An optimized algorithm for the prediction of the water emptying time on BPNN

BY

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• Introduction

Problem formulation

Xinhua 10:31, June 22, **2016**

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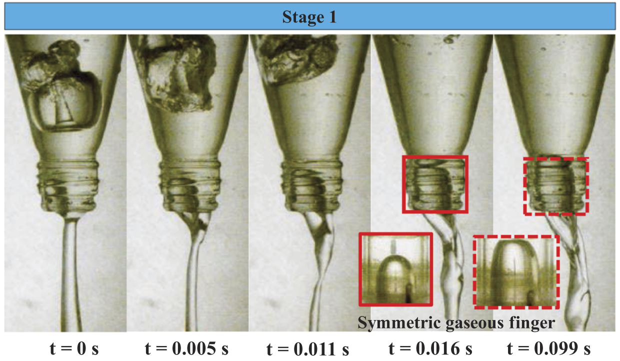
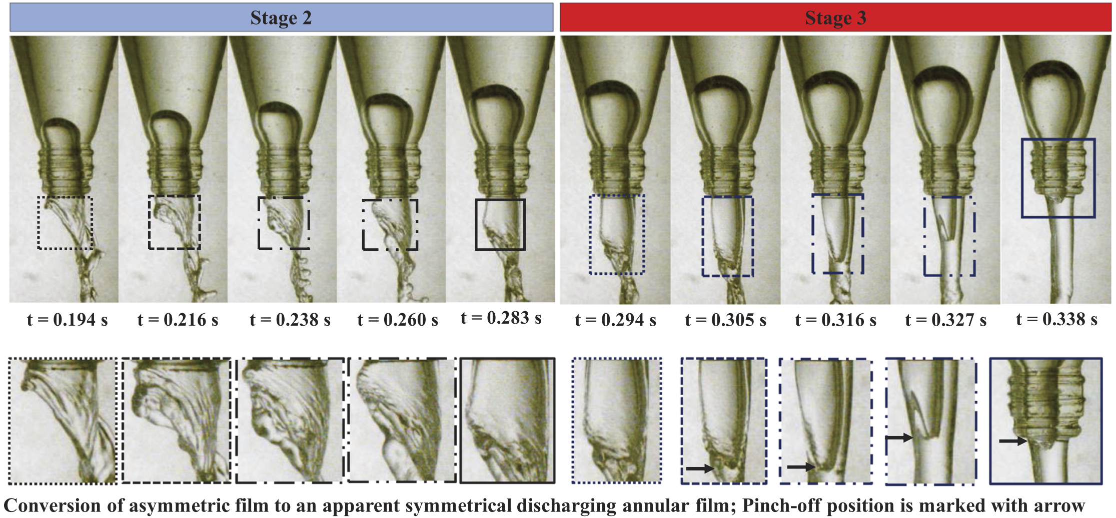
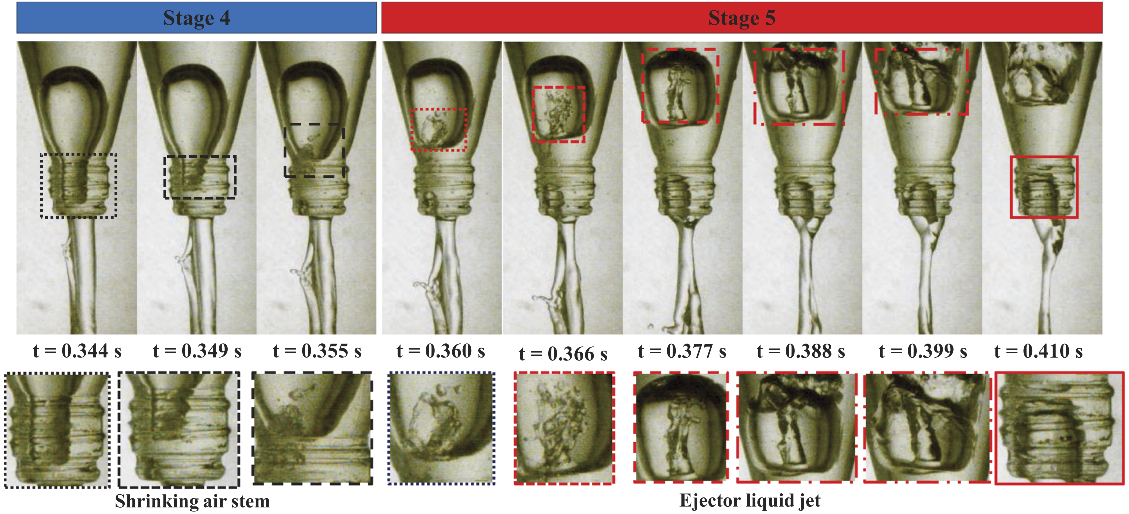
Water discharge problem is a classic fluid mechanics problem that is still commonly encountered nowadays.

- # Introduction

L. Rohilla and A. K. Das. *Phys. Fluids* **32**, 042102 (2020).







Previous study has showed that air bubble plays an essence role in the water discharge process.

F. Geiger *et al*. Journal of Food Engineering 109 (**2012**) 609–618

• Introduction

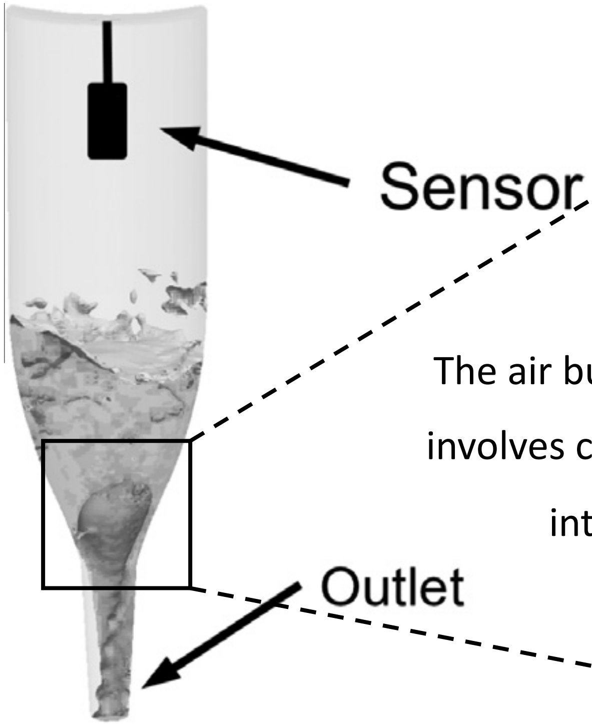


Fig. 4. Bottle with MSR 145 W pressure sensor.

The air bubble generation involves complex air-water interactions.

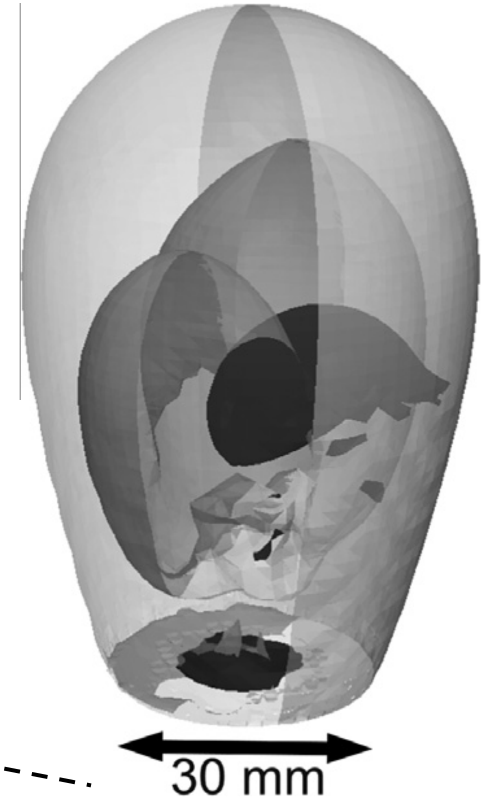


Fig. 11. Simulated bubble formation at different outlet diameters: 10 mm (black), 20 mm (dark grey), and 30 mm (light grey), corresponding to runs no. 3, 4, and 10 in Table 5.

• Method

How do we approach?

