TM Assignment 2

Class	TM
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Materials	
Reviewed	☐
Type	

Student Information

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- Group: BS18-RO
- Big HW2 - Assignment2

Tools

- Python (numpy, Matplotlib)
- LaTex
- Camera
- Caliper
- Ruller/measuring tape
- Markdown

- Notions

Description

In this assignment, the task is to create a mathematical model for Yo-yo with specific dimensions as it was specified in the assignment document.

In order to confirm that the created mathematical model is correct, it is required to compare our results with measurements from the real world.

Yo-yo dimensions:

- Small inner radius $r_0 = 0.65cm = 0.0065m$
- Big outer radius $R = 2.5cm = 0.025m$
- Inertia tensor is stated in the assignment description (J)
- Yo-yo mass is stated in the assignment description (m)
- Volume is stated in the assignment description

Experiments

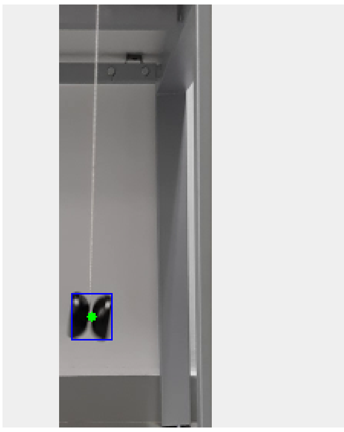
Five experiments has been done and each experiment has been repeated 3 times in order to remove some non-deterministic errors in the measurements.

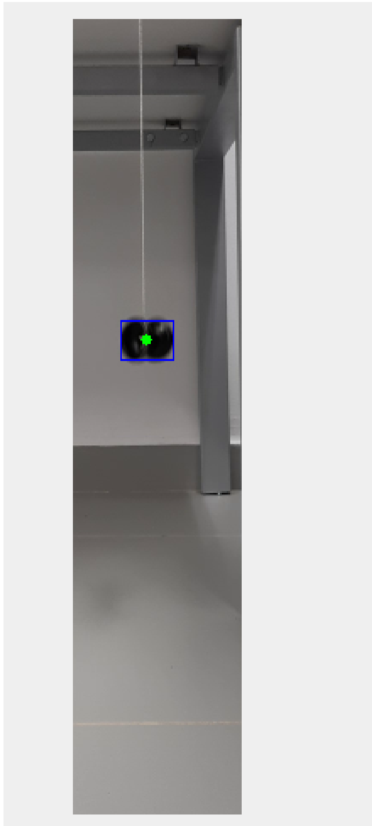
The experiments data has been measured using a code written in python using opencv and matplotlib libraries. The recording of the experiments made by phone's camera then with the code, we could identify the Yo-yo by converting to grayscale, threshold and then finding connected edges (contours) and then find the Yo-yo center of mass and then saving these data such that vertical position is measured in pixels manner then using the knowledge that we know the last length of the string, we can calculate the ratio between vertical position in pixels and vertical position in cm, and we can get the vertical position in cm along all the time steps.

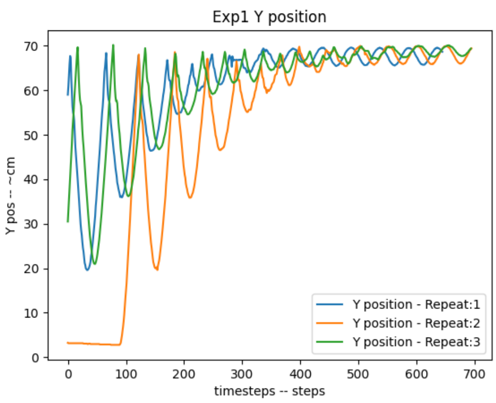
Then, using matplotlib, we can visualize the graphs of the motion and we can know the positions of minima and maxima with just pointing to the screen with the mouse.

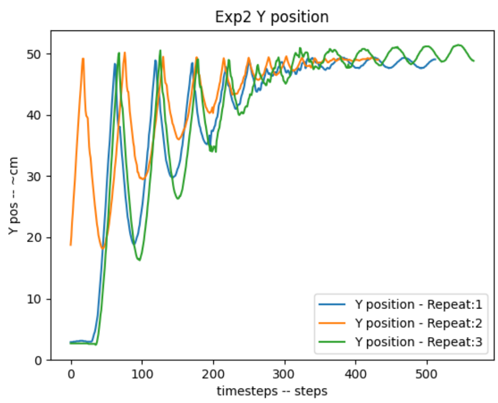
Note: if we could use motion capture systems it would be much better results and accurate results for the experiments [1]. Moreover, there was an interesting idea to use phone to capture the data [2].

