# House model FutureFactory

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# 1 Introduction

Building energy simulation is a vast field of research that started in the late 50's and that is still highly active nowadays. Building energy simulations are mainly used to help taking design decisions, to analyze current designs and to forecast future building energy use. Building energy modelling methods can mainly be divided into three categories:

- White box models (physics-based)
- Black box models (data-driven)
- Grey box models (hybrid)

White box models are based on the equations related to the fundamental laws of energy and mass balance and heat transfer. White box models can be differentiated in two types, *distributed* parameter models and *lumped* parameter models. Lumped parameter models simplify the description of distributed physical systems into discrete entities that approximate the behavior of a distributed system. The advantage of using lumped models is the decrease in simulation time **(Ramallo-González et al.**[1]**)**. White box models are of special interest for the design phase as they are used to predict and analyse the performance of the building envelope and building systems. Black box models are based on the statistical relation between input and output system values. The statistical relation between input and output is based on actual data. The relation between the parameters can differ based on the amount of data and the method used to analyze the relation. Currently, there is a large and active field of research about statistical models that are used on black box models **(Coacley et al.**[2]**)**. Black box models are of special interest when there is a large amount of actual input and output data available.

Grey box models are hybrid models that aim to combine the advantages of both approaches. In order to use them it is necessary to implement some equations and it is also required to have actual data of inputs and outputs.

## 2 White box lumped model: RC network

### 2.1 White box lumped model

The objective of the house model for this project is to serve as test environment for a heat pump model, which means that the house model is intended as a tool to help taking building systems design decisions. The house heating demand calculation model implemented for this project is a white box *lumped* model. Specifically, it is a RC network model consisting of resistances (R) and capacities (C). The RC network model is based on the analogy with electrical circuits. The simulation of thermodynamic systems characterizing building elements as resistances or capacities allows to simplify the model while maintaining a high simulation results accuracy (**Bagheri et al.**[3], **Bacher et al**[4]**.**).

There are several types of RC models, the most common being 3R4C models and 3R2C models which are applied on the outer and internal wall. For the simulation of simple house buildings 3R2C models perform as accurate as more complex 3R4C models (**Fraisse et al.**[5]). Considering that one of the objectives for this project is to obtain a fast but accurate simulation of a simple dwelling the 3R2C network model appeared a good starting point. In the 3R2C model two indoor temperature nodes are present in the dwelling. **with capacities (usually an air and a wall temperature) and a well-known outdoor temperature** . **Between these 3 temperature nodes 3 heat transfer resistances are present. However, the direct heat transfer between the inner walls and the outdoor air is low. Moreover, uncertainties are present about heat transfer coefficients between walls and indoor air, different indoor temperatures in the house rooms and the ground temperature which deviates from the outdoor temperature. In addition, occupancy behaviour varies strongly.** For that reason, we have made a further simplification to a 2R2C model. In section 4 it is shown that this dwelling model delivers a reliable annual energy consumption.

### 2.2 House Model R and C Values

This section presents the basic information for calculating a house model based on an RC network. This category of house models, analogous to electrical impedance networks, may have different numbers of R and C components and may have various component topologies. For the specific model properties, references will be given.

In heat transfer theory the basic thermal circuit contains thermal resistances. Heat transfer occurs via conduction, convection and radiation. In analogy with Ohm's Law for electricity, expressions can be derived for the heat transfer rate (analogous to electrical current) and the thermal resistances (analogous to ohmic resistances) in these three modes of heat transfer. The temperature difference plays a role analogous to the electrical voltage difference. These expressions are shown in Fig.1.

**Equations for different heat transfer modes and their thermal resistances.**

| Transfer Mode | Rate of Heat Transfer | Thermal Resistance |
|---|---|---|
| Conduction | $\dot{Q} = \dfrac{T_1 - T_2}{\left(\frac{L}{kA}\right)}$ | $\dfrac{L}{kA}$ |
| Convection | $\dot{Q} = \dfrac{T_{\text{surf}} - T_{\text{envr}}}{\left(\frac{1}{h_{\text{conv}} A_{\text{surf}}}\right)}$ | $\dfrac{1}{h_{\text{conv}} A_{\text{surf}}}$ |
| Radiation | $\dot{Q} = \dfrac{T_{\text{surf}} - T_{\text{surr}}}{\left(\frac{1}{h_r A_{\text{surf}}}\right)}$ | $\dfrac{1}{h_r A}$, where $h_r = \epsilon\sigma(T_{\text{surf}}^2 + T_{\text{surr}}^2)(T_{\text{surf}} + T_{\text{surr}})$ |

**Figure 1:** Heat transfer modes[6]

In [7] and [8] the expressions in Fig.1 are derived. For conduction, the expression for absolute thermal resistance is:

$$R = \frac{L}{kA} \qquad \left[\frac{K}{W}\right] \tag{1}$$

- $L$ is the distance over which heat transfer takes place, or the thickness of the material $[m]$.
- $k$ (also denoted with $\lambda$) is the thermal conductivity of the material. $[\frac{W}{mK}]$.
- $A$ is the conductive surface area $[m^2]$.
- Thermal resistivity is the reciprocal of thermal conductivity and can be expressed as $r = \frac{1}{k}$ in $[\frac{mK}{W}]$

For convection and radiation the expression for thermal resistance is: $R = \frac{1}{h \cdot A}$ $[\frac{K}{W}]$.

- $A$ is the surface area where the heat transfer takes place $[m^2]$.
- $h$ is the heat transfer coefficient $[\frac{W}{m^2K}]$

The $R$-value (in Dutch: $R$-waarde or $R_d$-waarde) of a building material [9] is the thermal resistance of a square meter surface. It can be calculated by multiplying the thermal *resistivity* with the thickness of the material in $m$. Alternatively it is calculated by dividing the material thickness by the thermal *conductivity* $k$ or $\lambda$.

$$\text{R-value} = r \cdot L \qquad \text{or} \qquad \text{R-value} = \frac{L}{k} \qquad \text{or} \qquad \text{R-value} = \frac{L}{\lambda} \qquad \left[m \cdot \frac{m \cdot K}{W}\right] = \left[\frac{m^2 \cdot K}{W}\right] \tag{2}$$

Some typical heat transfer $R$-values are: [10]:

- Static layer of air, 40 mm thickness (1.57 in) : R = 0.18 $[\frac{m^2K}{W}]$.
- Inside heat transfer resistance, horizontal current : R = 0.13 $[\frac{m^2K}{W}]$.
- Outside heat transfer resistance, horizontal current : R = 0.04 $[\frac{m^2K}{W}]$.
- Inside heat transfer resistance, heat current from down upwards : R = 0.10 $[\frac{m^2K}{W}]$.
- Outside heat transfer resistance, heat current from above downwards : R = 0.17 $[\frac{m^2K}{W}]$.

**Note**: in Dutch building physics, $R$-values with subscripts are used:

- $R_d$-waarde is used for the $R$-value of a homogeneous building material. $R = \frac{L}{\lambda}$
- $R_c$-waarde (compound, construction) is used for the $R$-value of a surface consisting of several building materials. $R_c$-waarden are calculated as the surface-area weighted sum of $R_d$-waarden of the building materials. For the simplest roof surface, $R_c$ is a linear combination of the $R$-values of the wooden joists and girders (spanten en gordingen) and the areas in between with a certain insulation material sandwich. The $R$-value of the insulation sandwich, in its turn, is the sum of the $R$-values of the materials in the sandwich. From inside out, this sandwich may consist of *e.g.* a 9.5 mm plaster board, a PIR/PUR insulation panel, an air gap and a wooden roof deck. All types of $R$-value have the dimension $[\frac{m^2 \cdot K}{W}]$.

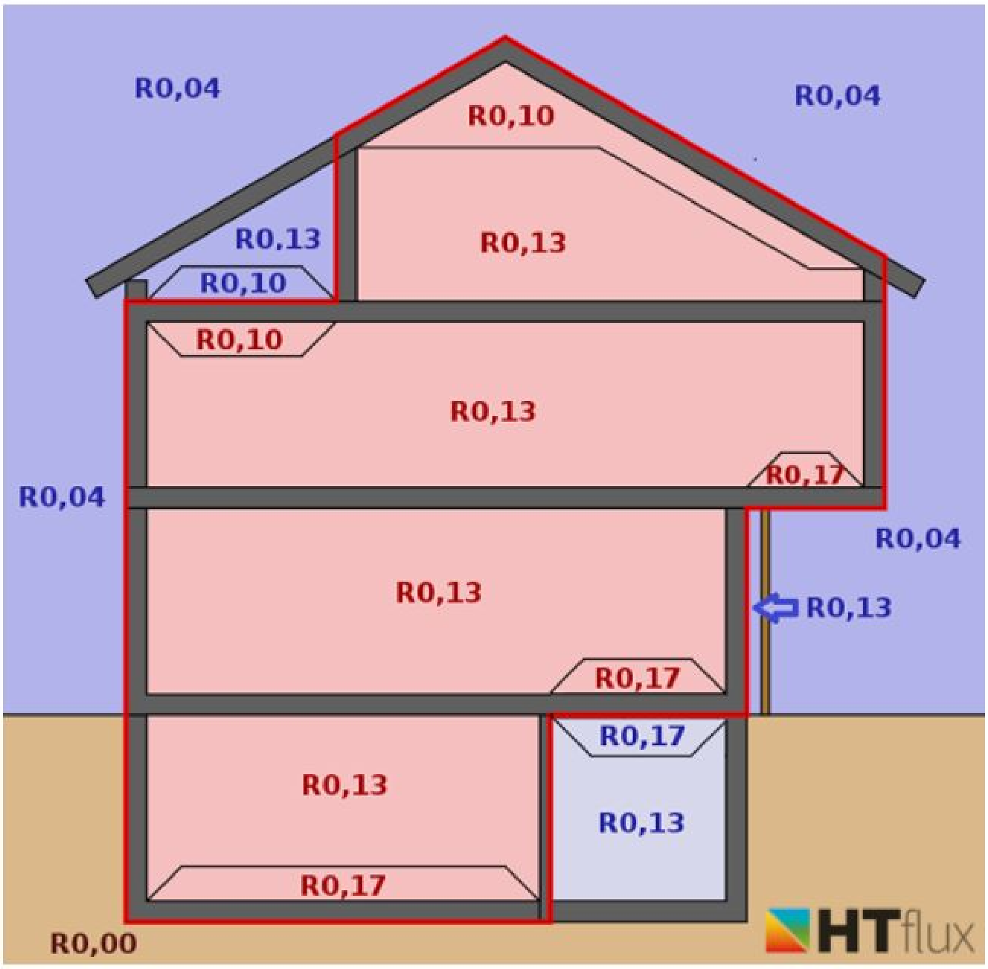


**Figure 2:** An overview of $R$-values for heat transfer [11].

The standard $R_c$-values that have been used for facades, roof and floor until 2020 are summarized in Fig.3:

| Construction | New construction | Renovation |
|---|---|---|
| Facades [1] | Rc 4.5 m2K / W | Rc 1.3 m2K / W |
| Roofs [2] | Rc 6.0 m2K / W | Rc 2.0 m2K / W |
| Floors [3] | Rc 3.5 m2K / W | Rc 2.5 m2K / W |

**Figure 3:** $R_c$ Values [12]

New standard values will be used from 1-1-2021, since the building standard NEN 1068 will be replaced by the NTA 8800 standard. The old and new situation is described in "EnergieVademecum Energiebewust ontwerpen van nieuwbouwwoningen", Hoofdstuk 5: Thermische isolatie, thermische bruggen en luchtdichtheid. [13].

