Theoretical Mechanics

Homework 2. Innopolis University, Fall 2020

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Task 1

Solution
The system consist of rigid body ABC that has the same angular velocity but different linear velocity in A,B and C. Also, it consists of slider D in the x-axis, and an attachment from E on the connection between the slider and the rigid body that supported using joints on F and O_3.

The origin point is at O_1 and x-axis positive is to the right and y-axis positive is to the left.

It can be seen as system of 4 bar linkage O_1, A, B and O_2 and with slider and RR manipulator system.

For the positions, we are using geometry to get all the positions using the intersection between circles of motions.

For the velocities, we are using relative motion analysis in order to get relations of velocities to the known velocities and using the vector components of the velocities in order to get the same number of equations and unknowns as we will have angular velocities and linear velocities as unknowns. The equations have been described in the code.

Draft solution for the velocities and angular velocities

TM, Innopolis University, Fall 2020
Author: Hany Hamed

$\rightarrow \vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$\hookrightarrow \left(\vec{v}_B^{\,x} ; \vec{v}_B^{\,y}\right) = \left(\vec{v}_A^{\,x} ; \vec{v}_A^{\,y}\right) + \left(\omega_{ABC}\hat{k}\right) \times \left(r^x_{B/A}\hat{i} ; r^y_{B/A}\hat{j}\right)$

ω_{ABC} is const for the whole body

$\rightarrow \vec{v}_B = \vec{\omega}_{BO} \times \vec{r}_{B/O_2}$

$\hookrightarrow \left(\vec{v}_B^{\,x} ; \vec{v}_B^{\,y}\right) = \left(\omega_{BO}\hat{k}\right) \times \left(r^x_{B/O_2}\hat{i} ; r^y_{B/O_2}\hat{j}\right)$

for $\hat{i}$

$$\Rightarrow \vec{v}_B^{\,x} = \underbrace{v_A^{x} - \omega_{ABC}\, r^y_{B/A} = -\omega_{BO}\, r^y_{B/O_2}}\quad (1)$$

? ?

for $\hat{j}$

$$\Rightarrow \vec{v}_B^{\,y} = \underbrace{\omega_{BO}\, r^x_{B/O_2} = v_A^{y} + \omega_{ABC}\, r^x_{B/A}}\quad (2)$$

? ?

v_A

from (2)

$$\omega_{BO} = \frac{v_A^{y}}{r^x_{B/O_2}} + \frac{\omega_{ABC}\, r^x_{B/A}}{r^x_{B/O_2}}$$

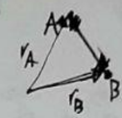
Substitute in (1)

$$-\frac{v_A^{y}}{r^x_{B/O_2}} r^y_{B/O_2} - \frac{\omega_{ABC}\, r^x_{B/A}\, r^y_{B/O_2}}{r^x_{B/O_2}} = -\omega_{ABC}\, r^y_{B/A}$$

$$-\frac{v_A^{y}\, r^y_{B/O_2}}{r^x_{B/O_2}} = \omega_{ABC}\left(-r^y_{B/A} + \frac{r^x_{B/A}\, r^y_{B/O_2}}{r^x_{B/O_2}}\right)$$

$\hookrightarrow \vec{v}_C = \vec{v}_A + \vec{\omega}_{ABC} \times \vec{r}_{C/A}$