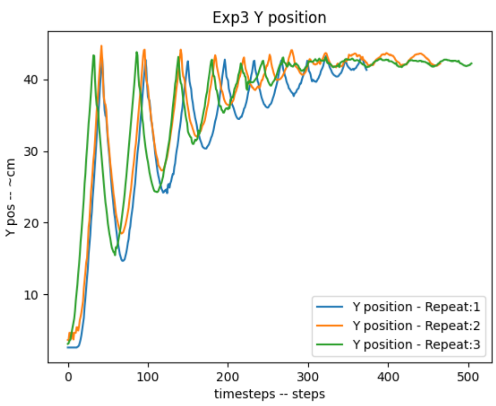
Fig. 1: Illustration for finding the Yo-yo using opencv

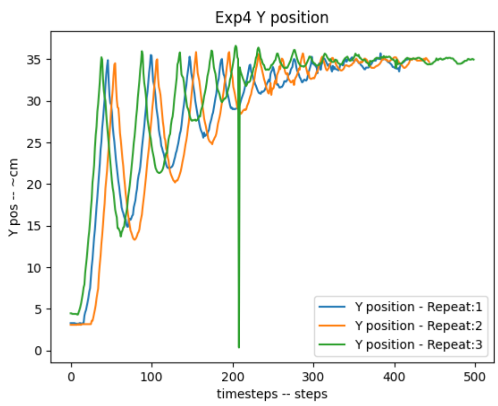












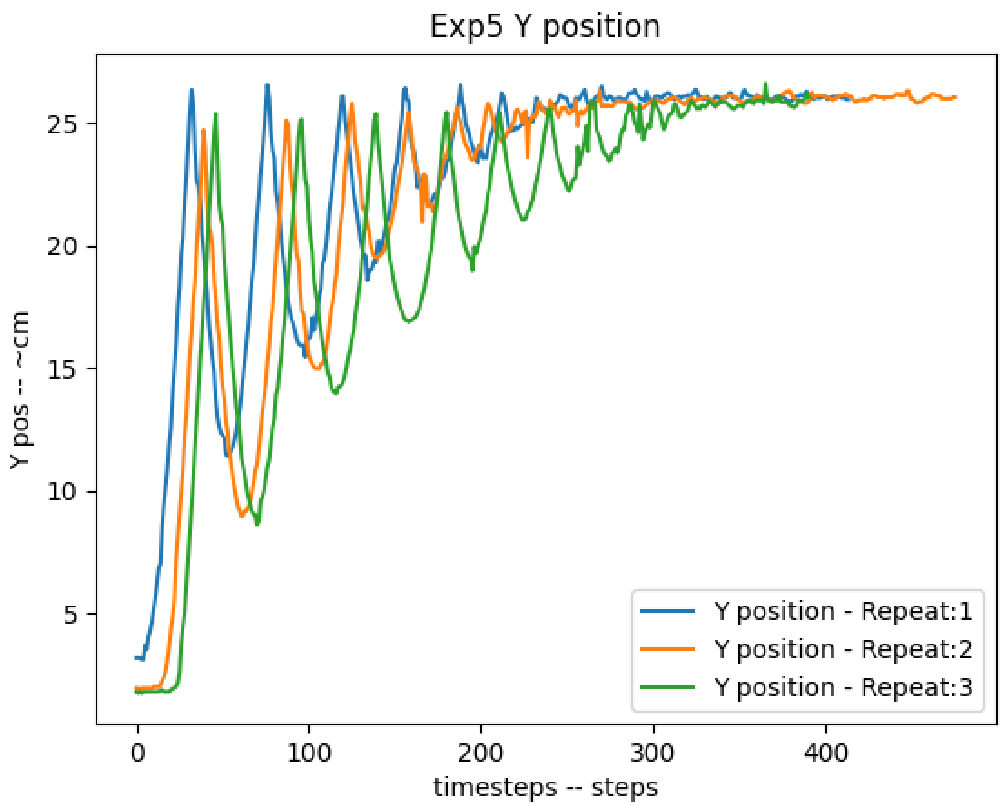


Fig 2. Position graphs from the experiments using opencv and matplotlib

In the following table, the measurements with the parameters for the experiments:

Table 1: Experiments details

Aa Experiment no.	Max string length	Num. oscillations
1	69cm	~14 (+/-2)
2	49cm	~11.667 (+/-1.527)
3	42cm	~10 (+/-1)
4	35cm	~7.66 (+/-0.577)
5	26cm	~6.33 (+/-0.577

Table 2: Number of oscillations

Aa Experiment no.	# 1st Repetition	# 2nd Repetition	# 3rd Repetition	# Mean	# Std
1	14	16	12	14	2
2	12	10	13	11.66666667	1.527525232

Aa Experiment no.	# 1st Repetition	# 2nd Repetition	# 3rd Repetition	# Mean	# Std
3	10	11	9	10	1
4	8	7	8	7.666666667	0.5773502692
5	6	6	7	6.333333333	0.5773502692
Mean	10	10	9.8	9.933333333	1.136445154
Std	3.16227766	3.937003937	2.588435821	3.067753431	0.6209794803

We can understand from the table that mean of initial string length over the number of oscillations = 9.933 oscillation cycles, standard deviation initial string length over the number of oscillations = 3.06

**Number of oscillations per different initial string length in the experiments = 9.933 (+/-3.06) oscillation cycles*

Table 3: Max distance from bottom position (oscillation amplitude) over oscillation

# Experiment no.	# 1	# 2	# 3	# 4	# 5	# 6	Aa 7	# Mean	# Std
1	19.5	35.6	47	54.6	61	66.7	69	34.03333333	13.81677724
2	19	29.8	35.1	43.7	45.2	49	Untitled	27.96666667	8.205079727
3	14.4	24.1	30.3	34.6	39.6	42	Untitled	22.93333333	8.013946177
4	15.3	21.9	26	33.6	35		Untitled	21.06666667	5.39845657
5	11.39	15.76	23.68	26			Untitled	16.94333333	6.229866237
							Mean	24.58866667	8.332825191
							Std	6.599400478	3.287274265

We can understand from the table that mean and the standard deviation for maximum distance from bottom position (oscillation amplitude) over oscillation from the last two columns.

Modeling

Yo-yo is research object that is doing a translation motion and rotational motion and both of them in most of the motion axis. Yo-yo is a research object that consists from two disks as approximation and cylindrical connection between the two disks.

Note: y-axis is the axis out from the face of the disks, x-axis is out from the split (connection between the two disks) of the Yo-yo and z-axis is in the same direction of the string. These axis follows from the figure from the assignment description.

Many models have been thought about such as:

1. Yo-yo can be modeled as a particle without that is rotating and moving in a vertical direction, and when it reaches the bottom, it is not bouncing but such model will not be sufficient for accurate model for yo-yo, but it is easy to implement.

   using the principle of conservation of energy:

   $$M.E = K_2 - K_1 = \pi_2 - \pi_1; \pi_1 = 0, K_1 = 0$$

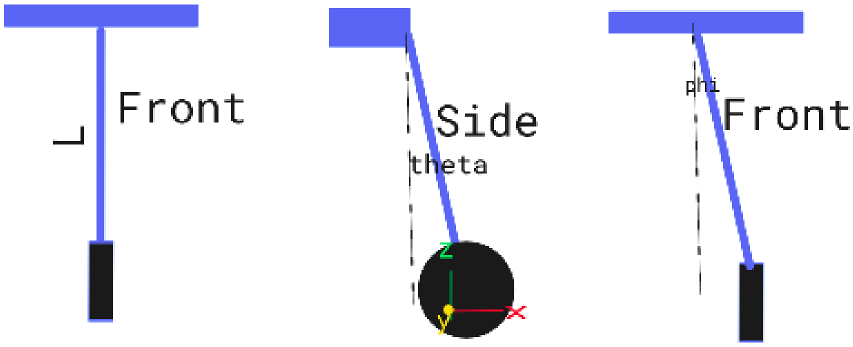
   $$0.5 * mv^2 + 0.5 * I\omega^2 - mgL = 0; v = \dot{L} = \omega * r_0;$$
   $$I\ is\ inertia\ around\ main\ axis\ of\ rotation\ (x - axis)$$

   by substitution, we get $\dot{L}^2 = \frac{2gL}{1 - I/(mr_0)}$ by solving the previous ODE, we can get L and its derivatives.