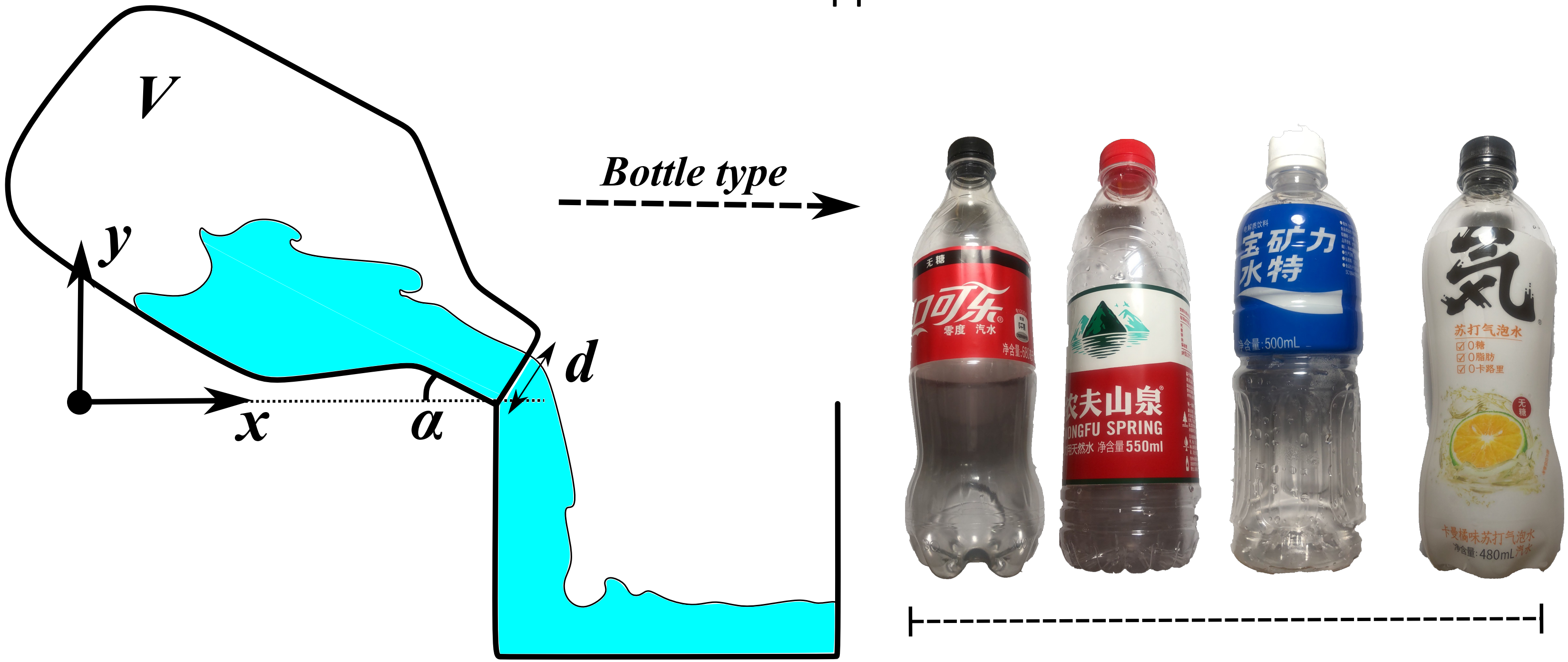


Fig. 1 Schematic illustration for the water discharge problem and the experimental method.

- **Method**

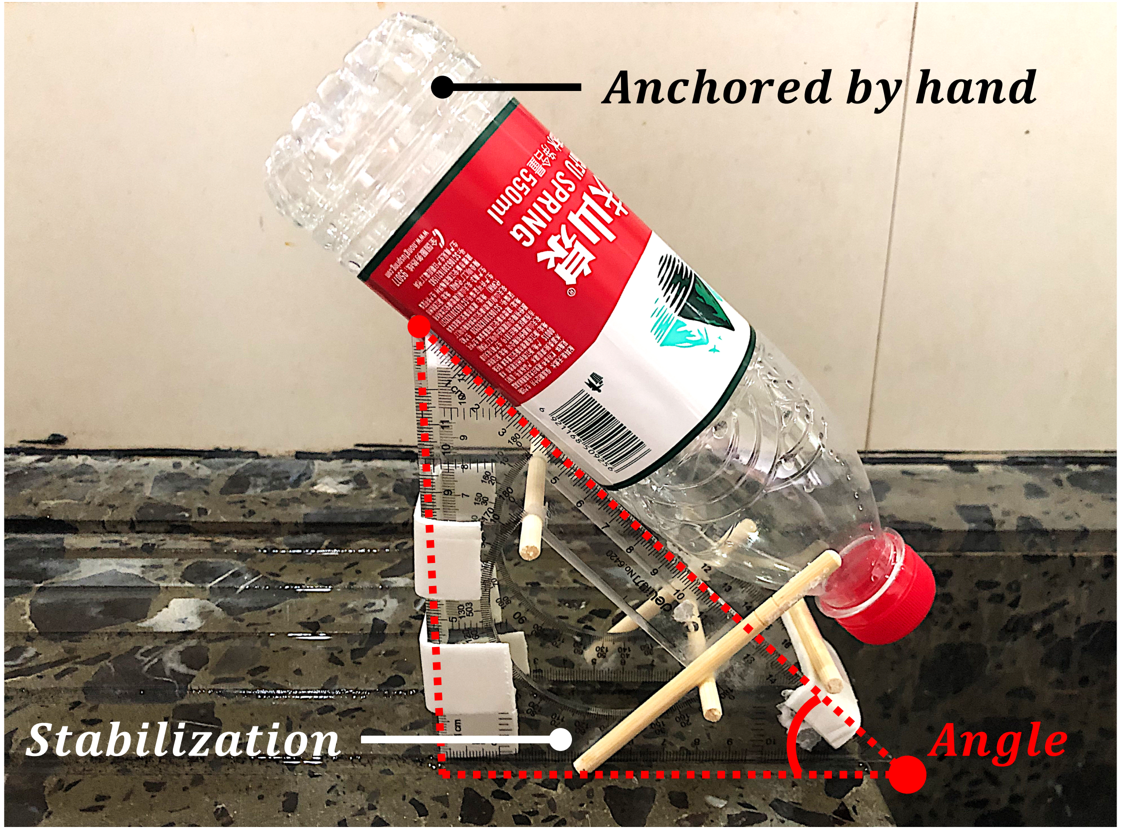


Fig. 2 Picture for the experimental setup.

• Method

Video representation for our experiment

Vid. 1 Experimental process for the water discharge.

- # Method

We present an analytical solution for a simplified water discharge problem with a simplified model.

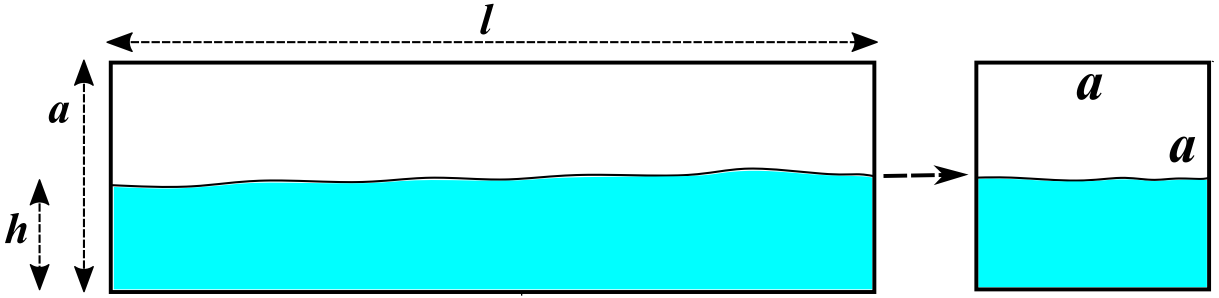


Fig. 3 Schematic view for the water flow problem, in assumed conditions.

$$\begin{cases} \rho g \dfrac{h}{2} \cdot ah \cdot dt = dm \cdot V = ahdx\rho \cdot \dfrac{dx}{dt} \\ dQ = dh \cdot l \cdot a = ah \cdot dx \end{cases} \Longrightarrow \quad t = -2\sqrt{\frac{2}{g}}\, lh^{-\frac{1}{2}} + C$$

$$\begin{cases} h = a \\ h = 0 \end{cases} \Longrightarrow \quad \Delta t = -2l\sqrt{\frac{2}{ga}} + \infty \xrightarrow{\text{yields}} \infty$$

