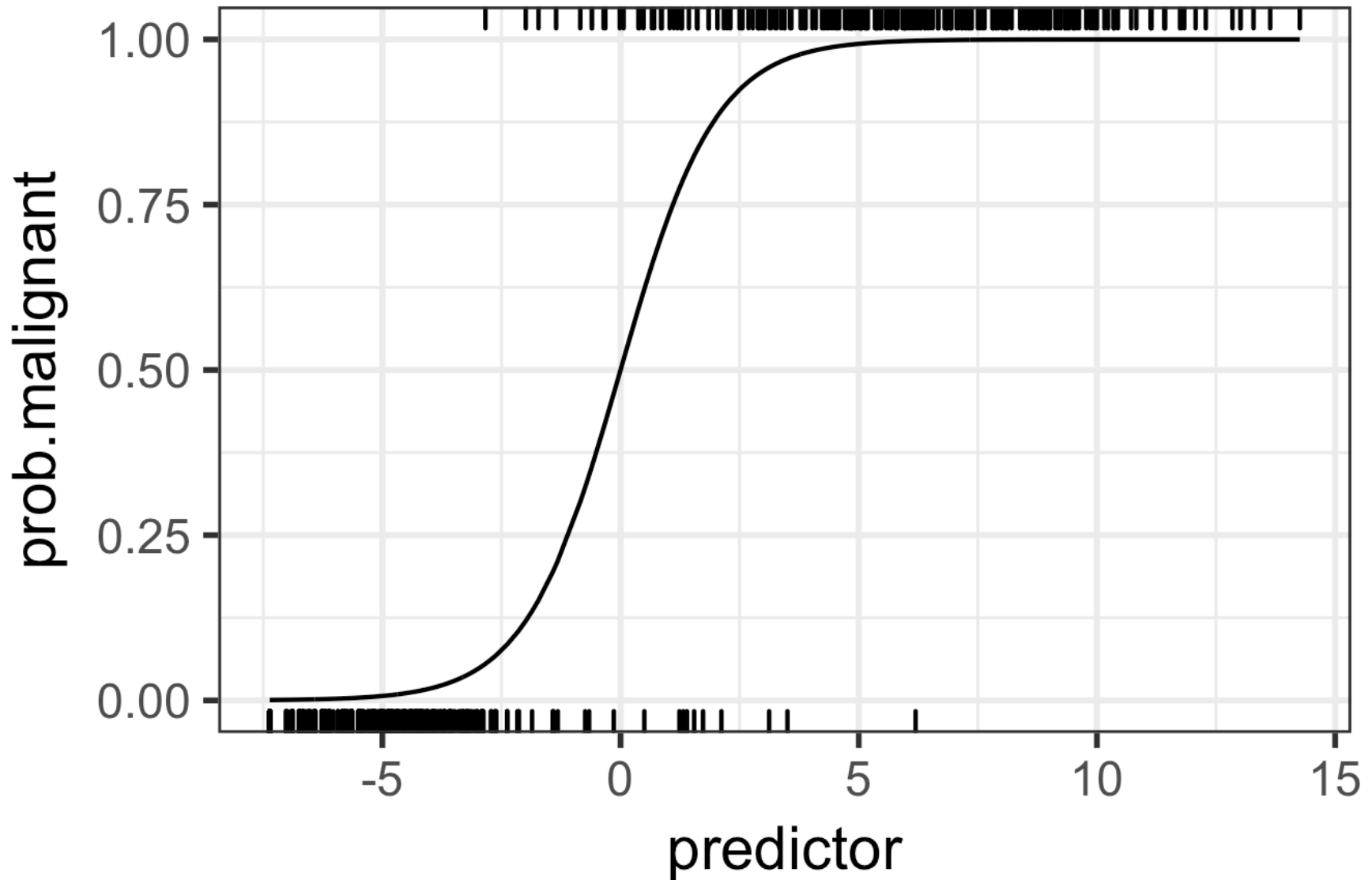$$t = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

(generalized linear model, GLM)

