# TM HW6

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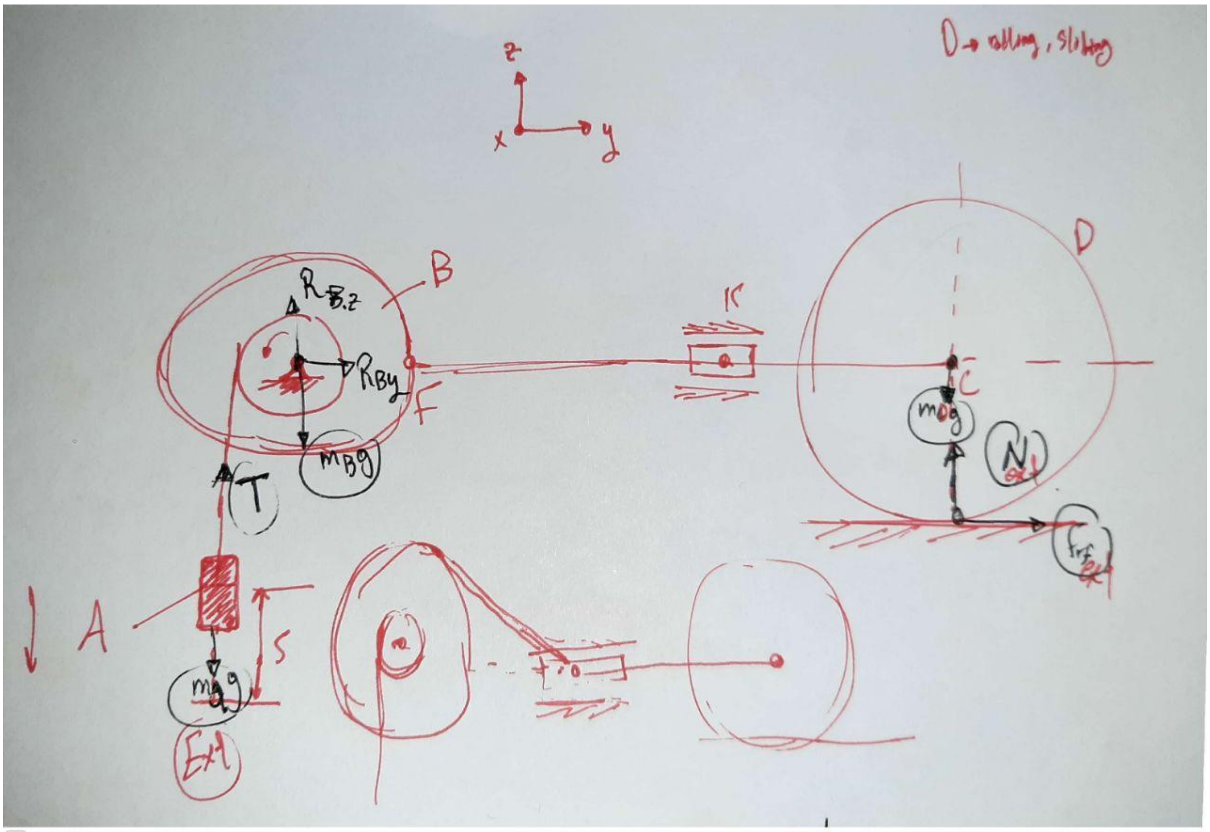
## Homework: HW6

## Task1:






### Research objects:

- System that consists of 6 bodies (3 main):
  - Solid cylinder (object D) → Rotational and translational motion in Y direction
  - Pulley (object B) → Rotational motion
    - Connected with rotational joint from the center for the rotational
  - Block (object A) → translational motion in Z direction
  - Piston on slider (object K) sliding in Y direction as a translational motion
  - FK link
  - KC link
- Axis as in the picture such that Z is to the up, Y is to the right and X is outward the screen.

## Givens:

- $m_A = 1kg$
- $m_B = 3kg$
- $m_D = 20kg$
- $R_B = 20cm$
- $R_D = 20cm$
- $i_{Bx} = 18cm$
- $r_B = 0.8 * R_B = 16$
- $FK = 4 * r_B = 64$
- $\phi = 0.6$ → rolling friction

## Initial conditions:

- System starts at rest, all the velocities are equal to zero
- System is starting to move due to the gravitational force on A

## Final conditions:

- Object A moves $s$ cm
- All other objects move according to this
- $v_A(s) = ?;\ such\ that:\ s = f(t)$

## Forces analysis:

- Gravitational forces on object A, B, D that works downward in the negative direction of Z-axis.
- Tension in the string that is connected between A and Pulley B with the small extension of the pulley.