- **Method**

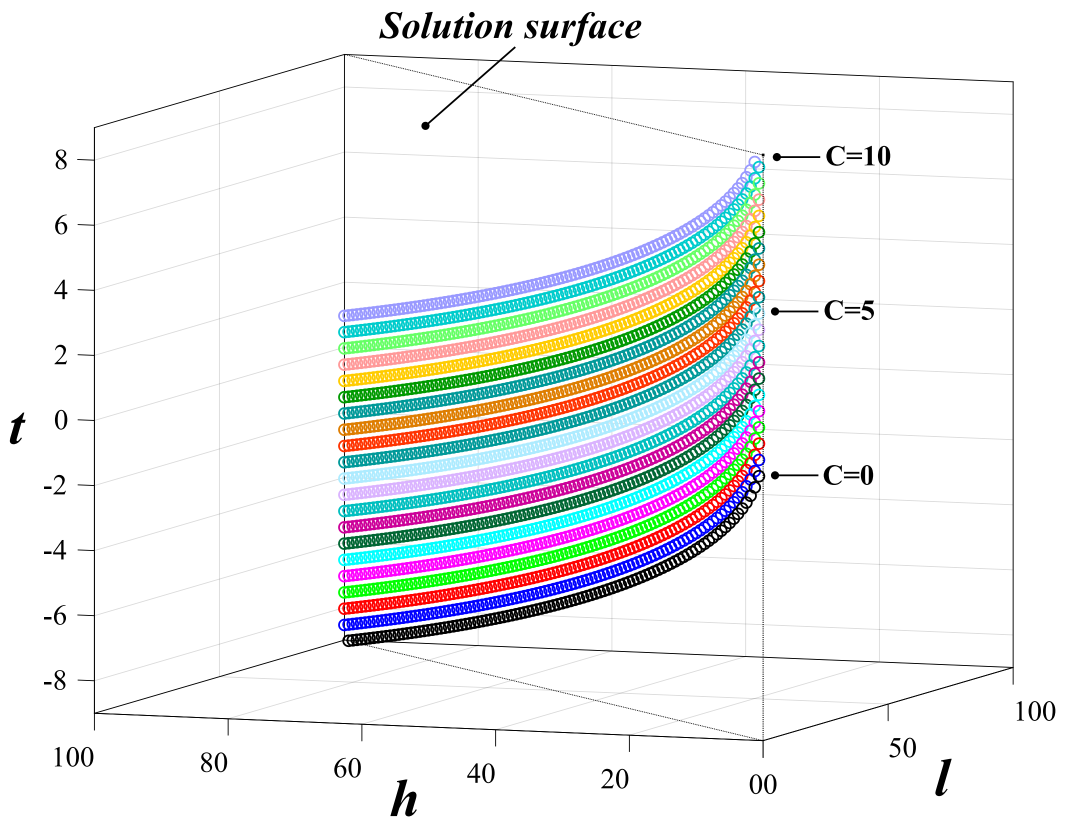


Fig. 4 Visualization of the analytical solution corresponding water emptying time with length and water height.

- **Method**

The database can be seen as a regression model.

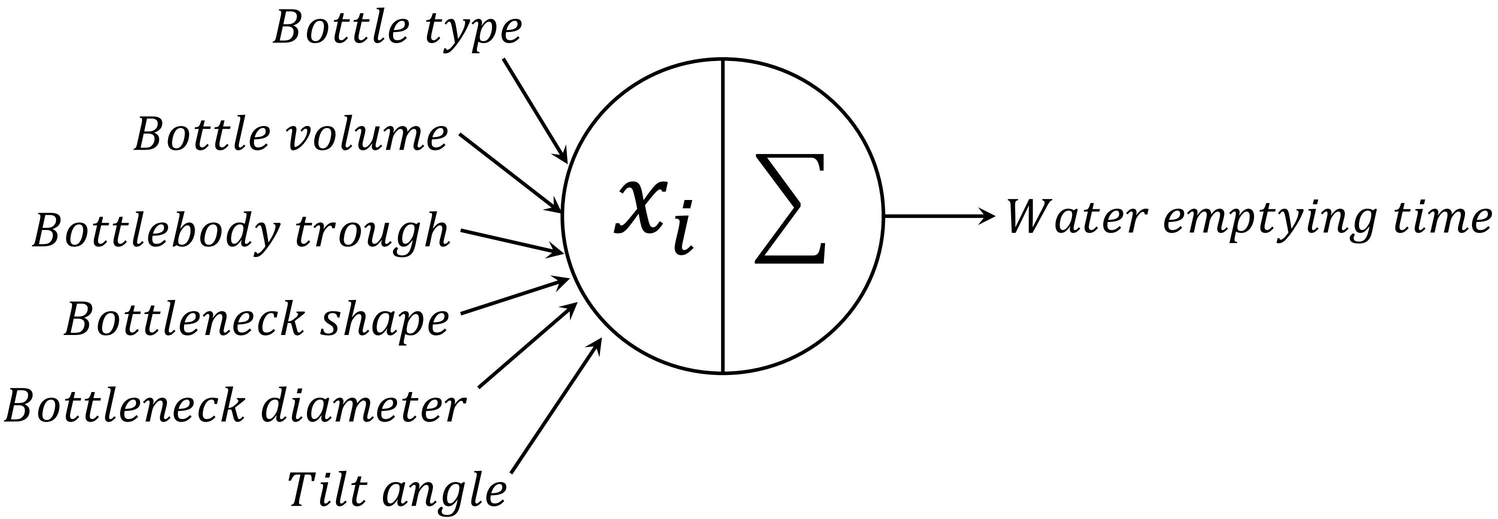


Fig. 5 Illustration for the regression problem based on the experimental data.

- **Method**

Here we give a BPNN algorithm for the regression model.

Algorithm 1: Determining the Parameters of Original BPNN

Set the initial value of sum of the determining factor $R_{\text{sum}}^2 = 0$; and sum of the relative error $\varepsilon_{\text{sum}} = 0$.

For a loop running with 10^3 iterations:

  Clear all the remaining values of the last iteration;

  1: Importing and operating the data.

  2: Generate the network:

    Change the specific parameters:

      i. Set the neuron numbers $\xi \rightarrow$ change from 1 to 20.

      ii. Set the training epochs $= 10^5$.

      iii. Set the training goals $= 10^{-3}$.

      iv. Set the learning rate $\alpha \rightarrow$ change from 10^{-1} to 10^{-5}.

    Training the data with given parameters and creating simulation.

  3: Calculate the results:

    The sum of determining factor $R_{\text{sum}}^2 = R_{\text{sum}}^2 + R^2$, the sum of the relative error $\varepsilon_{\text{sum}} = \varepsilon_{\text{sum}} + \varepsilon$.

    The mean value of R^2 and ε is calculated by $\frac{R_{\text{sum}}^2}{\text{Iterations}}, \frac{\varepsilon_{\text{sum}}}{\text{Iterations}}$.

    Calculate the CPU running time t_{run}

End the loop.

- **Method**

Henceforth, we need to find a network through BPNN approach by a loop that seems "satisfying", which I called the "*Lucky-Draw* Algorithm".

Algorithm 2: Obtain the BPNN model

For a loop running with 10^5 iterations:

  Clear all the remaining values of the last iteration;

  1: Operating the data:

    Import the water discharge experimental data to define the input and output data, and set the training and testing sets.

    Normalize the input dataset.

  2: Generate the network:

    Set the given parameters obtained from Algorithm 1.

    Training the data with given parameters and creating simulation.

  3: Calculate the results:

    Obtaining errors and deciding parameter R^2;

    Outputs the results as comparing the testing data and simulation results, and calculate the relative error ε.

  4: Generate a condition to break the loop:

    If a value of R^2 that beyond 0.95 is detected, then plot the errors distribution and break the loop.

End the loop.

- **Method**

The equation for the determining factor R^2:

$$R^2 = N \cdot \sum T_{sim} \cdot T_{test} - \frac{\left(\sum T_{sim} \cdot \sum T_{test}\right)^2}{\left(N \cdot \sum (T_{sim})^2 - \left(\sum T_{sim}\right)^2\right) \cdot \left(N \cdot \sum (T_{test})^2 - \left(\sum T_{test}\right)^2\right)}$$

The relative error ε is calculated as:

$$\varepsilon = \frac{\|T_{sim} - T_{test}\|}{T_{test}}$$

- # Results and discussion

Experimental data: Coke bottle

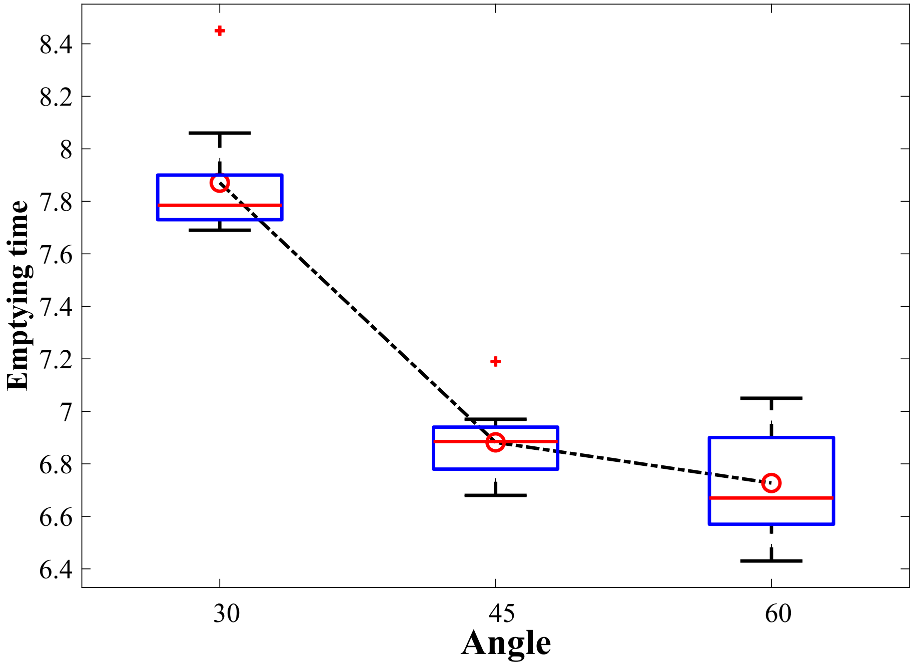


Fig. 6 The Emptying time-angle boxplot diagram of the coke bottle.