$\vec{r}_{B/A} = \vec{r}_{BA} = \vec{r}_B - \vec{r}_A$



$(\omega\hat{k}) \times (r_x\hat{i} ; r_y\hat{j})$

$$= \begin{bmatrix} i & j & k \\ 0 & 0 & \omega \\ r_x & r_y & 0 \end{bmatrix}$$

$= -\omega r_y \vec{i} + \omega r_x \vec{j}$

$\vec{v}_D = (v_D^x ; 0)$

$\vec{v}_D = \vec{v}_C + \vec{\omega}_{CD} \times \vec{r}_{D/C}$

$$\boxed{0 = v_C^y + \omega_{CD}\, r_{D/C}^x}$$

$$\omega_{CD} = \frac{-v_C^y}{r_{D/C}^x}$$

$$v_D^x = v_C^x - \omega_{CD}\, r_{D/C}^y$$

$\vec{v}_E = (v_E^x ; v_E^y)$

$= \vec{v}_C + \vec{\omega}_{CD} \times \vec{r}_{E/C}$

$\vec{r}_{E/C} = \vec{r}_E - \vec{r}_C$

$\vec{v}_F = (v_F^x , v_F^y)$

$\vec{v}_F = \vec{v}_E + \vec{\omega}_{EF} \times \vec{r}_{F/E}$

$\vec{v}_F = \vec{\omega}_{FO_3} \times \vec{r}_{FO_3}$

as previous page

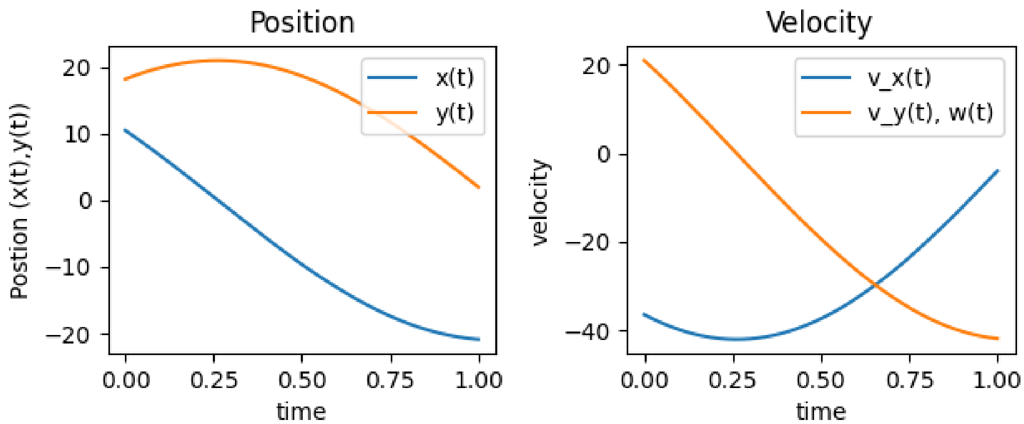
$F \to B$

$E \to A$

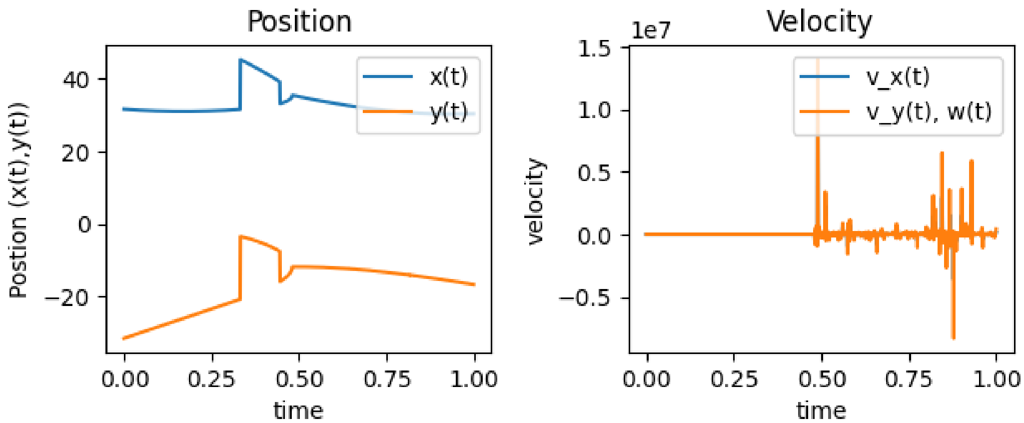
$FF \to ABC$

$FO_3 \to BO_2$

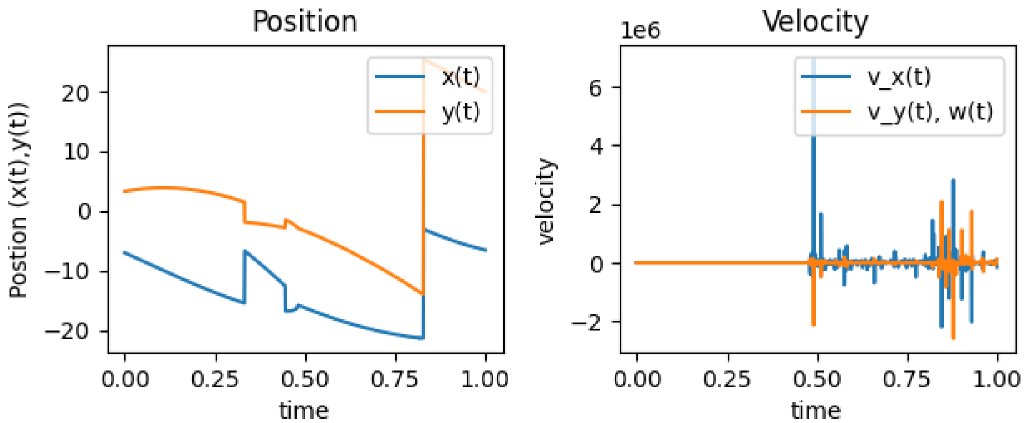
Plots


Position
x(t)
y(t)
Postion (x(t),y(t))
time
Velocity
v_x(t)
v_y(t), w(t)
velocity
time

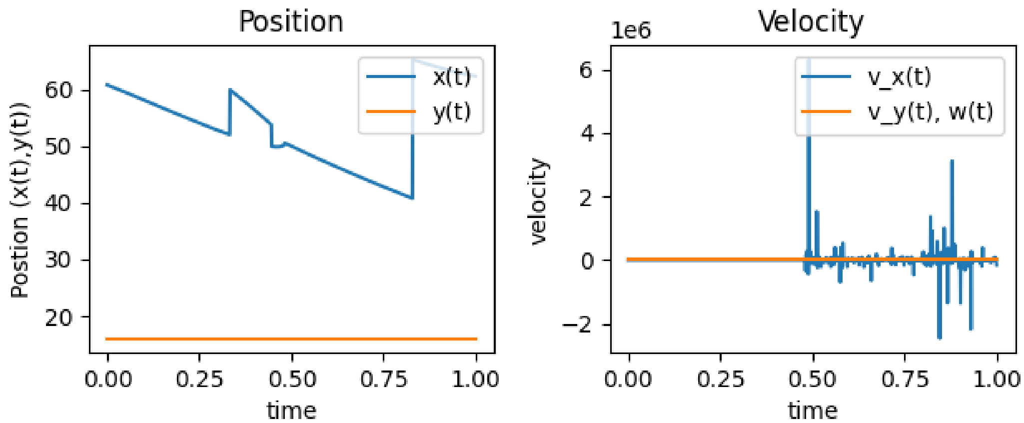
- **For A**


Position
Postion (x(t),y(t))
40
20
0
−20
0.00
0.25
0.50
0.75
1.00
time
x(t)
y(t)
Velocity
1e7
1.5
1.0
0.5
0.0
−0.5
velocity
0.00
0.25
0.50
0.75
1.00
time
v_x(t)
v_y(t), w(t)

