Homework4

Class	TM
Created	@Sep 12, 2020 7:11 PM
Materials	
Reviewed	☐
Type	Assignment

Student Information

- Name: Hany Hamed
- Group: BS18-RO
- Homework4

I have solved the tasks on paper, to get the number of equations that equal to the number of the unknowns and then transform it into matrix form to be used with numpy in python to be solved the equations and get the unknowns. Only the problem is to find the equations that will make the matrix of A is not Singular. So, we can find more equations than the number of the unknown but some of them are dependent on each other will not help us to get the unknowns, that's why wee need to find the n independent equations that will make A not singular (full rank matrix - n rank)

$Ax = B$

Such that A (nxn - n is the number of the unknowns) is the matrix of the coefficients of the variables and B (nx1) is the absolute terms without unknowns and x is the variables vector (nx1)

Solution

Task1:

We are using SI unit system

Givens: Note A2 is A` and vice versa, ...etc

A2B = 2m, AA2 = 2m, DE = 2m, E2F = 2m, BC = 3m, CD = 2.5m

q = 1.4N/m, P_1=12N, P_2=18N, M_1=36N.m

Research objects:

- AF is a Rigid Rod that is divided by connections (Bearings/Joints) in C and E.
- A, D and F are Roller supports.
- B is a pen connection (free rotating joint).
- C and E are Bearings/Joints that have 0 torques and can divide the analysis of the whole rod.

Force Analysis:

- Note: I am defining $theta = 90 - \alpha$ such that α is the angle of the surface that has the roller support of A with the x-axis and β is the angle of the force P_2 line with the x-axis
- $R_A -> R_{Ax} = R_A cos(\theta),\ R_{Ay} = RAsin(/theta)$
- $R_B -> -R_{Bx},\ +R_{By}$
- Q = q*BE $(-y$-axis direction)
- $R_{Dy} = R_D\ (+y)$
- $-M_1$ (Torque)
- $P_2 -> -P_2 cos(\beta),\ -P_2 sin(\beta)$
- $R_F = R_{Fy}$ (+y-axis direction)

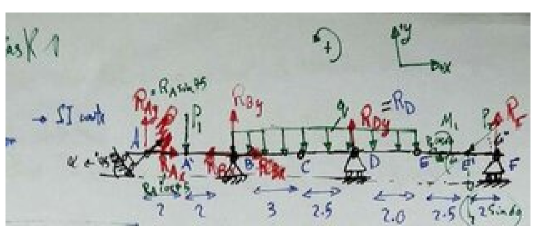


Illustration for the task

Solution

We have 5 unknown that's why we need 5 equations

- Dealing with the whole rod together

- $\sum F_x = 0$
- $\sum F_y = 0$
- $\sum T_A = 0$

- Dealing with only the separate body "AC"
  - $\sum F_x = 0$
  - $\sum F_y = 0$
  - $\sum T_A = 0$

We will get 6 equations, but we only need 5 of them. One of them will make the rank less than it should be. So, we will remove it to make the rank 5.

- Snippet from the code

```
AA2 = 2
A2B = 2
AB = AA2 + A2B
BC = 3
CD = 2.5
AD = AB + BC + CD
DE = 2
BE = BC + CD + DE
EE2 = 2.5
E2F = 2
EF = EE2 + E2F
AF = AB + BE + EF

P1 = 12
P2 = 18
M1 = 36
q = 1.4
Q = q*BE

alpha = 45
theta = to_rad(90-alpha)
beta = to_rad(60)

A = np.array([
                [-1, cos(theta), 0, 0, 0],
                [0, sin(theta), 1, 1, 1],
                [0, 0, AF, AB, AD],
                # [1, -cos(theta), 0, 0, 0],    This equation makes the rank of A not a full rank matrix
                [0, 0, 0, AB, 0],
                [0, sin(theta), 0, 1, 0]
             ])
B = np.array([
                P2*cos(beta),
                Q + P2*sin(beta) + P2,
                M1 + P1*(AA2) + Q*(BE/2+AB),
                # 0,
                q*BC*(BC/2+AB) + P1*AA2,
                q*BC + P1
```

```
             ])
# print(np.linalg.matrix_rank(A))
sol = np.linalg.solve(A,B)
```

