Theoretical Mechanics

Homework 1. Innopolis University, Fall 2020

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Task 1

Solution

The system is about particle that is moving in a specific trajectory and this trajectory appeared to be a parabola.

First, $t \in [-5, 5]$ from the description of the question with 1000 timestamp in that range (1) X:

$$\begin{cases} x = 3t \\ y = 4t^2 + 1 \end{cases} \tag{1}$$

if we transform the equation from the coordinate form:
$t = \dfrac{x}{3}$
$y = \dfrac{4}{9}x^2 + 1$
Then, we can notice it is a parabola in the following form: $4p(y-k) = (x-h)^2$, $k = 1, p = \dfrac{9}{16}, h = 0$

TM, Innopolis University, Fall 2020
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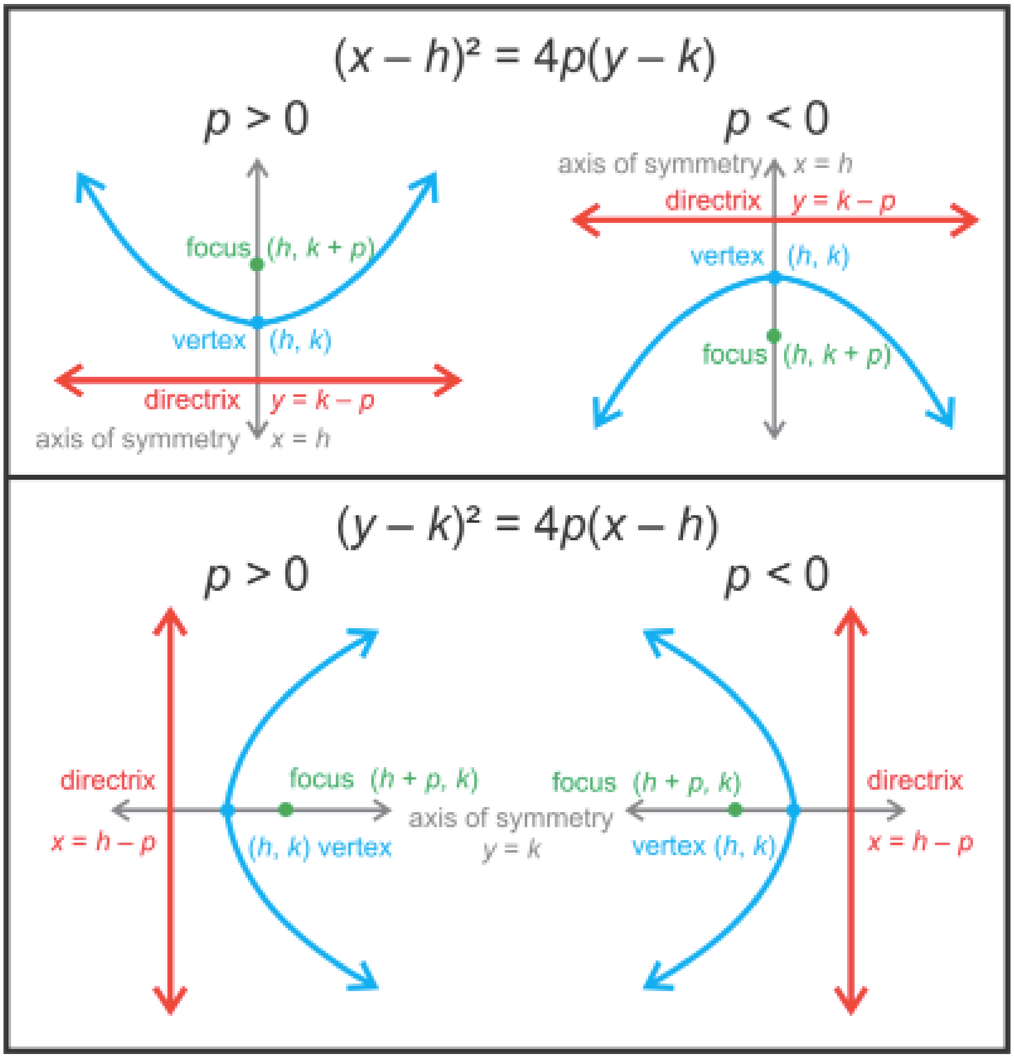


Figure1: Parabola equations and the positions of the focus.