From 2015, the following RC values apply to new construction in the Netherlands:

| *Location* | *RC value (NEN 1068, until 1-1-2021) [m2K / W]* | *Rc value (NTA 8800, from 1-1-2021) [m2K / W]* |
|---|---|---|
| **floor** | **> = 3.5** | **> = 3.7** |
| **facade** | **> = 4.5** | **> = 4.7** |
| **roof** | **> = 6.0** | **> = 6.3** |

**Figure 4:** $R_c$ Values [14]

The values used for different types of houses such as: row houses, detached houses and apartments can be found in the document "Voorbeeldwoningen 2011" [15]. An example with values for a common type of row house, built in the period from 1975 to 1991 is shown in Fig. 5:

| Bouwdelen | Huidig | | | Besparingspakket | | | Investeringskosten | |
|---|---|---|---|---|---|---|---|---|
| | Opp. (m²) | Rc-Waarde (m² K/W) | U-Waarde (W/m² K) | Opp. (m²) | Rc-Waarde (m² K/W) | U-Waarde (W/m² K) | Per m² | Totaal |
| **Begane grondvloer 3** | 51,0 | 0,52 | 1,28 | 51,0 | 2,53 | 0,36 | € 20 | € 1.020 |
| **Plat dak 3** | - | - | - | - | - | - | - | € 0 |
| **Hellend dak 3** | 68,6 | 1,30 | 0,64 | 68,6 | 2,53 | 0,36 | € 53 | € 3.640 |
| **Achter- en voorgevel** | | | | | | | | |
| – Gesloten 3 | 40,6 | 1,30 | 0,64 | 40,6 | 2,53 | 0,36 | € 21 | € 850 |
| – Enkelglas 3 | 3,1 | | 5,20 | - | | - | € 139 | € 430 |
| – Dubbelglas 3 | 16,2 | | 2,90 | - | | - | € 142 | € 2.300 |
| – HR++ glas | - | | - | 19,3 | | 1,80 | | |
| **Zijgevel** | | | | | | | | |
| – Gesloten | 58,4 | 1,30 | 0,64 | 58,4 | 2,53 | 0,36 | € 21 | € 1.230 |
| – Enkelglas | - | | - | - | | - | - | € 0 |
| – Dubbelglas | 1,8 | | 2,90 | - | | - | € 142 | € 260 |
| – HR++ glas | - | | - | 1,8 | | 1,80 | | |

**Figure 5:** $R_c$-values for a row house type built between 1975-1991 [15]

## 2.3 Dwelling (envelope) model analogous to a 2R-2C network

The heat flow will be modelled by analogy to an electrical circuit where heat transfer rate is analogous to by current, temperature difference is analogous to potential difference, heat sources are represented by constant current sources, absolute thermal resistances are represented by resistors and **thermal capacitance** heat capacity ? by capacitors [16]. Figure 6 summarizes the similar term use in different fields.

| type | structural analogy[1] | hydraulic analogy | thermal | electrical analogy[2] |
|---|---|---|---|---|
| quantity | impulse $J$ [N·s] | volume $V$ [m$^3$] | heat $Q$ [J] | charge $q$ [C] |
| potential | displacement $X$ [m] | pressure $P$ [N/m$^2$] | temperature $T$ [K] | potential $V$ [V = J/C] |
| flux | load or force $F$ [N] | flow rate $Q$ [m$^3$/s] | heat transfer rate $\dot{Q}$ [W = J/s] | current $I$ [A = C/s] |
| flux density | stress $\sigma$ [Pa = N/m$^2$] | velocity $\mathbf{v}$ [m/s] | heat flux $\mathbf{q}$ [W/m$^2$] | current density $\mathbf{j}$ [C/(m$^2$·s) = A/m$^2$] |
| resistance | flexibility (rheology defined) [1/Pa] | fluid resistance $R$ [...] | **thermal resistance** $R$ [K/W] | electrical resistance $R$ [Ω] |
| conductance | ... . . . [Pa] | fluid conductance $G$ [...] | thermal conductance $G$ [W/K] | electrical conductance $G$ [S] |
| resistivity | flexibility $1/k$ [m/N] | fluid resistivity | thermal resistivity [(m·K)/W] | electrical resistivity $\rho$ [Ω·m] |
| conductivity | stiffness $k$ [N/m] | fluid conductivity | thermal conductivity $k$ [W/(m·K)] | electrical conductivity $\sigma$ [S/m] |
| lumped element linear model | Hooke's law $\Delta X = F/k$ | Hagen–Poiseuille equation $\Delta P = QR$ | Newton's law of cooling $\Delta T = \dot{Q}R$ | Ohm's law $\Delta V = IR$ |
| distributed linear model | ... . . . | ... . . . | Fourier's law $\mathbf{q} = -k\nabla T$ | Ohm's law $\mathbf{J} = \sigma\mathbf{E} = -\sigma\nabla V$ |

**Figure 6:** Table of Analogies [16]

The 2R-2C house model structure is implemented as described below. The schematic of an envelope house model has been shown in figure 9 and the equivalent electrical 2R-2C network with components and topology is given in fig 10.