The logistic equation

$$f(t) = \frac{e^t}{1 + e^t}$$

Example: Pr(malignant) in biopsy data set

Let's do this step by step…

Recall the biopsy data set

	clump_thickness	uniform_cell_size	uniform_cell_shape	marg_adhesion
1	5	1	1	1
2	5	4	4	5
3	3	1	1	1
4	6	8	8	1
5	4	1	1	3
6	8	10	10	8

	epithelial_cell_size	bare_nuclei	bland_chromatin	normal_nucleoli	mitoses
1	2	1	3	1	1
2	7	10	3	2	1
3	2	2	3	1	1
4	3	4	3	7	1
5	2	1	3	1	1
6	7	10	9	7	1

	outcome
1	benign
2	benign
3	benign
4	benign
5	benign
6	malignant

We do logistic regression with the `glm()` function

```
> glm_out <- glm(
    outcome ~ clump_thickness +
      uniform_cell_size +
      uniform_cell_shape +
      marg_adhesion +
      epithelial_cell_size +
      bare_nuclei +
      bland_chromatin +
      normal_nucleoli +
      mitoses,
    data = biopsy,
    family = binomial
  )
```

```
> summary(glm_out)

Call:
glm(formula = outcome ~ clump_thickness + uniform_cell_size +
    uniform_cell_shape + marg_adhesion + epithelial_cell_size +
    bare_nuclei + bland_chromatin + normal_nucleoli + mitoses,
    family = binomial, data = biopsy)

Deviance Residuals:
    Min        1Q    Median        3Q       Max
-3.4841   -0.1153   -0.0619    0.0222    2.4698

Coefficients:
                       Estimate Std. Error z value Pr(>|z|)
(Intercept)           -10.10394    1.17488  -8.600  < 2e-16 ***
clump_thickness         0.53501    0.14202   3.767 0.000165 ***
uniform_cell_size      -0.00628    0.20908  -0.030 0.976039
uniform_cell_shape      0.32271    0.23060   1.399 0.161688
marg_adhesion           0.33064    0.12345   2.678 0.007400 **
epithelial_cell_size    0.09663    0.15659   0.617 0.537159
bare_nuclei             0.38303    0.09384   4.082 4.47e-05 ***
bland_chromatin         0.44719    0.17138   2.609 0.009073 **
normal_nucleoli         0.21303    0.11287   1.887 0.059115 .
mitoses                 0.53484    0.32877   1.627 0.103788
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> summary(glm_out)

Call:
glm(formula = outcome ~ clump_thickness + uniform_cell_size +
    uniform_cell_shape + marg_adhesion + epithelial_cell_size +
    bare_nuclei + bland_chromatin + normal_nucleoli + mitoses,
    family = binomial, data = biopsy)

Deviance Residuals:
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-3.4841   -0.1153   -0.0619    0.0222     2.4698

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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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      mitoses,
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> summary(glm_out)

Call:
glm(formula = outcome ~ clump_thickness + uniform_cell_shape +
    marg_adhesion + epithelial_cell_size + bare_nuclei +
bland_chromatin +
    normal_nucleoli + mitoses, family = binomial, data = biopsy)

Deviance Residuals:
    Min        1Q    Median        3Q       Max
-3.4823   -0.1154   -0.0620    0.0222    2.4694

Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
(Intercept)          -10.09765    1.15546  -8.739  < 2e-16 ***
clump_thickness        0.53456    0.14125   3.784 0.000154 ***
uniform_cell_shape     0.31816    0.17424   1.826 0.067847 .
marg_adhesion          0.32993    0.12115   2.723 0.006465 **
epithelial_cell_size   0.09612    0.15564   0.618 0.536876
bare_nuclei            0.38308    0.09384   4.082 4.46e-05 ***
bland_chromatin        0.44648    0.16986   2.628 0.008578 **
normal_nucleoli        0.21255    0.11174   1.902 0.057149 .
mitoses                0.53406    0.32761   1.630 0.103064