- Reaction in Z and Y directions on the rotational joint at B
- Normal forces from the ground to the up in the point of contact of D and the ground.
- Friction forces between D and the ball.
- These analyses with the consideration that masses of the links and the piston are ignored.

## Solution:

The solution is based on the Lagrangian dynamics by using the Kinetic Energy (T) and the work for all the external forces principles (A)

***Please note that inertia is represented as $I$ not $J$***

$T_2 - T_1 = \sum_i A_i$

As the system starts from the rest: $T_1 = 0$

$T_2 = T_{A2} + T_{B2} + T_{D2} + T_{K2}$

- $T_{A2} = 0.5 * m_A * v_A^2$
- $T_{B2} = 0.5 * I_{Bx} * w_B * 2$;
  - $I_{Bx} = m_B * i_{Bx}^2$;
  - $w_B = \frac{v_A}{r_B}$
  - → $T_{B2} = \frac{m_B * i_{Bx}^2 * v_A^2}{2r_b^2}$
- $T_{K2} = 0.5 * m_k * v_k^2$;
  - $v_k = v_A * R_B / r_B$
  - → $T_{K2} = 0.5 * m_k * v_A^2 * R_B^2 / r_B^2 = 0$
- $T_{D2} = 0.5 * m_D * v_C^2 + 0.5 I_{Dx} * w_D^2$;
  - $w_D = v_C / R_D = v_A R_B / (r_B * R_D)$;
  - $v_c = v_k = v_a * R_B / r_B$;
  - $I_{Dx} = 0.5 * m_D * R_D^2$
  - → $T_{D2} = 0.5 * m_D * v_a^2 * R_B^2 / r_B^2 + 0.25 * m_D * R_D^2 * v_A^2 * R_B^2 / (r_B^2 * R_D^@)$

Work done by external forces

- $A = \sum_i A_i$
- $A_A = m_a * g * s$
- $A_B = 0$ As it is non-moving object it is just fixed
- $A_K = 0$ As the internal forces that applied on object K are internal forces
- $A_D = m_D * g * 0 + A(F_{fr}) + \delta N * \phi_D$;

- $A(F_{fr}) = 0$ From rolling friction force. As it is applied in the center of the instantaneous velocity;
- $\phi_D = ?$;
  - $x_D = FK + R_B - \sqrt{(FK)^2 - R_B^2}$
  - $\phi_D = r_B/(R_B * R_D) * x_D$;
  - $\rightarrow \phi_D = \frac{r_B * x_D}{R_B * R_D}$

As in the work equation all of them are known and givens and now we can substitute in