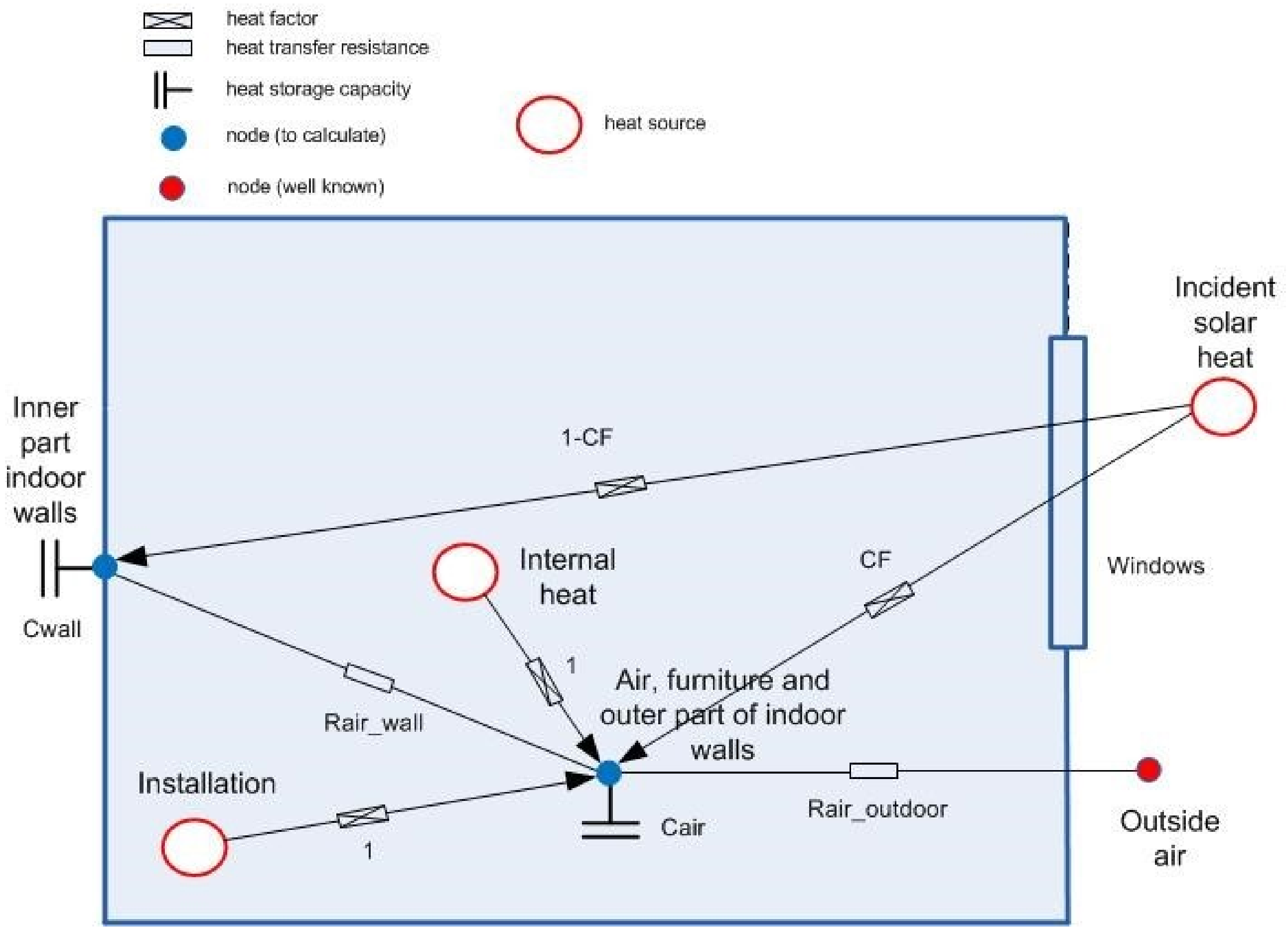


**Figure 7:** Schematic of envelope model

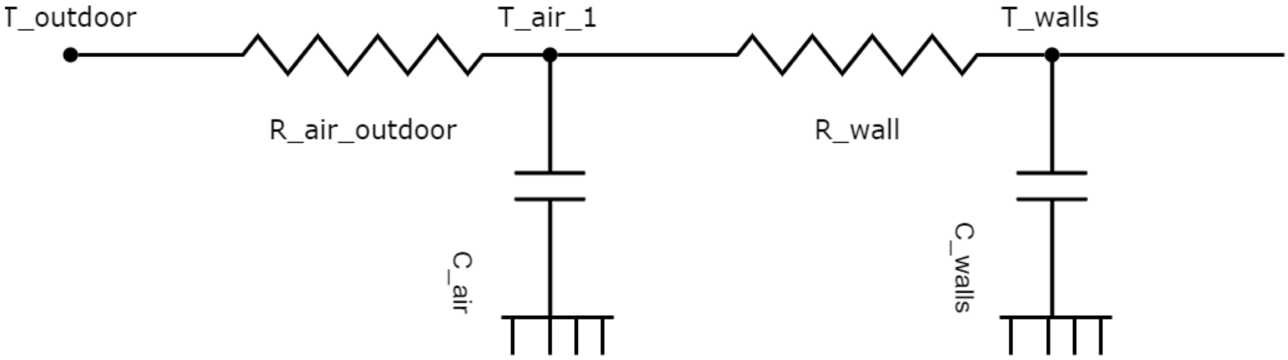


**Figure 8:** 2R-2C house model

The model consists of two heat capacities $C_{air,\ indoor}$ and $C_{wall}$ and two resistances $R_{wall}$ and $R_{air,\ outdoor}$. The incident solar energy is divided between $C_{wall}$ and $C_{air}$ through the convection factor CF. It is assumed that both internal heat (lighting, occupancy and electric devices) and supplied heat (installation) initially heat up the indoor air. In Fig. 9, they are fully released at the $T_{air}$ node.

It is also assumed that furniture and the **surface part** of the walls have the same temperature as the air **and the wall mass is divided between the air and wall mass**. Thus, the heat capacity of the air node consists of the air heat capacity, furniture heat capacity and the heat capacity **of a part of the walls**. **Appendix A** presents the coefficients in the dwelling model. In the resistance $R_{air,\ outdoor}$ the influence of heat transmission through the outdoor walls and natural ventilation is considered.

For the air and wall nodes the following power balances can be set up:

$$C_{air}\frac{dT_{air}}{dt} = \frac{T_{outdoor} - T_{air}}{R_{air\_outdoor}} + \frac{T_{wall} - T_{air}}{R_{air\_wall}} + \dot{Q}_{inst} + \dot{Q}_{internal} + CF \cdot \dot{Q}_{solar} \tag{3}$$

$$C_{wall}\frac{dT_{wall}}{dt} = \frac{T_{air} - T_{wall}}{R_{air\_wall}} + (1 - CF) \cdot \dot{Q}_{solar} \tag{4}$$

- $CF$: convection factor (solar radiation): the convection factor is the part of the solar radiation that enters the room and is released directly convectively into the room.
- $\dot{Q}_{inst}$: delivered heat from heating system (radiator) [W].
- $\dot{Q}_{inernal}$: internal heat [W].
- $\dot{Q}_{solar}$: heat from solar irradiation [W].
- $T_{air}$: indoor air temperature $^{o}$C.
- $T_{outdoor}$: outdoor temperature $^{o}$C.
- $T_{wall}$: wall temperature $^{o}$C.
- $R_{air\_wall}$: walls surface resistance $[\frac{K}{W}]$.
- $R_{air\_outdoor}$: outdoor surface resistance $[\frac{K}{W}]$.
- $C_{air}$: air thermal capacitance (heat capacity) $[\frac{J}{K}]$[17].
- $C_{wall}$: wall thermal capacitance (heat capacity) $[\frac{J}{K}]$[17].

Total heat transfer of solar irradiation through the glass windows.

$$\dot{Q}_{solar} = g. \sum (A_{glass}.\dot{q}_{solar}) \tag{5}$$

- $\dot{q}_{solar}$: solar radiation on the outdoor walls $[\frac{W}{m^2}]$.
- g: g value of the glass (ZTA in dutch) [0..1][18]
- A: glass surface $[m^2]$.

# 3 Dwelling (envelope) model analogous to a 2R-2C network

The 2R-2C house model structure is implemented as described below:

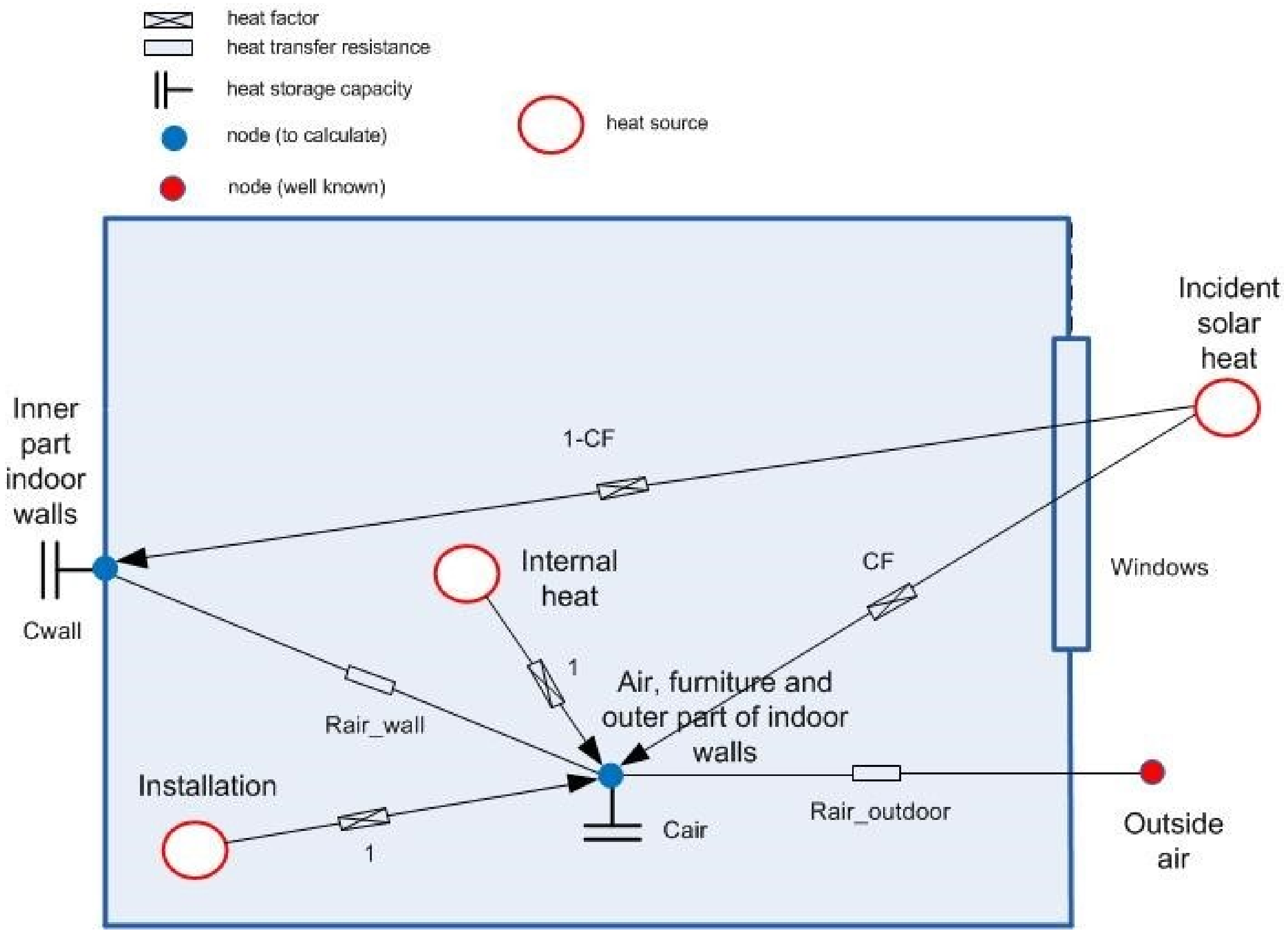


**Figure 9:** Schematic of envelope model

The equivalent electrical 2R-2C network with components and topology is given in Fig. 10.

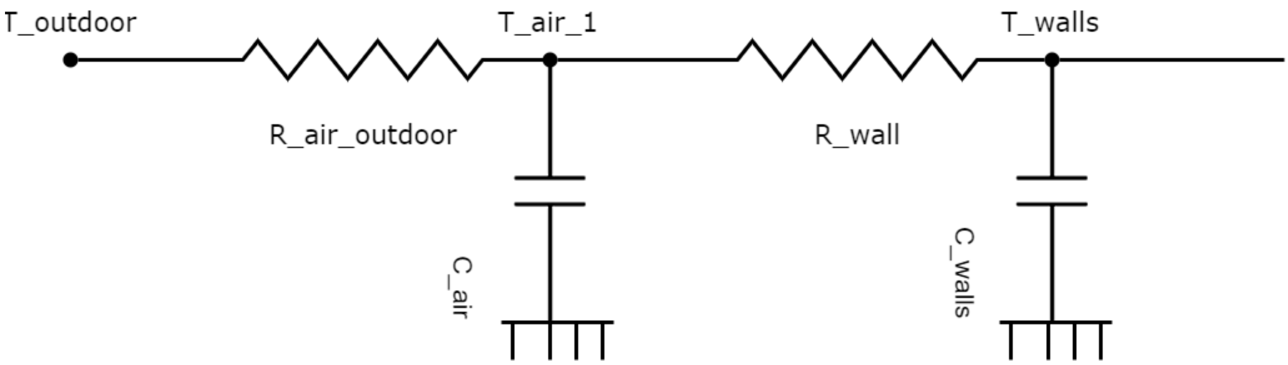


**Figure 10:** 2R-2C house model

The model consists of two capacitances $C_{air,\ indoor}$ and $C_{wall}$ and two resistances $R_{wall}$ and $R_{air,\ outdoor}$. The incident solar energy is divided between $C_{wall}$ and $C_{air}$ through the convection factor CF. It is assumed that both internal heat (lighting, occupancy and electric devices) and supplied heat (installation) initially heat up the indoor air. In Fig. 10, they are fully released at the $T_{air}$ node.

It is also assumed that furniture and the surface part of the walls have the same temperature as the air and the wall mass is divided between the air and wall mass. Thus, the capacity of the air node consists of the air capacity, furniture capacity and capacity of a part of the walls. **Appendix A** presents the coefficients in the dwelling model. In the resistance $\mathrm{R}_{\mathrm{air,\ outdoor}}$ the influence of heat transmission through the outdoor walls and natural ventilation is considered.