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> summary(glm_out)

Call:
glm(formula = outcome ~ clump_thickness + uniform_cell_shape +
    marg_adhesion + epithelial_cell_size + bare_nuclei +
bland_chromatin +
    normal_nucleoli + mitoses, family = binomial, data = biopsy)

Deviance Residuals:
    Min       1Q    Median        3Q        Max
-3.4823   -0.1154   -0.0620    0.0222     2.4694

Coefficients:
                       Estimate Std. Error z value Pr(>|z|)
(Intercept)           -10.09765    1.15546  -8.739  < 2e-16 ***
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    outcome ~ clump_thickness +
      uniform_cell_shape +
      marg_adhesion +
      bare_nuclei +
      bland_chromatin +
      normal_nucleoli +
      mitoses,
    data = biopsy,
    family = binomial
  )
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Call:
glm(formula = outcome ~ clump_thickness + uniform_cell_shape +
    marg_adhesion + bare_nuclei + bland_chromatin +
normal_nucleoli +
    mitoses, family = binomial, data = biopsy)

Deviance Residuals:
    Min        1Q    Median        3Q       Max
-3.5235   -0.1149   -0.0627    0.0219    2.4115

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         -9.98278    1.12610  -8.865  < 2e-16 ***
clump_thickness      0.53400    0.14079   3.793 0.000149 ***
uniform_cell_shape   0.34529    0.17164   2.012 0.044255 *
marg_adhesion        0.34249    0.11922   2.873 0.004068 **
bare_nuclei          0.38830    0.09356   4.150 3.32e-05 ***
bland_chromatin      0.46194    0.16820   2.746 0.006025 **
normal_nucleoli      0.22606    0.11097   2.037 0.041644 *
mitoses              0.53119    0.32446   1.637 0.101598
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> summary(glm.out)

Call:
glm(formula = outcome ~ clump_thickness + uniform_cell_shape +
    marg_adhesion + bare_nuclei + bland_chromatin +
normal_nucleoli +
    mitoses, family = binomial, data = biopsy)

Deviance Residuals:
    Min        1Q    Median        3Q       Max
-3.5235   -0.1149   -0.0627    0.0219    2.4115

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         -9.98278    1.12610  -8.865  < 2e-16 ***
clump_thickness      0.53400    0.14079   3.793 0.000149 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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```
> glm_out <- glm(
    outcome ~ clump_thickness +
      uniform_cell_shape +
      marg_adhesion +
      bare_nuclei +
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      normal_nucleoli,
    data = biopsy,
    family = binomial
)
```

```
> summary(glm_out)

Call:
glm(formula = outcome ~ clump_thickness + uniform_cell_shape +
    marg_adhesion + bare_nuclei + bland_chromatin +
normal_nucleoli,
    family = binomial, data = biopsy)

Deviance Residuals:
    Min        1Q   Median        3Q        Max
-3.5201   -0.1186  -0.0570    0.0250    2.4055

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         -9.76708    1.08506  -9.001  < 2e-16 ***
clump_thickness      0.62253    0.13712   4.540 5.62e-06 ***
uniform_cell_shape   0.34951    0.16503   2.118  0.03419 *
marg_adhesion        0.33753    0.11561   2.920  0.00350 **
bare_nuclei          0.37855    0.09381   4.035 5.45e-05 ***
bland_chromatin      0.47134    0.16612   2.837  0.00455 **
normal_nucleoli      0.24317    0.10855   2.240  0.02509 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```
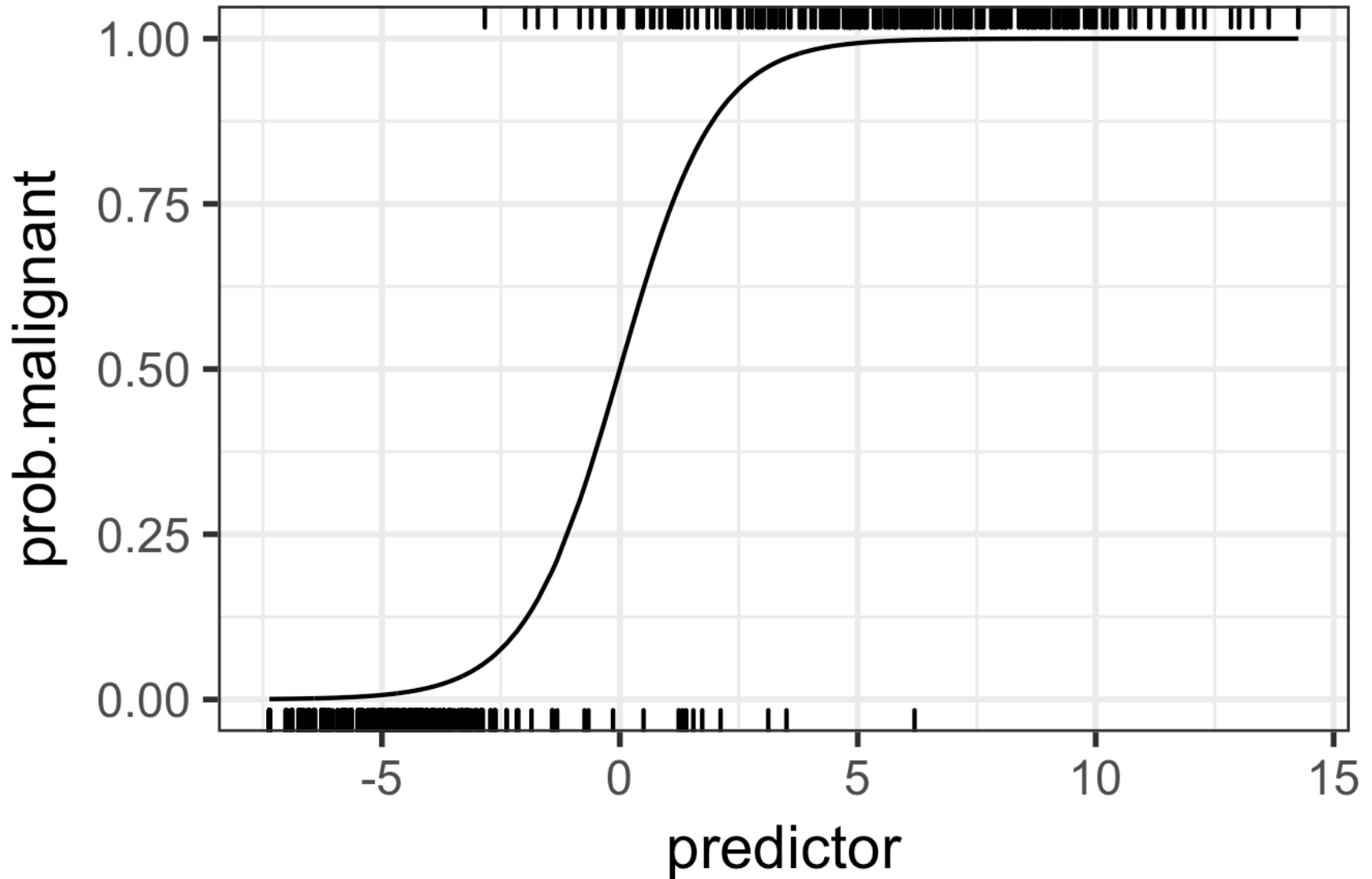
The fitted logistic model

We can extract fitted probabilities from `glm_out$fitted.values`