$$T_2 = \sum_i A_i$$

and get $v_A(s) = v_A(s(t))$

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v_a_2 = (m_a*s + m_d*g*((fk+R_b-sqrt(fk**2 - R_b**2))))
v_a_2 /= (0.5*m_a+0.5*m_b*i_bx/r_b**2 + 0.5*m_d*R_b**2/r_b**2 + 0.25*m_d*R_b**2/((R_d**2)*(r_b**2)))
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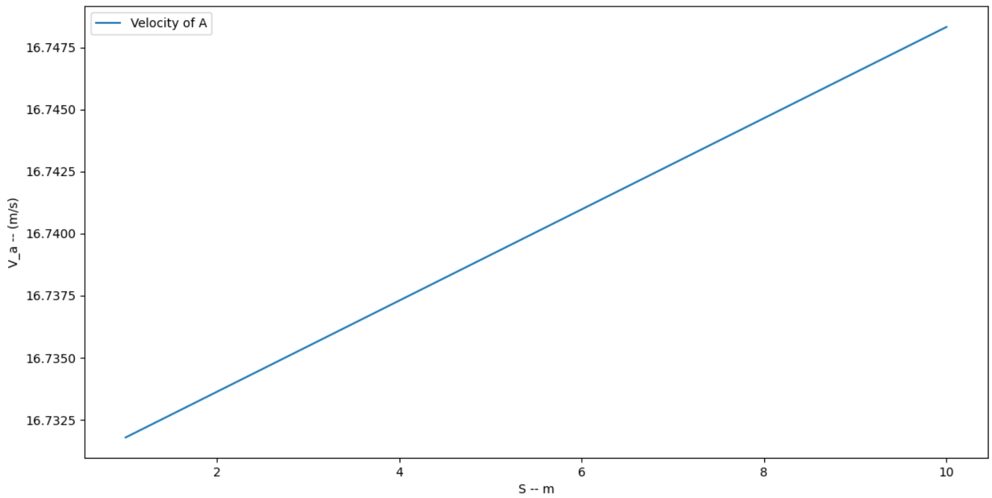
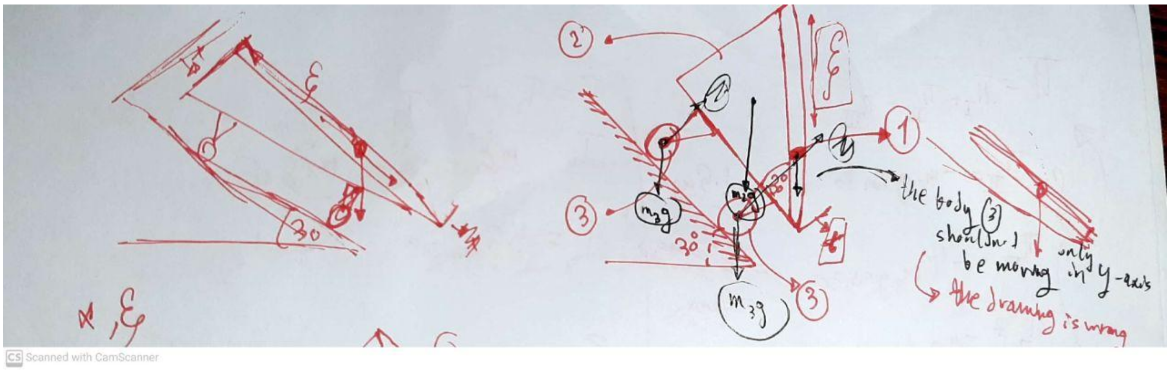



In the above graph, I have introduced S as the displacement that the block A did with respect to the starting position.

## For the second need:

- If the masses are not neglected that will lead to having forces in the center of masses of the links and the mass of the piston which will lead to take into consideration the pressure on the surface of the slider and the masses of the links will lead to work in the equations which will hardly make to be solved as $A_k \neq 0$ and there will be more unknowns than the number of equations. Also, it will affect the motion of the cylinder as it will add pressure to the cylinder and it will have reaction as normal forces and will take into account the friction.

# Task2:




The task is using generalized coordinates $x, \epsilon$ and it will be solved using Lagrangian dynamics

From the following equations:

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{x}}) - \frac{\partial T}{\partial x} = -\frac{\partial \pi}{\partial x} + Q_1$$

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\epsilon}}) - \frac{\partial T}{\partial \epsilon} = -\frac{\partial \pi}{\partial \epsilon} + Q_2$$

## Research objects:

- Mechanical system that is consisted of:
  - 4 wheels (3)
  - Body (2)
  - Particle (1)

## Givens:

- $m = 1kg$
- $m_1 = 1m$
- $m_2 = 3m$
- $m_3 = 2m$
- m in the above is mass = 1kg
- $b = 0.001$