For the air and wall nodes the following energy balances can be set up:

$$C_{air}\frac{dT_{air}}{dt} = \frac{T_{outdoor} - T_{air}}{R_{air\_outdoor}} + \frac{T_{wall} - T_{air}}{R_{air\_wall}} + \dot{Q}_{inst} + \dot{Q}_{internal} + CF \cdot \dot{Q}_{solar} \tag{6}$$

$$C_{wall}\frac{dT_{wall}}{dt} = \frac{T_{air} - T_{wall}}{R_{air\_wall}} + (1 - CF) \cdot \dot{Q}_{solar} \tag{7}$$

- CF: Convection factor (solar radiation): the convection factor is the part of the solar radiation that enters the room and is released directly convectively into the room
- $\dot{Q}_{inst}$: delivered heat from heating system (radiator) [W].
- $\dot{Q}_{solar}$: heat from solar irradiation [W].
- $T_{air}$: indoor air temperature $^{o}$C.
- $T_{outdoor}$: outdoor temperature $^{o}$C.
- $T_{wall}$: wall temperature $^{o}$C.
- $R_{air\_wall}$: walls surface resistance $[\frac{K}{W}]$.
- $R_{air\_outdoor}$: outdoor surface resistance $[\frac{K}{W}]$.
- $C_{air}$: air capacity $[\frac{J}{K}]$.
- $C_{wall}$: wall capacity $[\frac{J}{K}]$.

Total heat transfer of solar irradiation through the glass windows.

$$Q_{solar} = g.\sum(A_{glass}.q_{solar}) \tag{8}$$

- $q_{solar}$: solar radiation on the outdoor walls $[\frac{W}{m^2}]$.
- g: g value of the glass (ZTA in dutch) [0..1][18]
- A: glass surface $[m^2]$.

# 4 2 Zones house model 7R4C network

The 4R-7C house model structure is implemented as described below:

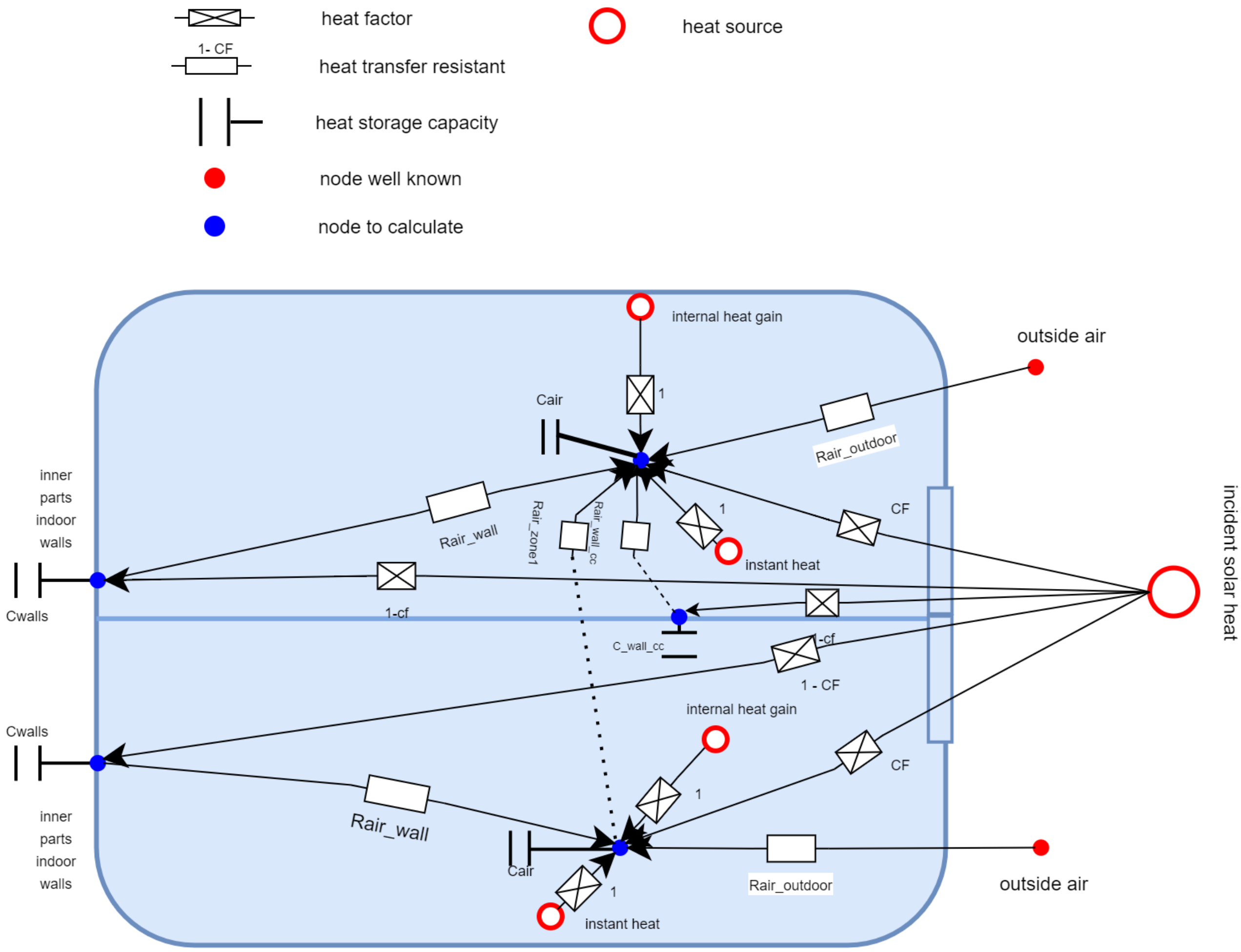


**Figure 11:** Schematic of a 2 zones house model

The equivalent electrical 7R-4C network with components and topology is given in Fig. 12.

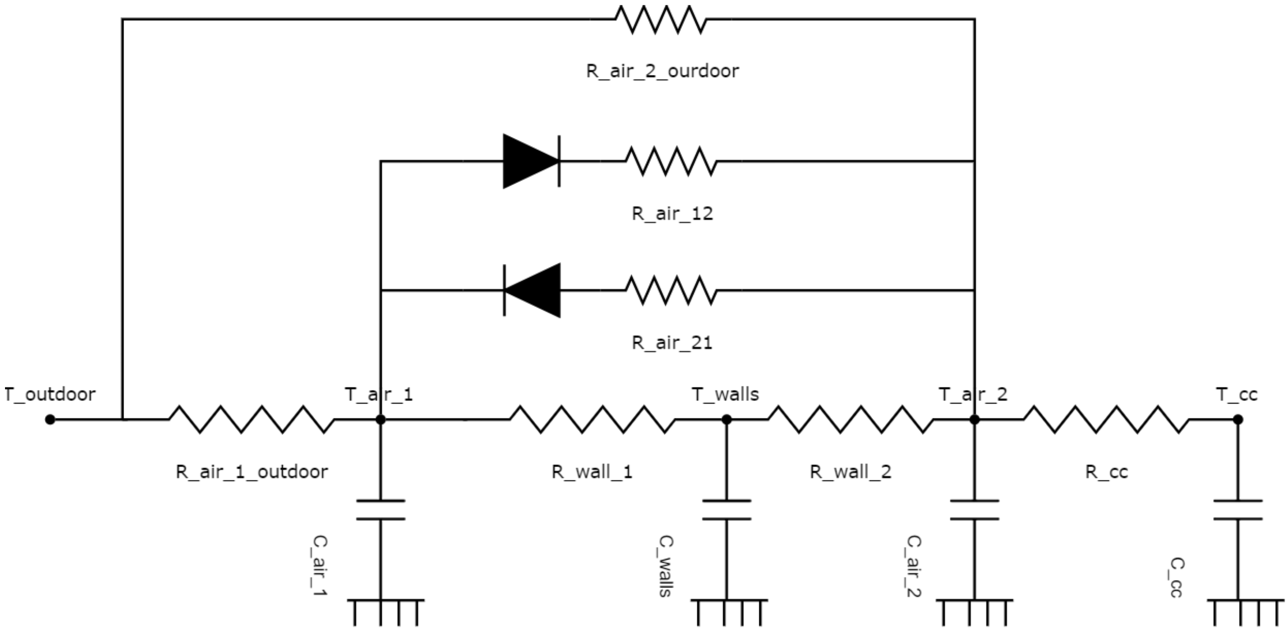


**Figure 12:** R-C circuits of 2 zones house model

with:

- `T_outdoor` : outdoor temperature [$°C$]
- `T_air_1` : zone 1 air temperature [$°C$]
- `T_walls` : wall temperature [$°C$]
- `T_air_2` : zone 2 air temperature [$°C$]
- `T_cc` : temperature of the concrete layer between zone 1 and zone 2 [$°C$]
- `R_air_1_outdoor` : outdoor resistance valus.
- `R_wall_1` : walls resistance value.
- `R_wall_2` : walls resistance value.
- `R_cc` : concrete resistance value.
- `R_air_12` : resistance value of air flow from zone 1 to zone 2.
- `R_air_21` : resistance value of air flow from zone 2 to zone 1.

# 5 Lumped-element thermal model of a building

Heat generation and transport inside a building, with heat loss to the surrounding outdoor environment is governed by the same laws of conduction, convection and radiation as elsewhere. A number of approximations is made, however, which will be treated below:

## 5.1 Heat Conduction: Fourier's Law

Heat transport *within* a solid material is governed by conduction, according to Fourier's Law, illustrated in Figure 13. One side of a rectangular solid is held at temperature $T_1$, while the opposite side is held at a lower temperature, $T_2$. The other four sides are insulated so that heat can flow only in the $x$-direction. For a given material, it is found that the rate, $\dot{Q}_x$ , at which heat (thermal energy) is transferred from the hot side to the cold side (the *heat transfer rate*) is proportional to the cross-sectional area, $A$, across which the heat flows; the temperature difference, $T_1 - T_2$; and inversely proportional to the thickness, $\Delta x$, of the material. That is:

$$\dot{Q}_x = -kA\frac{\Delta T}{\Delta x} \tag{9}$$

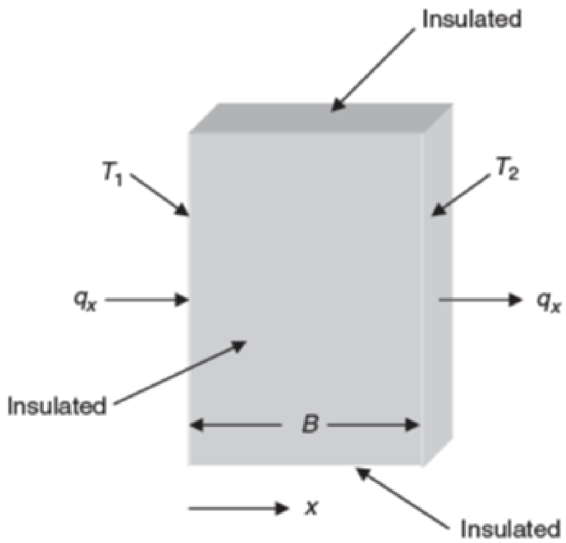


**Figure 13:** One-dimensional heat conduction in a solid

The constant of proportionality, $k$, is called the *thermal conductivity*. Equation (9) is also applicable to heat conduction in liquids and gases. However, when temperature differences exist in fluids, convection currents tend to be set up, so that heat is generally not transferred solely by the mechanism of conduction. The thermal conductivity is a property of the material. Values may be found in various handbooks and compendiums of physical property data.