- # Results and discussion

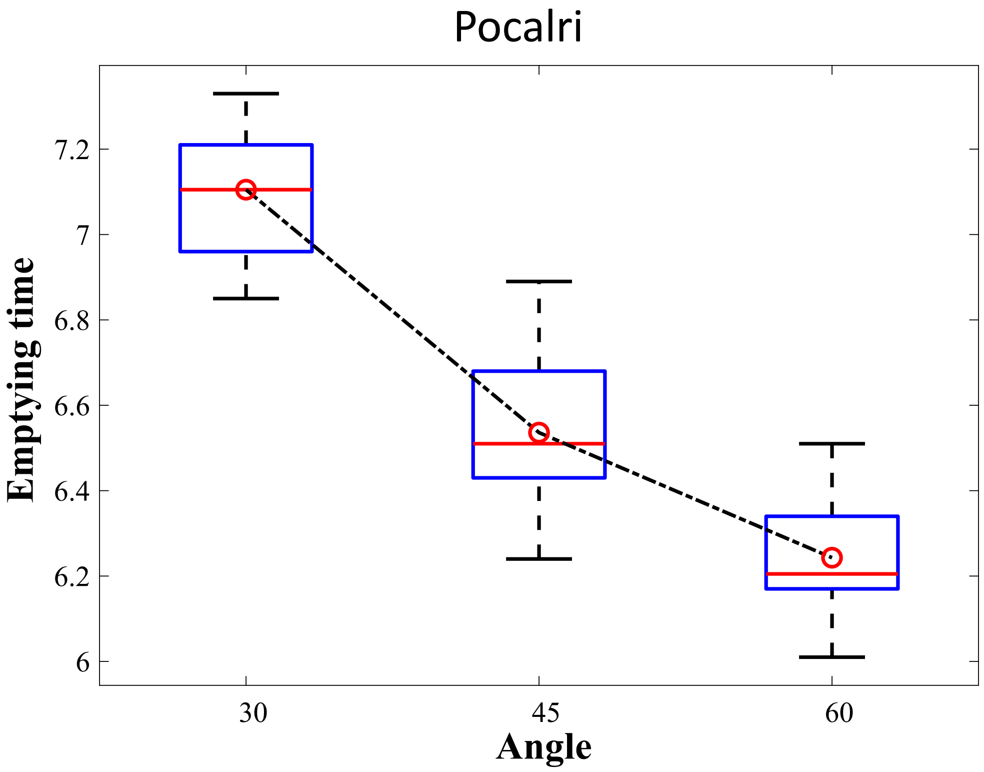


Fig. 7 The Emptying time-angle boxplot diagram of the Pocalri bottle.

- # Results and discussion

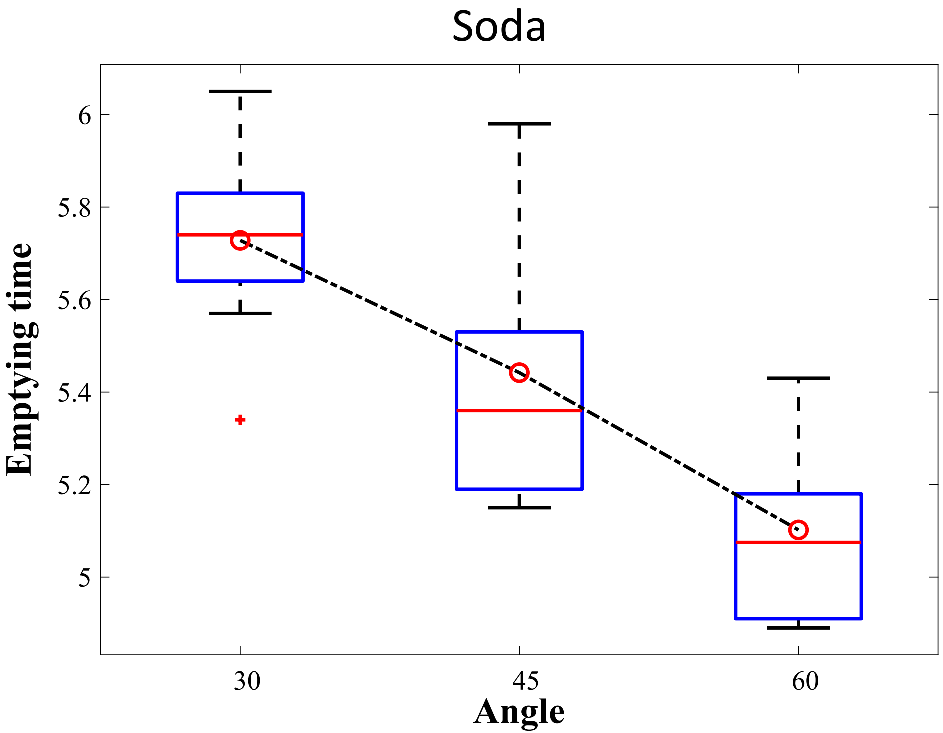


Fig. 8 The Emptying time-angle boxplot diagram of the soda bottle.

- # Results and discussion

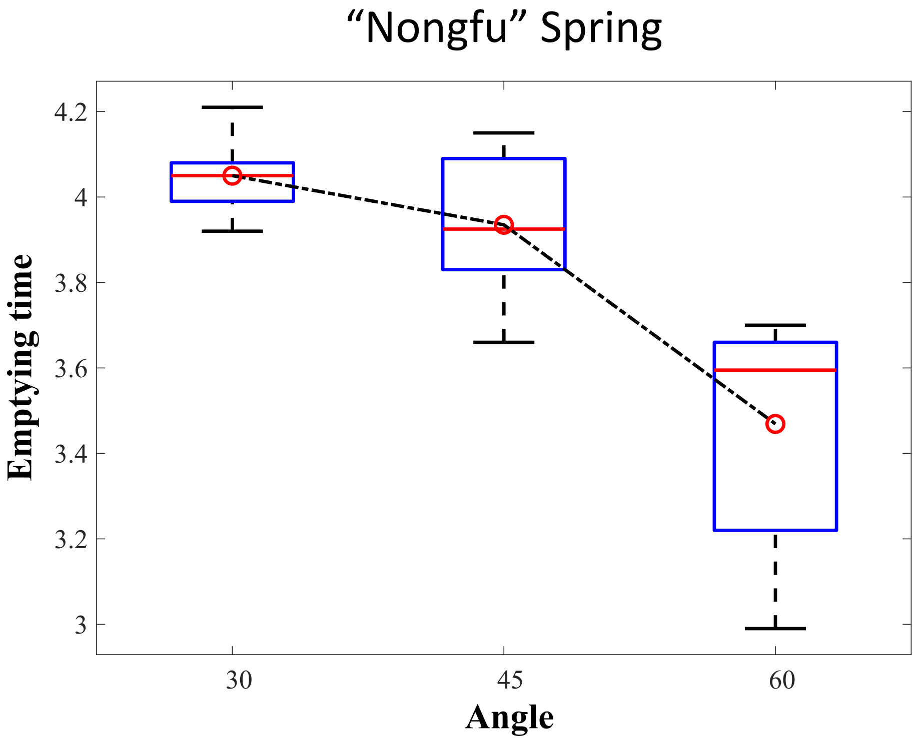


Fig. 9 The Emptying time-angle boxplot diagram of the Nongfu spring bottle.

- # **Results and discussion**

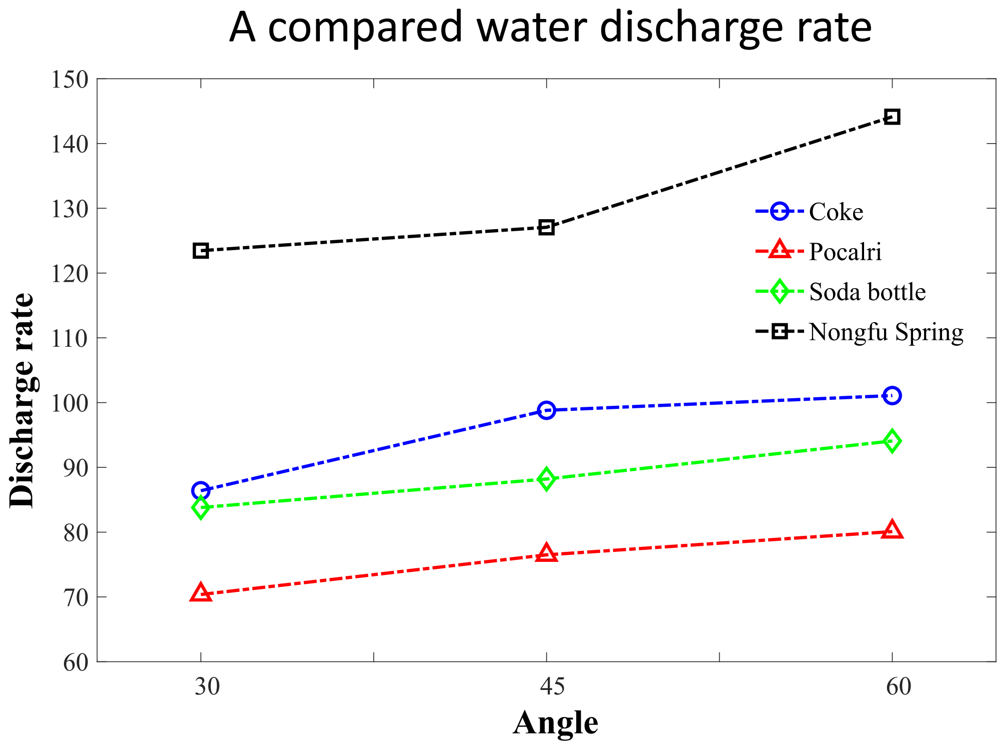


Fig. 10 The discharging rate calculated as a mean value comparing the four bottles.