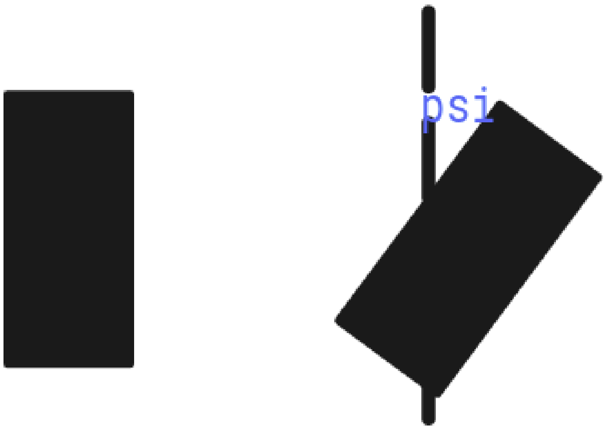
2. Yo-yo can be thought as 2 DoF research object: only vertical translation motion and rotation around y-axis. It will be similar to a problem of rotating wheel without slipping on an incline plane with 90 degree, however, we have to take into consideration the bouncing to the up when the yo-yo reaches the bottom. In this case we model the translational motion with bouncing and rotation motion.

3. More complex model is possible by taking into account that the yo-yo is not only moving in vertical direction but it has angles between the string and other axis ϕ, θ . Then the system can be described by $\phi, \theta, L, \omega.\ \phi\ and\ \theta$ can be modeled as damped pendulum with choosing small values for damping and small values for the initial angles.

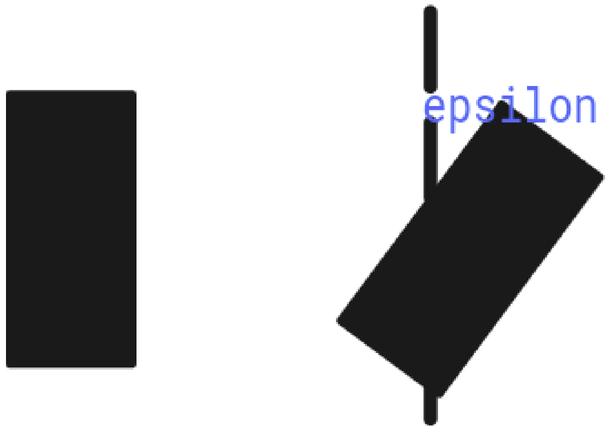
   $\ddot{\theta} = -\frac{g}{L} sin\theta - d * \dot{\theta}$ such that d is damping coefficient and sin could be replaced by the angle if the the angle is very small. By solving the previous ODE, we can get the angle the same for ϕ



4. Moreover, still some disturbances can be happened due to vibrations, friction from the string and the yo-yo disk. Such disturbances could be modeled as random noise. These disturbances create some angles between the yo-yo itself and its axis ψ, ϵ angles.



Front (around x-axis)



Top (around z-axis)

5. Yo-yo's string can be modeled as a spring.
6. The model can be determined by collecting the data from the experiments, assuming a polynomial model for the position and then fitting the data into the model to get the coefficients

I have chosen to implement the 2nd math model, for its sufficiency for the results and not hard relative to the 3rd model.

There are some assumptions that will be made:

1. Translation motion is only in the vertical direction.
2. String is flexible but not extensible
3. String's diameter and mass are negligible
4. Friction between string & surfaces of the yo-yo are proportional to the rotational velocity, thus, it will be small and may be ignored.

The motion is separated into four phases that are being changed by time:

- Going down
- The yo-yo rotate with pi degrees and winding up to the opposite directions. The time for this phase is considered negligible and will not be simulated only the need for distinguish between which phase is the yo-yo in.