The form of Fourier's law given by Equation (9) is valid only when the thermal conductivity can be assumed constant. A more general result can be obtained by writing the equation for an element of differential thickness. in the limit as $\Delta x$ approaches zero, $\frac{\Delta T}{\Delta x} \to \frac{dT}{dx}$. Thus, substituting in Equation (9) gives:

$$\dot{Q}_x = -kA\frac{dT}{dx} \tag{10}$$

Equation (10) is not subject to the restriction of constant $k$. Furthermore, when $k$ is constant, it can be integrated to yield Equation (9). Hence, Equation (10) is the general one-dimensional form of Fourier's law. The negative sign is necessary because heat flows in the positive $x$-direction when the temperature decreases in the $x$-direction. Thus, according to the standard sign convention that $\dot{Q}_x$ is positive when the heat flow is in the positive $x$-direction, $\dot{Q}_x$ must be positive when $dT/dx$ is negative.

### 5.1.1 More than one dimension

It is often convenient to formulate Fourier's Law in the original phrasing: the *heat flux* $\dot{\varphi}$ is proportional to the *temperature gradient*. We divide (10) by the area to give:

$$\dot{\varphi}_x \equiv \frac{\dot{Q}_x}{A} - k\frac{dT}{dx} \tag{11}$$

where $\dot{\varphi}_x$ is the heat flux. It has units of $\frac{J}{s \cdot m^2} = \frac{W}{m^2}$. Thus, the units of $k$ are $\frac{W}{m \cdot K}$.

Equation (11) is restricted to the situation in which heat flows in the $x$-direction only. In the general case in which heat flows in all three coordinate directions, the total heat flux is obtained by vector addition of adding the fluxes in the coordinate directions. Thus,

$$\dot{\boldsymbol{\varphi}} = \dot{\varphi}_x \mathbf{i} + \dot{\varphi}_y \mathbf{j} + \dot{\varphi}_z \mathbf{k} \tag{12}$$

where $\dot{\boldsymbol{\varphi}}$ is the heat flux vector and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors in the x-, y-, z-directions, respectively.

Each of the component fluxes is given by a one-dimensional Fourier expression as follows:

$$\dot{\varphi}_x = -k\frac{\partial T}{\partial x} \qquad \dot{\varphi}_y = -k\frac{\partial T}{\partial y} \qquad \dot{\varphi}_z = -k\frac{\partial T}{\partial z} \tag{13}$$

Partial derivatives are used here since the temperature now varies in all three directions. Substituting the above expressions for the fluxes into Equation (12) gives:

$$\dot{\boldsymbol{\varphi}} = -k\left(\frac{\partial T}{\partial x}\mathbf{i} + \frac{\partial T}{\partial y}\mathbf{j} + \frac{\partial T}{\partial z}\mathbf{k}\right) \tag{14}$$

The vector in parenthesis is the temperature gradient vector, and is denoted by $\nabla T$. Hence,

$$\dot{\boldsymbol{\varphi}} = -k\nabla T \tag{15}$$

Equation (15) is the three-dimensional form of Fourier's law. It is valid for homogeneous, isotropic materials for which the thermal conductivity is the same in all directions. Fourier's law states that heat flows in the direction of greatest temperature decrease.

### 5.1.2 The Heat Conduction Equation

The solution of problems involving heat conduction in solids can, in principle, be reduced to the solution of a single differential equation, the *heat conduction equation*. The equation can be derived by making a thermal power balance on a differential volume element in the solid. For the case of conduction in the $x$-direction only, such a volume element is illustrated in Figure 14.

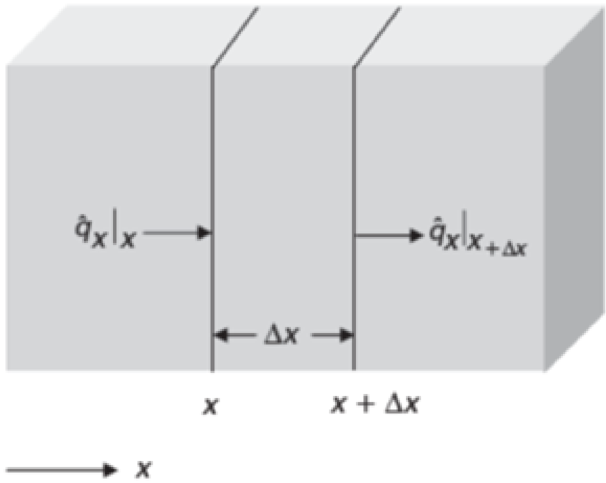


**Figure 14:** Differential element for 1D heat conduction

The rate at which thermal energy enters the volume element across the face at $x$ is given by the product of the heat flux and the cross-sectional area, $\dot{\varphi}_x|_x \cdot A$. Similarly, the rate at which thermal energy leaves the element across the face at $x + \Delta x$ is $\dot{\varphi}_x|_{x+\Delta x} \cdot A$.

A heat generation term appears in the equation because the balance is made on thermal energy, not total energy. For example, thermal energy may be generated within a solid by an electric current or by decay of a radioactive material.

For a homogeneous heat source of strength $\dot{q}$ *per unit volume*, the net rate of generation is $\dot{q}A\Delta x$. Finally, the rate of accumulation of heat in the material is given by the time derivative of the thermal energy content of the volume element, which is $\rho c(T - T_{ref})A\Delta x$, where $T_{ref}$ is an arbitrary reference temperature. Thus, the balance equation becomes:

$$\left(\dot{\varphi}_x|_x - \dot{\varphi}_x|_{x+\Delta x}\right) A + \dot{q}A\Delta x = \rho c \frac{\partial T}{\partial t} A\Delta x \tag{16}$$

It has been assumed here that the density, $\rho$, and heat capacity, $c$, are constant.

Dividing by $A\Delta x$ and taking the limit as $\Delta x \to 0$ yields:

$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial \dot{\varphi}_x}{\partial x} + \dot{q} \tag{17}$$

Using Fourier's law as given by Equation (11), the balance equation becomes:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k\partial T}{\partial x}\right) + \dot{q} \tag{18}$$

When conduction occurs in all three coordinate directions, the balance equation contains y- and z-derivatives analogous to the x-derivative. The balance equation then becomes:

$$\rho c \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\frac{k\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{k\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{k\partial T}{\partial z}\right) + \dot{q} \tag{19}$$

When $k$ is constant, it can be taken outside the derivatives and Equation (19) can be written as:

$$\frac{\rho c}{k}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} \tag{20}$$

or

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k} \tag{21}$$

where $\alpha \equiv k/\rho c$ is the *thermal diffusivity* and $\nabla^2$ is the Laplacian operator. The thermal diffusivity has units of $m^2/s$.

### 5.2 Convection: Newton's Law of cooling

When a solid is *immersed* in a fluid or atmospheric gas, heat transfer on the interface occurs by convection. This phenomenon is governed by Newton's Law of cooling:

"The rate of heat lost by a body is directly proportional to the temperature difference of a body and its surroundings"

$$\dot{Q}_x = -hA\Delta T \tag{22}$$

### 5.3 Radiation

### 5.4 Approximations: A Simplified Model

In building physics, it is often assumed that Fourier's Law is valid in the form of Eq. (9). This can be done under the condition that

$$\nabla^2 T \equiv 0 \rightarrow \quad \frac{\partial T}{\partial \mathbf{r}} = constant \tag{23}$$

### 5.5 Lumped-element matrix representation

We take the 2R-2C lumped-element model from Section 2:

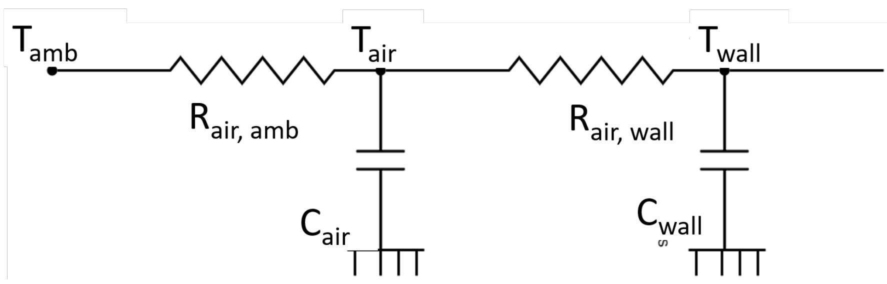


**Figure 15:** 2R-2C house model revisited

The differential equations are:

$$\begin{aligned}
C_{air}\frac{dT_{air}}{dt} &= \frac{T_{amb} - T_{air}}{R_{air,amb}} + \frac{T_{wall} - T_{air}}{R_{air,wall}} + \dot{Q}_{heat,air} + \dot{Q}_{int,air} + \dot{Q}_{solar,air} \\
C_{wall}\frac{dT_{wall}}{dt} &= \frac{T_{air} - T_{wall}}{R_{air,wall}} + \dot{Q}_{solar,wall}
\end{aligned} \tag{24}$$

Writing out the differential equations in the classical notation:

$$
\begin{aligned}
C_{air}\frac{dT_{air}}{dt} &= \left[\frac{-1}{R_{air,amb}} + \frac{-1}{R_{air,wall}}\right] \cdot T_{air} + \frac{1}{R_{air,wall}} \cdot T_{wall} + \frac{1}{R_{air,amb}} \cdot T_{amb} + \dot{Q}_{heat,air} + \dot{Q}_{int,air} + \dot{Q}_{solar,air} \\
C_{wall}\frac{dT_{wall}}{dt} &= \frac{1}{R_{air,wall}} \cdot T_{air} + \frac{-1}{R_{air,wall}} \cdot T_{wall} + \dot{Q}_{solar,wall}
\end{aligned} \tag{25}
$$