Then focus, the center of rotation, is positioned at $(0, \frac{25}{16})$

Thus, the radius of the rotation is $\vec{r} = \vec{focus} - \vec{x} = (0 - 3t, \frac{25}{16} - 4t^2 + 1)$
(2) v:

$$\begin{cases} v_x = 3 \\ v_y = 8t \end{cases} \tag{2}$$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{9 + 64t^2}$

(3) a, a_n, a_t: Reference

$a_n = \frac{dv}{dt} = \frac{4096t}{\sqrt{9+64t^2}}$

$a_t = \frac{v^2}{r} = (9+64t^2)*(\frac{3}{128}+\frac{41}{2}t^2)$

$a = \sqrt{a_n^2 + a_t^2}$

(4) ρ: Reference1, Reference2

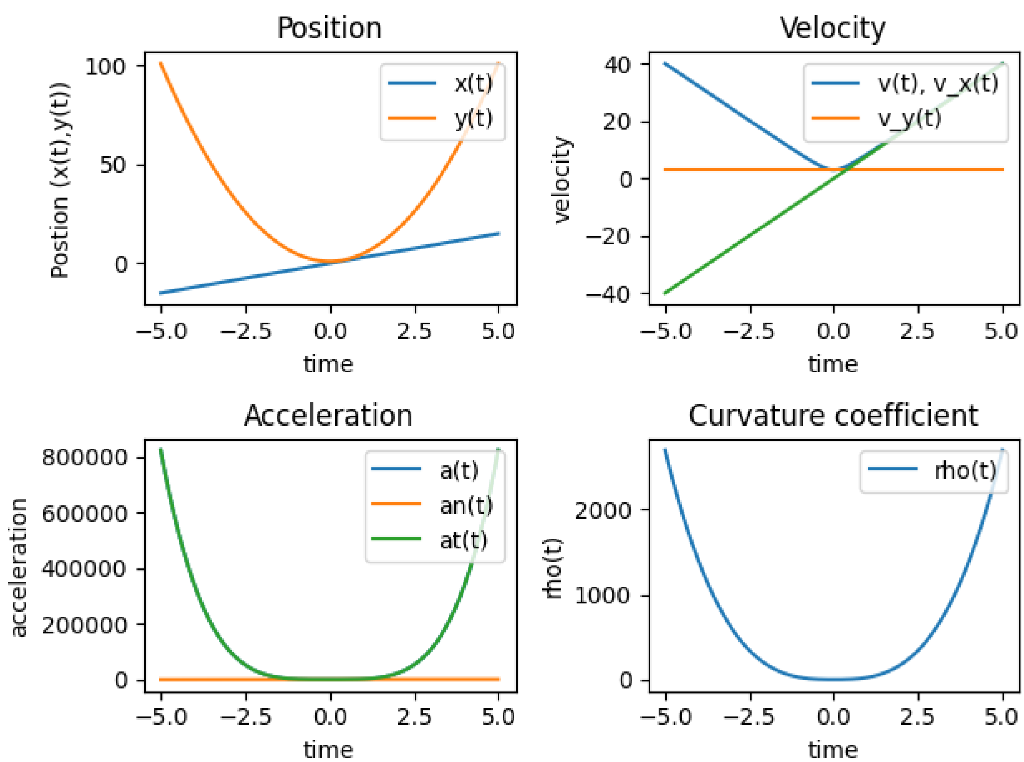
We find κ the curvature:

$k = \frac{24}{(9+64t^2)^{\frac{3}{2}}}$

and $\rho = \frac{1}{\kappa} = \frac{(9+64t^2)^{\frac{3}{2}}}{24}$

Plots

For the particle



Task 2.1

Solution

The system is about 2 revolute joints with one constraint that is the angle OPB and it is physically constructed with walls to slide in a specific direction. The rotations is powered from rotating the joint on O CCW (Positive rotation). The positive for x-axis is for left, the positive for y-axis is for up.

This will be solved using geometry, to find point B, it is identified by the intersection between PB line and a circle that centered in A with radius BA. And to find point C, it is the intersection between PB line and a circle that centered in A with radius CA.

First, we noted that $\phi = \omega.t$ such that $t \in [0, 1]$ with 1000 timestamp in that range

For A:

$$\begin{cases} x_a = OA.sin(\phi) \\ y_a = OA.cos(\phi) \end{cases} \tag{3}$$

1^{st} derivative with respect to t in order to get the velocity, and 2^{nd} derivative with respect to t in order to get the acceleration

$$\begin{cases} vx_a = OA.\omega.cos(\phi) \\ vy_a = -OA..sin(\phi) \end{cases} \tag{4}$$

For B: Equation of circle that centered in A with radius AB:

$(x - x_a)^2 + (y - y_a)^2 = (AB)^2$

Equation of PB line, such that the y-axis intercept (b) = PO, slope(m) = $tan(30)$

$y = mx + b$

And solving them together using "fsolver" or analytically to get the intersection point which is point B.

Then, for now, trying to get the Instantaneous point of zero velocity(D):

First, we can get from the triangle OAB, the angle of OBA

$$(\psi) = arcsin(\frac{OA}{AB}.sin(\pi/2 - \phi)$$

, such that phi is the angle between AO and PO the horizontal line.

