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WY EY Addrive closure. u+v=v+uAdditive Commutativity (u+v)+w=u+(v+w)Additive Associationy u + Oi = u VueV Sero. For evry u, sxists w u+w=0Additive Inverse Multiplicative Closure C.V EV $(c+d)\cdot v = c\cdot v + d\cdot v$. Diseribulivity $C \cdot (\mathcal{U} + \mathcal{V}) = C \cdot \mathcal{U} + C \cdot \mathcal{V}$ Distributionty $(ed) \cdot v = c \cdot (d \cdot v)$ Associatively 1.V = V V V E F Unity : Sebastian Tomaskovie-Moore, UPenn. Examples

(D 3(a,b) ER2: b=3a+1}.

counter-example: . Not closed addition & multiplication.

(1+5)(a,b) = ((1+5)a,b) = (ra+5a,b) (r+5)(a,b) = ((r+5)a,b) = (ra+5a,b)

$$\Gamma(a,b) + S(a,b) = (\Gamma a,b) + (Sa,b) = (\Gamma a + Sa, 2b).$$

$$Violates the distributivity \(\text{off} \)
$$S(a,b) \in IR^2 \(Y \) \(W \) \(Sax \) \(Ia,b) = (ka,0).$$

$$I(a,b) = (Ia,0) = (a,0) \neq (a,b).$$

$$Violates \ \ \text{both} \ \ Mul. elseure } \(Unity \) \(mul. \)$$

$$Euler \ \ \text{Lagrange} = \frac{Squatfon}{Squatfon} \ \ \text{credit: Norbert Stoop, MIT}.$$

$$Let \ \ \text{us define an "Energy functional".}$$

$$P(u) = \int_0^b F(u,u') dx \ \ \omega / \left(\frac{U(0)}{S} = a \right).$$

$$P(u) = \int_0^b h(x,f(x),f(x)) dx.$$

$$If = \frac{\partial h}{\partial f} - \frac{\partial x}{\partial x^2} = \int_0^b \left(\frac{\partial h}{\partial f} \) \int_0^b f(x) + \frac{\partial h}{\partial f} \frac{\partial x}{\partial x} \int_0^b f(x) \right) dx$$

$$First \ \ \text{uaritation} \qquad (not \ \ \text{Lagrange} \) \int_0^b \left(\frac{\partial F}{\partial u} + \frac{\partial F}{\partial u} \right) dx \ \ \text{for every } 0$$$$

our old friend, test function !

Weak fam:
$$\int_{0}^{1} V(x) \left(\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u} \right) \right) dx + \left[v \frac{\partial F}{\partial u} \right]_{0}^{1} = 0$$

integral boundary terms

Note that this is satisfied

for ALL text functions

Fuller - Lagrange squation for u ;

 $\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u} \right) = 0$.

Example $(4x. 2.20; P. 36)$

function u Satisfies

$$\int_{0}^{1} u'v' dx + u'(0) v(0) + u(0) v'(0) + u(u) v'(0)$$

$$-\int_{0}^{1} \int_{0}^{1} v(x) dx - dx v(1) - g_{0} v'(0) - u(0) v'(0)$$

for all $v \in V = \{v: v(1), J \to R \text{ Smooth}\}$

For general procedure, See P. 35 ~36 Step 1: eliminate the definative on ve W(L) V(L) - W(0) V(0) - (W" V dx + W(0) V(0) + U(0) V(0) + NUW) V(0) = \int \text{f v dx + d, V(L)} + 9, 2(0) + Mg, 2(0) Step 2: Collect & terms $\int_{0}^{L} \left(u'' + f \right) v \, dx = \left(u'lL \right) - d_{L} \right) v \, lL \right)$ + (u10)-90) v(0) + u(u10)-90) v(0) For v∈2: → v(0) = v(1) = v'(0) = 0 Leer implying RHS =0 We conclude: $\int_{0}^{L} (u'' + \int_{0}^{L}) v dx = 0$ Step 3: Obtain PDE & B.C.s \Rightarrow $u'(x) + f(x) = 0 \chi \in (0, 1).$ u needs to satisfy this PDE

For such
$$U$$
, the previous PHS should be satisfied for all V not just $V(0) = V(L) = V(0) = 0$ $V \in U$.

$$0 = (u(1) - d_1) v(1) + (u(0) - g_0) v(0)$$

$$- u(u(0) - g_0) v(0)$$

if
$$V(L) \neq 0 \Rightarrow V(L) - dL = 0$$
.

