Article

Linear Stability Analysis of a Nonlinear Valve System

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Abstract: Feedback control systems are widely studied and applied in various engineering disciplines. The feed-forward process usually encompasses controller decisions, control processes, and transfer functions, executing the command of the system input. Valve is an ideal model that possesses such characteristics. Here we study the dynamics of the nonlinear valves by comparing the linear and nonlinear systematic responses and the root locus. The system is stabilized with the initial value equals to 1.49. Results indicate that the linearized system behaves in a very similar way and the nonlinear system generates a very different response based on different inputs.

Keywords: Nonlinear system; Control; Linearization; Nonlinear valve

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1. Introduction

Valves are devices that are able to restrict or allow the passage of a fluid in response to a control signal [Petkov *et al.*, 2018]. Valves encompass a wide range of applications, including controlling water for irrigation, industrial uses for controlling processes, residential uses such as on/off and pressure control to dish and clothes washers and taps in the home [Wikipedia]. Specifically, multiple types of research on the valve are strongly related to biomechanics since the heart possesses the structure of a valve [Stanford Children's Health], and similar structures occur in aortic [Thubrikar, 2017], lymph ducts, and many related fields in anatomy [Britannica, 2017]. Numerical simulations were among the primary methods selected to study the behaviors of "bio-valves". Kunzelman *et al.* [1993] developed a finite element model to examine deformation and stress patterns in the mitral valve under systolic loading conditions. Kuan *et al.* [2014] using 3D simulations to study bileaflet mechanical heart valves and found out that valve orientation was found not to affect the shear stress distribution significantly in the downstream aorta, and this was in agreement with the findings of earlier studies. Similar methods were also employed to facilitate the designing of artificial heart [Kuan *et al.*, 2015]. Besides, Particle Image Velocimetry was also employed to study the mechanical behavior of fluid dynamics of aortic valves [Barakat *et al.*, 2015]. In fine, numerical studies of heart and vein pumps is a main research direction of the valve.

Valve exhibits nonlinear dynamical behaviors, which are of the key essence studied in the field of system and control [Astrom and Wittenmark, 2008]. A nonlinear model-based adaptive control approach can also be adopted for a solenoid-valve system [Lee *et al.*, 2012]. Liaw *et al.* [1990] use an energy-based method to develop a nonlinear model of an electrohydraulic flapper-nozzle valve. Schimitt *et al.* [2018] proposes a fully nonlinear model for a pneumatic process control valve for developing accurate control schemes for the whole process and/or facilitate the tuning of control parameters in "smart" process control valves. Wu *et al.* [2021] proposed a novel direct proportional pressure-regulating valve, and show that a fixed relationship between the orifice diameters of the valve can be achieved.

Although the dynamical behavior of nonlinear valve is well studied, we are focus on the scheme difference of nonlinear valve and the linearized form under a feed-back control system, following the classical control model of nonlinear valves [Astrom and Wittenmark, 2008]. Of the two forms, the root locus and responses of the systems are analyzed and the differences are elaborated.

In this mini-manuscript, we first formulate and define the nonlinear valve problem in Section 2. We then present the methods and mathematical derivations of the nonlinear valve systems as in Section 4. We briefly conclude this manuscript in Section 5.

2. Problem Formulation

Valve is ubiquitous in control sciences and system engineering, specifically for flow control. Usually, the system involved for a controlled exhibits nonlinear behavior termed as a nonlinear valve. Here, we simply the nonlinear valve problem to a simple Proportional Integral (PI) Control problem, as illustrated in Figure 1 (**A**). Within the valve control system, the valve function can be written into the form:

$$v = f(u) = u^4 \qquad u \geq 0$$

Linearizing this system around a steady-state operating point indicates the incremental gain of the valve is $f'(u)$. Therefore, the loop gain is proportional to $f'(u)$. Furthermore, the valve function $f(\cdot)$ determines whether the system is linear or not. Such feedback control loop PI system is a classic system in dynamics and system control having wide applications in the automobile industry, spacecraft design, etc.

