Topics on survival analysis

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Outline

1. Introduction

2. Definition

3. Statistical Methods

4. Related Topics

5. Applications

Outline

1. Introduction

2. Definition

3. Statistical Methods

4. Related Topics

5. Applications

Introduction[1]

▶ The goal of survival analysis is to analyze and model data where the outcome is the time until an event of interest occurs, T.

▶ T can always be transformed into an occurrence of the event within a specified period.

▶ The advantages and disadvantages of using either survival analysis methods or other rudimentary regression methods should be weighed.

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[1]The following contents are primarily based on Wang, Li, and Reddy (2019, ACM Computing Surveys).

Introduction

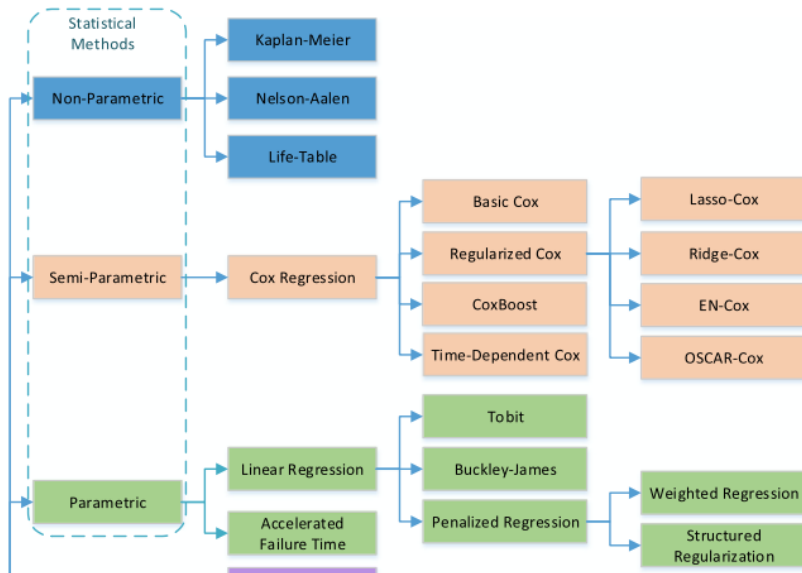Survival analysis can deal with

- ▶ Censoring
- ▶ Time-dependent covariates
- ▶ Competing risks

Introduction
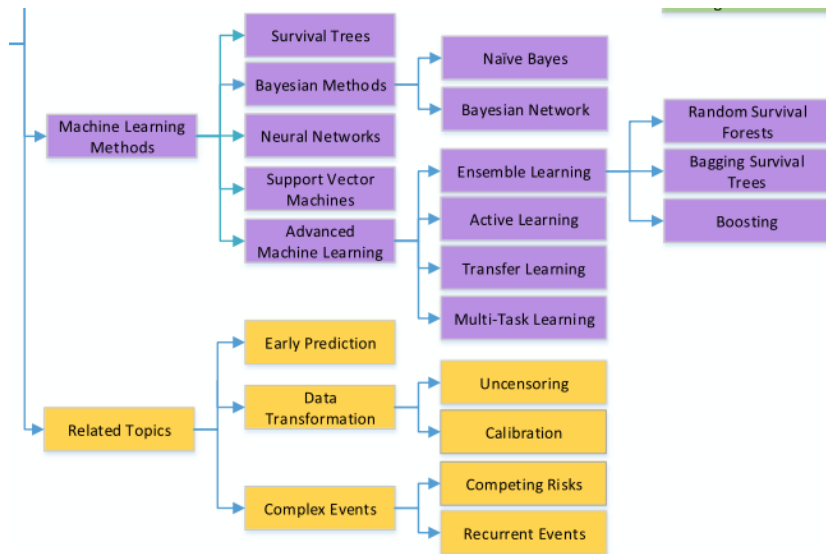
However, the costs are that

▶ Assumptions for causal inference are strong

▶ Models are complicated and not so intuitive

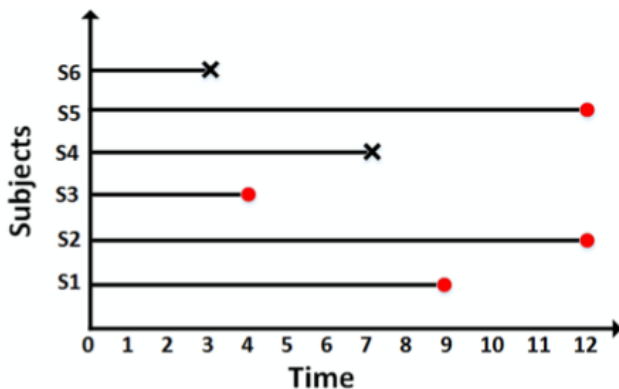▶ Machine learning methods are difficult to apply