The differential equations can be written in matrix notation as:

$$\mathbf{C} \cdot \dot{\boldsymbol{\theta}} = -\mathbf{K} \cdot \boldsymbol{\theta} + \dot{\mathbf{q}} \tag{26a}$$

$$\mathbf{C} \cdot \dot{\boldsymbol{\theta}} + \mathbf{K} \cdot \boldsymbol{\theta} = \dot{\mathbf{q}} \tag{26b}$$

with:

$$\mathbf{C} \cdot \dot{\boldsymbol{\theta}} = \begin{bmatrix} C_{air} & 0 \\ 0 & C_{wall} \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_{air}}{dt} \\ \frac{dT_{wall}}{dt} \end{bmatrix} \tag{27}$$

$$\mathbf{K} \cdot \boldsymbol{\theta} = \begin{bmatrix} \frac{1}{R_{air,amb}} + \frac{1}{R_{air,wall}} & \frac{-1}{R_{air,wall}} \\ \frac{-1}{R_{air,wall}} & \frac{1}{R_{air,wall}} \end{bmatrix} \cdot \begin{bmatrix} T_{air} \\ T_{wall} \end{bmatrix} \tag{28}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \frac{1}{R_{air,amb}} \cdot T_{amb} + \dot{Q}_{heat,air} + \dot{Q}_{int,air} + \dot{Q}_{solar,air} \\ \dot{Q}_{solar,wall} \end{bmatrix} \tag{29}$$

Written out, the differential equation according to (26) becomes:

$$
\begin{aligned}
\begin{bmatrix} C_{air} & 0 \\ 0 & C_{wall} \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_{air}}{dt} \\ \frac{dT_{wall}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-1}{R_{air,amb}} + \frac{-1}{R_{air,wall}} & \frac{1}{R_{air,wall}} \\ \frac{1}{R_{air,wall}} & \frac{-1}{R_{air,wall}} \end{bmatrix} \cdot \begin{bmatrix} T_{air} \\ T_{wall} \end{bmatrix} + \\
\begin{bmatrix} \frac{1}{R_{air,amb}} \cdot T_{amb} + \dot{Q}_{heat,air} + \dot{Q}_{int,air} + \dot{Q}_{solar,air} \\ \dot{Q}_{solar,wall} \end{bmatrix}
\end{aligned} \tag{30}
$$

In the alternative notation:

$$
\begin{aligned}
\begin{bmatrix} C_{air} & 0 \\ 0 & C_{wall} \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_{air}}{dt} \\ \frac{dT_{wall}}{dt} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_{air,amb}} + \frac{1}{R_{air,wall}} & \frac{-1}{R_{air,wall}} \\ \frac{-1}{R_{air,wall}} & \frac{1}{R_{air,wall}} \end{bmatrix} \cdot \begin{bmatrix} T_{air} \\ T_{wall} \end{bmatrix} = \\
\begin{bmatrix} \frac{1}{R_{air,amb}} \cdot T_{amb} + \dot{Q}_{heat,air} + \dot{Q}_{int,air} + \dot{Q}_{solar,air} \\ \dot{Q}_{solar,wall} \end{bmatrix}
\end{aligned} \tag{31}
$$

The lumped-element equations above are systems of *first-order ordinary differential equations* (ODE). The first order derivative is with respect to *time*. The (silent) assumption that heat conduction within the air and the wall of the previous 2R-2C model is *faster* than the exchange of heat at the *interfaces* between air and wall and air and ambient surroundings has replaced all spatial information from the *second-order partial differential equations* (PDE) that govern conductive heat transport *within* materials.

Therefore, the lumped-element equations can be solved by:

- the odexxx in Matlab., preferrably ode45.
- the state-space module in Simulink, after conversion to a state-space representation.
- the scipy.integrate.solve_ivp function in Python. In older code, scipy.integrate.odeint is still encountered.
- in C++ several options exist, similar to the options in Python.

The routines in Matlab, Simulink and Python need a *model function* that provides the vector $\dot{\boldsymbol{\theta}}$ for evaluation at any time instance chosen by the algorithm. The equations (26) then should be cast in the following form by left multiplication with $\mathbf{C^{-1}}$.

$$\mathbf{C}^{-1} \cdot \mathbf{C} \cdot \dot{\boldsymbol{\theta}} = -\mathbf{C}^{-1} \cdot \mathbf{K} \cdot \boldsymbol{\theta} + \mathbf{C}^{-1} \cdot \dot{\mathbf{q}} \tag{32a}$$

$$\dot{\boldsymbol{\theta}} = -\mathbf{C}^{-1} \cdot \mathbf{K} \cdot \boldsymbol{\theta} + \mathbf{C}^{-1} \cdot \dot{\mathbf{q}} \tag{32b}$$

Since $\mathbf{C}$ is a *diagonal* matrix with positive elements only, its inverse exists and contains the reciprocal elements on its diagonal:

$$\mathbf{C^{-1}} = \begin{bmatrix} \frac{1}{C_{air}} & 0 \\ 0 & \frac{1}{C_{wall}} \end{bmatrix} \tag{33}$$

This provides the division by the lumped thermal capacitances of the air and wall compartments in the model, necessary for the calculating the derivative vector $\dot{\boldsymbol{\theta}}$ in the model functions.

## 5.6 Extension of the method to larger lumped-element networks

Take a house model with two stories. Each level in the building is described with a 2R-2C model. Heat transfer occurs between the ground floor and the 1st floor.

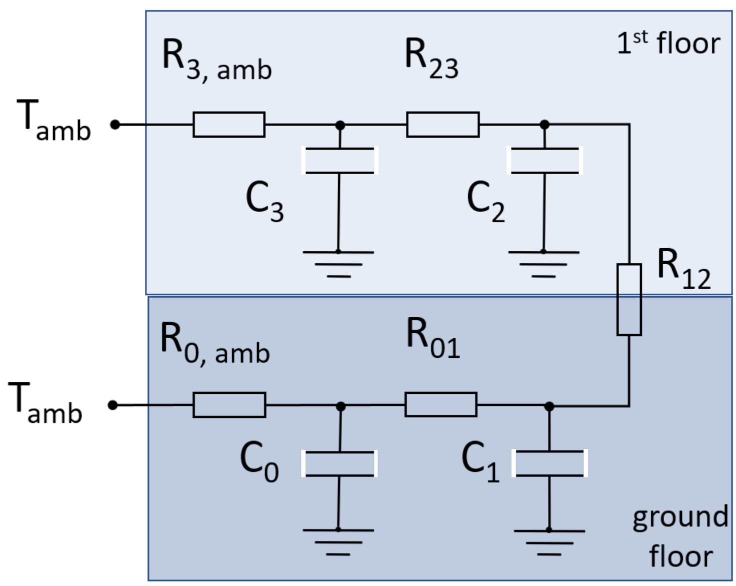


**Figure 16:** 5R-4C house model

$$\mathbf{C} \cdot \dot{\boldsymbol{\theta}} = \begin{bmatrix} C_0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & C_3 \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \\ \frac{dT_3}{dt} \end{bmatrix} \tag{34}$$

$$\mathbf{K} \cdot \boldsymbol{\theta} = \begin{bmatrix} \frac{1}{R_{0,amb}} + \frac{1}{R_{01}} & \frac{-1}{R_{01}} & 0 & 0 \\ \frac{-1}{R_{01}} & \frac{1}{R_{01}} + \frac{1}{R_{12}} & \frac{-1}{R_{12}} & 0 \\ 0 & \frac{-1}{R_{12}} & \frac{1}{R_{12}} + \frac{1}{R_{23}} & \frac{-1}{R_{23}} \\ 0 & 0 & \frac{-1}{R_{23}} & \frac{1}{R_{3,amb}} + \frac{1}{R_{23}} \end{bmatrix} \cdot \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix} \tag{35}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \frac{1}{R_{0,amb}} \cdot T_{amb} + \dot{Q}_{heat,0} + \dot{Q}_{int,0} + \dot{Q}_{solar,0} \\ \dot{Q}_{solar,1} \\ \dot{Q}_{solar,2} \\ \frac{1}{R_{3,amb}} \cdot T_{amb} + \dot{Q}_{heat,3} + \dot{Q}_{int,3} + \dot{Q}_{solar,3} \end{bmatrix} \tag{36}$$

### 5.7 Alternative representation of 5R-4C model

The 5R4C model of the previous section can be built from two 2R2C models, one for the ground floor and one for the first floor. The thermal resistance between the construction nodes of the ground and first floor is then added, $R_{13}$ in the figure:

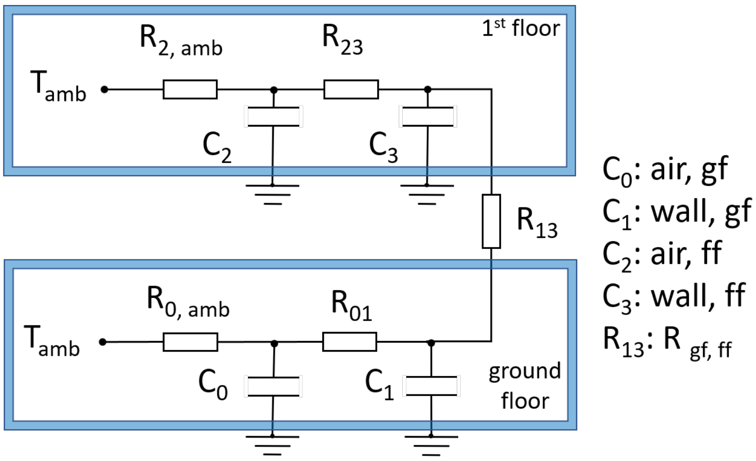


**Figure 17:** 5R-4C house model, alternative representation

As can be seen in the matrices below, adding $R_{13}$ to the ground floor and first floor "chains" results in a non-symmetric matrix. It has to determined if this disadvantage outweighs the benefit of adding "chains".