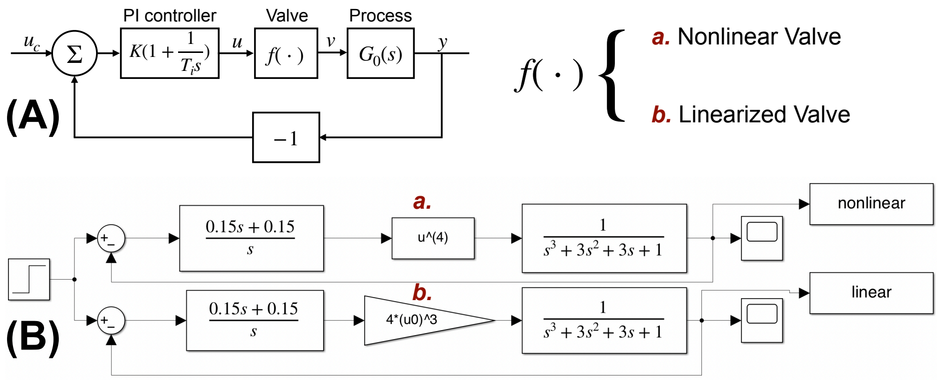

Figure 1. The schematic for the nonlinear valve in the PI control system, reproduced from *Astrom and Wittenmark*. **A** control schematic in theories. **B** the nonlinear system implementation in MATLAB.

3. Methodology

Based on the theoretical model in Figure 1 (**A**), we can generate the linearized and nonlinear system as in Figure 1 (**B**), in MATLAB Simulink. Here, a `Step` function is adopted for initializing and input the command for the system. Two `Sum` function was connected with it for the nonlinear (**a.**) and linearized (**b.**) system respectively. Then the PI Controller is signified with a `Transfer Fcn`, writes: $\frac{0.15s+0.15}{s}$. For the valve part, the nonlinear valve was specified with the function in *Astrom and Wittenmark* as a `Fcn`: $f(u) = u^4$. While for the linearized form, it was signified as a `Gain`: $f(u_0) = 4u_0^3$, where u_0 is the initial value of the valve. Then the process function was $G_0(s)$ is also signified as a `Tranfer Fcn`:

$$G_0(s) = \frac{1}{s^3 + 3s^2 + 3s + 1}$$

. With the linearization mentioned, Jacobian linearization was performed using the Taylor series expansion neglecting second–order and higher terms. The Taylor series evaluates an approximate derivative at a nominal input point (We nominate as u_0). Note u_0 corresponds to an initial valve position, which can be written as $v_0 = f(u_0) = u_0^4$. Evaluate u_0 with Taylor Series, the reference output y_{ref} can then be determined at steady state. When the system reaches steady state, $y_{ref} = u_c = v_{ref} = u_0^4$.

3.1. Taylor series linearization

For the linearization process mentioned, the given function $f(u)$ can be expanded via Taylor series:

$$f(u) = u^4 = f(u_0) + \frac{df(u)}{du}|_{u_0}(u - u_0) + \frac{1}{2!}\frac{d^2y}{dx^2}|_{u_0}(u - u_0)^2 + H.O.T.$$

Neglecting H.O.T., and reducing to linear approximation:

$$\begin{aligned} f(u) &= f(u_0) + \frac{f(u)}{u}|_{u_0}(u - u_0) \\ &= f(u_0) + 4u^3|_{u_0}(u - u_0) = f(u_0) + 4u_0^3(u - u_0) \end{aligned}$$

Such a form can be rewriting into:

$$f(u) - f(u_0) = 4u_0^3(u - u_0) \rightarrow \frac{f(u) - f(u_0)}{(u - u_0)} \quad = 4u_0^3$$

Therefore, the nonlinear valve function can be linearized as $4u_0^3$.

3.2. Transfer functions

In Figure 1, the initial value u_0 undergoes a control process to be outputted as a targeted value, whose process we summarized as a conbination of Transfer functions. Within the process, the PI controller obeys the function $C(s) = K\left(1 + \frac{1}{T_i s}\right)$, and the process writes $G_0(s) = \frac{1}{(s+1)^3} = \frac{1}{s^3+3s^2+3s+1}$. In the control system, we defined *Plant* as $Plant = Valve + Process$. We can therefore write the equation representing the *Plant* process:

$$F(s)G_0(s) = G(s) = 4u_0^3 \frac{1}{(s+1)^3} = \frac{4u_0}{s^3 + 3s^2 + 3s + 1}$$

3.3. Open-loop transfer function

To generate the bode plots and analyze the system through root locus, we need to compute the open-loop transfer function.

$$\begin{aligned} \frac{Y(s)}{U_0(s)} &= \left(K\left(1 + \frac{1}{T_i s}\right)\right)\left(4u_0^3\right)\left(\frac{1}{(1+s)^3}\right) \\ &= \frac{K(T_i s + 1)(4u_0^3)}{T_i s(s+1)^3} = \frac{4u_0^3 K T_i s + 4u_0^3 K}{T_i s^4 + 3T_i s^3 + 3T_i s^2 + T_i s} \end{aligned}$$

As shown in Figure 1 in the PI controller, we know that in our case $T_i = 1$ and $K = 0.15$, so that the open-loop transfer function can be written into the final form:

$$\frac{Y(s)}{U_0(s)} = \frac{0.6u_0^3 s + 0.6u_0^3}{T_i s^4 + 3s^3 + 3s^2 + s} \tag{1}$$

We can thence analyze the root locus with the confined transfer function in Eq. 1.