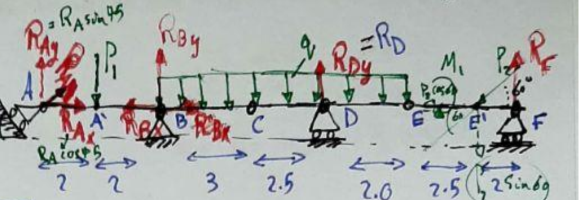
- Results from the python code

```
RA: 6.257895013500952,
RBx: -4.574999999999997,
RBy: 11.774999999999999,
RDy=RD: 54.14466404460282,
RF=RFy: -26.256206776482927
Notice that the negative sign means that the force is in the opposite direction regarding the assumed one
```

Handwritten solution with image

* Week HW4 → Task 1

* units are in Meters → SI units
, Newton
, sec



$\boxed{A'B = AA' = 2\,m = DE = E'F}$

, $q = 1.4$ N/m
$P_1 = 12$ N, $P_2 = 18$ N
$M_1 = 36$ N.m

BC = 3 , CD = 2.5

Draft

Research Objects
→ $\overline{AF}$ → Rigid Rod
→ A is a Roller support
→ B is a Pin connection → free ~~move~~ rotating joint
→ C is a bearing/joint → 0 torque
→ D is a Roller Support
→ E is a bearing/joint
→ F is a Roller support

*Extra forces
→ P_1 at A'
→ q on $\overline{BE}$ uniformally distributed
→ $-M_1$ torque
→ P_2 at E'

$(90 - \ldots) = \theta$

→ Force Analysis

? → R_A → $R_{Ax} = R_A \cos 45°$ $(+\vec{x})$; $R_{Ay} = R_A \sin 45°$ $(+\vec{y})$ → ~~Analyzed~~ Decomposed Reaction forces

?? → R_B → R_{Bx} $(+\vec{x})$; $+R_{By}$ $(+\vec{y})$ → Reaction force

✓ → $\vec{Q} = q \cdot 7.5 = -q \cdot (BC + CD + DE)$ $(-\vec{y})$

? → $+R_{Dy}$ $(+\vec{y}) = R_D$

✓ → $-M_1$ → torque

✓ → P_1 → $(-\vec{y})$

✓ → P_2 → $-P_2 \cos 60$ $(+\vec{x})$; $-P_2 \sin 60$ $(-\vec{y})$

? → $+R_F$ $(+\vec{y})$

Solution:

↳ we can work with the whole Rod (Rigid) or we can work with pieces due to the bearings

↳ starting with the whole Rod.

$\theta = (90-\alpha) = \theta \to \alpha = 45$

$45° = \alpha$ = Angle of incline of A obj.

$60° = \beta$ = Angle of P_2 with $\vec{x}$

→ Applying the conditions for equilibrium

* $\sum F_x = 0$

↳ $-R_{Bx} + R_A\cos\theta - P_2\cos\beta + \cancel{R_{Fx}} = 0$

$l_1 = BC$, $l_2 = CD$, $l_3 = DF$.

unknowns ↳ we need 3 eq.

* $\sum F_y = 0$

↳ $+R_A\sin\theta + R_{By} - q(\overbrace{l_1+l_2+l_3}^{BE}) + R_{Dy} - P_2\sin\beta + R_F - P_1 = 0$

* $\sum T_A = 0$ → $+R_{By}(\underbrace{A'B + AA'}_{AB}) + R_{Dy}(\underbrace{DC+CB+BA'+A'A}_{AD}) - M_1 + R_F\cdot AF - P_1 AA'$

$- q\left(\frac{BE}{2} + AB\right) = 0$

* Dealing with separate bodies → AC Body

↳ $\sum F_x = 0$
↳ $-R_{Bx} + R_A\cos\theta = 0 \to \boxed{R_{Bx} = R_A\cos\theta}$

↳ $\sum F_y = 0$ ↳ $R_A\sin\theta + R_{By} - q(BC) - P_1 = 0$

↳ $\sum T_A = 0$
↳ $R_{By}\cdot(AB) - (q\cdot(BC))\cdot\left(\frac{BC}{2} + AB\right) - P_1\, AA' = 0$