```
> glm_out$fitted.values
            1             2             3             4             5             6
0.0192341317  0.8925583864  0.0081774737  0.8496854505  0.0202506282  0.9999854554
            7             8             9            10            11            12
0.0467606911  0.0042790664  0.0011789931  0.0065253423  0.0016231293  0.0018875638
           13            14            15            16            17            18
0.3544332567  0.0034543023  0.9993353305  0.7371582761  0.0065253423  0.0104135504
           19            20            21            22            23            24
0.9989353409  0.0352597948  0.9969203982  0.9994994519  0.0035120154  0.0016231293
           25            26            27            28            29            30
0.7802514369  0.0035120154  0.0120927435  0.0018875638  0.0012725934  0.0035120154
           31            32            33            34            35            36
0.0030206952  0.9977220579  0.0042283384  0.0049740412  0.0018875638  0.9998755391
           37            38            39            40            41            42
0.1940709471  0.9954253327  0.6691128086  0.9536389392  0.9974078013  0.3002866244
           43            44            45            46            47            48
0.9996235802  0.0010137236  0.9583091930  0.0010137236  0.0202506282  0.9836985106
           49            50            51            52            53            54
0.7842860362  0.4122043566  0.9956800184  0.9922376046  0.9988895968  0.9870508267
           55            56            57            58            59            60
0.9927513406  0.6585108620  0.7534314353  0.8341431018  0.9032183182  0.0014795146
           61            62            63            64            65            66
0.9921570845  0.5158282353  0.0010137236  0.7040691331  0.0104135504  0.9498144607
```

We can extract linear predictors from `glm_out$linear.predictors`