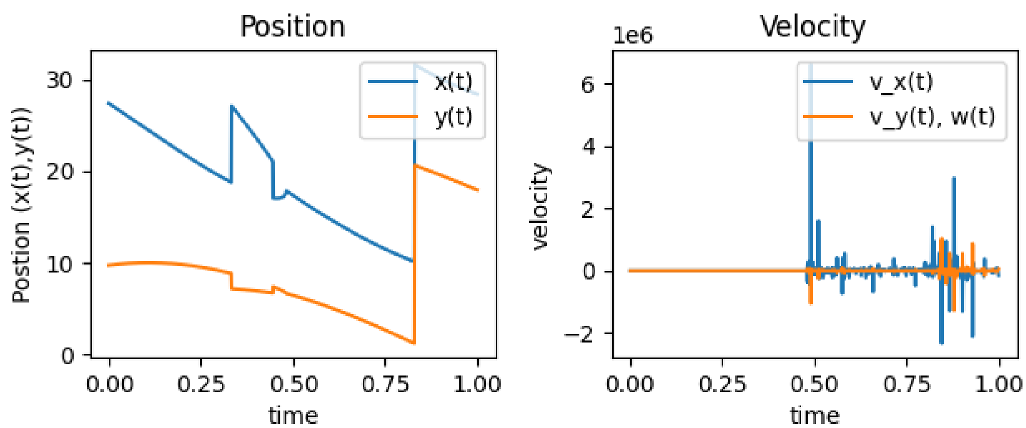
- **For B**


Position
x(t)
y(t)
Postion (x(t),y(t))
20
10
0
−10
−20
0.00
0.25
0.50
0.75
1.00
time
Velocity
1e6
v_x(t)
v_y(t), w(t)
velocity
6
4
2
0
−2
0.00
0.25
0.50
0.75
1.00
time

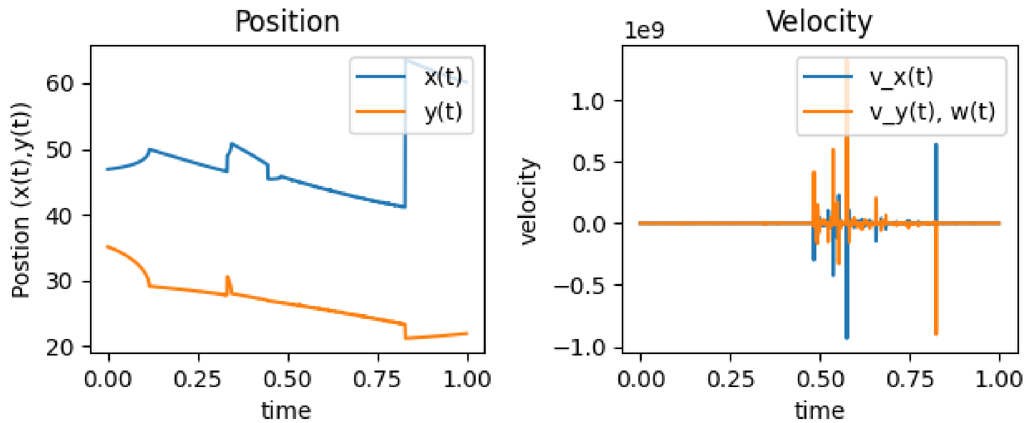
- **For C**


Position
Postion (x(t),y(t))
60
50
40
30
20
0.00
0.25
0.50
0.75
1.00
time
x(t)
y(t)
Velocity
1e6
velocity
6
4
2
0
−2
0.00
0.25
0.50
0.75
1.00
time
v_x(t)
v_y(t), w(t)

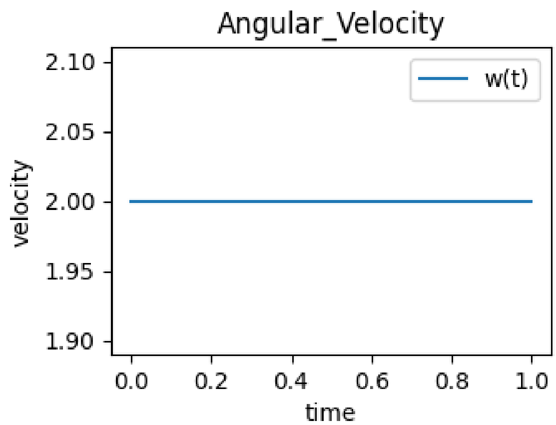
- **For D**


Position
Postion (x(t),y(t))
time
x(t)
y(t)
Velocity
1e6
velocity
time
v_x(t)
v_y(t), w(t)