~~$R_A = x_1$ $R_{By} = z$ $R_{Dy} = z$~~

$R_{Bx} = x_1$
$R_A = x_2$
$R_F = x_3$
$R_{By} = x_4$
$R_{Dy} = x_5$

$$-x_1 + x_2\cos\theta - P_2\cos\beta = 0$$

$$+x_2\sin\theta + x_4 - Q + x_5 - P_2\sin\beta + x_3 - P_1 = 0$$

$$+x_4(AB) + x_5(AD) - M_1 + x_3\cdot AF - P_1AA' - Q\ldots = 0$$

$$\boxed{x_1 = x_2\cos\theta}$$

$$x_2\sin\alpha + x_4 - q\cdot BC - P_1 = 0$$

$$x_4AB - q(BC)\cdot\left(\frac{BC}{2} + AB\right) - P_1AA' = 0$$

$5\times5 \cdot 5\times1$

$$\begin{array}{c} 1\to \\ 2\to \\ 3\to \\ 4\to \\ 5\to \end{array}\begin{bmatrix} -1 & \cos\theta & 0 & 0 & 0 \\ 0 & \sin\theta & 1 & 1 & 1 \\ 0 & 0 & AF & AB & AD \\ \cancel{1} & -\cos\theta & 0 & 0 & 0 \\ 0 & \sin\theta & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} P_2\cos\beta \\ Q + P_2\sin\beta + P_1 \\ M_1 + P_1(AA') + Q\left(\frac{BE}{2} + AB\right) \\ \cancel{0} \\ +q(BC) + P_1 \end{bmatrix}$$

$$+q(BC)\cdot\left(\frac{BC}{2} + AB\right) + P_1AA'$$

$0 \quad 0 \quad 0 \quad AB \quad 0$

Task2:

Note: in the handwritten solution I was used ksi greek letter but in latex it is not defined, so I am using zeta greek letter instead of it.

We are using SI unit system

Givens:

- G = 18N (weight of ABCD is a thin plate), P =30N, a = 4m, b = 4.5m, c = 3.5m

Research objects:

- ABCD is a thin plate in a rectangular shape in the horizontal orientation.
- 6 Rods: 1,2,3,4,5,6 → connected to the plate by welding most probably as the bearing or joints don't seem suitable solutions for connection, unless the joints are some magnetic elements and connect easy with the rods and the thin plate
- $\alpha, \beta, \gamma, \zeta$ are free rotational joints.

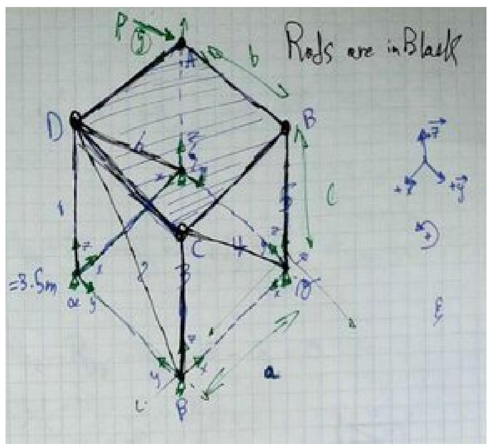


Illustration for the task

Force Analysis:

- $R_\alpha -> -R_{\alpha x},\ +R_{\alpha y},\ +R_{\alpha z}$ → $[-R_{\alpha x},\ +R_{\alpha y},\ +R_{\alpha z}]$
- $R_\beta -> -R_{\beta x},\ -R_{\beta y},\ +R_{\beta z}$ → $[-R_{\beta x},\ -R_{\beta y},\ +R_{\beta z}]$
- $R_\gamma -> +R_{\gamma x},\ -R_{\gamma y},\ +R_{\gamma z}$ → $[+R_{\gamma x},\ -R_{\gamma y},\ +R_{\gamma z}]$
- $R_\zeta -> +R_{\zeta x},\ +R_{\zeta y},\ +R_{\zeta z}$ → $[+R_{\zeta x},\ +R_{\zeta y},\ +R_{\zeta z}]$
- $G = -G_z -> [0, 0, -G_z]$ in the intersection of DB with AC (the center of mass of the thin plate)
- $P = +P_y -> [0, P_y, 0]$