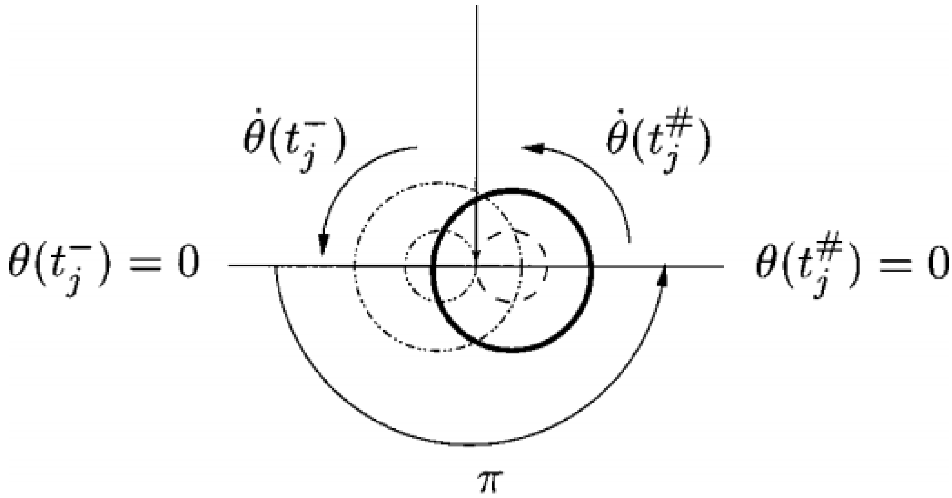


Figure 2 from Source [3]

- Going up with the negative signs
- Stabilize into specific height due to not having enough velocity in the vertical axis

We will model based on the conservation of mechanical energy with some dissipation of the kinetic energy. Such that d is the coefficient for the dissipation of the kinetic energy that results from the friction.

$K_2 + \pi_2 = (1-d)K_1 + \pi_1$

$0.5 * mv^2 + 0.5 * I * w^2 - (1-d)mv_0^2 - (1-d)Iw_0^2 = mgL$

Such that: $v = \dot{L} = w * r_0$; $w = \dot{L}/r_0$;

L is the length of the string that changes with time and r_0 is the inner diameter of the yo-yo.

Substitute in the above equation:

$0.5 * m\dot{L}^2 + 0.5 * I * \dot{L}^2/r_0^2 - (1-d)m * v_0^2 - (1-d)Iw_0^2 = mgL$

by multiply both sides by 2/m and get $\dot{L}^2$ in one side

$\dot{L}^2 = \frac{2gL + 2*(1-d)Iw_0^2/m + 2*(1-d)v_0^2}{1+(I/(mr_0^2))}$

Then by solving this 1st ODE with initial condition $L_0 = 0$ we can get L formula.

$v = \begin{cases} v_0, & \text{if it is the same phase} \\ v, & \text{otherwise} \end{cases} w = \begin{cases} w_0, & \text{if it is the same phase} \\ -w, & \text{otherwise} \end{cases}$

But the question here, is how to know if we are in the same phase or not?

We can find it, by identify the end of each phase.

$$\begin{cases} \text{end of phase 1 when L = string length} \\ \text{end of phase 3 when } \dot{L} = 0 \end{cases}$$

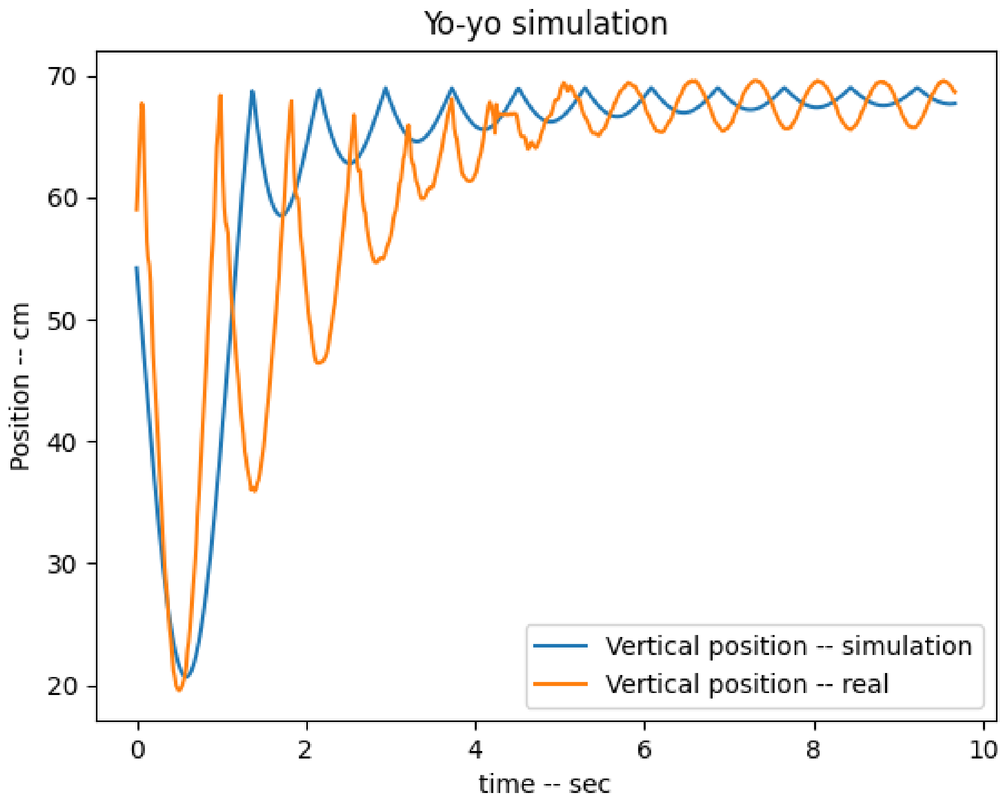
We can substitute these conditions in terms of indicator i_{cond}