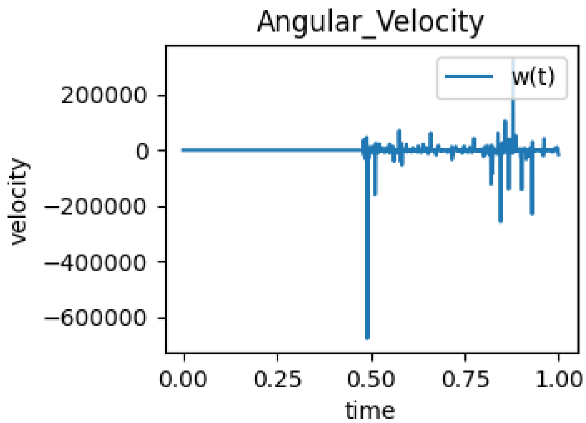
- **For E**


Position
x(t)
y(t)
Postion (x(t),y(t))
60
50
40
30
20
0.00
0.25
0.50
0.75
1.00
time
Velocity
1e9
v_x(t)
v_y(t), w(t)
velocity
1.0
0.5
0.0
−0.5
−1.0
0.00
0.25
0.50
0.75
1.00
time

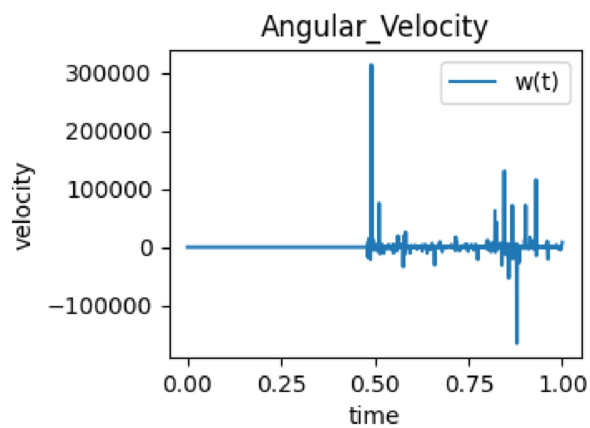
- **For F**


Angular_Velocity
w(t)
velocity
time
2.10
2.05
2.00
1.95
1.90
0.0
0.2
0.4
0.6
0.8
1.0

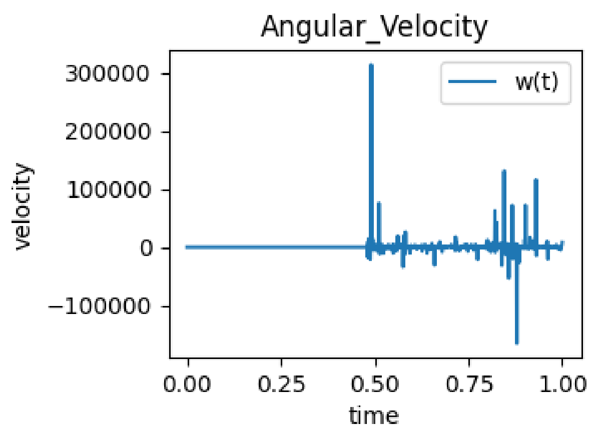
- **For AO1**


Angular_Velocity
w(t)
200000
0
−200000
−400000
−600000
velocity
0.00
0.25
0.50
0.75
1.00
time

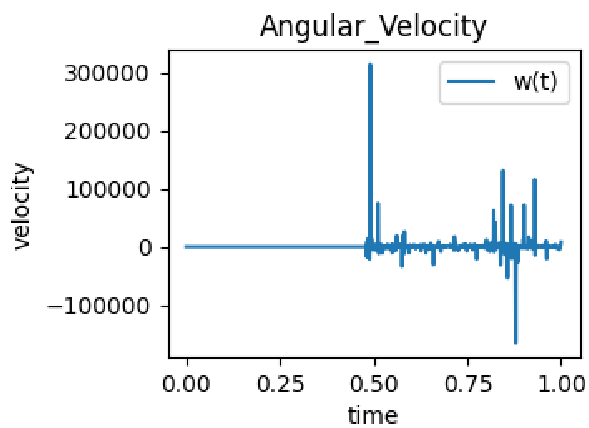
- **For BO2**


Angular_Velocity
w(t)
velocity
300000
200000
100000
0
−100000
time
0.00
0.25
0.50
0.75
1.00