Solution

Note that $\vec{T} = \vec{F} \times \vec{r}$

We have 12 unknown that's why we need 12 equations

- $\sum F_x = \vec{0}$

- $\sum \vec{F}_y = \vec{0}$
- $\sum \vec{T}_\zeta = \vec{0}$
- $\sum \vec{T}_\alpha = \vec{0}$
- $\sum \vec{T}_\gamma = \vec{0}$
- $\sum \vec{T}_A = \vec{0}$

From each equation we will get 3 equations, from here we can get 18 equations but some of them will make matrix A singular that's why we needed more equations than the unknowns as we need 12 independent equation with respect to the variables' coefficients.

- Snippet from the code

```
G = 18
P = 30
a,b,c = 4, 4.5, 3.5

A = np.array([
                [0,0,0,0,0,1,0,0,-1,0,0,0],
                [0,0,1,0,0,1,0,0,0,0,0,0],
                [0,-a,0,-b,a,0,0,0,0,0,0,0],
                [0,0,0,0,0,1,0,0,1,0,0,0],
                [0,0,0,0,0,0,0,1,0,0,0,1],
                [0,0,0,0,b,0,-b,a,0,0,a,0],
                # [0,0,1,0,0,0,0,0,0,0,0,0],
                [0,0,1,0,0,1,0,0,0,0,0,-b/a],
                [b,-a,0,0,-a,0,0,0,0,0,b,0],
                [-1,0,0,-1,0,0,1,0,0,1,0,0],
                [0,1,0,0,-1,0,0,-1,0,0,1,0],
                # [0,0,1,0,0,1,0,0,1,0,0,1],
                # [0,0,1,0,0,0,0,0,0,0,0,1],
                # [0,0,0,0,0,0,0,0,1,0,0,1],
                [b,0,0,0,0,0,0,a,0,b,-a,0],
                # [0,-c,0,0,c,-b,0,c,-b,0,-c,0]
                [1,0,a/c,-1,0,a/c,1,0,0,1,0,0],
             ])
B = np.array([
                G/2-P*c/b,
                G/2,
                0,
                P*c/b+G/2,
                G/2,
                P*a,
                # P*c/b+G/2,
                G/2,
                0,
                0,
                -P,
                # G
                # G/2-P*c/b,
                # G*a/2
                P*a,
                G*a/(c*2)
             ])
# print(np.shape(A))
# print(np.linalg.matrix_rank(A))
sol = np.linalg.solve(A,B)
```

- Results from the python code

```
R_alpha_x: -1.578983857244667e-15,
R_alpha_y: -10.23843416370107,
R_alpha_z: 6.217248937900877e-15,
R_beta_x: 22.386714116251486,
R_beta_y: 14.946619217081851,
R_beta_z: 8.999999999999996,
R_gamma_x: -4.440892098500626e-16,
R_gamma_y: 9.0,
R_gamma_z: 23.33333333333333,
R_ksi_x: 22.386714116251483,
R_ksi_y: 4.185053380782917,
R_ksi_z: 0.0,
Notice that the negative sign means that the force is in the opposite direction regarding the assumed one
```