Second, we can get from the triangle CBO, the angle ODB

$$(\theta) = \phi - \psi$$

Third, get

$$AD = \frac{AB}{sin(\theta)}$$

$$BD = \sqrt{AD^2 - AB^2}$$

Fourth, we can get the angular velocity of link AB

$$w_{AB} = \frac{}{AD}$$

Fifth, we can get now the linear velocity of B:

$$v_b = w_{AB}.BD$$

Sixth, we can get the tangential and normal acceleration and calculate the acceleration for B:

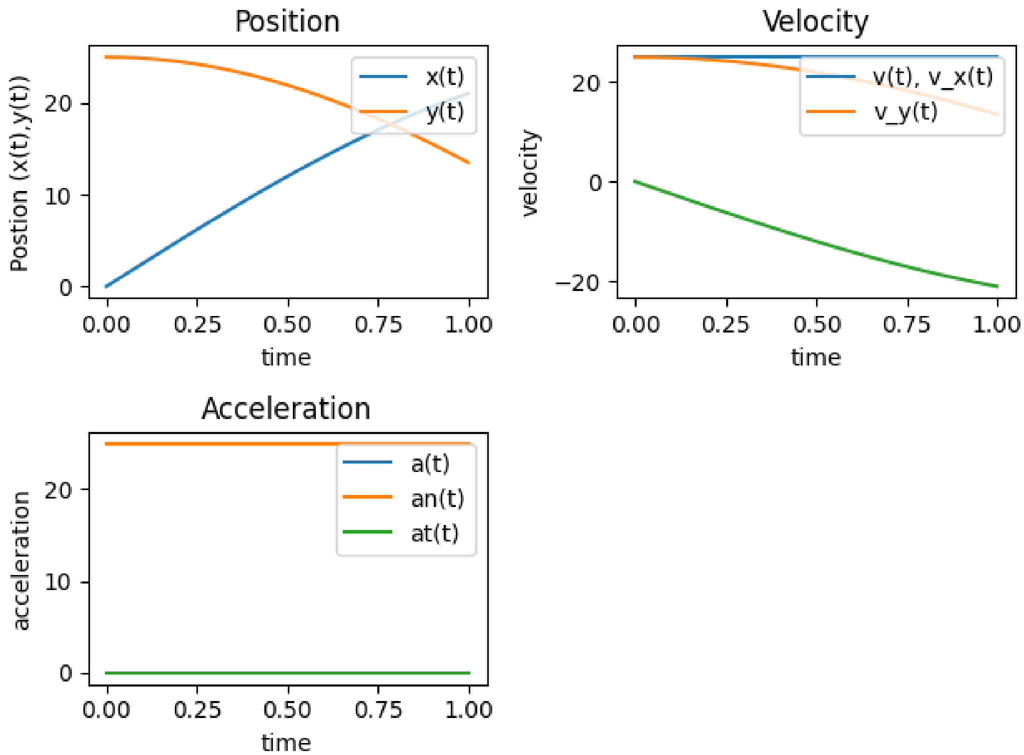
$$an_b = w_{AB}^2.BD$$
$$at_b = \alpha.BD^2$$

such that $\alpha = \epsilon = angular\ acceleration = 0$

$$a = \sqrt{at^2 + an^2}$$

For point C, we can do the same, to solve the line of PB as C is on the same line but with the intersection with the circle of A as a center and radius of AC, or it can be solved as relative to point B or A.

Plots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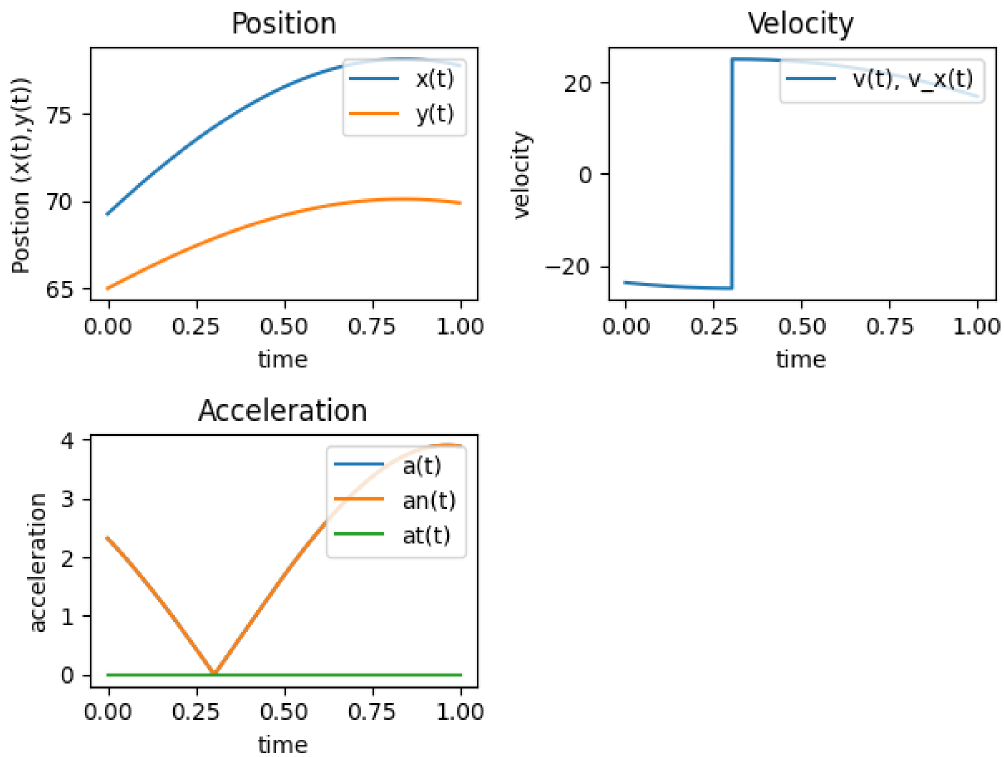
Postion (x(t),y(t))
time
x(t)
y(t)
Velocity
velocity
time
v(t), v_x(t)
v_y(t)
Acceleration
acceleration
time
a(t)
an(t)
at(t)