- # Results and discussion

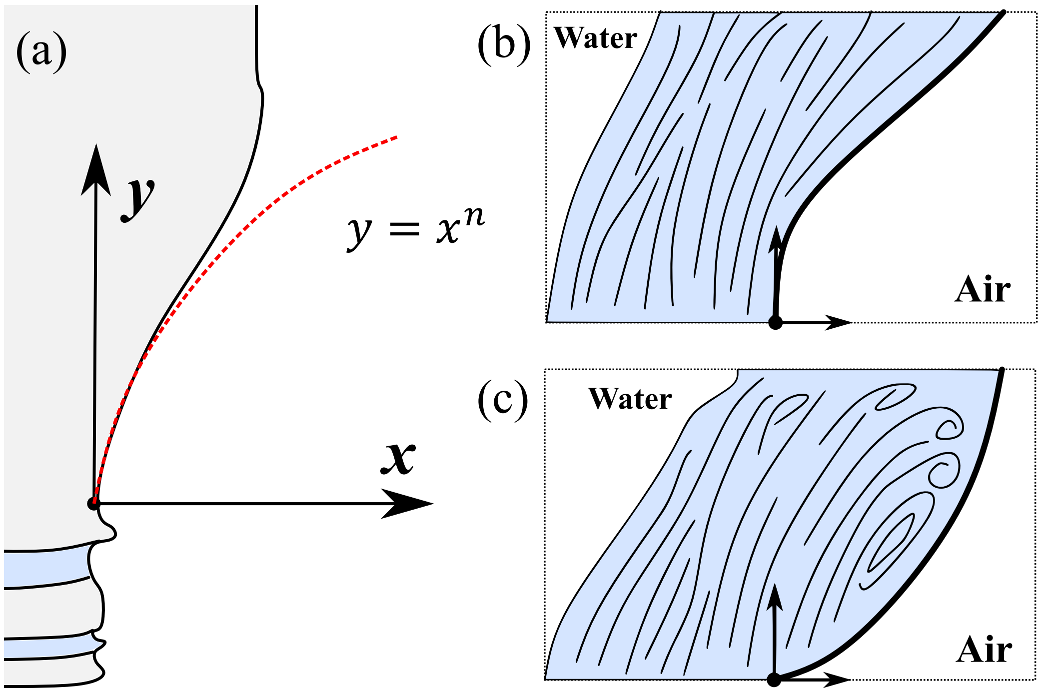


Fig. 11 Schematic of the provided explanation of the water discharge rate considering the bottleneck shape. (a) Basic strategy for mathematical description of the bottle shape. (b) The schematic of water emptying process when $n < 1$. (c) The schematic of water emptying process when $n > 1$.

• Results and discussion

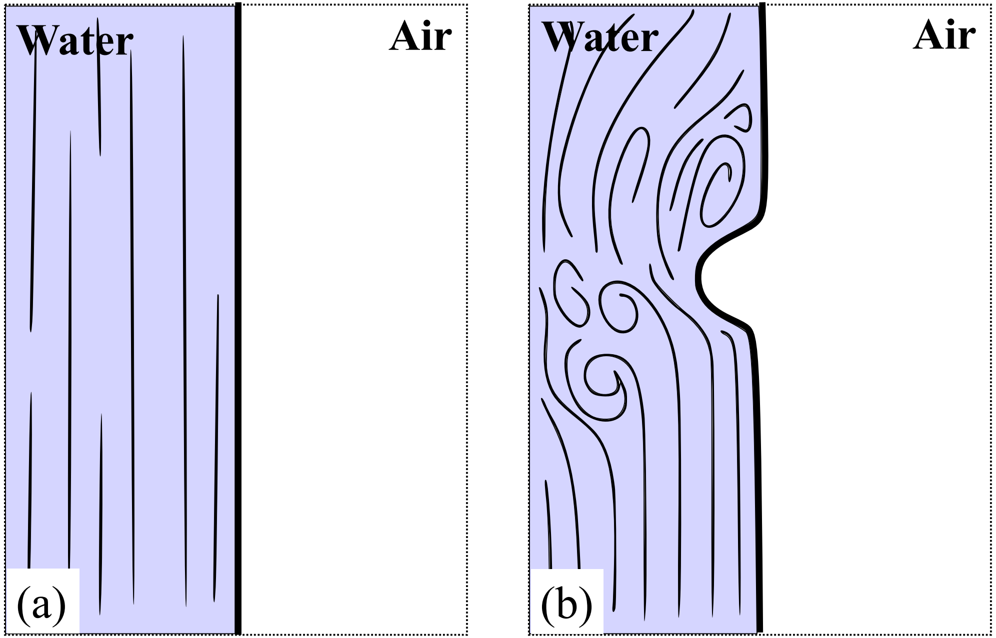


Fig. 12 Schematic of the provided explanation of the water discharge rate considering the bottle-body shape. (b) The schematic of water emptying process when the bottle-body has no trough. (b) The schematic of water emptying process when the bottle-body has a trough.

- **Results and discussion**

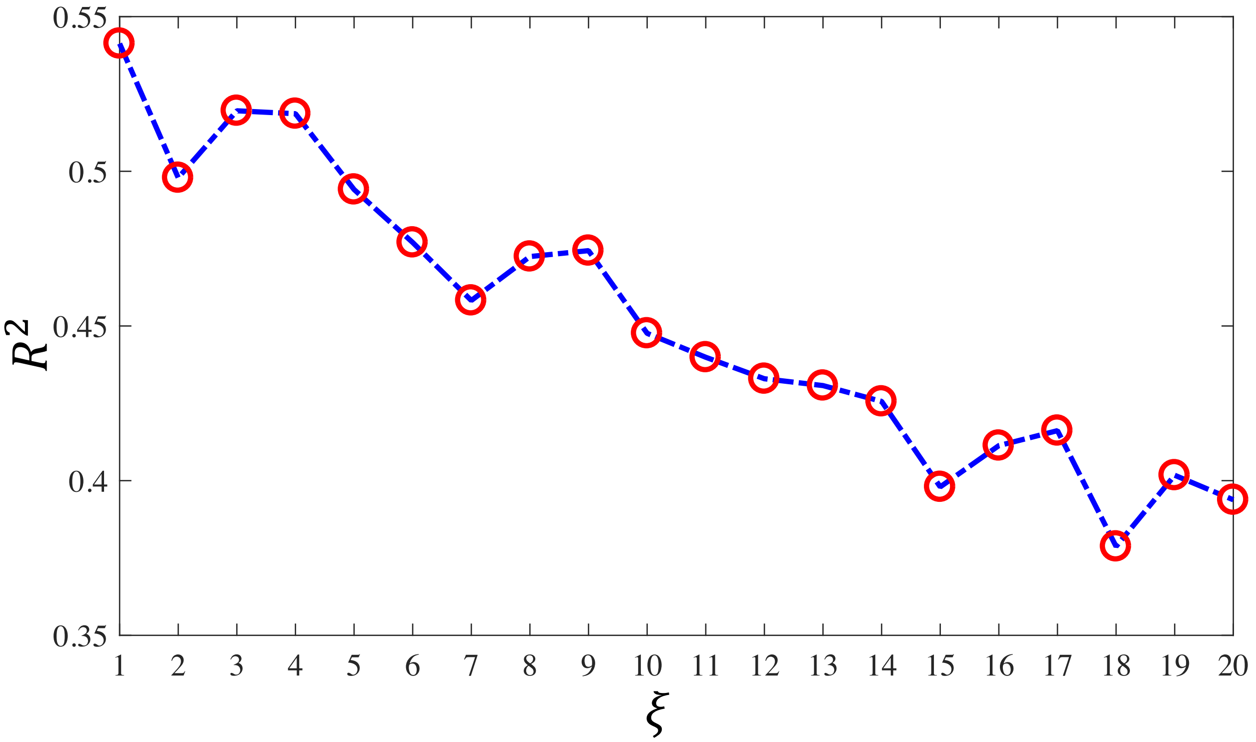


Fig. 13 The relations between the neuron numbers ξ to determining factor R^2.