$$\mathbf{C} \cdot \dot{\boldsymbol{\theta}} = \begin{bmatrix} C_0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & C_3 \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \\ \frac{dT_3}{dt} \end{bmatrix} \tag{37}$$

$$\mathbf{K} \cdot \boldsymbol{\theta} = \begin{bmatrix} \frac{1}{R_{0,amb}} + \frac{1}{R_{01}} & \frac{-1}{R_{01}} & 0 & 0 \\ \frac{-1}{R_{01}} & \frac{1}{R_{01}} + \frac{1}{R_{13}} & 0 & \frac{-1}{R_{13}} \\ 0 & 0 & \frac{1}{R_{2,amb}} + \frac{1}{R_{23}} & \frac{-1}{R_{23}} \\ 0 & \frac{-1}{R_{13}} & \frac{-1}{R_{23}} & \frac{1}{R_{23}} + \frac{1}{R_{13}} \end{bmatrix} \cdot \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix} \tag{38}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \frac{1}{R_{0,amb}} \cdot T_{amb} + \dot{Q}_{heat,0} + \dot{Q}_{int,0} + \dot{Q}_{solar,0} \\ \dot{Q}_{solar,1} \\ \frac{1}{R_{2,amb}} \cdot T_{amb} + \dot{Q}_{heat,2} + \dot{Q}_{int,2} + \dot{Q}_{solar,2} \\ \dot{Q}_{solar,3} \end{bmatrix} \tag{39}$$

Renumbering restores the matrices to a symmetric representation:

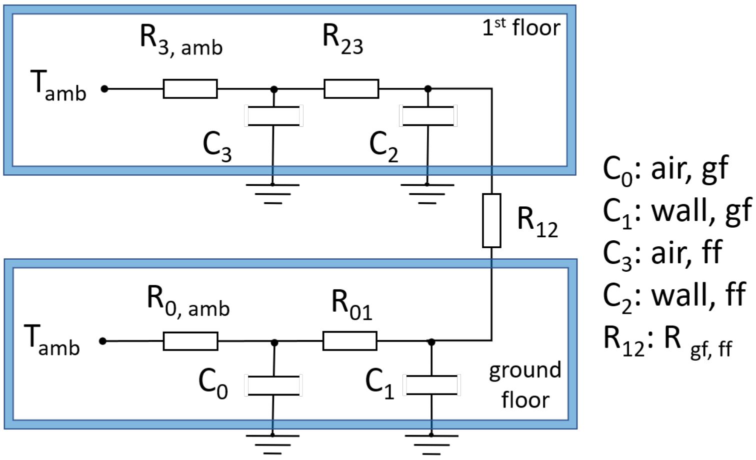


**Figure 18:** 5R-4C house model, alternative representation, renumbered

$$\mathbf{C} \cdot \dot{\boldsymbol{\theta}} = \begin{bmatrix} C_0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & C_3 \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \\ \frac{dT_3}{dt} \end{bmatrix} \tag{40}$$

$$\mathbf{K} \cdot \boldsymbol{\theta} = \begin{bmatrix} \frac{1}{R_{0,amb}} + \frac{1}{R_{01}} & \frac{-1}{R_{01}} & 0 & 0 \\ \frac{-1}{R_{01}} & \frac{1}{R_{01}} + \frac{1}{R_{12}} & \frac{-1}{R_{12}} & 0 \\ 0 & \frac{-1}{R_{12}} & \frac{1}{R_{23}} + \frac{1}{R_{12}} & \frac{-1}{R_{23}} \\ 0 & 0 & \frac{-1}{R_{23}} & \frac{1}{R_{3,amb}} + \frac{1}{R_{23}} \end{bmatrix} \cdot \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix} \tag{41}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \frac{1}{R_{0,amb}} \cdot T_{amb} + \dot{Q}_{heat,0} + \dot{Q}_{int,0} + \dot{Q}_{solar,0} \\ \dot{Q}_{solar,1} \\ \dot{Q}_{solar,2} \\ \frac{1}{R_{3,amb}} \cdot T_{amb} + \dot{Q}_{heat,3} + \dot{Q}_{int,3} + \dot{Q}_{solar,3} \end{bmatrix} \tag{42}$$

### 5.8 2R-2C model with buffervessel

The ”air” and ”wall” nodes of the 2R2C model can be extended with ”radiator” node. The radiator has a finite heat capacity of itself. Instead of a thermal resistance, the radiator heat exchange in $W/K$ is entered in the model. The radiator emits heat to the ”air” node only. In its turn, the radiator is fed from a ”buffervessel” node. The buffervessel loses heat to the radiator and is heated up by a gas boiler or alternatively a heat pump. The gas boiler does not heat the house directly, as was the case in the simplest model. A schematic view is given in Fig. ??.

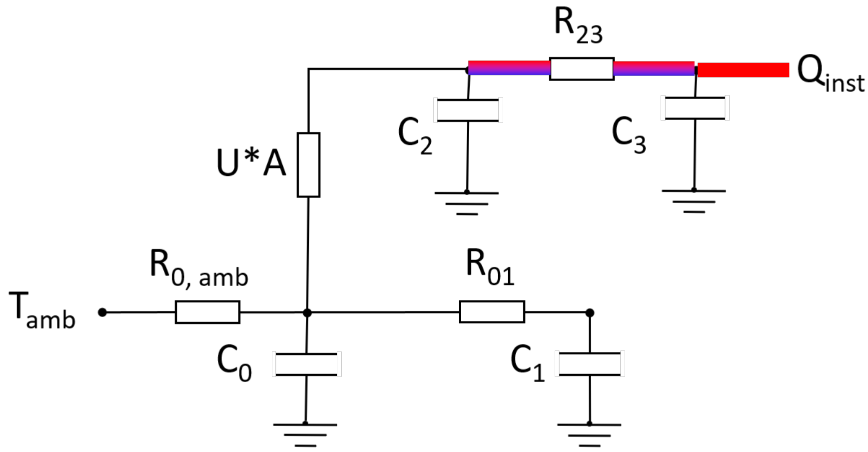


**Figure 19:** 2R-2C house model with radiator and buffer vessel

The differential equations for heat transport in the model of Fig. **??** are:

$$
\begin{aligned}
C_{air}\frac{dT_{air}}{dt} &= \frac{T_{outdoor}-T_{air}}{R_{air\_outdoor}} + \frac{T_{wall}-T_{air}}{R_{air\_wall}} + U_{rad}\cdot A_{rad}\cdot(T_{return}-T_{air}) + \dot{Q}_{internal} + \dot{Q}_{solar,0} \\
C_{wall}\frac{dT_{wall}}{dt} &= \frac{T_{air}-T_{wall}}{R_{air\_wall}} + \dot{Q}_{solar,1} \\
C_{rad}\frac{dT_{return}}{dt} &= \dot{m}\cdot c_{p,water}\cdot(T_{buffer}-T_{return}) + U_{rad}\cdot A_{rad}\cdot(T_{air}-T_{return}) \\
C_{buffer}\frac{dT_{buffer}}{dt} &= \dot{m}\cdot c_{p,water}\cdot(T_{return}-T_{buffer}) + \dot{Q}_{inst} \\
\frac{dE}{dt} &= \dot{Q}_{inst}
\end{aligned}
\tag{43}
$$

A fifth equation, integrating the heat source energy is sometimes added. Re-arranging the terms in the equation gives:

$$
\begin{aligned}
C_{air}\frac{dT_{air}}{dt} &= \left[\frac{-1}{R_{air\_outdoor}} + \frac{-1}{R_{air\_wall}} + -1\cdot U_{rad}\cdot A_{rad}\right]\cdot T_{air} + \frac{T_{wall}}{R_{air\_wall}} + U_{rad}\cdot A_{rad}\cdot T_{return} + \\
&\quad \frac{T_{outdoor}}{R_{air\_outdoor}} + \dot{Q}_{internal} + \dot{Q}_{solar,0} \\
C_{wall}\frac{dT_{wall}}{dt} &= \frac{1}{R_{air\_wall}}\cdot T_{air} + \frac{-1}{R_{air\_wall}}\cdot T_{wall} + \dot{Q}_{solar,1} \\
C_{rad}\frac{dT_{return}}{dt} &= U_{rad}\cdot A_{rad}\cdot T_{air} + [-U_{rad}\cdot A_{rad} - \dot{m}\cdot c_{p,water}]\cdot T_{return} + \dot{m}\cdot c_{p,water}\cdot T_{buffer} \\
C_{buffer}\frac{dT_{buffer}}{dt} &= \dot{m}\cdot c_{p,water}\cdot T_{return} - \dot{m}\cdot c_{p,water}\cdot T_{buffer} + \dot{Q}_{heat,3} \\
\frac{dE}{dt} &= \dot{Q}_{inst}
\end{aligned}
\tag{44}
$$

Conversion of the equations to a matrix equation yields:

$$
\mathbf{C}\cdot\dot{\boldsymbol{\theta}} = \begin{bmatrix} C_0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & C_3 \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \\ \frac{dT_3}{dt} \end{bmatrix}
\tag{45}
$$

$$\mathbf{K}\cdot\boldsymbol{\theta}=\begin{bmatrix}\frac{1}{R_{0,amb}}+\frac{1}{R_{01}}+U\cdot A & \frac{-1}{R_{01}} & -U\cdot A & 0\\ \frac{-1}{R_{01}} & \frac{1}{R_{01}} & 0 & 0\\ -U\cdot A & 0 & U\cdot A+\frac{1}{R_{23}} & \frac{-1}{R_{23}}\\ 0 & 0 & \frac{-1}{R_{23}} & \frac{1}{R_{23}}\end{bmatrix}\cdot\begin{bmatrix}T_0\\T_1\\T_2\\T_3\end{bmatrix} \tag{46}$$

$$\dot{\mathbf{q}}=\begin{bmatrix}\frac{1}{R_{0,amb}}\cdot T_{amb}+\dot{Q}_{int,0}+\dot{Q}_{solar,0}\\ \dot{Q}_{solar,0}\\ 0\\ \dot{Q}_{heat,3}\end{bmatrix} \tag{47}$$