Handwritten solution with image

Week HW4 , task ②

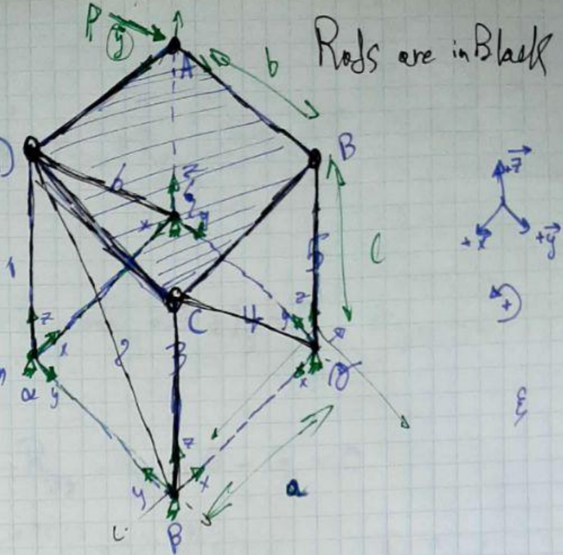
SI

$G = 18\ N$ $= mg$

ABCD is thin plate

$P = 30\,N$

$a = 4m$, $b = 4.5m$, $c = 3.5m$



* Research Objects
  - ABCD is a thin plate in rectangular shape horizontal direction
  - 6 Rods
    - 1, 2, 3, 4, 5, 6 → connected to the thin plate ~~by joints that has zero torque in~~ → ~~to treat as different objects~~
  - α, β, γ, ξ → are pin connectors/joints

* Force Analysis

12 unknown → 12 eq

$\vec{R}_\alpha \to -R_{\alpha x},\ +R_{\alpha y},\ +R_{\alpha z}$ | $\vec{R}_\beta \to -R_{\beta x},\ -R_{\beta y},\ +R_{\beta z}$ | $\vec{R}_\gamma \to +R_{\gamma x},\ -R_{\gamma y},\ +R_{\gamma z}$ | $\vec{R}_\xi \to +R_{\xi x},\ +R_{\xi y},\ +R_{\xi z}$

$[0, 0, -G]$ → $G \equiv -G_z$ affect in $-\vec{z}$ in intersection of $\overline{DB}$ with $\overline{AC}$

→ $P \equiv +P_y$ $+\vec{y}$

* Solution

$\vec{F} \times \vec{r}$

1 $\to \sum F_x = 0$

$\hookrightarrow -R_{\alpha x} - R_{\beta x} + R_{\gamma x} + R_{\xi x} = 0$

$$\vec{F} \times \vec{r} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ f_x & f_y & f_z \\ r_x & r_y & r_z \end{bmatrix}$$

$= (f_x \cdot r_y - f_y \cdot r_x)\hat{k}$ 3

$+ (f_y r_z - f_z r_y)\hat{i}$ 1

$+ (f_z r_x - f_x r_z)\hat{j}$ 2

2 $\to \sum F_y = 0$

$\hookrightarrow R_{\alpha y} - R_{\beta y} - R_{\gamma y} + R_{\xi y} \cancel{= 0} + P_y = 0$

3 $\to \sum F_z = 0$

$\hookrightarrow R_{\alpha z} + R_{\beta z} + R_{\gamma z} + R_{\xi z} \cancel{= 0} - G_z = 0$

4, 5, 6 $\to \sum \vec{M}_{\xi} = \vec{0} \to -P \cdot c - G \cdot \frac{d}{2} + R_{\gamma z} \cdot b - R_{\gamma x} \cdot b$

$+ R_{\beta x} \cdot b + R_{\beta z} \cdot \frac{a}{2} - R_{\beta y} \cdot a$

$+ R_{\alpha y} \cdot a - R_{\alpha z} \cdot a = 0$

$0, +P_y, 0$ $\quad 0, 0, c$ $\quad 0, 0, -G$ $\quad [a/2, b/2, c]$ $\quad (a, b, 0)$ $\quad 0, 0, c$

$\frac{d}{2} = \frac{\sqrt{a^2+b^2}}{2} = \frac{AC}{2}$

a, b, B, C, A, D

$[a, b, 0]$

direction is directly inside the vector

$\frac{a}{2}$, $\frac{b}{2}$, AC

$[Pc, 0, 0] + [\frac{Gb}{2}, \frac{-Ga}{2}, 0] + [R_{\gamma z} b, 0, 0] + [0, 0, -R_{\gamma x} b]$

$+ [0, 0, R_{\beta x} \cdot b] + [R_{\beta z} \frac{b}{2}, R_{\beta z}, 0] + [R, 0, \quad]$