- **Results and discussion**

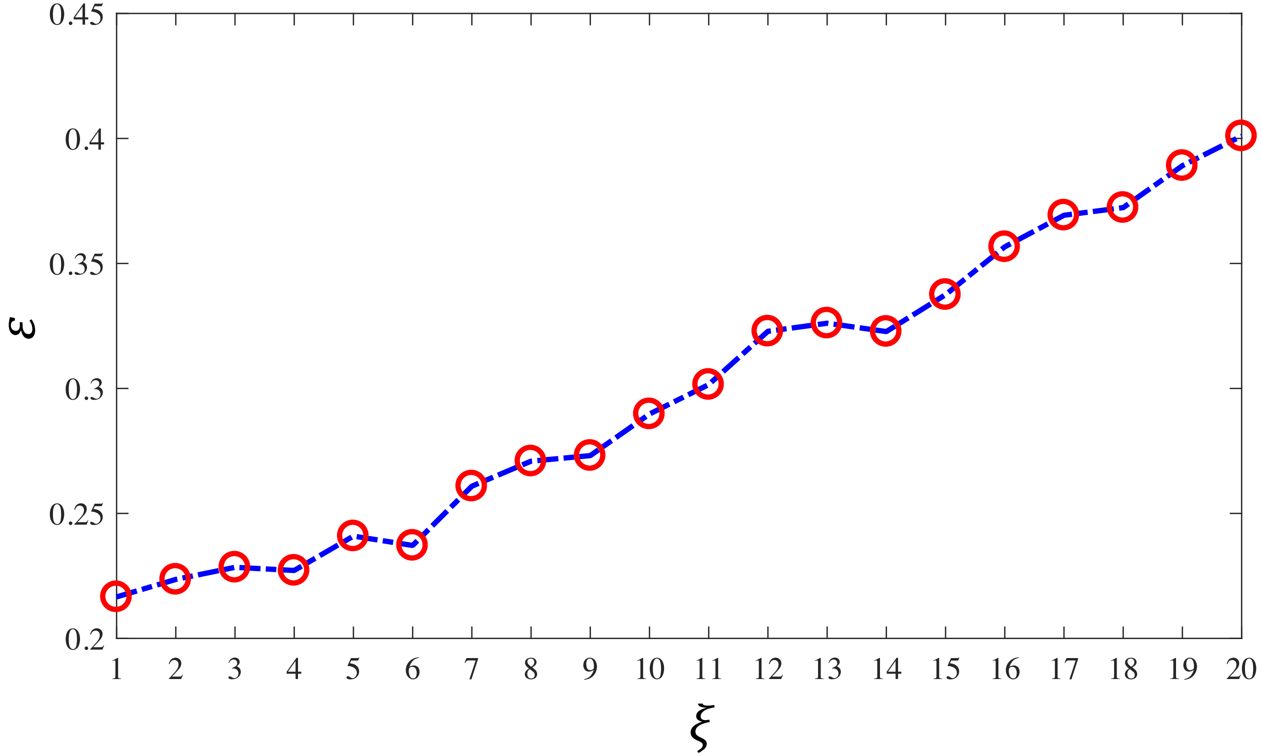


Fig. 14 The relations between the neuron numbers ξ to relative error ε.

- **Results and discussion**

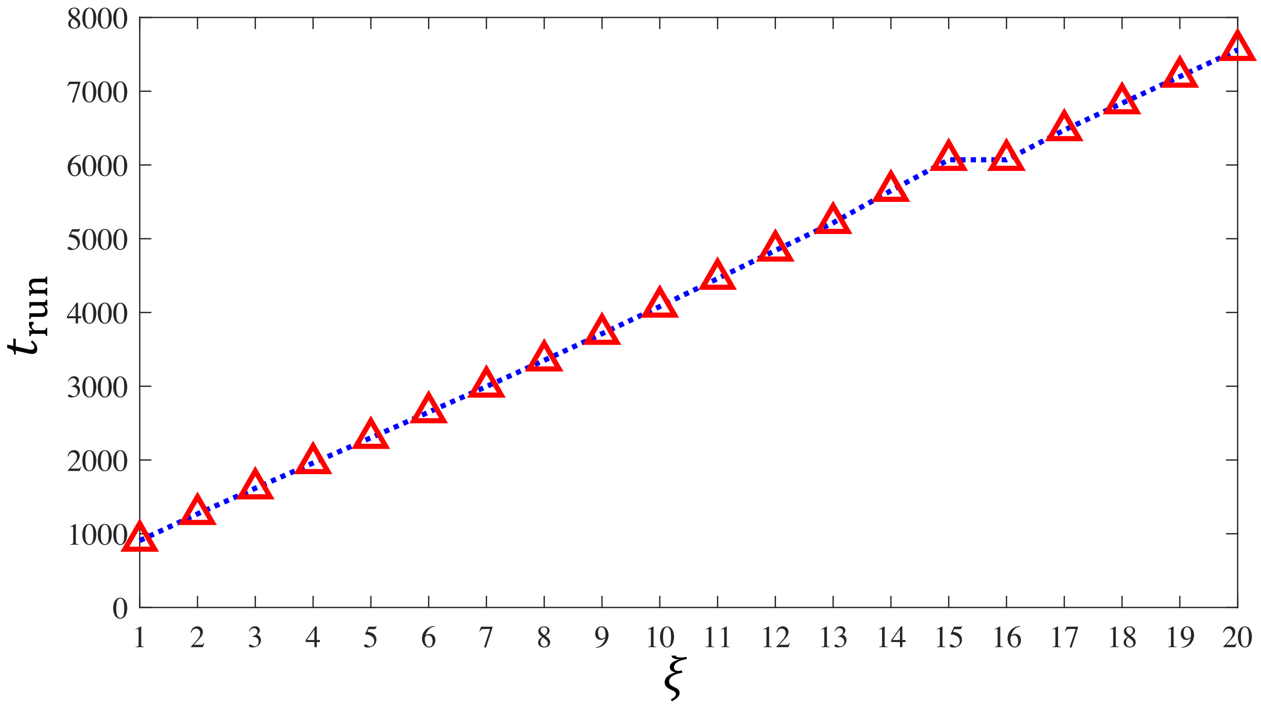


Fig. 15 The relations between the neuron numbers ξ to CPU time t_{run}.

- # Results and discussion

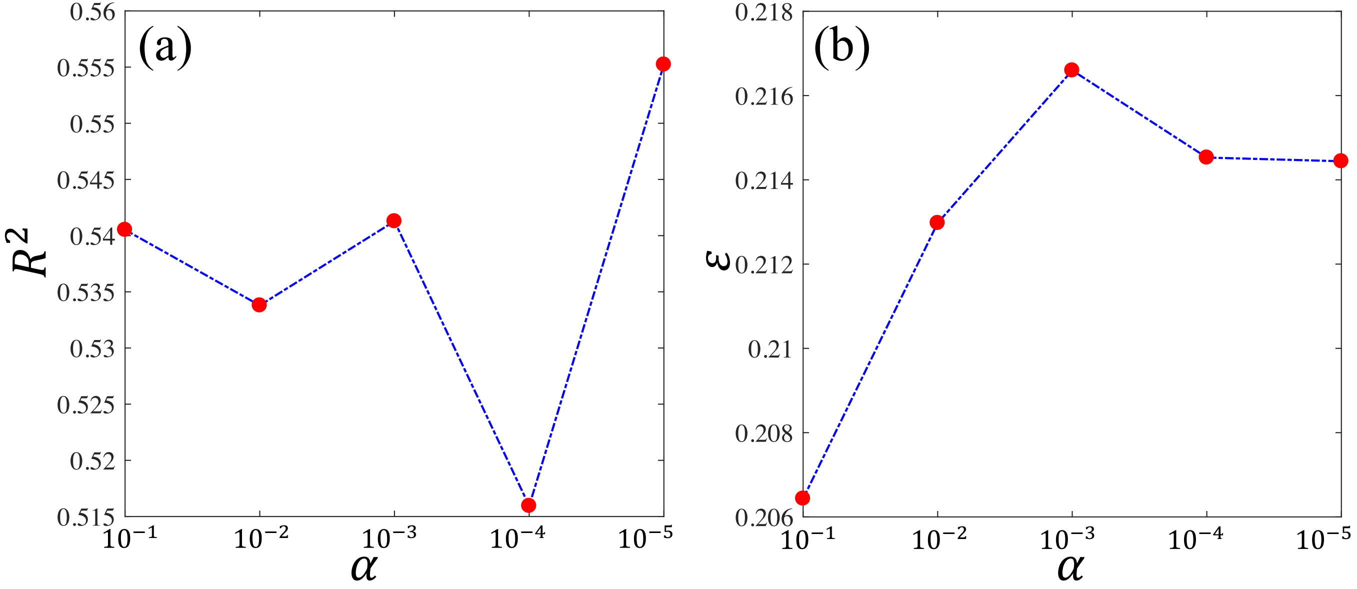


Fig. 16 The learning rate α effect on the network's performance. (a) The learning rate (α) – determining factor (R^2) diagram. (b) The learning rate (α) – relative error (ε) diagram.