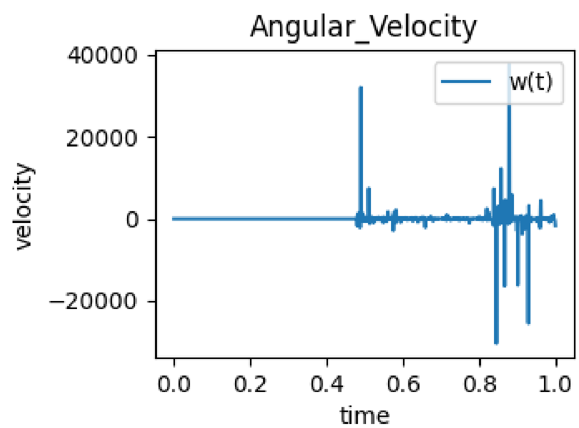
- **For AB**


Angular_Velocity
w(t)
velocity
time
300000
200000
100000
0
−100000
0.00
0.25
0.50
0.75
1.00

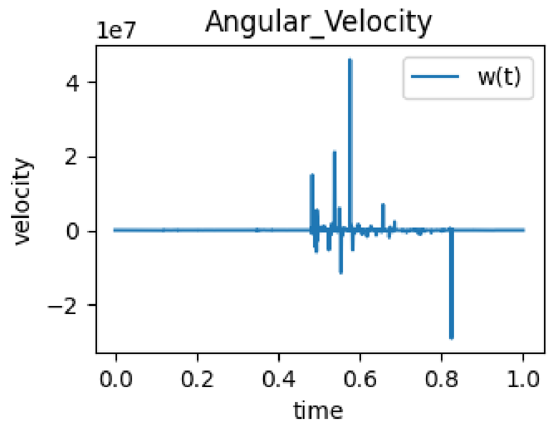
- **For AC**


Angular_Velocity
w(t)
300000
200000
100000
0
−100000
velocity
0.00
0.25
0.50
0.75
1.00
time

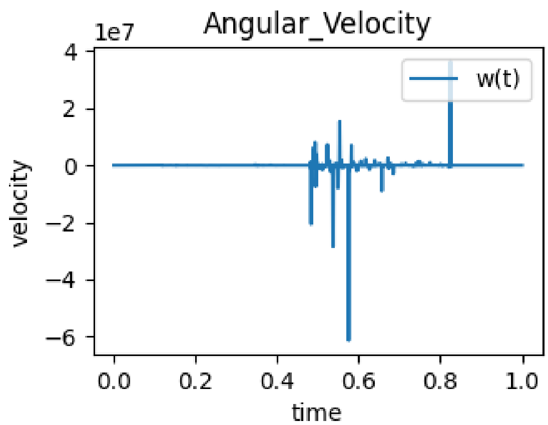
- **For CB**


Angular_Velocity
w(t)
velocity
40000
20000
0
−20000
0.0
0.2
0.4
0.6
0.8
1.0
time

- **For CD**



- **For EF**



- **For FO3**

Task 2

Solution
It is a small cone that rotates inside a bigger cone that is rotating with angular velocity and angular acceleration.

The axis of rotation of the smaller cone is the intersected line between the small cone and the big cone.

The equations are specified inside the code.

Draft solution

ω_B ✓ ϵ_B ✓

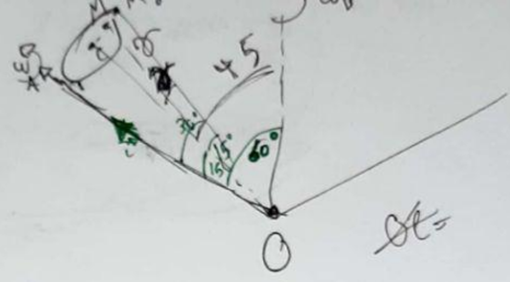
ω_A ? ϵ_A ?

