TM Assignment 1

Class	TM
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Materials	
Reviewed	☐
Type	

Student Information

- Name: Hany Hamed
- Group: BS18-RO
- Big HW1 - Assignment1

Tools

- Python (numpy, Matplotlib)
- LaTex
- Markdown
- Notions

Source

- Lecture about Trajectory Planning
- Lab1 slides

Description

In this assignment, the task is to make a mobile robot that is represented as particle follow a specific trajectory and reach the end point as fast as possible. The trajectory planning is under some constraint:

- Maximum Velocity = 1.5 m/s
- Maximum Tangential Acceleration (Max power on the motor) = 10m/s^2
- Maximum Normal Acceleration (Road Adhesion) = 6m/s^2

A Trapezoidal Velocity profile control will be used with some modification in the following way:

- Having period of time (t_a) to accelerate by the maximum acceleration to reach the maximum velocity.
- Having period of time (t_{vel}) to have zero acceleration, to stay at the maximum velocity.
- Having period of time (t_d) to decelerate by the maximum acceleration to reach the zero velocity.

```
# acceleration time
self.t_a = (self.MAX_VELOCITY_var - abs(self.vel))/self.MAX_ACCELERATION
# deceleration time
self.t_d = (self.vel)/self.MAX_ACCELERATION
self.total_displacement = abs(pos_ref - self.pos)
# total time of acceleration, zero acceleration and deceleration
self.T = (self.total_displacement*self.MAX_ACCELERATION+self.MAX_VELOCITY_var**2)/(self.MAX_ACCELERATION*self.MAX_VELOCITY_va
# total time + current_time_step
self.t_f = self.T + time
```

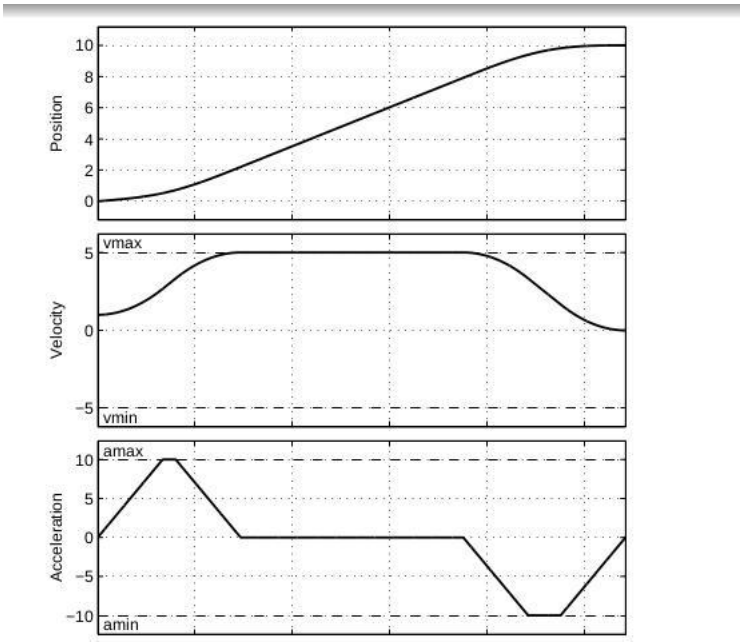


Figure1: Illustration for Trapezoidal Velocity Profile with s-spline from the shared material.

There are some other planners like: 3^{rd} order polynomial, 5^{th} order polynomial and there are modification that can be implemented with Trapezoidal Velocity profile trajectory planning which is S-spline to make the edges in the planning for velocity much smoother.

Applications

- Such trajectory planners can be found inside motors like Dynamixal.
- Simple trajectory planner for wheeled mobile robotics.
- Simple illustration for follow trajectory task.

Solution

In the normal planners and the description of the planner inside the lecture from prof. C. Melchiorri, there are no constraints on the tangential acceleration. However, in our problem, there is a constraint on the maximum tangential acceleration as well as on the maximum velocity.

Due to the dependency of the velocity on the tangential acceleration and the curvature which is changing by time, we need to limit the velocity in order not to exceed the constraint on the maximum velocity.