7, 8, 9 $\to \sum \vec{M} = \vec{0}$

10, 11, 12 $\hookrightarrow \sum \vec{M}_B = \vec{0}$

$$\sum \vec{T}_{\gamma} = \vec{0}$$

$$\hookrightarrow [0, P_y, 0] \times [0, -b, c]$$
$$+ [0, 0, -G] \times \left[\tfrac{+a}{2}, \tfrac{-b}{2}, c\right]$$
$$+ [R_{\beta x}, R_{\beta y}, R_{\beta z}] \times [a, 0, 0]$$
$$+ [R_{\alpha x}, R_{\alpha y}, R_{\alpha z}] \times [a, -b, 0]$$
$$+ [R_{\xi x}, R_{\xi y}, R_{\xi z}] \times [-b, 0, 0]$$

$$\left[P_y c - \frac{Gb}{2} + R_{\alpha z} b + R_{\xi z} b,\ \frac{Ga}{-2} - R_{\alpha z} a - R_{\xi z} a,\ P_y a - R_{\alpha x} b - R_{\alpha y} a - R_{\xi x} b + R_{\xi y} a\right] = [0, 0, 0]$$

$$\sum \vec{T}_{\beta} = \vec{0}$$

$$[0, P_y, 0] \times \left[-a, -b, c\right] = [P_y c, 0, +P_y a]$$
$$+ [0, 0, -G] \times \left[\tfrac{-a}{2}, \tfrac{-b}{2}, c\right] + \left[-\frac{Gb}{2}, +\frac{Ga}{2}, 0\right]$$
$$+ [-R_{\alpha x}, R_{\alpha y}, R_{\alpha z}] \times [0, -b, 0] + [+R_{\alpha z} b, 0, -R_{\alpha x} b]$$
$$+ [R_{\gamma x}, -R_{\gamma y}, R_{\gamma z}] \times [-a, 0, 0] + [0, -R_{\gamma z} a, -R_{\gamma y} a]$$
$$+ [R_{\xi x}, R_{\xi y}, R_{\xi z}] \times [-a, -b, 0] + [+R_{\xi z} b, -R_{\xi z} a, -R_{\xi x} b + R_{\xi y} a]$$

Scanned with CamScanner

$(f_y r_z - f_z r_y)\hat{i} + (f_z r_x - f_x r_z)\hat{j} + (f_x r_y - f_y r_x)\hat{k}$

$\overset{?}{\to} \sum \vec{T}_\xi = \vec{0}$

$\hookrightarrow [0, P_y, 0] \times [0, 0, c]$

$+ [0, 0, -G] \times [\frac{a}{2}, \frac{b}{2}, c]$

$+ [0, 0, R_{\gamma z}] \times [0, b, 0]$

$+ [R_{\gamma x}, 0, 0] \times [0, b, 0]$ $\quad \rightarrow [R_{\gamma x}, -R_{\gamma y}, R_{\gamma z}] \times [0, b, 0]$

$+ [-R_{\beta x}, 0, 0] \times [a, b, 0]$

$+ [0, 0, R_{\beta z}] \times [a, b, 0]$ $\quad \rightarrow [-R_{\beta x}, -R_{\beta y}, R_{\beta z}] \times [a, b, 0]$

$+ [0, -R_{\beta y}, 0] \times [a, b, 0]$

$+ [0, R_{\alpha y}, 0] \times [a, 0, 0]$ $\quad \rightarrow [-R_{\alpha x}, R_{\alpha y}, R_{\alpha z}] \times [a, 0, 0]$

$+ [0, 0, R_{\alpha z}] \times [a, 0, 0] = 0$

Coordinate frame at α

$\sum \vec{T}_\alpha = \vec{0}$

$\hookrightarrow [0, P_y, 0] \times [-a, 0, c]$

$+ [0, 0, -G] \times [-\frac{a}{2}, +\frac{b}{2}, c]$

$+ [0, 0, +R_{\beta z}] \times [0, b, 0]$

$+ [R_{\beta x}, 0, 0] \times [0, b, 0]$ $\quad \rightarrow [R_{\beta x}, -R_{\beta y}, R_{\beta z}] \times [0, b, 0]$

$+ [R_{\gamma x}, 0, 0] \times [-a, b, 0]$

$+ [0, R_{\gamma}, 0] \times [-a, b, 0]$ $\quad \rightarrow [R_{\gamma x}, -R_{\gamma y}, +R_{\gamma z}] \times [-a, b, 0]$