```
> glm_out$linear.predictors
          1           2           3           4           5           6
-3.93164737  2.11714436 -4.79816093  1.73213613 -3.87911098 11.13827708
          7           8           9          10          11          12
-3.01482307 -5.44973218 -6.74191480 -5.02551514 -6.42177489 -6.27057890
         13          14          15          16          17          18
-0.59960855 -5.66467448  7.31555568  1.03125059 -5.02551514 -4.55417925
         19          20          21          22          23          24
 6.84403543 -3.30911549  5.77987063  7.59930618 -5.64804702 -6.42177489
         25          26          27          28          29          30
 1.26713222 -5.64804702 -4.40298326 -6.27057890 -6.66542501 -5.64804702
         31          32          33          34          35          36
-5.79924301  6.08220228 -5.46170888 -5.29853619 -6.27057890  8.99139484
         37          38          39          40          41          42
-1.42377192  5.38263613  0.70417516  3.02382523  5.95265328 -0.84593335
         43          44          45          46          47          48
 7.88442916 -6.89311078  3.13488983 -6.89311078 -3.87911098  4.10006298
         49          50          51          52          53          54
 1.29082051 -0.35486010  5.44017479  4.85067163  6.80192104  4.33368959
         55          56          57          58          59          60
 4.91966368  0.65666514  1.11699791  1.61527962  2.23350656 -6.51456058
         61          62          63          64          65          66
 4.84027081  0.06333410 -6.89311078  0.86675068 -4.55417925  2.94053974
```
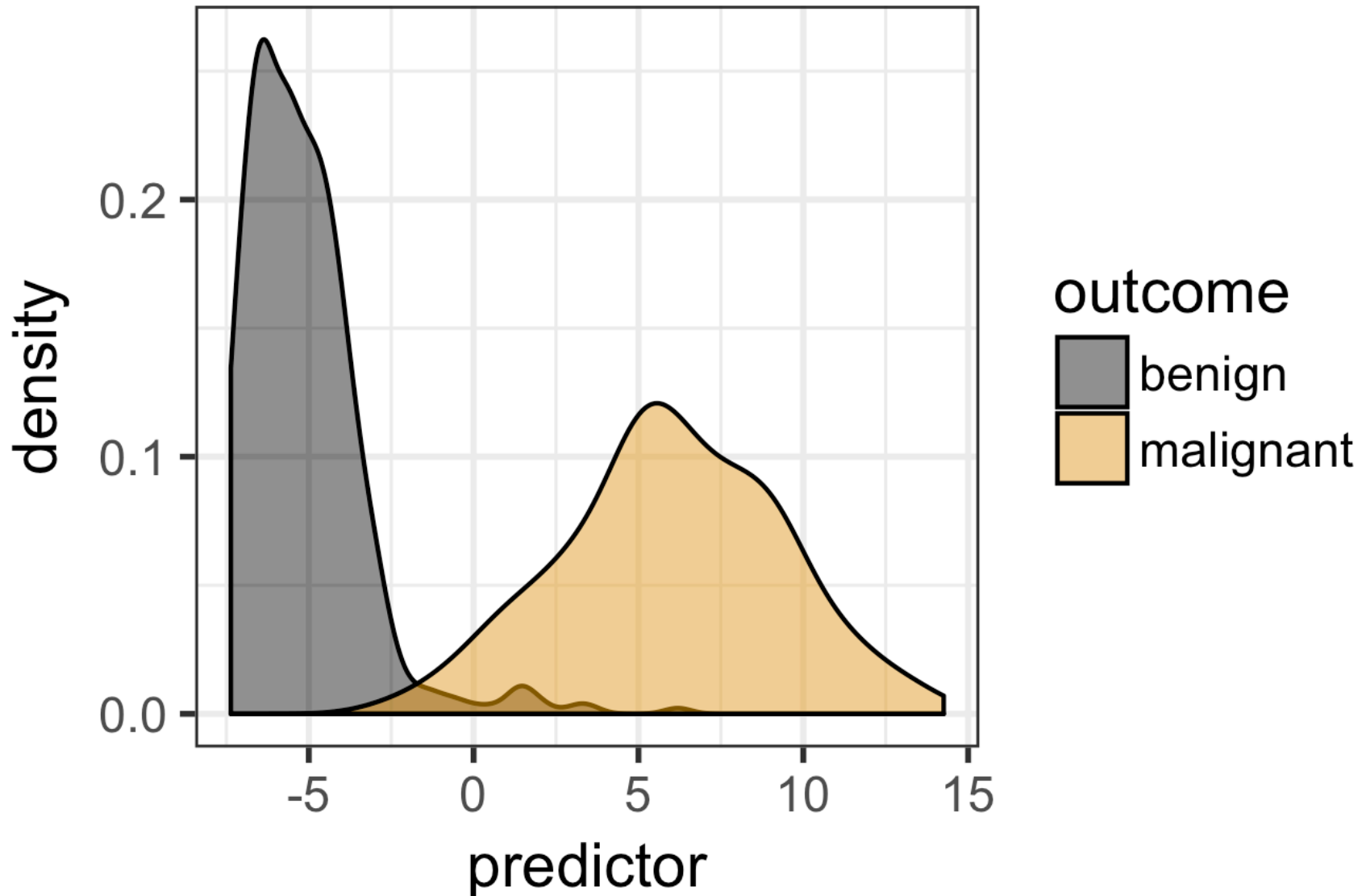
The linear predictor clearly separates benign and malignant outcomes
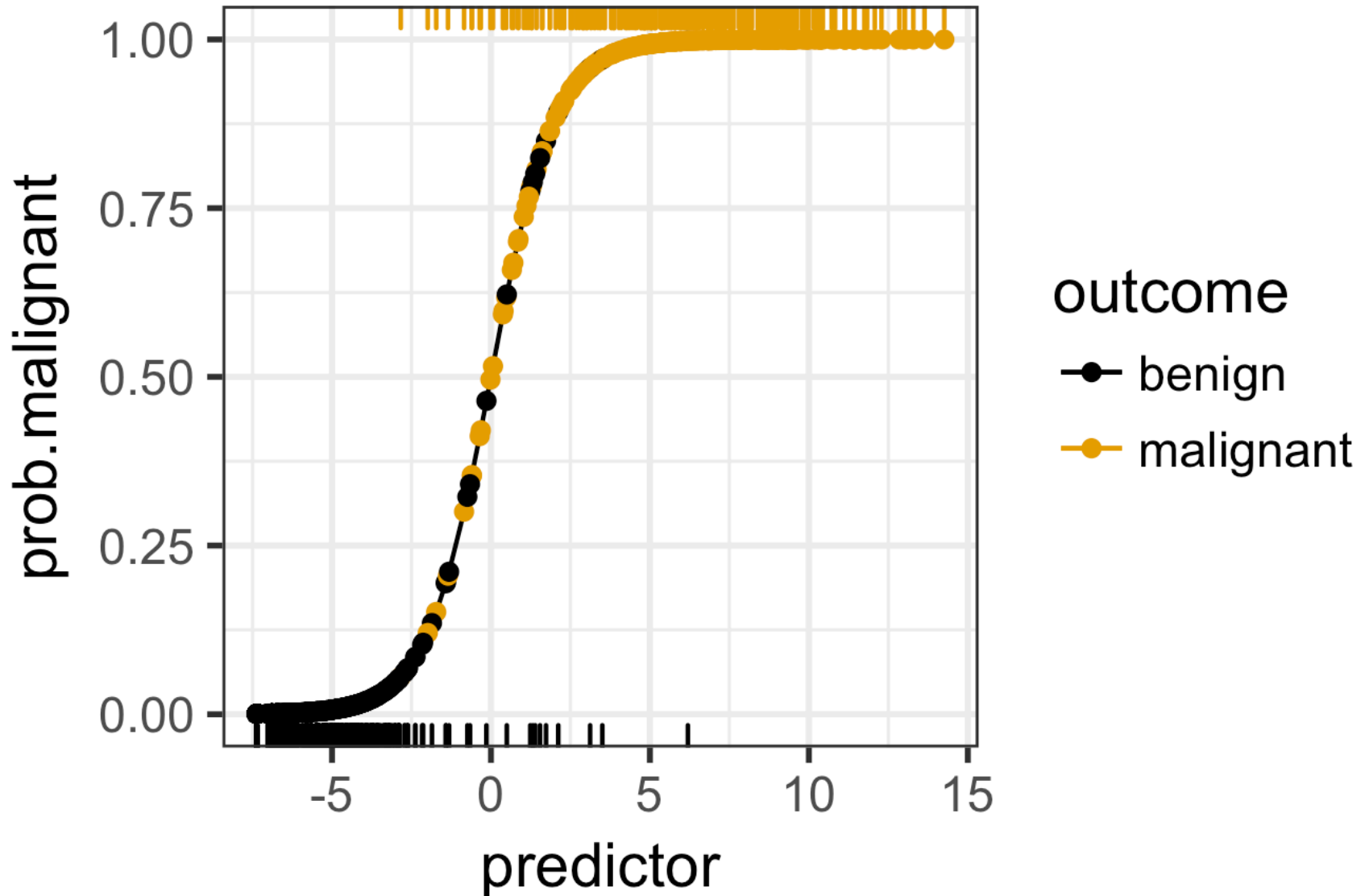
The linear predictor clearly separates benign and malignant outcomes

Predicting outcome for new data with the `predict()` function