Curvature (k) is calculated as following

$$k = |\frac{\ddot{y}(x)}{(1+\dot{y}(x)^2)^{\frac{3}{2}}}|$$

And the tangential acceleration is calculated as following

$$a_t = v^2 * k$$

$$a_t^{(max)} = v_{(max)}^2 * k$$

$$v_{(max)}{}' = min(v_{(max)}, a_t^{(max)}/k)$$

And $v_{(max)}{}'$ is the new maximum value for the velocity under the constraint of tangential acceleration and this value is being updated with each time step in the simulation.

From the constraint of the maximum tangential and normal acceleration we can get the maximum acceleration from:

$$a_{(max)} = \sqrt{a_{t(max)}^2 + a_{n(max)}^2}$$

So, the planner now is the same planner in the lecture with limiting the maximum velocity with the original maximum and the function of the curvature each time step.

Then after calculating the corresponding velocities according to the planned acceleration, we can get the velocity

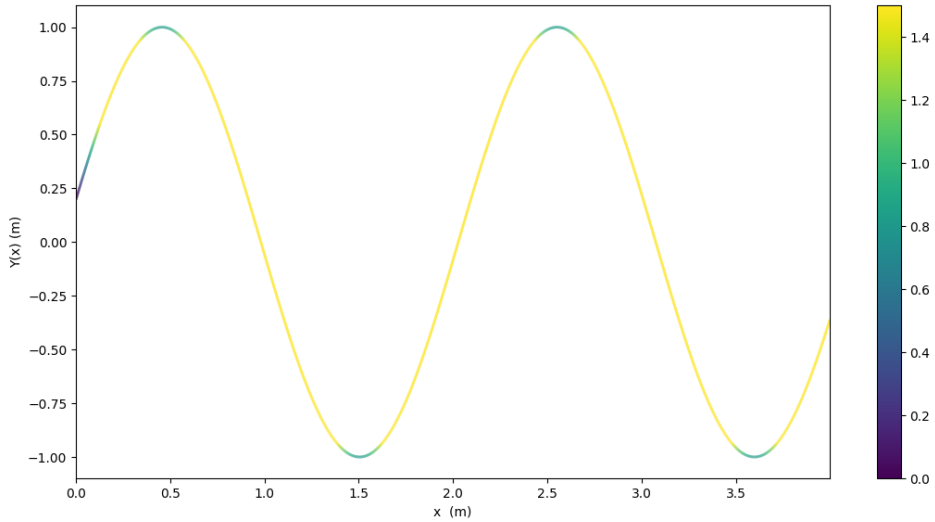


Figure: The velocity with the trajectory in heatmap figure.

You can see that in the maxima and minima of the curves, the velocity approaches the zero due to the curvature but the acceleration will be maximum to return again to maximum velocity.

The robot finished the trajectory in 7.822447706010614 sec,

and the real trajectory length = 8.72824057216428m it has been calculated with the following formula:

$L = \int_0^4 \sqrt{(1 + OM * A * cos(OM * x + \theta_0)^2)}$

and the achieved length = 8.728712732851724m due to the precision in the step.

The simulation was running with while loop until fulfill the following criteria: the summation of distances each time step ≥ the real trajectory

The step size in the distance was calculated from this simple rule for motion:

$s = \frac{(v_f^2 - v_i^2)}{2a}$ Such that v_i is from the previous time step and v_f is the velocity from the trapezoidal velocity profile and the same as acceleration (a) and the total achieved length is the summation of these small distances every time step