4. Results and discussion

Input the loop transfer function as simplified in Eq. 1 we can generate the root locus as in the right subfigure in Figure 2, corresponding to the bode plot as in the left subfigure. Here, the solution is located between 0.14 to 0.28 for the set loop transfer function, and the system is overdamped since the horizontal solution is beyond 1. As can be observed from the left subfigure we see that the magnitude is below the stable line and the phase value is bigger than $-180°$.

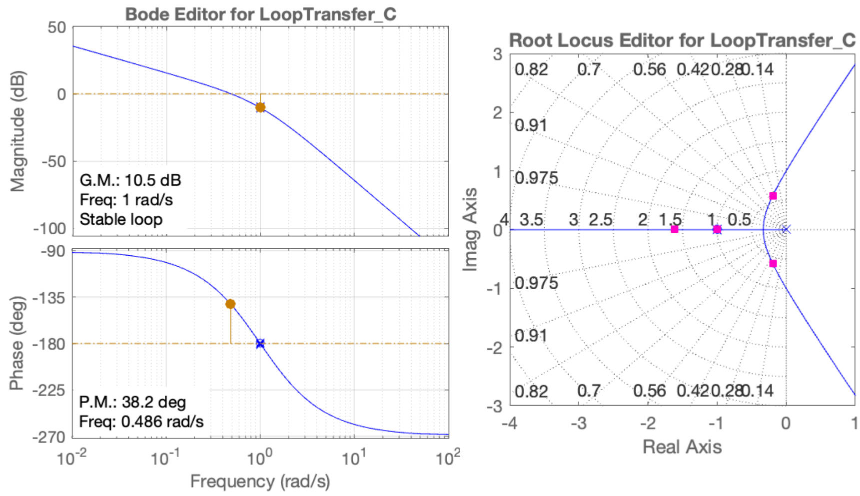

Figure 2. The bode plot (*left*) and the root locus (*right*) for the nonlinear valve control system.

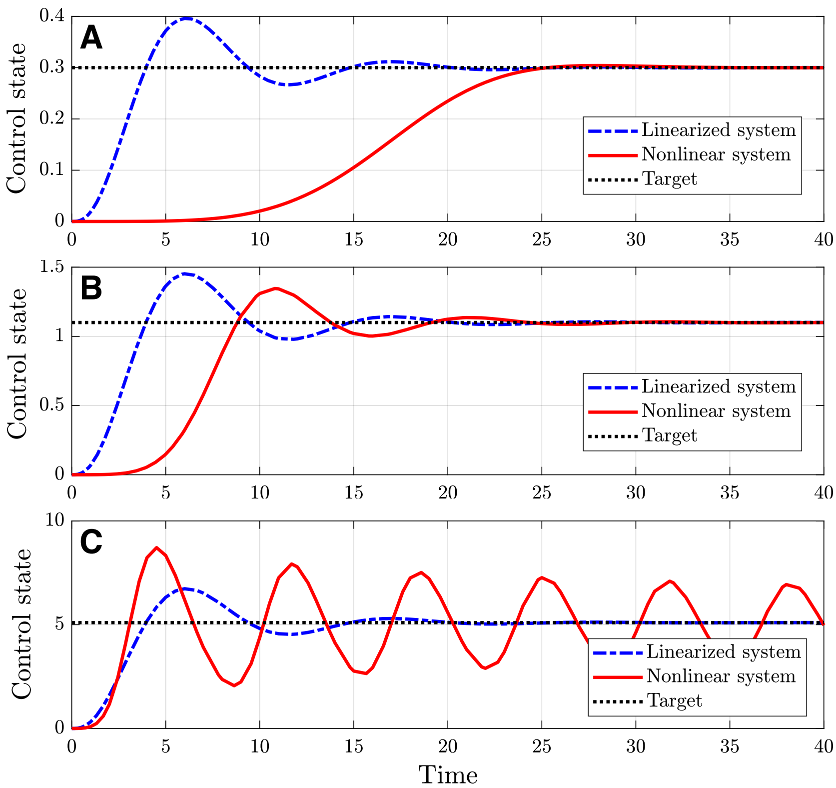

Figure 3. The response of the system with regards to different targeted values for the nonlinear valve system. (**A**) Initial value: 0.2, target value: 0.3. (**B**) Initial value: 1, target value: 1.1. (**C**) Initial value: 5, target value: 5.1.