```
> patient1 <- data.frame(
    clump_thickness = 1,
    uniform_cell_size = 1,
    uniform_cell_shape = 1,
    marg_adhesion = 1,
    epithelial_cell_size = 4,
    bare_nuclei = 3,
    bland_chromatin = 1,
    normal_nucleoli = 1,
    mitoses = 1
  )
```

Predicting outcome for new data with the `predict()` function

```
> patient1 <- data.frame(
    clump_thickness = 1,
    uniform_cell_size = 1,
    uniform_cell_shape = 1,
    marg_adhesion = 1,
    epithelial_cell_size = 4,
    bare_nuclei = 3,
    bland_chromatin = 1,
    normal_nucleoli = 1,
    mitoses = 1
  )

> predict(glm_out, patient1) # linear predictor
        1
-6.607346
```

Predicting outcome for new data with the `predict()` function

```
> patient1 <- data.frame(
    clump_thickness = 1,
    uniform_cell_size = 1,
    uniform_cell_shape = 1,
    marg_adhesion = 1,
    epithelial_cell_size = 4,
    bare_nuclei = 3,
    bland_chromatin = 1,
    normal_nucleoli = 1,
    mitoses = 1
  )

> predict(glm_out, patient1) # linear predictor
        1
-6.607346

> predict(glm_out, patient1, type="response") # probability
        1
0.00134859
```

Predicting outcome for new data with the `predict()` function

```
> patient2 <- data.frame(
    clump_thickness = 4,
    uniform_cell_size = 5,
    uniform_cell_shape = 5,
    marg_adhesion = 10,
    epithelial_cell_size = 4,
    bare_nuclei = 10,
    bland_chromatin = 7,
    normal_nucleoli = 5,
    mitoses = 8
  )
```

```
> patient2 <- data.frame(
    clump_thickness = 4,
    uniform_cell_size = 5,
    uniform_cell_shape = 5,
    marg_adhesion = 10,
    epithelial_cell_size = 4,
    bare_nuclei = 10,
    bland_chromatin = 7,
    normal_nucleoli = 5,
    mitoses = 8
  )

> predict(glm_out, patient2) # linear predictor
       1
6.14665
```

Predicting outcome for new data with the `predict()` function

```
> patient2 <- data.frame(
    clump_thickness = 4,
    uniform_cell_size = 5,
    uniform_cell_shape = 5,
    marg_adhesion = 10,
    epithelial_cell_size = 4,
    bare_nuclei = 10,
    bland_chromatin = 7,
    normal_nucleoli = 5,
    mitoses = 8
  )

> predict(glm_out, patient2) # linear predictor
        1
6.14665

> predict(glm_out, patient2, type = "response") # probability
        1
0.9978639
```