For calculating the time,

$t = \frac{v_f - v_i}{a}$

and the total time is the summation of these small-time every time step

Snippets from the code for illustration:

```
OM = 3
A = 1
theta_0 = 0.2
dx = 0.001

def motion_equation(x):
    y = A*sin(OM*x+theta_0)
    return x,y

MAX_VELOCITY = 1.5
MAX_ACCELERATION_TANGENTIAL = 10
MAX_ACCELERATION_NORMAL = 6
MAX_ACCELERATION = sqrt(MAX_ACCELERATION_TANGENTIAL**2 + MAX_ACCELERATION_NORMAL**2)

traj_planner = TrapezoidalMotion_mod(max_velocity=MAX_VELOCITY, max_acceleration_normal=MAX_ACCELERATION_NORMAL, max_acceleratio

L_sum = 0
L_sum_list = [0]
t_time = 0
# for s in range(1, total_steps):
s = 0
while(L_sum < total_length):
    s+=1
    pos = motion_equation(s*dx)
    traj_planner.update(pos[1], s*dx)
    curvature = traj_planner._calc_curvature()
    acc = sqrt(traj_planner._calc_normal_acceleration()**2 + traj_planner._calc_tangential_acceleration()**2)
    L_sum_part = abs((traj_planner.vel**2 - plot_data["velocity"][-1]**2))/(2*acc)
    t_time += abs((abs(traj_planner.vel) - abs(plot_data["velocity"][-1])))/(acc)
    L_sum += L_sum_part
    L_sum_list.append(L_sum_part)   plotter_data_append(traj_planner)

print("Total length to cover: ", total_length)
print("Achieved distance: ", L_sum)
print("Total time that is taken to achieve the above mentioned distance:", t_time)
plotter(plot_data)
```

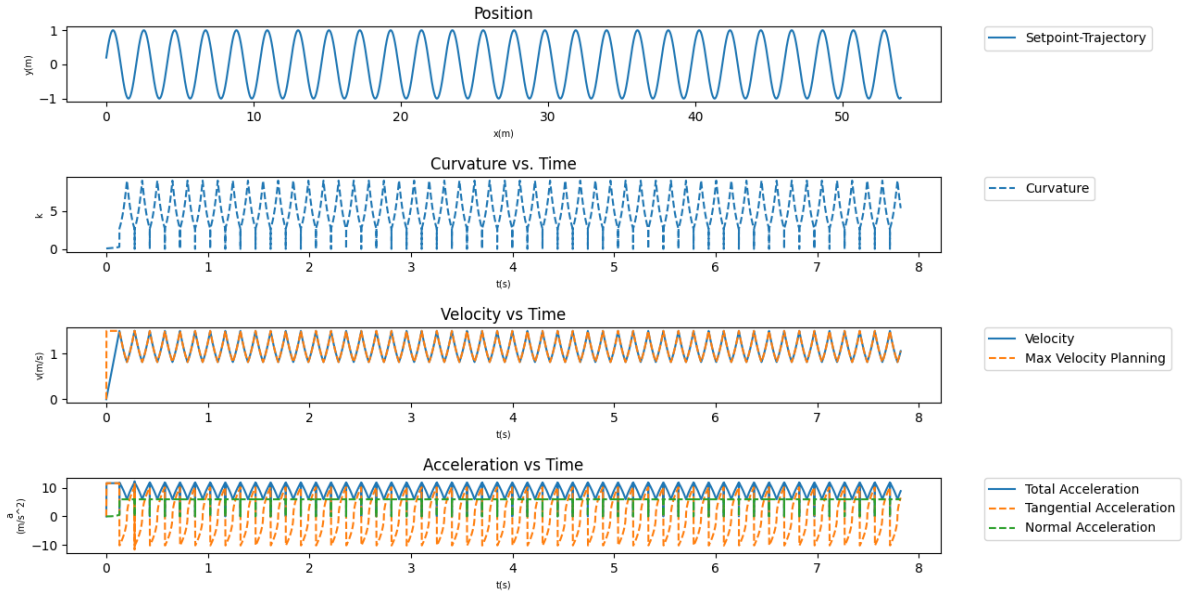
Note: the code has been written in OOP way.