$+ [0, 0, +R_{\gamma z}] \times [-a, b, 0]$

$+ [0, R_{\xi y}, 0] \times [-a, 0, 0]$

$+ [0, 0, +R_{\xi z}] \times [-a, 0, 0]$ $\quad [R_{\xi x}, R_{\xi y}, R_{\xi z}] \times [-a, 0, 0]$

$\sum \vec{T}_\xi = \vec{0}$

$-R_{\beta x} b + R_{\beta y} a$

$[P_y c, 0, 0] + [-\frac{Gb}{2}, -\frac{Ga}{2}, 0] + [-R_{\gamma z} b, \cancel{R_{\gamma y}}\, 0, R_{\gamma x} b] + [R_{\beta z} b, R_{\beta z} a, \ ]$

$+ [0, R_{\alpha z} a, -R_{\alpha y} a] = [0, 0, 0]$

$= [P_y c - \frac{Gb}{2} - R_{\gamma z} b + R_{\beta z} b, -\frac{Ga}{2} + R_{\beta z} a + R_{\alpha z} a, R_{\gamma x} b - R_{\beta x} b + R_{\beta y} a - R_{\alpha y} a]$

$= [0, 0, 0]$

$\sum \vec{T}_\alpha = \vec{0}$

$[P_y c, 0, +P_y a] + [+\frac{Gb}{2}, +\frac{Ga}{2}, 0] + [-R_{\beta z} b, 0, -R_{\beta x} b] + [-R_{\gamma z} b, -R_{\gamma y} a, \ ]$

$R_{\gamma x} b - R_{\gamma y} a$

$+ [\cancel{P_y c, 0,}\ 0, -R_{\xi z} a, -R_{\xi y} a] = [0, 0, 0]$

$[P_y c + \frac{Gb}{2} - R_{\beta z} b - R_{\gamma z} b, \frac{Ga}{2} - R_{\gamma y} a - R_{\xi z} a, P_y a - R_{\beta x} b + R_{\gamma x} b - R_{\gamma y} a - R_{\xi y} a] = [0, 0, 0]$

$\sum \vec{T}_\gamma = \vec{0}$

$[P_y c, 0, 0] + [+\frac{Gb}{2}, -\frac{Ga}{2}, 0] + [0, R_{\beta z} a, -R_{\beta y} a]$

$+ [-R_{\alpha z} b, R_{\alpha z} a, +R_{\alpha x} b - R_{\alpha y} a] + [0, -R_{\xi z} b, +R_{\xi y} b] = [0, 0, 0]$

$\Rightarrow [P_y c + \frac{Gb}{2} - R_{\alpha z} b, -\frac{Ga}{2} + R_{\beta z} a + R_{\alpha z} a - R_{\xi z} b, -R_{\beta y} a + R_{\alpha x} b - R_{\alpha y} a + R_{\xi y} b]$

$= [0, 0, 0]$

$$\sum \vec{T}_A = \vec{0}$$

$$[0, 0, -G] \times [a/2, b/2, 0] = \left[\frac{+Gb}{2}, \frac{-Ga}{2}, 0\right]$$

$$+ [R_{\alpha x}, R_{\alpha y}, R_{\alpha z}] \times [a, 0, -c] + [-R_{\alpha y}c, R_{\alpha z}a + R_{\alpha x}c, -R_{\alpha y}a]$$

$$+ [-R_{\beta x}, -R_{\beta y}, R_{\beta z}] \times [a, b, -c] + [+R_{\beta y}c - R_{\beta z}b, aR_{\beta z} - R_{\beta x}c, -R_{\beta x}b + R_{\beta y}a]$$

$$+ [R_{\gamma x}, -R_{\gamma y}, R_{\gamma z}] \times [0, b, -c] + [+R_{\gamma y}c - R_{\gamma z}b, +R_{\gamma x}c, +R_{\gamma x}b]$$

$$+ [R_{\xi x}, R_{\xi y}, R_{\xi z}] \times [0, 0, -c] + [R_{\xi y}c, +cR_{\xi x}, 0]$$

$$\left[\frac{Gb}{2} - R_{\alpha y}c + R_{\beta y}c - R_{\beta z}b + R_{\gamma y}c - R_{\gamma z}b - R_{\xi y}c = 0\right] = [0, 0, 0]$$