(Neumann B.C.s)

if
$$v(0) \neq 0$$

$$v(0) = 0$$

$$v(0) = 0$$
(Direlial B.C.s)

Euler-Logrange squatton:

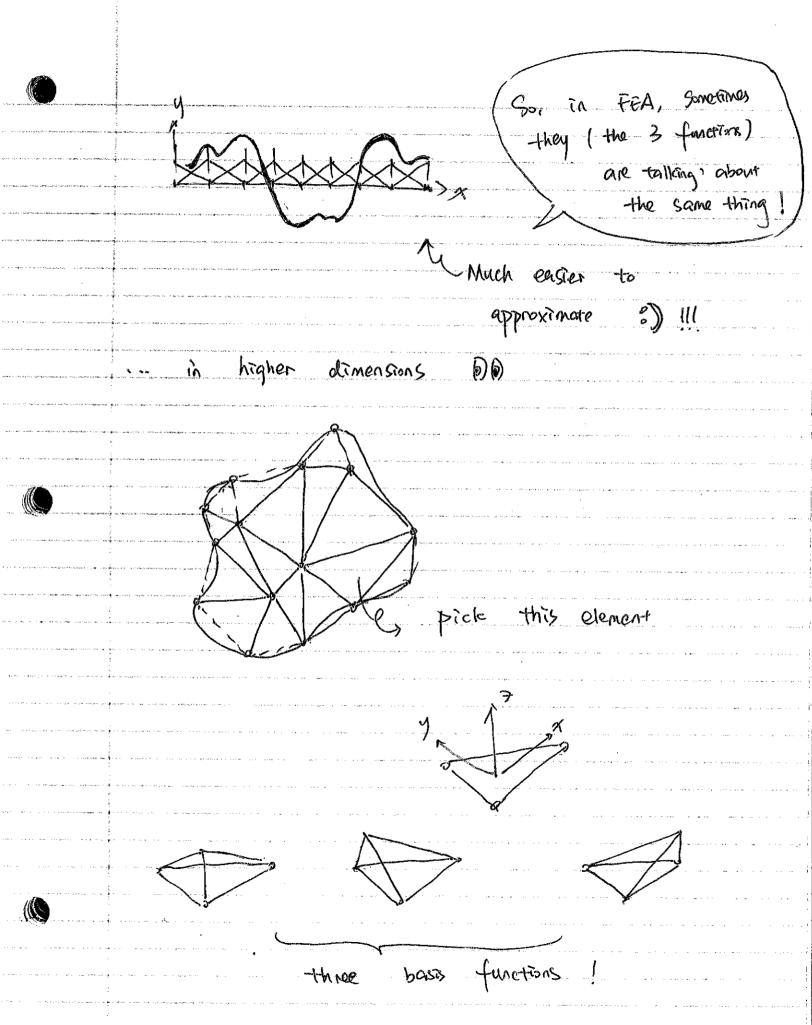
$$u''(x) + f(x) = 0$$
, $x \in (0, L)$
 $u(0) = g_0$,
 $u'(L) = dL$

Conceptual Clarifications test functions, ----N test how well trial functions satisfy sola how do (Neeall last P.S.) we use "A test function is an infinitely differentiable function how does !! Of compact Support! (NIST. North. Func) it look like A function has compact support if (Workan) it is zero ourside of a compact set." (-topological space) 1D sxample Example $\int_{0}^{\infty} U''v dA < \infty$ test k trial func.

U must be twice differentiable same. even have to be continuous basis functions approximation in FEA U(X) = Si Ci Yi

"an element of a particular basis for a function space." sovery function in the function space can be represented on a linear combination of basis functions. Grample 4= SEN (K) use basis function probably trying to interpolate k a good choice a werld-shaped function of basis function. Shape functions ----(usually referred specifically in FEA) The shape function is the function which Interpolates the Solution between the discrete values obtained on the the solution botween mech nodes

(Credit: Roberto Lacarda de Orio



Grample 2.40 base space Wh = span (12. x, x2, x37) test spare $2h = span(\{x, x^2, x^3\})$ Sn = 33+ Un 1 Un = 2h 5 trial space NICED NZCON NACKO MUCA) m=4 In=3.

Un (x) - 3N4(x) = 3.

Neall definition of consistency

 $P_h(u, v_h) = 0$ $Q_h(u_h, v_h) = l_h(v_h)$

the solves ancu, vn = lulva

(Ohlun, NI) = In(NI) an (Un, Nz) = la (Nz) an (Un, Nz) = la (Nz)

Witte some vodes (MAUAR & Python) to Golde for the numerical values in Kij

Solving for
$$KU = F \rightarrow U = K'F$$

$$U = \begin{cases} u_1 \\ u_2 \\ u_4 \end{cases} \longrightarrow U_A(x) = U_1 N_1(x) + U_2 N_2(x) + U_3 N_3(x) + U_4 N_4(x) + U_4 N_4$$

Gome values !