The full class for the planner:

```
class TrapezoidalMotion_mod():
    def __init__(self, max_velocity, max_acceleration_tangential, max_acceleration_normal, initial_pos=0, initial_vel=0, initial_
        self.MAX_ACCELERATION_TANGENTIAL = max_acceleration_tangential
        self.MAX_ACCELERATION_NORMAL = max_acceleration_normal
        self.MAX_ACCELERATION = sqrt(self.MAX_ACCELERATION_NORMAL**2 + self.MAX_ACCELERATION_TANGENTIAL**2)
        self.MAX_VELOCITY = max_velocity

        self.MAX_VELOCITY_var = max_velocity

        self.old_pos = initial_pos
        self.old_pos_ref = initial_pos
        self.old_vel = initial_vel
        self.pos = initial_pos
        self.vel = initial_vel
        self.acc = 0

        self.total_displacement = self.old_pos_ref - self.old_pos

        self.t_a = 0
        self.T = 0
        self.t_f = 0
        self.t_i = 0
        self.delta_time = dt
        self.current_time = 0

        self.curvature = self._calc_curvature()


    def _calc_limited_vel(self):
        velocity = sqrt(self.MAX_ACCELERATION_NORMAL/self._calc_curvature())
        return velocity

    def _calc_max_vel(self):
        # print(self._calc_limited_vel(), self.MAX_VELOCITY)
        return min(self.MAX_VELOCITY, self._calc_limited_vel())


    def _calc_curvature(self):
        x = self.current_time
        dot_f = A*cos(OM*x+theta_0)*OM
        ddot_f = -A*sin(OM*x+theta_0)*OM*OM
        k = abs(ddot_f/(pow((1+dot_f*dot_f),(3/2))))
        return k

    def _calc_normal_acceleration(self):
        return self.vel**2*self._calc_curvature()

    def _calc_tangential_acceleration(self):
        return (self.vel-self.old_vel)/self.delta_time

    def update(self, pos_ref, time):
        self.current_time = time
        self.old_vel = self.vel
        if(self.old_pos_ref != pos_ref):
            self.old_pos = self.pos
            self.old_pos_ref = pos_ref
            self.t_i = time

            self.MAX_VELOCITY_var = self._calc_max_vel()
            self.t_a = (self.MAX_VELOCITY_var - abs(self.vel))/self.MAX_ACCELERATION
            self.t_d = (self.vel)/self.MAX_ACCELERATION
            self.total_displacement = abs(pos_ref - self.pos)
            # self.T = t_a + t_d
            self.T = (self.total_displacement*self.MAX_ACCELERATION+self.MAX_VELOCITY_var**2)/(self.MAX_ACCELERATION*self.MAX_VEL
            self.t_f = self.T + time

        self.calculateTrapezoidalProfile()


    def calculateTrapezoidalProfile(self):
        # if(self.t_i <= self.current_time and self.current_time <= self.t_i+self.t_a):
        if(self.delta_time <= self.t_a):
            # print("+ Acc")
            self.pos = self.pos + self.vel*self.delta_time
            # self.pos = self.old_pos + 1/2*self.MAX_ACCELERATION*((self.current_time - self.t_i)**2)
            self.vel = self.vel + self.MAX_ACCELERATION*self.delta_time
            # print(self.vel)
            self.acc = self.MAX_ACCELERATION
```

```
        # elif(self.t_i + self.t_a < self.current_time and self.current_time <= self.t_f - self.t_a):
        elif(self.delta_time > self.t_a and self.delta_time < self.T - self.t_a - self.t_d):
            # print("0 Acc")
            self.pos = self.pos + self.vel*self.delta_time
            # self.pos = self.old_pos + self.MAX_ACCELERATION*((self.current_time-self.t_i-(self.t_a/2))**2)
            self.vel = self.MAX_VELOCITY_var
            self.acc = 0

        # elif(self.t_f - self.t_a < self.current_time and self.current_time <= self.t_f):
        elif(self.delta_time >= self.T - self.t_a - self.t_d and self.delta_time <= self.t_d):
            # print("- Acc")
            self.pos = self.pos + self.vel*self.delta_time
            # self.pos = self.old_pos_ref - (0.5*self.MAX_ACCELERATION*((self.t_f-self.current_time-self.t_i)**2))
            self.vel = self.vel - self.MAX_ACCELERATION*self.delta_time
            self.acc = -self.MAX_ACCELERATION
```

Plot to the whole data until the robot finished the trajectory:



For better quality for the plots and for part of the orignial plots

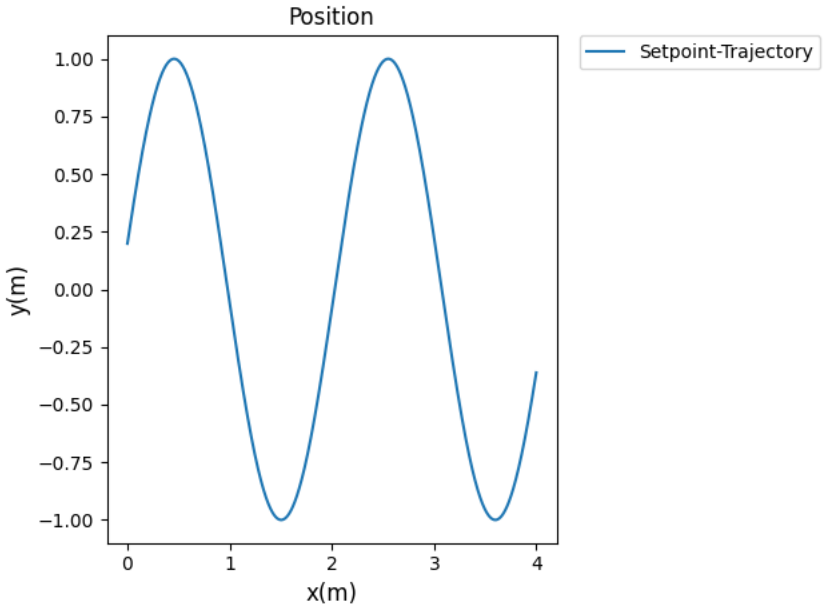


Figure3: Position equation: x(t), y(t)=y(x) they are in meters.

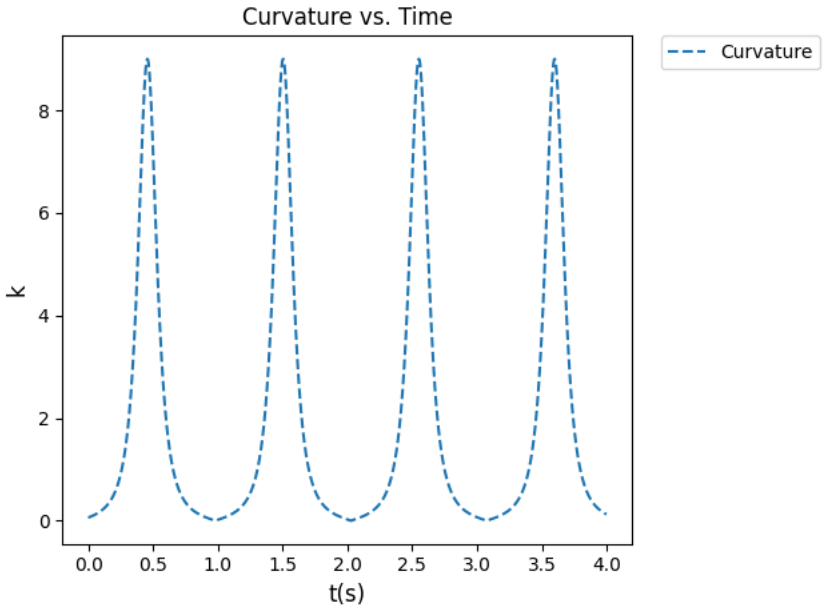


Figure4: Curvature vs. time.

