

# COMP3411/9414: Artificial Intelligence

## 4b: Motion Planning

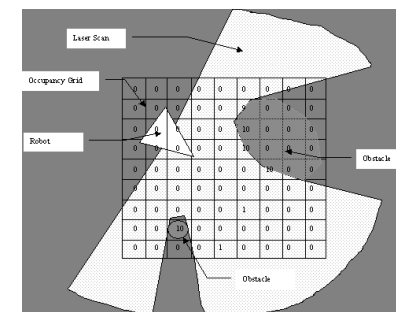
## Motion Planning Approaches

- Partially Observable Environments (on-board sensors only)
  - ▶ Occupancy Grid
  - ▶ Potential Field
  - ▶ Vector Field Histogram
- Fully Observable Environments (overhead cameras)
  - ▶ Delaunay Triangulation
  - ▶ Parameterized Cubic Splines

## Robots



## Occupancy Grid



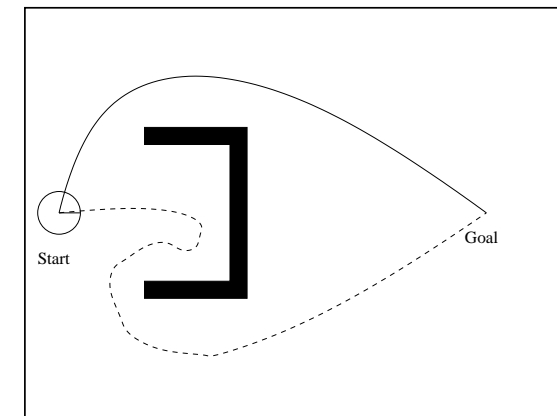
- divide environment into a Cartesian grid
- for each square in the grid, maintain an estimate of the probability of an obstacle in that square

## Potential Field

Using an occupancy grid for its map, an agent can plan a path using a Potential Field.

- treat robot's configuration as a point in a potential field that combines attraction to the goal, and repulsion from obstacles
- very rapid computation, but can get stuck in local optima, thus failing to find a path

## Problem - Local Optima



## Vector Field Histogram

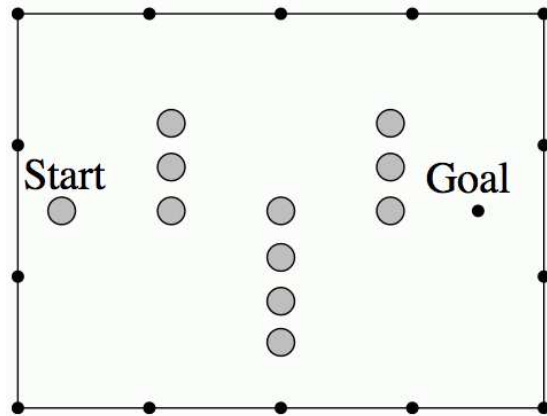
A more sophisticated approach - less likely to get stuck in local optima - is a Vector Field Histogram

- Cartesian histogram grid, continuously updated
- Polar histogram, based on current position/orientation of robot
- candidate valleys (contiguous sectors with low obstacle density)
- select candidate valleys, based on proximity to target direction

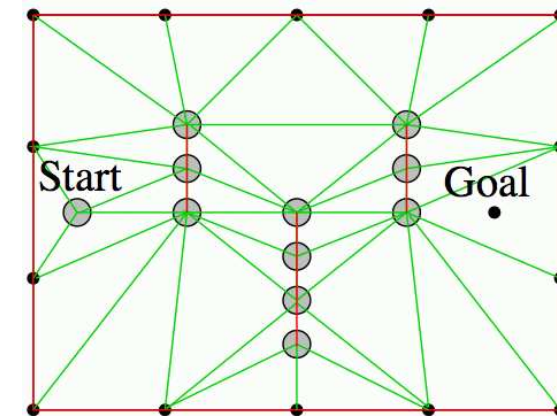
## Delaunay Triangulation

- applicable in situations where the environment is well mapped by overhead cameras (museums, shopping centres, robocup small league)
- add line segments between (closest points of) obstacles, sorted according to length (shortest segments first)
- do not add any segment that crosses an existing segment

## Delaunay Triangulation



## Delaunay Triangulation



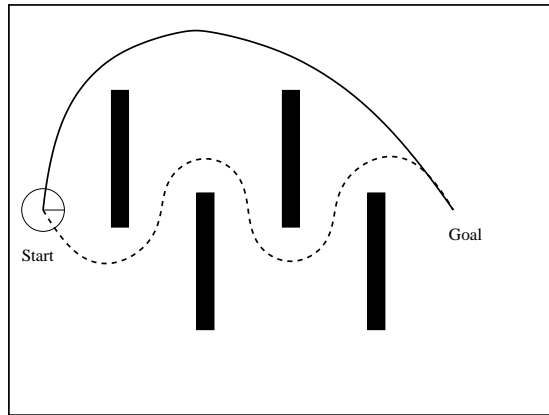
## Voronoi Diagram

- dual of the Delaunay triangulation is called a Voronoi diagram
- prune arcs that are too small for the robot to traverse
- A\*Search can then be applied on the resulting graph
- path can be converted to a trajectory for the robot

## Minimizing Time instead of Distance

- For the problem of a soccer robot getting to the ball, or a wheeled robot navigating a maze, the “shortest distance” path might not be the same as the “shortest time” path.
- By speeding up and then slowing down, the robot could traverse a path with long straight stretches faster than a shorter path with lots of twists and turns.