$\omega_A = \frac{\omega_B \sin 45}{\sin 15}$

1.1

$\vec{v}_C = \vec{\omega}_B \times \vec{OC} = \vec{\omega}_A \times \vec{OC}$ → $\cos(15) * OM_0$

$\|\vec{v}_C\| = \underbrace{\omega_B OC \sin(60-15)}$ $= \omega_A OC \sin(15)$



$\delta t =$

$\vec{\epsilon}_A = \frac{d\vec{\omega}_A}{dt}$

$\vec{\omega}_A = \omega_A \vec{i}$

$\vec{\epsilon}_A = \frac{d\omega_A}{dt}\vec{i} + \frac{d\vec{i}}{dt}\omega_A$ → $\frac{d\vec{i}}{dt} = \vec{\omega}_B \times \vec{i} = \omega_B \sin(60)\vec{j}$

$\vec{\epsilon}_\parallel = \frac{d\omega_A}{dt}\vec{i}$

↓

$\frac{d\omega_A}{dt} = \epsilon_B \frac{\sin 45}{\sin 15}$

$\vec{\epsilon}_\tau = \epsilon_B \frac{\sin 45}{\sin 15}\vec{i}$

$\vec{\epsilon}_\perp = \frac{d\vec{i}}{dt}\omega_A = \omega_A \omega_B \sin(60)\vec{j}$

↓

$\vec{\epsilon}_A = \sqrt{\epsilon_\tau^2 + \epsilon_n^2}$

1.2

cosine rule for OM

$OM = \frac{OC\sqrt{2}}{\cos(15-60)}$.

$\frac{\sin 75}{OM} = \frac{\sin ?}{MM_0} = \frac{\sin ?}{OM_0}$

2.1

$$\vec{v}_M = \vec{\omega}_A \times \vec{OM} = \omega_A \, MM_0 \sin 30$$

↳ Or

$$\gamma = \arcsin\left(\frac{MM_0}{OM_0}\right) \rightarrow v_M = \omega_B \times OM$$

$$= \omega_B \cdot OM \sin(30 + \gamma)$$

$\cancel{OM = \sqrt{40^2 + \ldots}}$

?

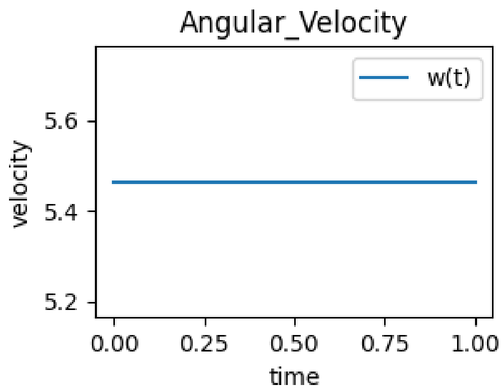
$$\vec{a}_M = \vec{a}_C + \vec{a}_{MC}^{(T)} + \vec{a}_{MC}^{(N)}$$

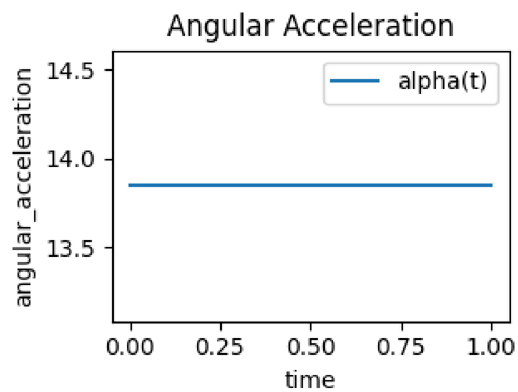
$$\begin{cases} \|\vec{a}_{MC}^{(T)}\| = \epsilon_{II} \, OM \sin\gamma & \rightarrow \|\vec{a}_C\| = \omega_A \, v_M \\ \|\vec{a}_{MC}^{(N)}\| = \epsilon_I \, OM \end{cases}$$

$$\vec{a}_M = \sqrt{(a_C)^2 + \left(a_{MC}^{(N)}\right)^2 - 2(a_C)\left(a_{MC}^{(N)}\right)\cos\gamma + \left(a_{MC}^{(T)}\right)^2}$$