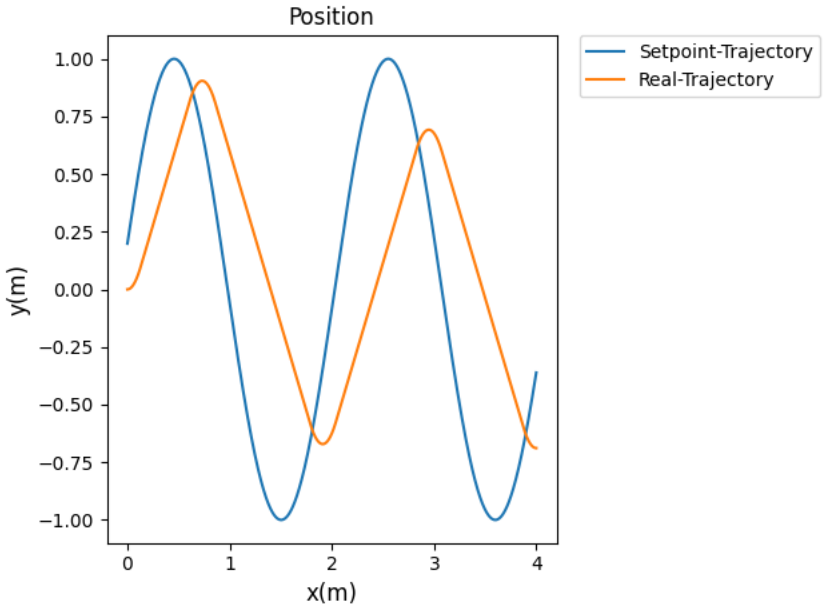


Figure5: Goal positions and Real Trajectory.

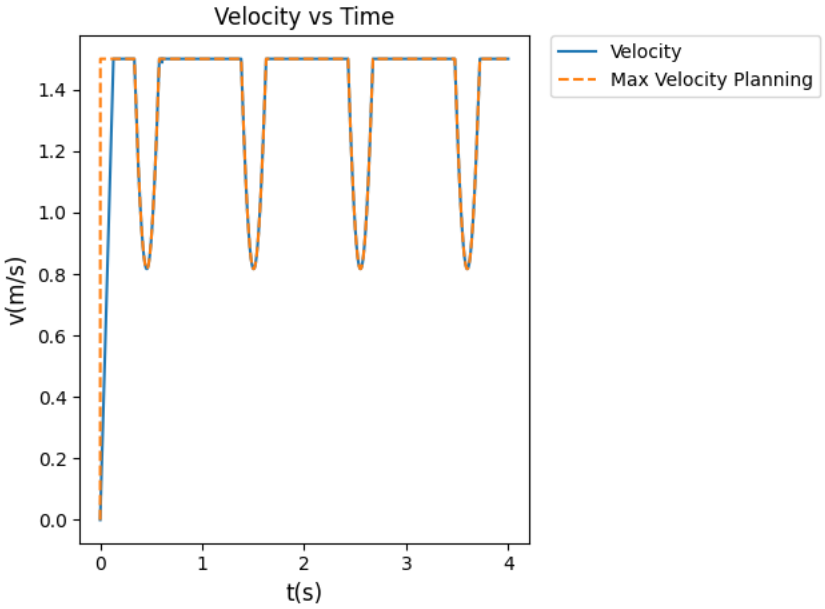


Figure6: Planned Velocity and max velocity calculation based on curvature.

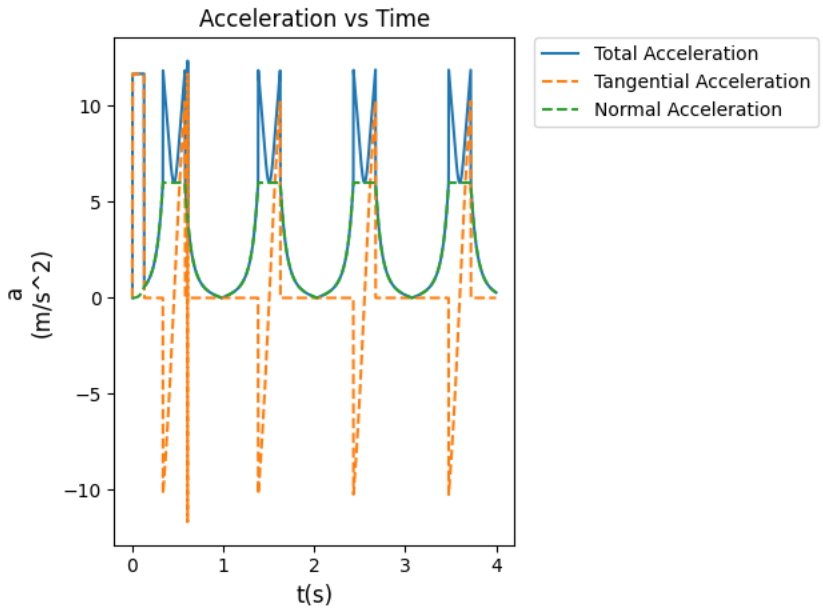


Figure7: Planned acceleration