- **Results and discussion**

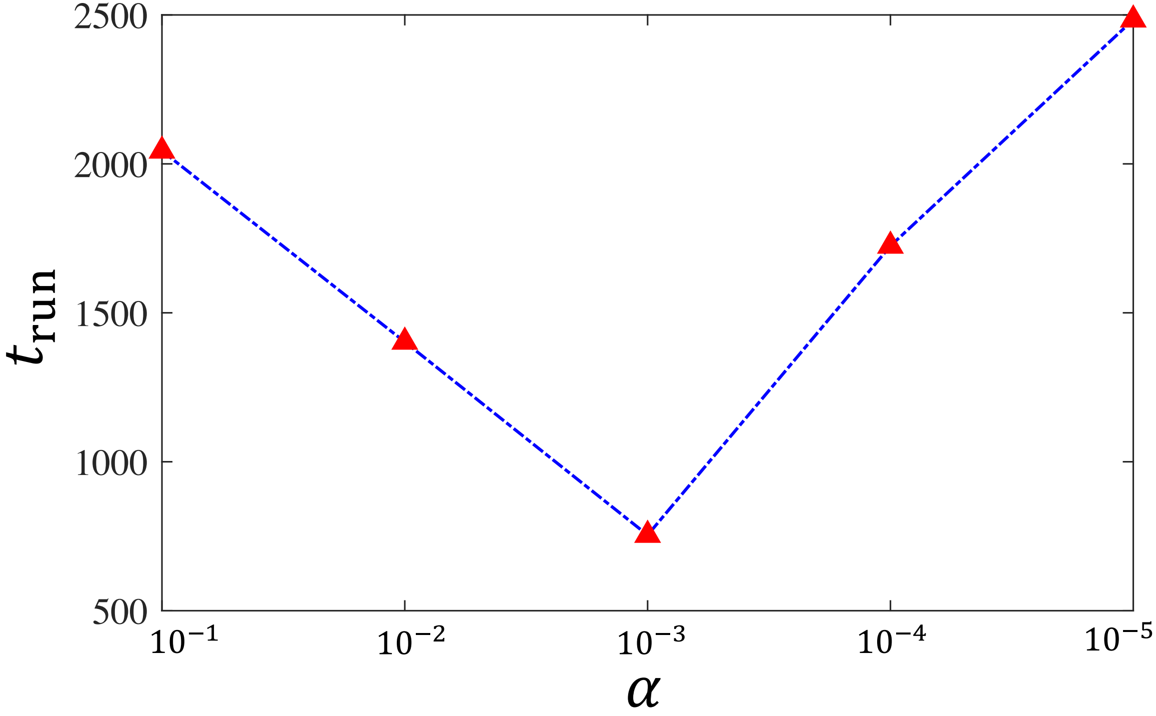


Fig. 17 The relations between the learning rate α to CPU time t_{run}.

- # Results and discussion

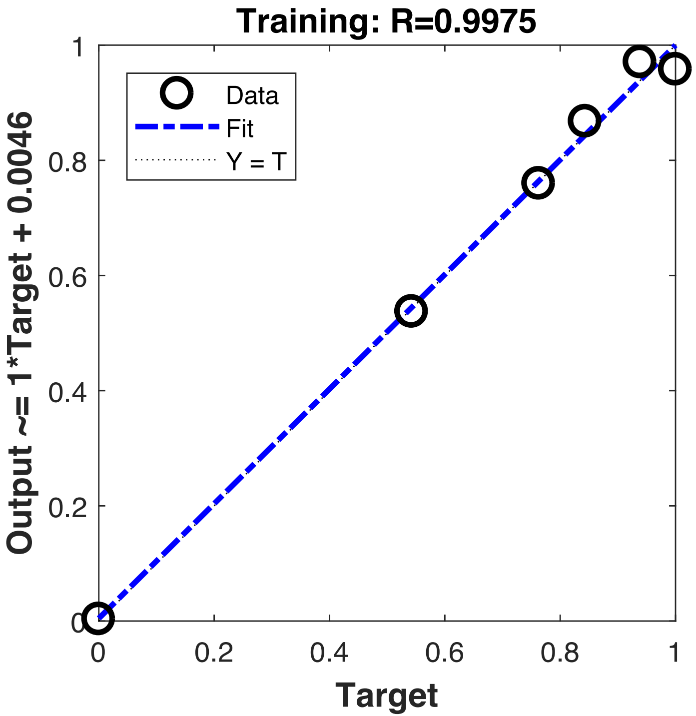


Fig. 18 The regression model of the generated network.

- # Results and discussion

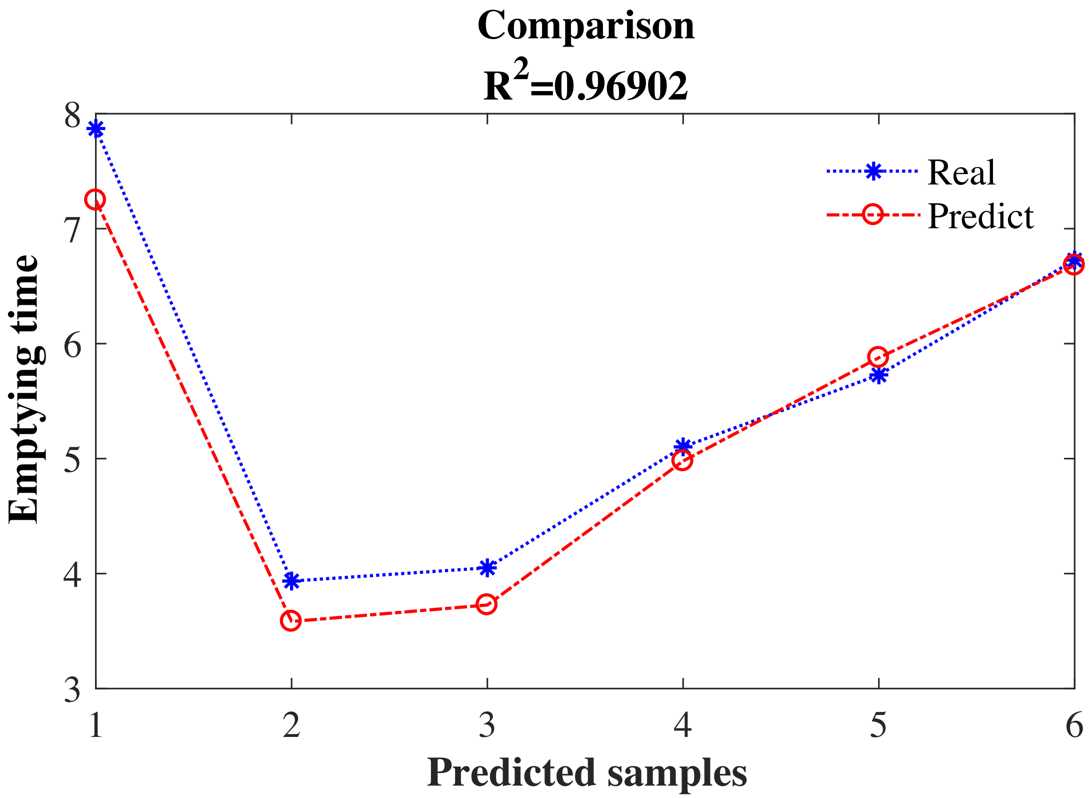


Fig. 19 The comparison between the testing samples T_{test} and the neural network's simulation results T_{sim}.