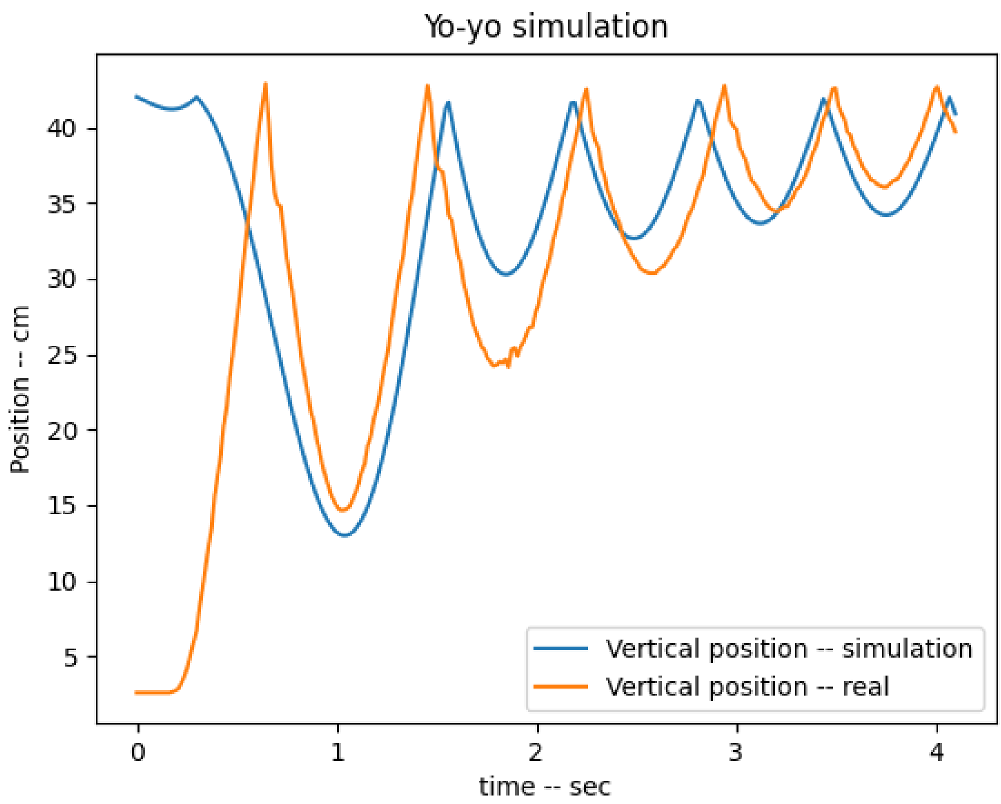
$$i_{cond} = \begin{cases} 1, & \text{condition is true} \\ 0, & \text{otherwise} \end{cases}$$

$$v = v_o * i_{same\ phase} - v * i_{different\ phase}$$

$$w = w_o * i_{same\ phase} - w * i_{different\ phase}$$

The following graph represents the vertical position of the yo-yo. The differences have occurred because of the inaccuracy in image processing as in some frames the contour was not computed correctly. However, the model has the same behavior under different initial conditions.





Source

1. Sasaki, A., Hashimoto, H., Yokota, S. and Ohyama, Y., 2007, September. Investigation of human motion under conditions with sensory feedbacks restricted in yo-yo control. In SICE Annual Conference 2007 (pp. 3052-3055). IEEE.
2. Salinas, I., Monteiro, M., Marti, A.C. and Monsoriu, J.A., 2019. Dynamics of a yoyo using a smartphone gyroscope sensor. arXiv preprint arXiv:1903.01343.
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6. McDonald, K., 2014. [online] Kirkmcd.princeton.edu. Available at: http://kirkmcd.princeton.edu/examples/yoyo.pdf [Accessed 11 October 2020