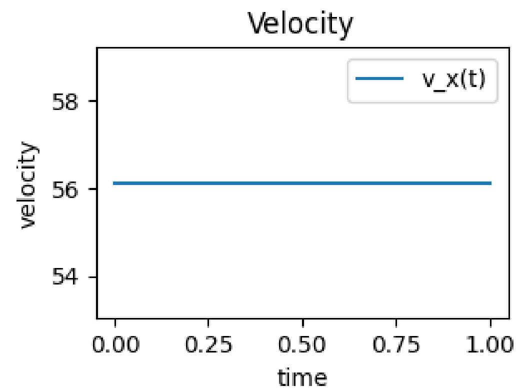
Scanned with CamScanner

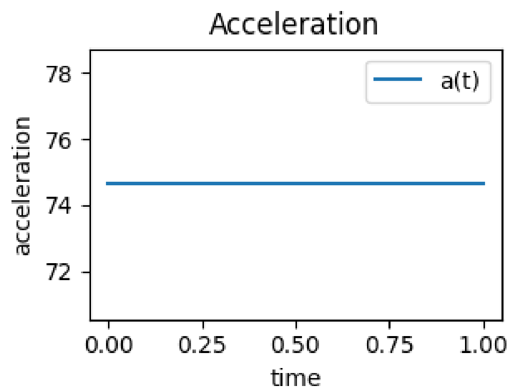
Plots


Angular_Velocity
w(t)
velocity
5.6
5.4
5.2
0.00
0.25
0.50
0.75
1.00
time

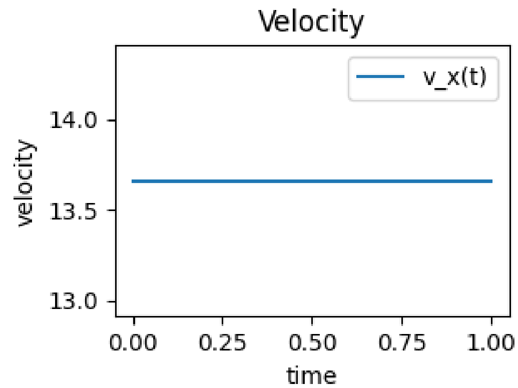

Angular Acceleration
alpha(t)
angular_acceleration
14.5
14.0
13.5
0.00
0.25
0.50
0.75
1.00
time

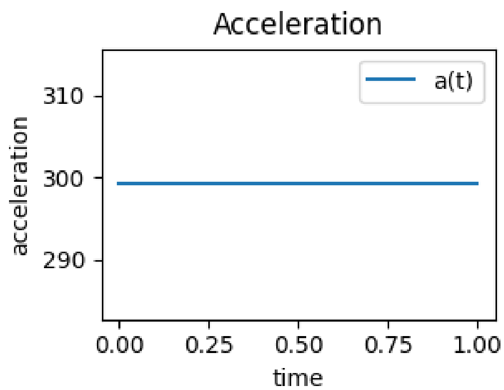
- **For A**


Velocity
v_x(t)
velocity
58
56
54
0.00
0.25
0.50
0.75
1.00
time


Acceleration
a(t)
acceleration
78
76
74
72
0.00
0.25
0.50
0.75
1.00
time

- **For C**


Velocity
v_x(t)
velocity
14.0
13.5
13.0
0.00
0.25
0.50
0.75
1.00
time


Acceleration
a(t)
acceleration
310
300
290
0.00
0.25
0.50
0.75
1.00
time

- **For M**