- **For A**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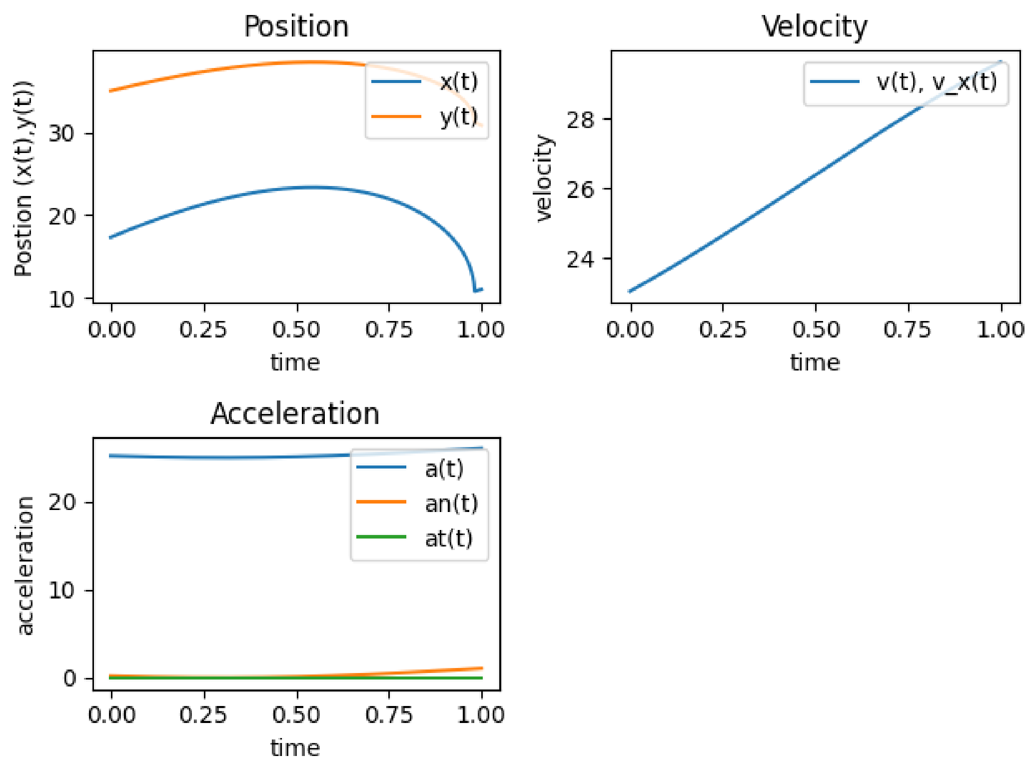
time

- **For B**



- **For C**

Task 2.2

Solution
The system is constructed from 2 revolute joints with one constraint that object B is moving in x-axis only and it is physically with walls to slide in a specific direction. The rotations is powered from rotating the joint on O CCW (positive rotation). The positive for x-axis is for right, the positive for y-axis is for up.

First, we noted that $\phi = \omega.t$ such that $t \in [0, 1]$ with 1000 timestamp in that range

For A:

$$\begin{cases} x_a = -OA.sin(\phi) \\ y_a = OA.cos(\phi) \end{cases} \tag{5}$$

Then, we can get the velocity, tangential acceleration, normal acceleration and acceleration:

$$v_a = \omega.OA$$
$$an_a = \omega^2.OA$$
$$at_a = \alpha.OA^2$$
$$a_a = \sqrt{an_a^2 + at_a^2}$$

For B:

First, we get angle ABO β, angle BAO α:

$$\beta = arcsin(\frac{OA.cos(\phi)}{AB}$$
$$\alpha = \pi/2 - \beta + \phi$$

such that ϕ is the angle between the horizontal axis and OA link

$$\begin{cases} x_b = \sqrt{(}AB^2 - y_a^2) - |x_a| \\ y_b = 0 \end{cases} \tag{6}$$

$$v_b = OA.sin(phi).\omega$$

Now, we can determine the point of IC (P) by the intersection of the perpendiculars on the velocity from AB link.

$$PB = \frac{x_b}{tan(\phi)}$$
$$w_{AB} = v_b/PB$$
$$an_b = w_{AB}^2.PB$$
$$at_b = \alpha.PB^2$$

such that $\alpha = \epsilon = angular\ acceleration = 0$

$$a = \sqrt{at^2 + an^2}$$

For point C:

$$\begin{cases} x_c = x_b - \dfrac{BC * x_B}{AB} \\ y_b = \dfrac{BC.y_a}{AB} \end{cases} \tag{7}$$

using $\vec{v_c} = \vec{v_b} + \vec{v_{c/b}} = (v_b, 0) + \omega_{AB}\ x\ \vec{r_b}c$, such that $\omega_{AB}s = (0, 0, w), \vec{r_{bc}} = (BC.cos(\beta), -BC.sin(\beta))$

$$\begin{cases} vx_c = w_{AB} * BC * sin(beta) + v_b \\ vy_b = -w_{AB} * BC * cos(beta) \end{cases} \tag{8}$$