$$, \frac{-Ga}{2} + R_{\alpha z}a + R_{\alpha x}c + aR_{\beta z} - R_{\beta x}c + R_{\gamma x}c + cR_{\xi x} = 0$$

$$, 0 = -R_{\alpha y}a - R_{\beta x}b + R_{\beta y}a + R_{\gamma x}b = 0$$

$P_y = x_0$

$G = x_1$

$R_{\alpha x} = x_8$, $R_{\alpha y} = x_3$, $R_{\alpha z} = x_2$

$R_{\beta x} = x_3$, $R_{\beta y} = x_4$, $R_{\beta z} = x_5$

$R_{\gamma x} = x_6$, $R_{\gamma y} = x_7$, $R_{\gamma z} = x_8$

$R_{\xi x} = x_9$, $R_{\xi y} = x_{10}$, $R_{\xi z} = x_{11}$

$$\frac{P_y c}{2} + \frac{-Gb}{2} - b x_8 + b x_5 + \frac{-Ga}{2} + a x_5 + a x_2 + x_0 = 0$$

$$x_5 - x_8 = \frac{G}{2} - \frac{P_y c}{b} \quad (1)$$

$$x_2 + x_5 = \frac{G}{2} \quad (2)$$

$$-a x_1 - b x_3 + a x_4 + b x_6 = 0 \quad (3)$$

$$x_5 + x_8 = \frac{P_y c}{b} + \frac{G}{2} \quad (4)$$

$$x_7 + x_{11} = \frac{G}{2} \quad (5)$$

$$b x_4 - b x_6 + a x_7 + a x_{10} = P_y a \quad (6)$$

$$x_2 = \frac{P_y c}{b} + \frac{G}{2} \quad (7)$$

$$x_5 + x_2 - \frac{b x_{11}}{a} = \frac{G}{2} \quad (8)$$

$$-a x_4 + b x_0 - a x_1 + b x_{10} = 0 \quad (9)$$

$$-x_0 - x_3 + x_6 + x_9 = 0 \quad (10)$$

$$x_1 - x_4 - x_7 + x_{10} = -P_y \quad (11)$$

$$x_2 + x_5 + x_8 + x_{11} = G_z \quad (12)$$

$$x_2 + x_{11} = \frac{G}{2} - \frac{Pc}{b} \quad (13)$$

$$x_8 + x_{11} = \frac{Ga}{2} \quad (14)$$

$$b x_0 + a x_7 + b x_9 - a x_{10} = P_y a$$

$$-c x_1 + c x_4 - b x_5 + c x_7 - b x_8 - c x_{10} = -\frac{Gb}{2} \quad (16)$$

$$x_0 + \frac{a}{c} x_2 - x_3 + \frac{a}{c} x_5 + x_6 + x_9 = \frac{Ga}{c2} \quad (17)$$

Scanned with CamScanner

$$
\begin{array}{c|cccccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & -a & 0 & -b & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
6 & 0 & 0 & 0 & 0 & b & 0 & -b & a & 0 & 0 & a & 0 \\
7 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & b/a \\
9 & b & -a & 0 & 0 & -a & 0 & 0 & 0 & 0 & 0 & b & 0 \\
10 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
11 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
12 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1
\end{array}
\times
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \end{bmatrix}
=
\begin{bmatrix} \frac{G}{2} - \frac{P_y C}{b} \\ G/2 \\ 0 \\ PC/b + G/2 \\ G/2 \\ Pa \\ PC/b + G/2 \\ G/2 \\ 0 \\ 0 \\ -P_y \\ G \end{bmatrix}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array}
$$

12, 7

Redundant

$$
\begin{array}{c|cccccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
\hline
13 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
15 & b & 0 & b & 0 & 0 & 0 & 0 & a & 0 & b & -a & 0 \\
16 & 0 & -c & 0 & 0 & c & -b & 0 & c & -b & 0 & -c & 0 \\
17 & 1 & 0 & a/c & -1 & 0 & a/c & 1 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

$\frac{G}{2} - \frac{PC}{b}$

$Ga/2$

$P_y a$

$-Gb/2$

$Ga/(c2)$

Scanned with CamScanner