## Minimizing Time instead of Distance



How can we approach the problem of finding the “shortest time” path?

## Optimal Trajectory Planning

Steps in the Algorithm:

- Delaunay Triangulation
- A\*Search, using paths composed of Parametric Cubic Splines
- Smooth entire curve with Waypoint Tuning by Gradient Descent

## Parameterized Cubic Splines

Assume each path segment is of the form

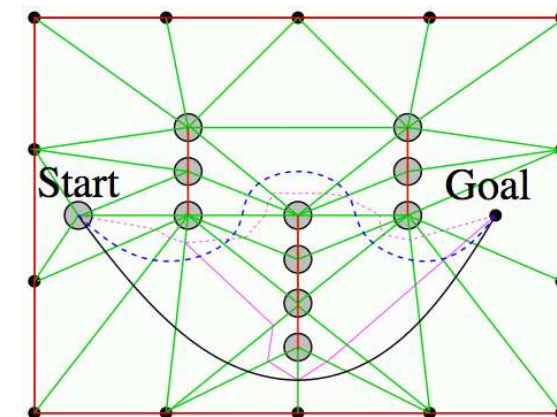
$$P(t) = \begin{pmatrix} P_x(t) \\ P_y(t) \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} t^3 + \begin{pmatrix} b_x \\ b_y \end{pmatrix} t^2 + \begin{pmatrix} c_x \\ c_y \end{pmatrix} t + \begin{pmatrix} d_x \\ d_y \end{pmatrix}$$

Solve these equations for a specified position and velocity at the beginning ( $t = 0$ ) and the end ( $t = s$ ) of the segment. Traditional method was to set  $s = 1$ . We instead try to minimize  $s$  (total time for segment) while satisfying the kinematic constraints:

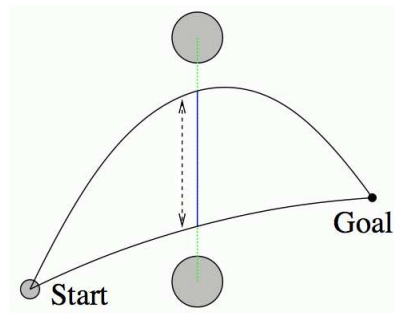
$$\left| \frac{P''(t)}{A} + \frac{P'(t)}{V} \right|^2 \leq 1, \quad \text{for } 0 \leq t \leq s,$$

where  $A$  and  $V$  are the maximal acceleration and velocity of the robot.

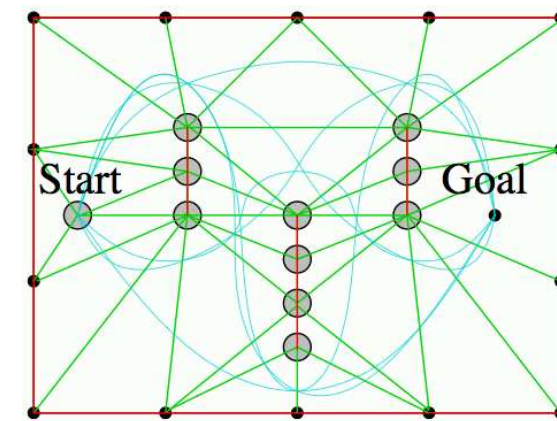
## A\* Search With and Without Smoothing



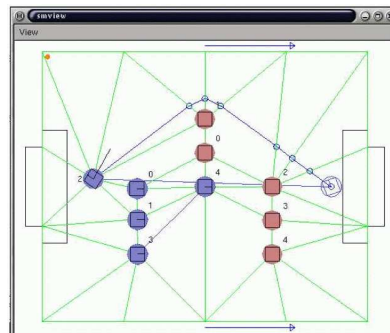
## Waypoint Tuning by Gradient Descent



## Various Paths Explored



## Robocup



This system has been successfully deployed in the Robocup F180 League.

## References

- S. Thrun, W. Burgard & D. Fox, [Probabilistic Robotics](#), MIT Press, 2005.
- J. Thomas, A. Blair & N. Barnes, "Towards an Efficient Optimal Trajectory Planner for Multiple Mobile Robots", [2003 International Conference on Robotics and Systems \(IROS'03\)](#), 2291–2296.