and $v_c = \sqrt{vx_c^2 + vy_c^2}$

$$PC = v_c / w_{AB}$$

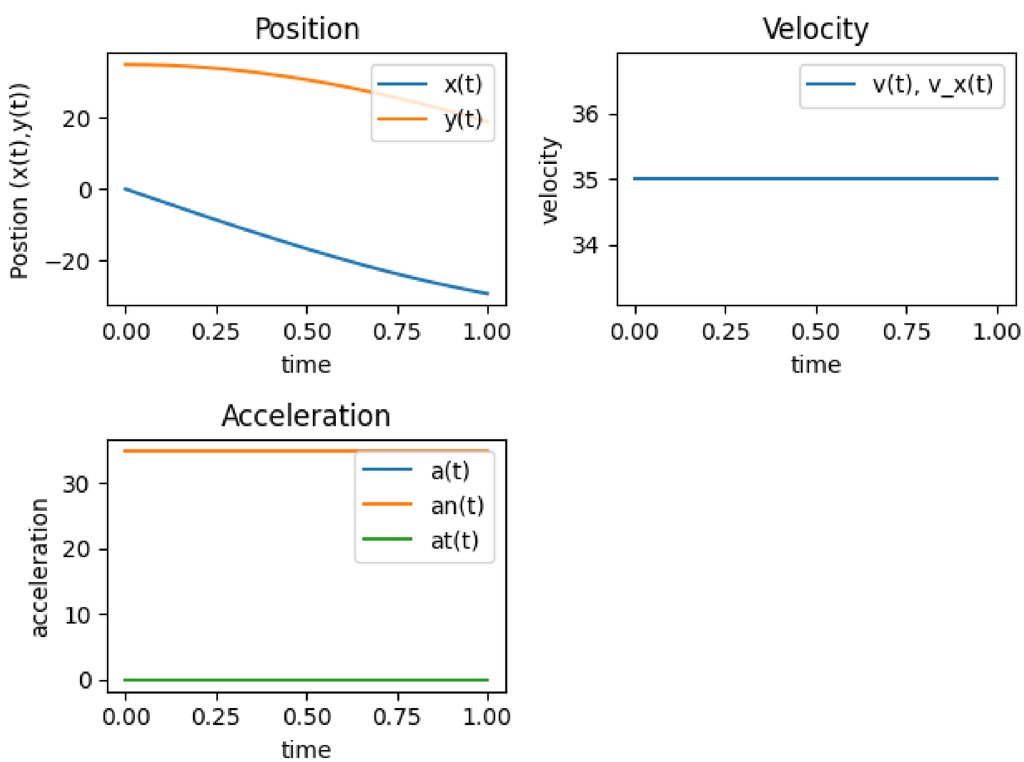
$$an_c = w_{AB}^2 . PC$$

$$at_c = \alpha . PC^2$$

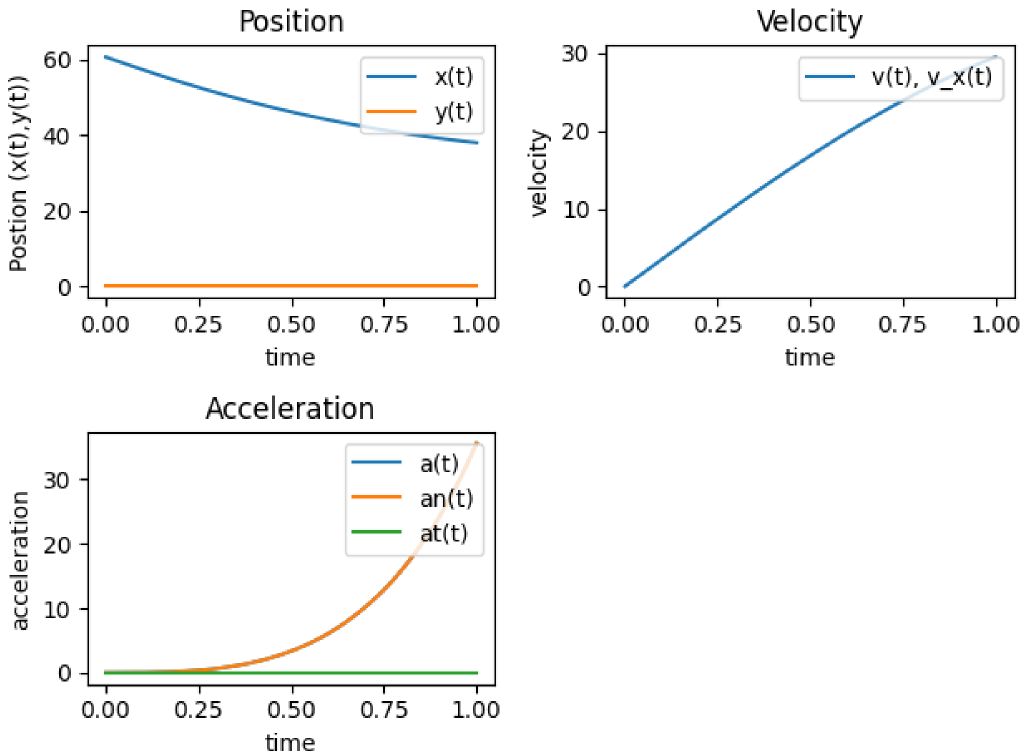
such that $\alpha = \epsilon = angular\ acceleration = 0$

$$a = \sqrt{at^2 + an^2}$$

Plots



- **For A**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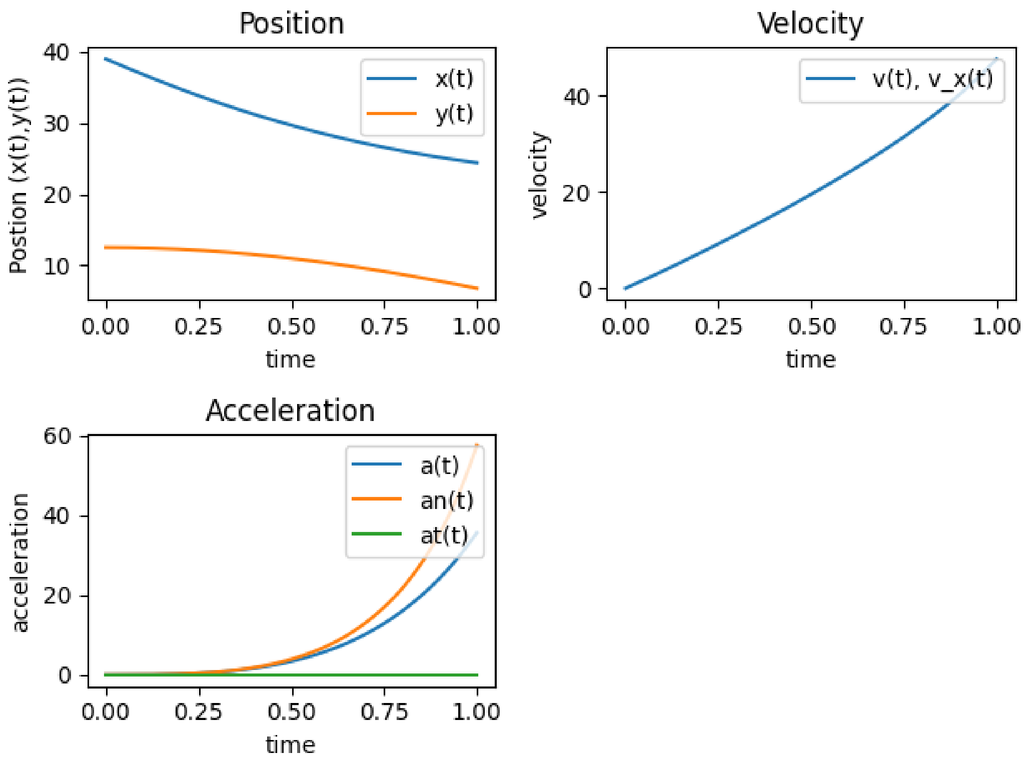
- **For B**



- **For C**

Task 3

Solution
This system is about two moving objects that sliding in x and y axis respectively, with joints on top of them. Object A is moving in y-axis and object B is following the movement in its direction in x-axis. The positive for x-axis is for left, the positive for y-axis is for up.