• Results and discussion

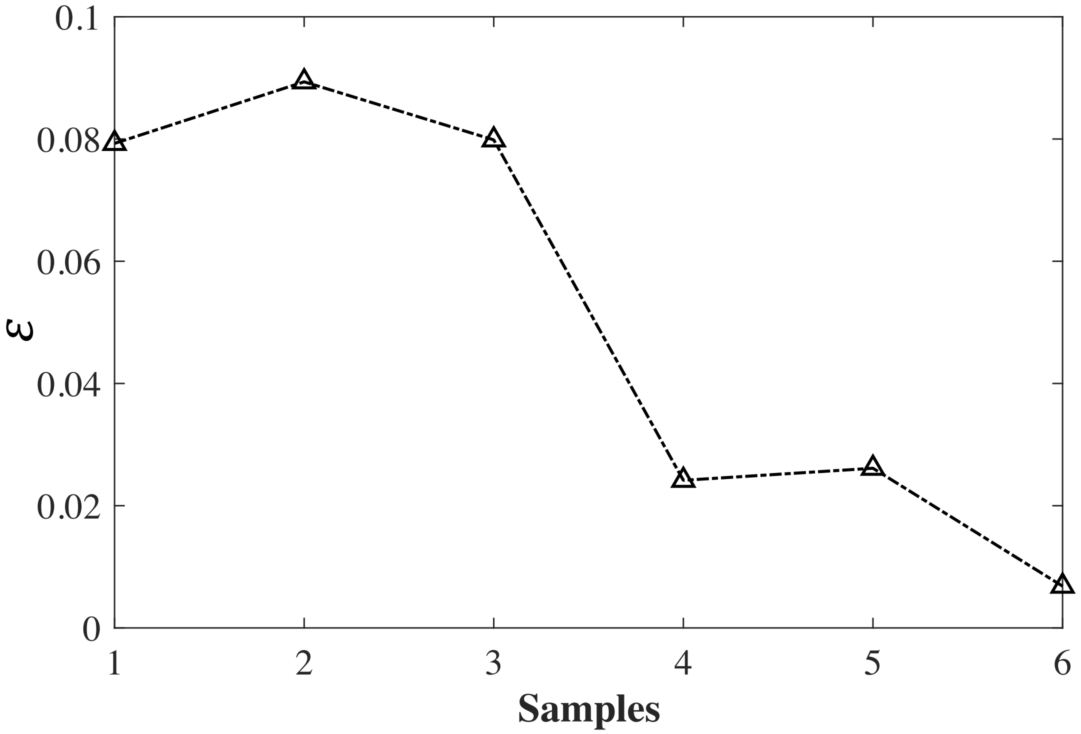
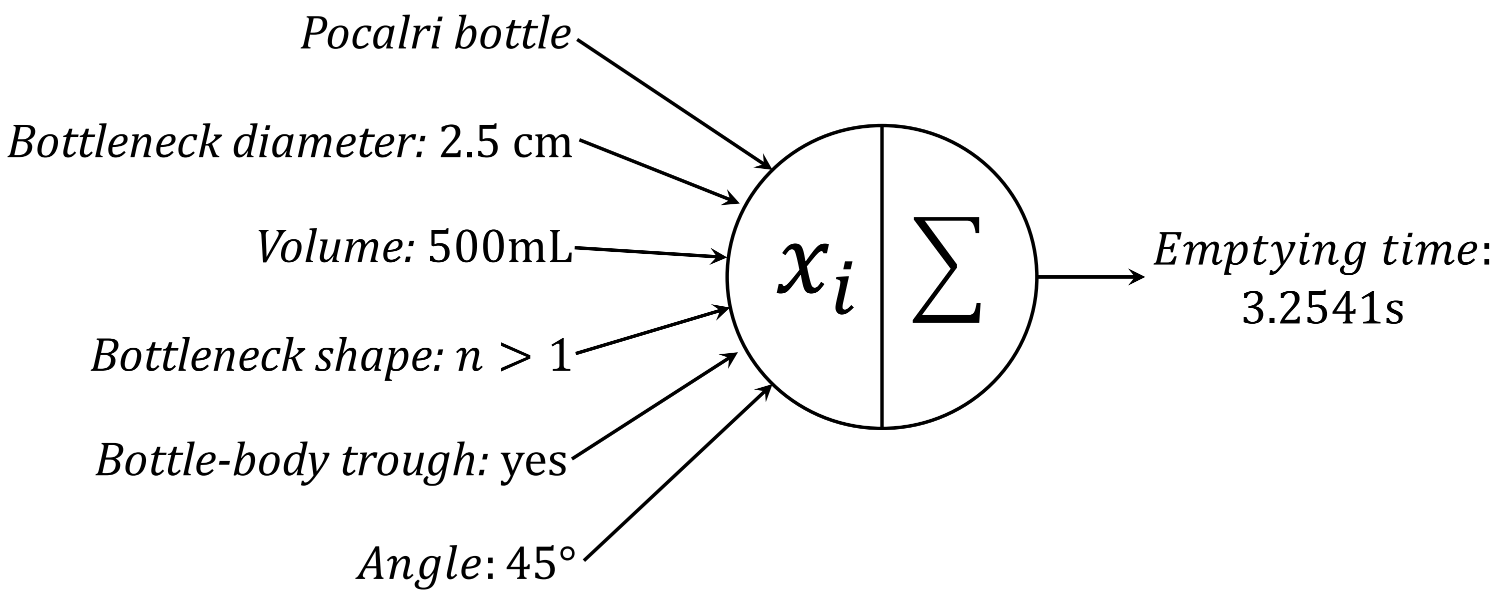


Fig. 20 The distribution of the relative errors ε with the testing samples.

- # Results and discussion

Prediction example



- **Conclusion**

✓ Provide an analytical solution for a simplified water discharge problem.

✓ Experimentally show how the tilted angle influence the water discharge rate.

✓ Elucidate the bottle shape influence on the water discharge rate

✓ Provide a statistical method for the prediction of the water emptying time.

- **Supplementary materials**

Code, data, and supplementary materials:

https://github.com/fzhai0/code

Running system: Windows 10

Software: MATLAB R2017A

Computer: Lenovo ThinkPad E480

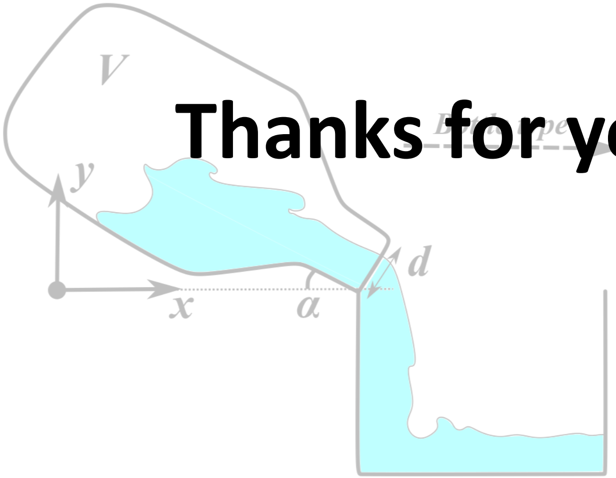
CPU: intel CORE i5 8th Gen

• Acknowledgement & Authorship Contribution

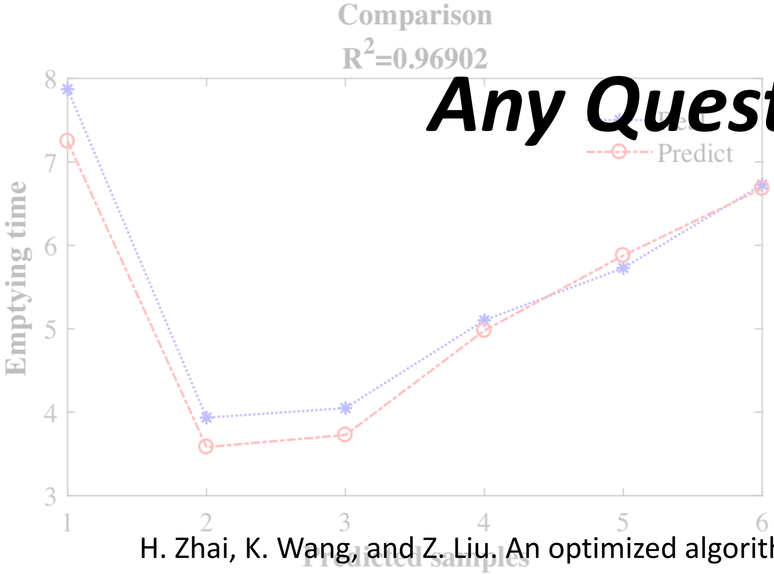
The authors would like to thank R. Alam, W. Liang, S. Tan, and D. Kong for the valuable discussion and insightful suggestions.

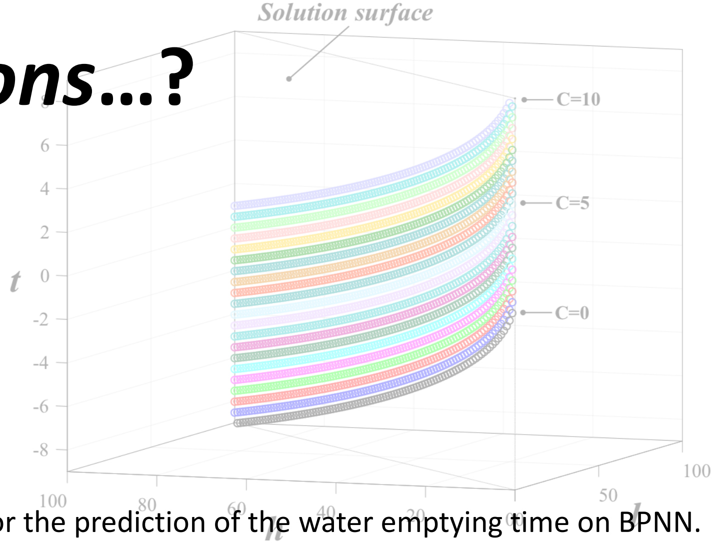
Frank Hanfeng Zhai	Coordinate the team and divide the works; Conceive the process and draft the report; Providing the algorithm (BPNN); Generating the results and plotted as graphs; Making the slides; Report editing and writing (the rest parts).
Alexander Kai Wang	Designing the experiment; Making the experimental equipment; Carrying out the experiment; Collecting the data; photo’ing the experimental process.
Fred Zhenyu Liu	Mathematical model for the simplified water discharge problem; Carrying out the experiment; Collecting the data; Report writing for the mathematical model.

Thanks for your patience!



Any Questions…?





H. Zhai, K. Wang, and Z. Liu. An optimized algorithm for the prediction of the water emptying time on BPNN.