## Initial conditions:

- $x_o = 0m$
- $\dot{x}_o = 0m/s$

- $\epsilon_0 = 0m$
- $\dot{\epsilon} = 3m/s$

## Solution:

- Finding the total kinetic energy:
    - $T = \sum_i T_i = T_1 + T_2 + 4 * T_3$
        - $T_1 = 0.5 * m_1 * v_1^2;$
            - $v_1 = \sqrt{(\dot{\epsilon} sin30)^2 + (\dot{\epsilon} cos30 + \dot{x})^2}$
            - → substitute in $T_1 = 0.5 * m * \dot{\epsilon}^2 + 0.5 * dotx^2 + 0.5m\dot{x}\dot{\epsilon} * \sqrt{3}$
        - $T_2 = 0.5 * m_2 * \dot{x}^2 = 1.5m\dot{x}^2$
        - $T_3 = 0.5 * m_3 * \dot{x}^2 + 0.5 * I_3 * w_r^2;$
            - $I_3 = 0.5 * m_3 * R^2;$
            - $w_3 = \dot{x}/R$
            - → substitute in $T_3 = 1.5m\dot{x}^2$
- Finding the potential energy:
    - $\pi = \pi_1 + \pi_2 + \pi_3$
        - $\pi_1 = -mg(xsin30 + \epsilon sin30) = -mgx - mg\epsilon\sqrt{3}/2$
        - $\pi_2 = -m_2 gsin30x = -1.5mgx$
        - $\pi_3 = -4m_3 gsin30 = -4mgx$
- Finding $Q_1, Q_2$:
    - $Q_1 = \frac{N_1}{\dot{x}_1} = 0$ as $N_1 = 0$
    - $Q_2 = -b\dot{\epsilon}$
- Finding the derivatives for the main equation:
    - $\frac{\partial T}{\partial x} = 0$
    - $\frac{\partial T}{\partial \epsilon} = 0$
    - $\frac{\partial \pi}{\partial x} = -6mg$
    - $\frac{\partial \pi}{\partial \epsilon} = -0.5 * mg\sqrt{3}$
    - $\frac{\partial T}{\partial \dot{x}} = 16m\dot{x} + \frac{\sqrt{3}}{2}m\dot{\epsilon}$
    - $\frac{d}{dt}(\frac{\partial T}{\partial \dot{x}}) = 16m\ddot{x} + \frac{\sqrt{3}}{2}\ddot{\epsilon}$
    - $\frac{\partial T}{\partial \dot{\epsilon}} = m\dot{\epsilon} + \frac{\sqrt{3}}{2}m\dot{x}$
    - $\frac{d}{dt}(\frac{\partial T}{\partial \dot{\epsilon}}) = m\ddot{\epsilon} + \frac{\sqrt{3}}{2}m\ddot{x}$
- The algorithm will be as following:

- Substitute in the main equation
- Solve them together
- Get one of the generalized coordinates as a function of the second coordinates
- Get the second coordinate in initial value problem in differential equation form and solve it and get the coordinate with the initial values to get the constant values.
- Then use it to get the first coordinate by integrating with respect to t two times and get the constants using the initial values.
- Such steps it will be easily to be done using numerical integration. I have here solved it analytically and find the constant of the integration using the initial values.

part of the analytical solution:

$$16 m\ddot{x} + \frac{\sqrt{3}}{2} m\ddot{\xi} - 0 = +6mg + 0 \quad (2)$$

$$m\ddot{\xi} + \frac{\sqrt{3}}{2} m\ddot{x} - 0 = +mg\frac{\sqrt{3}}{2} - b\dot{\xi} \quad (3)$$

$$\rightarrow \frac{2m}{\sqrt{3}}\ddot{\xi} + \ddot{x} = g - \frac{2b}{m\sqrt{3}}\dot{\xi}$$

$$\downarrow$$

$$\ddot{x} = g - \frac{2b}{m\sqrt{3}}\dot{\xi} - \frac{2\ddot{\xi}}{\sqrt{3}}$$

$\downarrow$ in (2.1)

(2.1)

$$16\ddot{x} + \frac{\sqrt{3}}{2}\ddot{\xi} = 6g$$

$$16g - \frac{32}{m\sqrt{3}} b\dot{\xi} - \frac{32}{\sqrt{3}}\ddot{\xi} + \frac{\sqrt{3}}{2}\ddot{\xi} = 6g$$