First, we noted that $\phi = \omega.t$ such that $t \in [0, 10]$ with 10000 timestamps in that range

$$\begin{cases} x_a = 0 \\ y_a = 22.5 + 10sin(\pi t/5) \end{cases} \tag{9}$$

$$v_a = 2(\pi t/5)$$

only in the direction of y.

$$\begin{cases} x_b = \sqrt{AB^2 - y_a^2} \\ y_b = 0 \end{cases} \tag{10}$$

Now, we can determine the point of IC (P) by the intersection of the perpendiculars on the velocities from AB link. PAOB will be like square with x_b, y_a as sides.

$$PA = x_b$$
$$w_{AB} = \frac{v_a}{PA}$$
$$an_a = w_{AB}^2 PA$$
$$at_a = \frac{dv_a}{dt} = -2/5 * (\pi^2) * sin(7pi/5 * t)$$
$$a_a = \sqrt{at_a^2 + an_a^2}$$

$$at_a/PA^2 = \alpha$$

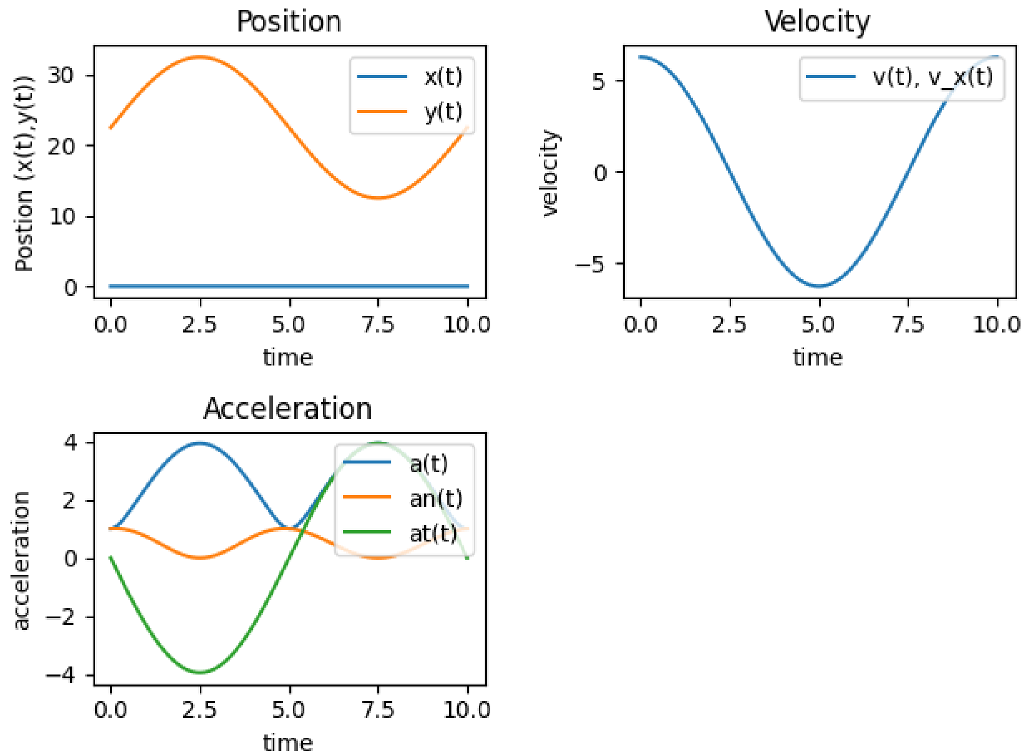
such that $\alpha = \epsilon = angular\ acceleration = 0$

$$PB = y_a$$
$$v_b = w_{AB} * PB$$
$$an_b = w_{AB}^2 * PB$$
$$at_b = \alpha^2 * PB$$
$$a_b = \sqrt{an_b^2 + at_b^2}$$

$$\begin{cases} x_c = BC * x_b/AB \\ y_c = BC * y_a/AB \end{cases} \tag{11}$$

$$PC = \sqrt{(}(y_a - y_c)^2 + (x_b - x_c)^2)$$
$$v_c = PC * w_{AB}$$
$$an_c = w_{AB}^2 * PC$$
$$at_c = (\alpha^2) * PC$$
$$a_c = \sqrt{(}an_c^2 + at_c^2)$$