Measuring the response from different initial values we generate results as in Figure 3 with three different system settings from **A** to **C**, where the black dotted lines are the targeted values, the blue dotted lines are a systematic response from the linearized system, and the red solid lines are the nonlinear systematic response. Among all the subfigures we can observe that with the linearized system behave in a similar way, which is pretty straightforward according to the theory of linearization, and the nonlinear systems exhibit evident difference on systematic responses. For the **A** system it approaches the targeted value smoothly with a longer response time comparing with the nonlinear system. In system **B** both the linearized and nonlinear valve behave similarly but the nonlinear system responses slower. For system **C** the nonlinear system response quicker yet oscillates into an unstable state as approaching the targetted value 5. We can then deduce that the targeted value strongly influence the behavior of nonlinear system while doesn't variate such of linearized systems.

5. Conclusion

In this mini-manuscript, we introduce a classical control system of a non-linear valve, where the response of the linearized and nonlinear valve function is studied and compared. We formulated an open-loop transfer function and then based upon analyzed the bode plots and root locus. We thence study the systematic response of the two different systems considering different targetted values. Results indicate that the targeted value strongly influence the behavior of nonlinear system while doesn't variate such of linearized systems.

References

Petkov *et al.*, 2018. Petkov P, Araújo M.E.U.J., Gadelha J.R.T., et al. Nonlinear Predictive Control System for Stiction Compensation in Electropneumatic Control Valves. *Journal of Control Science and Engineering* **2018**, *1510705*, 1687-5249.

Wikipedia. Wikipedia. *Valve*. Available online: https://en.wikipedia.org/wiki/Valve.

Stanford Children's Health. Stanford Children's Health. *Anatomy and Function of the Heart Valves*. Available online: https://www.stanfordchildrens.org/en/topic/default?id=anatomy-and-function-of-the-heart-valves-90-P03059.

Thubrikar, 2017. Thubrikar M. The Aortic Valve. Boca Raton. **2017**.

Britannica, 2017. Britannica, The Editors of Encyclopaedia. Valve. Encyclopedia Britannica, 12 Dec. **2017**.

1993. Kunzelman KS, Cochran RP, Chuong C, et al. Finite element analysis of the mitral valve. *The Journal of Heart Valve Disease*. **1993** May;2(3):326-340. PMID: 8269128.

2014. Kuan YH, Nguyen VT, Kabinejadian F, et al. Numerical analysis of the hemodynamic performance of bileaflet mechanical heart valves at different implantation angles. *J Heart Valve Dis.* **2014** Sep;23(5):642-50.

Kuan *et al.*, 2015. Kuan YH, Nguyen VT, Kabinejadian F, et al. Computational Hemodynamic Investigation of Bileaflet and Trileaflet Mechanical Heart Valves. *J Heart Valve Dis.* **2015** May;24(3):393-403.

Barakat *et al.*, 2015. Barakat M, Dvir D, Azadani AN. Fluid Dynamic Characterization of Transcatheter Aortic Valves Using Particle Image Velocimetry. *Artif Organs.* **2018** Nov;42(11):E357-E368.

Astrom and Wittenmark, 2008. Astrom KJ and Wittenmark B. Adaptive Control (2nd Edition). Dover Publications, Inc. **2008.**

Slotine and Li, 1997. Slotine JJE and Li W. Applied Nonlinear Control. Prentice Hall. **1997**.

Lee *et al.*, 2012. Lee DB, Naseradinmousavi P, Nataraj C. Nonlinear Dynamic Model-Based Adaptive Control of a Solenoid-Valve System. *Journal of Control Science and Engineering*. vol. 2012, Article ID 846458, 13 pages, **2012**.

1990. Liaw, C., and Brown, F. T. (June 1, 1990). "Nonlinear Dynamics of an Electrohydraulic Flapper Nozzle Valve." *ASME. J. Dyn. Sys., Meas., Control.* June **1990**; 112(2): 298–304.

2018. R. Schmitt and M. R. Sobczyk Sobrinho, "Nonlinear Dynamic Modeling of a Pneumatic Process Control Valve," in *IEEE Latin America Transactions*, vol. 16, no. 4, pp. 1070-1075, April **2018**.

2021. Wu, W., Wei, C., Zhou, J. et al. Numerical and experimental nonlinear dynamics of a proportional pressure-regulating valve. *Nonlinear Dyn* 103, 1415–1425 (**2021**).