$$-17.6\ddot{\xi} - 18.47 b\dot{\xi} = -10g$$

$$\downarrow$$

$$\boxed{\ddot{\xi} + 1.049 b\dot{\xi} = 0.568g}$$

$\downarrow$

solve differential equation → characteristic equation

$$r^2 + 1.049b\, r = 0.568g$$

$$\boxed{r_1 = 0,\ r_2 = -1.049b}$$

$$\xi = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

get $\xi$ with solving differential equation

get $\dot{\xi}$

$\ddot{\xi}$ → get $\ddot{x}$ → integrate → get $\dot{x}$ → $x$



Plots for the generalized coordinates

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epsilon = lambda t: A*(exp(-1.049*b*t) - 1) + 5.41*t/b
x = lambda t: 0.9087 *E/b*exp(-1.049*b*t) + D*t*t/2 + 0.953*E/b*t - 0.908*E/(b*b)
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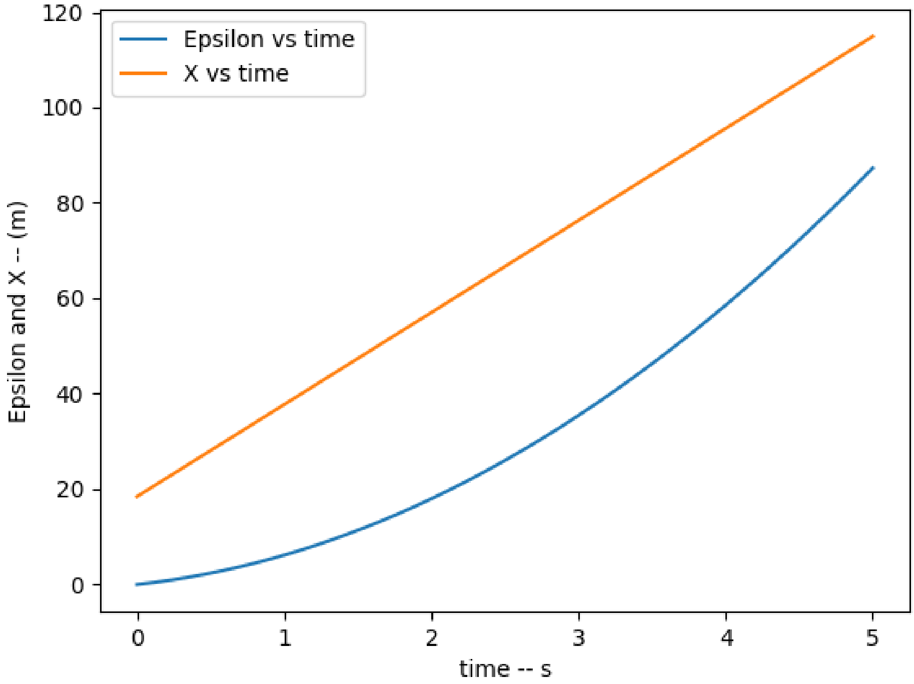
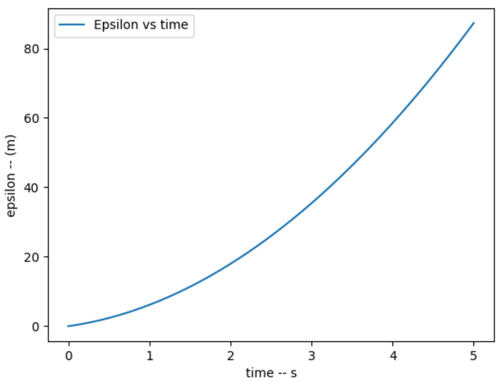
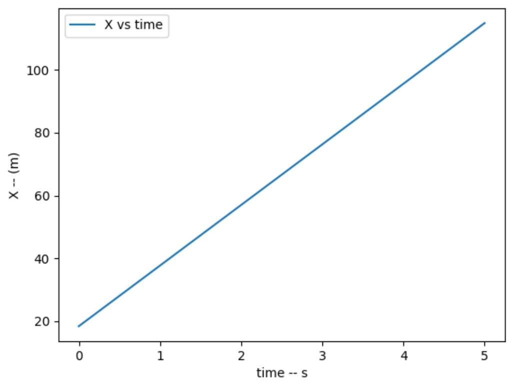
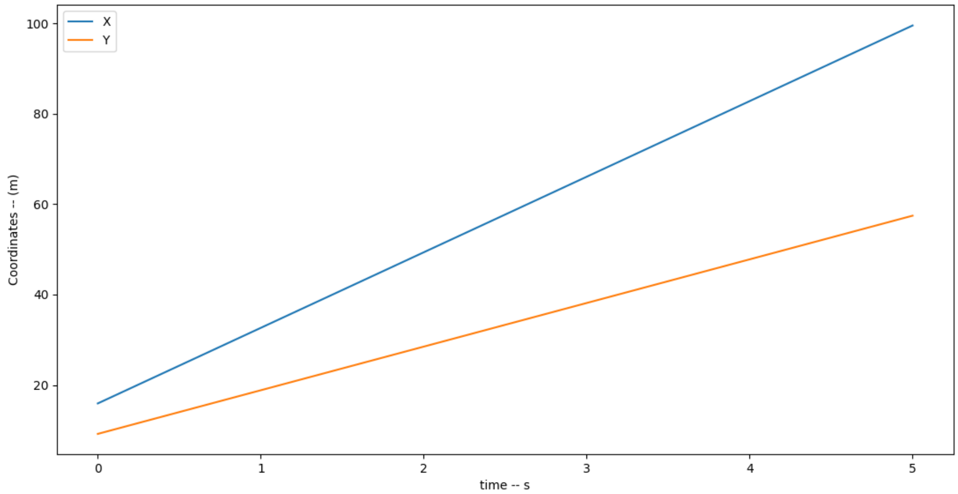



To find X and Y of each object we need to project each motion equation of the generalized coordinates on x-axis (cos30) and y-axis(sin30) and we will get x and y motion equations for each

- For the whole Body




- For the particle