Plots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v(t), v_x(t)
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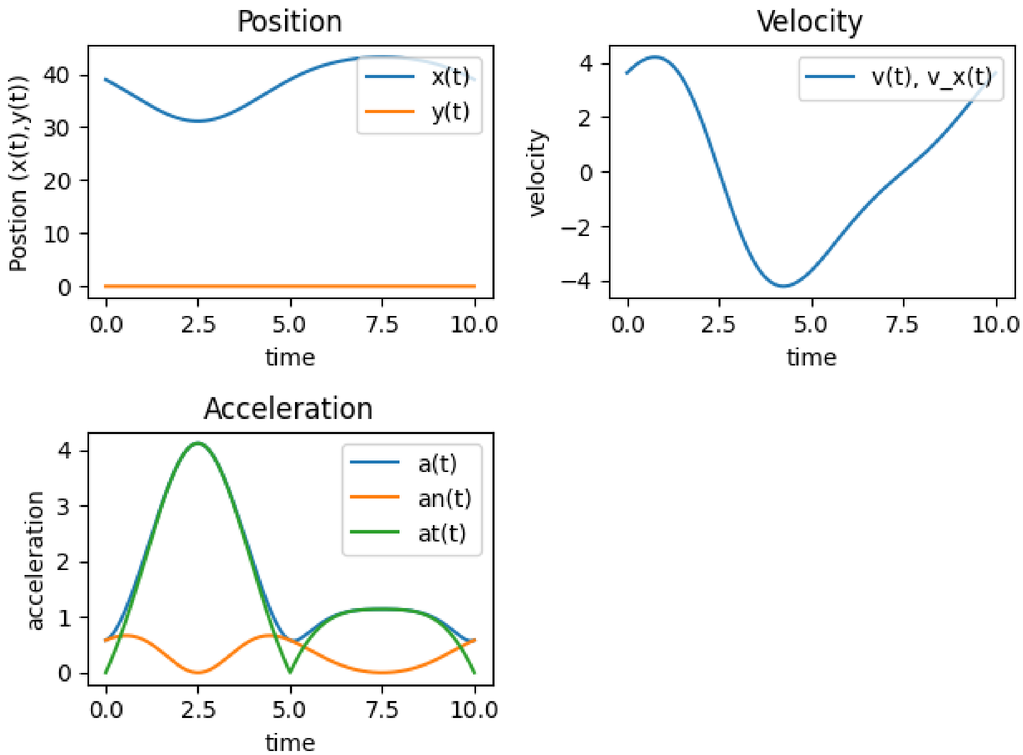
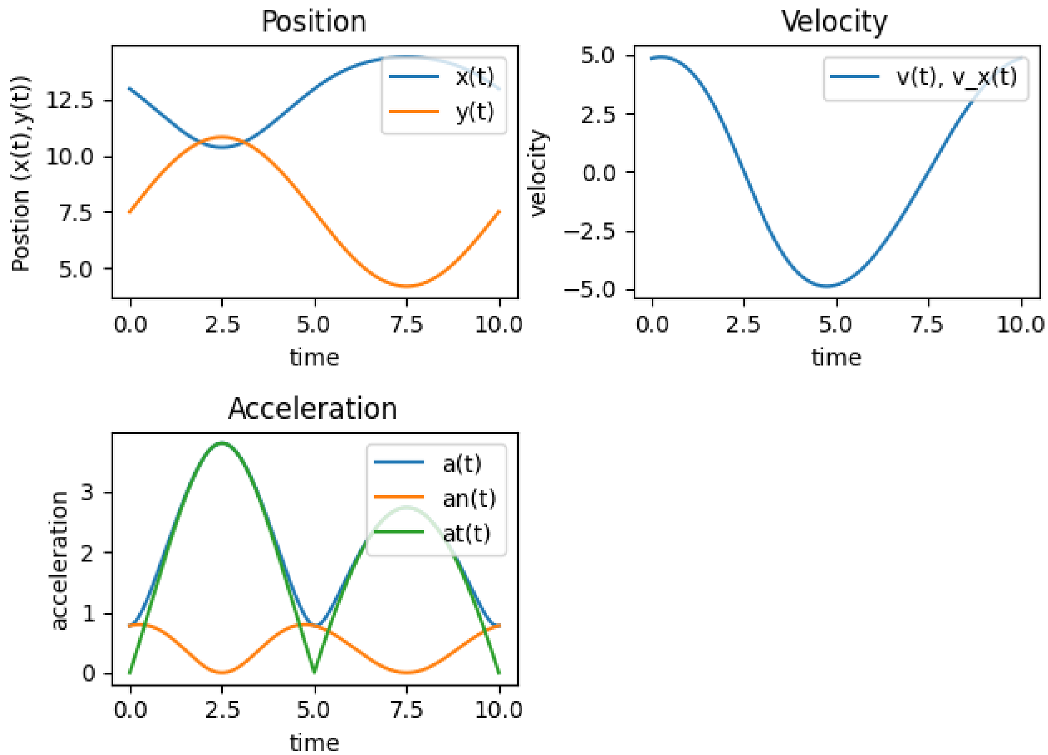
Acceleration
acceleration
a(t)
an(t)
at(t)
time

- **For A**


Position
x(t)
y(t)
Postion (x(t),y(t))
time
Velocity
v(t), v_x(t)
velocity
time
Acceleration
a(t)
an(t)
at(t)
acceleration
time

- **For B**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velocity
time
Acceleration
a(t)
an(t)
at(t)
acceleration
time

- **For C**