Topics

Topics

Outline

Survival Data and Censoring

X marks: event occurrence
Red dots: censored

Problem Statement

▶ Data: a triplet (X_i, y_i, δ_i)
  - $i \in \{1 \ldots N\}$: individual
  - $X_i \in \mathbb{R}^p$: independent variables
  - δ_i: 0 if censored and 1 if otherwise
  - y_i: T_i if $\delta_i = 1$ and C_i if otherwise
  - T_i: survival time
  - C_i: censored time

▶ Goal: estimate the effect of some elements of X on T or predict T with X
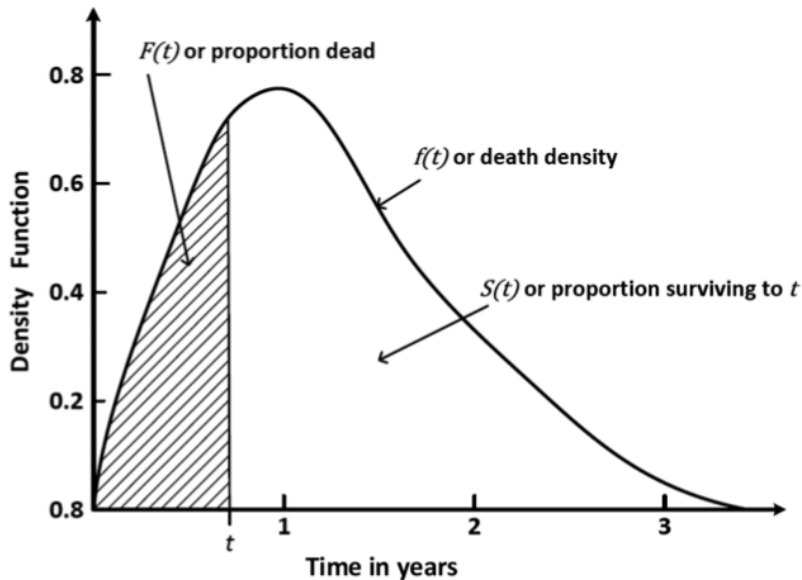
Survival and Hazard Function

▶ Survival function: $S(t) = \Pr(T \geq t)$

▶ Cumulative distribution function: $F(t) = 1 - S(t)$

▶ Density function:

$$f(t) = \frac{d}{dt}F(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \leq T < t + \Delta t)}{\Delta t}$$

▶ Hazard function:

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \leq T < t + \Delta t | T \geq t)}{\Delta t} = \frac{f(t)}{S(t)}$$

Survival and Hazard Function

Identification

▶ Consider a simple nonparametric estimator of $f(t)$:

$$\hat{f}(t) = \frac{1}{N} \sum_{i=1}^{N} I(t \leq T_i < t + \Delta t)$$

$$\rightarrow \lim_{\Delta t \rightarrow 0} \frac{\Pr(t \leq T < t + \Delta t, \delta = 1)}{\Delta t}$$

▶ Unfortunately,

$$\Pr(t \leq T < t + \Delta t, \delta = 1) < \Pr(t \leq T < t + \Delta t)$$

unless $\delta_i = 1$ for all i, no censoring.

Identification

- Thus, we cannot get a consistent estimator for $f(t)$.
- How about $h(t)$?
- $R(t)$: set of individuals considered to be "at risk" at time t
- $r(t)$: number of individuals considered to be "at risk" at time t, $|R(t)|$

Identification

Then, a simple nonparametric estimator of $h(t)$ is

$$
\begin{aligned}
& \hat{h}(t) \\
= \ & \frac{1}{r(t)} \sum_{i \in R(t)} I(t \leq T_i < t + \Delta t) \\
\rightarrow \ & \lim_{\Delta t \rightarrow 0} \frac{\Pr(t \leq T < t + \Delta t, \delta = 1 | T \geq t, \delta = 1)}{\Delta t}
\end{aligned}
$$

What is the condition to be

$$
\begin{aligned}
& \Pr(t \leq T < t + \Delta t, \delta = 1 | T \geq t, \delta = 1) \\
= \ & \Pr(t \leq T < t + \Delta t | T \geq t)?
\end{aligned}
$$

Identification

If $T \perp\!\!\!\perp \delta$, then

$$\Pr(t \leq T < t + \Delta t, \delta = 1 | T \geq t, \delta = 1)$$
$$= \frac{\Pr(t \leq T < t + \Delta t, \delta = 1)}{\Pr(T \geq t, \delta = 1)}$$
$$= \frac{\Pr(t \leq T < t + \Delta t) \Pr(\delta = 1)}{\Pr(T \geq t) \Pr(\delta = 1)}$$
$$= \Pr(t \leq T < t + \Delta t | T \geq t).$$

Thus, we can get a consistent estimator for $h(t)$.