### 5.9 2R-2C model with radiator only

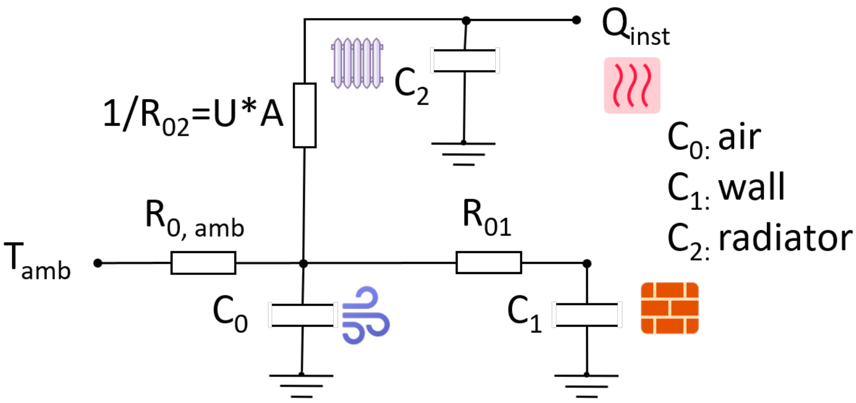


**Figure 20:** 2R-2C house model with radiator only

The rate of heat transfer from a radiator to the ambient(room) air can be calculated as follows [19]:

$$\begin{aligned}P&=P_{50}\cdot\left[\Delta T_{LMTD}\cdot\frac{1}{49.32}\right]^n\\ \Delta T_{LMTD}&=\frac{T_{inlet}-T_{return}}{\ln\frac{T_{inlet}-T_{ambient}}{T_{return}-T_{ambient}}}\\ n&=1.33\end{aligned} \tag{48}$$

This is sometimes simplified to:

$$\begin{aligned}P&=U\cdot A\cdot\Delta T_{LMTD}\\ \Delta T_{LMTD}&=\frac{T_{inlet}-T_{return}}{\ln\frac{T_{inlet}-T_{ambient}}{T_{return}-T_{ambient}}}\end{aligned} \tag{49}$$

or simplified to [20, 21]:

$$\begin{aligned}P&=K_m\cdot\Delta T^n\\ \Delta T&=\frac{T_{inlet}+T_{return}}{2}-T_{ambient}\end{aligned} \tag{50}$$

The differential equations for heat transport in the model of Fig. 20 are:

$$
\begin{aligned}
C_{air}\frac{dT_{air}}{dt} &= \frac{T_{outdoor}-T_{air}}{R_{air\_outdoor}} + \frac{T_{wall}-T_{air}}{R_{air\_wall}} + U_{rad}\cdot A_{rad}\cdot(T_{rad}-T_{air}) + \dot{Q}_{internal} + \dot{Q}_{solar,0} \\
C_{wall}\frac{dT_{wall}}{dt} &= \frac{T_{air}-T_{wall}}{R_{air\_wall}} + \dot{Q}_{solar,1} \\
C_{rad}\frac{dT_{rad}}{dt} &= \dot{Q}_{inst} + U_{rad}\cdot A_{rad}\cdot(T_{air}-T_{rad})
\end{aligned} \tag{51}
$$

Re-arranging the terms in the equation gives:

$$
\begin{aligned}
C_{air}\frac{dT_{air}}{dt} &= \left[\frac{-1}{R_{air\_outdoor}} + \frac{-1}{R_{air\_wall}} + -1\cdot U_{rad}\cdot A_{rad}\right]\cdot T_{air} + \frac{T_{wall}}{R_{air\_wall}} + U_{rad}\cdot A_{rad}\cdot T_{rad} + \\
&\quad \frac{T_{outdoor}}{R_{air\_outdoor}} + \dot{Q}_{internal} + \dot{Q}_{solar,0} \\
C_{wall}\frac{dT_{wall}}{dt} &= \frac{1}{R_{air\_wall}}\cdot T_{air} + \frac{-1}{R_{air\_wall}}\cdot T_{wall} + \dot{Q}_{solar,1} \\
C_{rad}\frac{dT_{rad}}{dt} &= U_{rad}\cdot A_{rad}\cdot T_{air} - U_{rad}\cdot A_{rad}\cdot T_{rad} + \dot{Q}_{heat,2}
\end{aligned} \tag{52}
$$

Conversion of the equations to a matrix equation yields:

$$
\mathbf{C}\cdot\dot{\boldsymbol{\theta}} = \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_1 & 0 \\ 0 & 0 & C_2 \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} \tag{53}
$$

$$
\mathbf{K}\cdot\boldsymbol{\theta} = \begin{bmatrix} \frac{1}{R_{0,amb}} + \frac{1}{R_{01}} + U\cdot A & \frac{-1}{R_{01}} & -U\cdot A \\ \frac{-1}{R_{01}} & \frac{1}{R_{01}} & 0 \\ -U\cdot A & 0 & U\cdot A \end{bmatrix} \cdot \begin{bmatrix} T_0 \\ T_1 \\ T_2 \end{bmatrix} \tag{54}
$$

$$
\dot{\mathbf{q}} = \begin{bmatrix} \frac{1}{R_{0,amb}}\cdot T_{amb} + \dot{Q}_{int,0} + \dot{Q}_{solar,0} \\ \dot{Q}_{solar,1} \\ \dot{Q}_{heat,2} \end{bmatrix} \tag{55}
$$

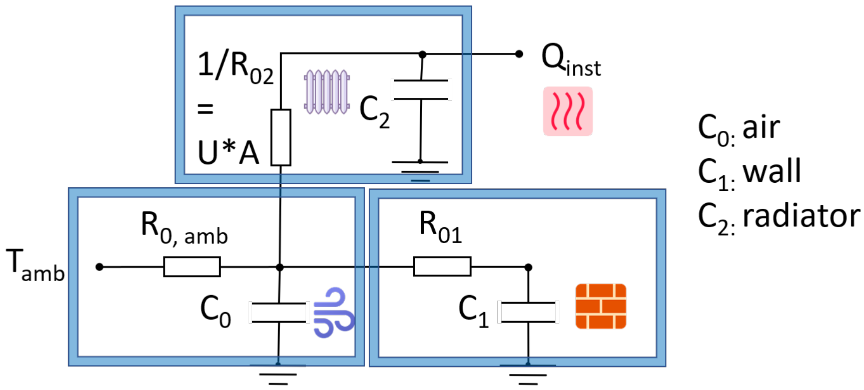


**Figure 21:** 2R-2C house model with radiator in 3 chains

Starting with the basic 2R2C model we write down the matrices. Note that the heat source for the house is omitted at first. Solar energy entering the house is partitioned between air and wall, Heat generated due to the presence and activities of inhabitants is added to the air node:

$$
\mathbf{C} \cdot \dot{\boldsymbol{\theta}} = \begin{bmatrix} C_0 & 0 \\ 0 & C_1 \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \end{bmatrix} \tag{56}
$$

$$
\mathbf{K} \cdot \boldsymbol{\theta} = \begin{bmatrix} \frac{1}{R_{0,amb}} + \frac{1}{R_{01}} & \frac{-1}{R_{01}} \\ \frac{-1}{R_{01}} & \frac{1}{R_{01}} \end{bmatrix} \cdot \begin{bmatrix} T_0 \\ T_1 \end{bmatrix} \tag{57}
$$

$$
\dot{\mathbf{q}} = \begin{bmatrix} \frac{1}{R_{0,amb}} \cdot T_{amb} + \dot{Q}_{int,0} + \dot{Q}_{solar,0} \\ \dot{Q}_{solar,1} \end{bmatrix} \tag{58}
$$

As a third link in the chain, a radiator is added, with a heat capacity $C_{rad}$ and a heat delivery $U \cdot A \cdot (T_{rad} - T_{air})$ to the air node. The heat source $\dot{Q}_{inst}$ is now connected to the radiator.

$$
\mathbf{C} \cdot \dot{\boldsymbol{\theta}} = \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_1 & 0 \\ 0 & 0 & C_2 \end{bmatrix} \cdot \begin{bmatrix} \frac{dT_0}{dt} \\ \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} \tag{59}
$$

$$
\mathbf{K} \cdot \boldsymbol{\theta} = \begin{bmatrix} \frac{1}{R_{0,amb}} + \frac{1}{R_{01}} + U \cdot A & \frac{-1}{R_{01}} & -U \cdot A \\ \frac{-1}{R_{01}} & \frac{1}{R_{01}} & 0 \\ -U \cdot A & 0 & U \cdot A \end{bmatrix} \cdot \begin{bmatrix} T_0 \\ T_1 \\ T_2 \end{bmatrix} \tag{60}
$$

$$
\dot{\mathbf{q}} = \begin{bmatrix} \frac{1}{R_{0,amb}} \cdot T_{amb} + \dot{Q}_{int,0} + \dot{Q}_{solar,0} \\ \dot{Q}_{solar,1} \\ \dot{Q}_{heat,2} \end{bmatrix} \tag{61}
$$

In this example, it becomes visible (in red) that the rank of the $C$- and $K$-matrix, and the $\dot{q}$-vector is extended by 1. The heat capacity of the radiator is included as an extra *diagonal* element in the $C$-matrix. The heat delivery from the radiator to the indoor air is added to or subtracted from the 00, 22, 02 and 20 elements of th $K$-matrix, so that it remains a *symmetric* matrix. The heater is connected to the radiator, represented by element 2 of the $\dot{q}$-vector.

## 5.10 Package "housemodel"

The repository "twozone_housemodel-git" contains the modules for the house model. The customary way to organize the modules is to make a *Python package* with *subpackages*. This opens up the possibility of publishing the package on PyPi, so that it can be imported.

See: https://pypi.org/

From commit e74ce58 the files in the twozone_housemodel-git repository are organized as a package. The proposed structure, implemented in this commit, is:

```
twozone_housemodel-git
└── housemodel
    ├── __init__.py
    ├── controls
    │   └── __init__.py
    ├── solvers
    │   └── __init__.py
    └── sourcesink
        └── __init__.py
```

```
│
│   │
│   └── tools
│       └── __init__.py
└── tests
    ├── __init__.py
    ├── context1.py
    └── test_*.py
```

- the *repository root* twozone_housemodel-git contains the simulation scripts and configuration files (for now)
- the *package root* housemodel contains the complete package. This can be seen since it contains an (empty) __init__.py module.
- the *subpackage* folders contain the modules with common functions and classes for all simulations. They each contain an (empty) __init__.py module.
- a tests folder is placed carefully as a subfolder of the *repository root*. See: https://docs.python-guide.org/writing/structure/ for the underlying philosophy. Here, testing modules (scripts) can be placed. If the names of the test scripts start with test_, they can be automatically run with the pytest Python package.

*Note*: Running the simulations and tests is best done from the *repository root*. All simulations and tests have been updated to find the package, subpackages and configuration files from this directory.

# 6 NEN and ISO

The list of NEN and ISO standard used in the calculation:

- NTA 8800
- NEN 1068
- ISO 6946
- ISO 10077-2
- NEN 7120

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