From $h(t)$ to $f(t)$

As

$$h(t) = \frac{f(t)}{1 - F(t)} = -\frac{d\log\{1 - F(t)\}}{dt}$$

$$\Leftrightarrow \ F(t) = 1 - \exp\left\{-\int_0^t h(\tau)d\tau\right\}$$

$$\Leftrightarrow \ f(t) = h(t)\exp\left\{-\int_0^t h(\tau)d\tau\right\},$$

we can calculate $f(t)$ from $h(t)$.

Outline

Nonparametric Models: Kaplan-Meier Curve

Let event times be $T_1 < T_2 < \cdots < T_K$ for N individuals and consider a specific event time T_j:

- $d(T_j)$: observed events
- $c(T_j)$: censored individuals between T_j and T_{j+1}
- then, $r(T_j) = r(T_{j-1}) - d(T_{j-1}) - c(T_{j-1})$

Kaplan-Meier Curve

▶ Conditional probability of surviving beyond T_j:

$$p(T_j) = \frac{r(T_j) - d(T_j)}{r(T_j)}$$
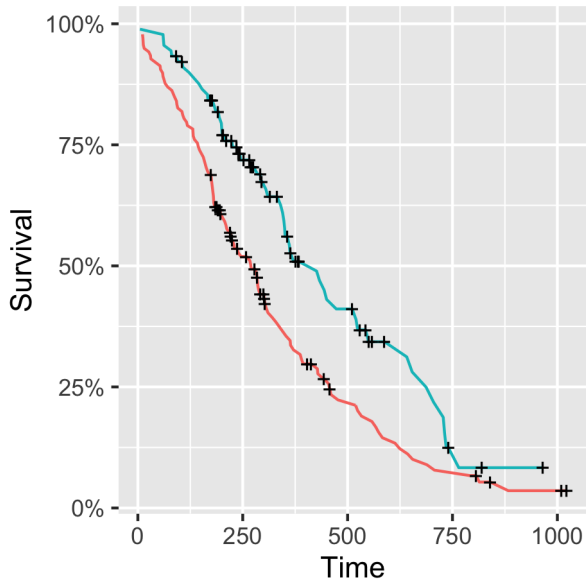
▶ The estimator of $S(t) = \Pr(T \geq t)$ is given as

$$\hat{S}(t) = \prod_{j:T_j < t} p(T_j)$$

▶ Nonparametric: no functional form specification

Kaplan-Meier Curve

▶ The effect of X can be quantified by stratification.
▶ Time-dependent covariates $X(t)$ can be incorporated as well.
▶ Set of individuals considered to be "at risk" at time t will be $R(t, X(t))$.

Kaplan-Meier Curve

Semiparametric Models: Cox Model

▶ Semiparametric models can give more efficient estimates than nonparametric models

▶ Hazard function $h(t, X_i)$ follows the proportional hazards assumption:

$$h(t, X_i) = h_0(t) \exp(X_i \beta)$$

▶ Proportional hazards:

$$\frac{h(t, X_i)}{h(t, X_j)} = \frac{h_0(t) \exp(X_i \beta)}{h_0(t) \exp(X_j \beta)} = \frac{\exp(X_i \beta)}{\exp(X_j \beta)}$$

Cox Model

▶ Semiparametric: baseline hazard function $h_0(t)$ can be left unspecified in the estimation of β

▶ $h_0(t)$ can be estimated nonparametrically

Cox Model

▶ Likelihood of event time T_i:

$$\frac{h(T_i, X_i)\Delta t}{\sum_{j \in R(T_i)} h(T_i, X_j)\Delta t}$$
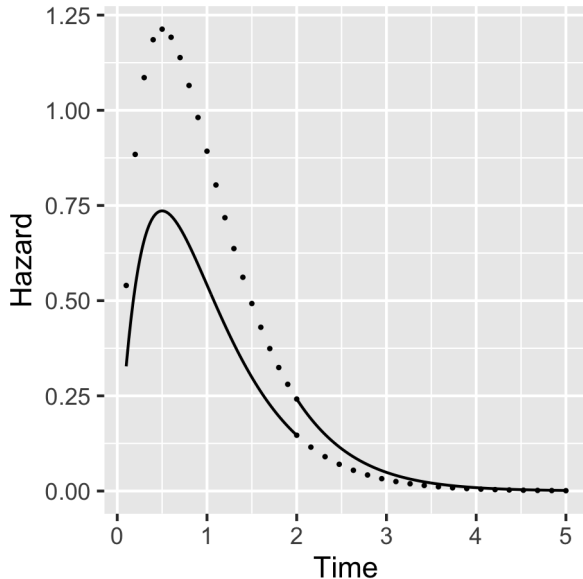
▶ Maximum partial likelihood estimator:

$$\hat{\beta} = \arg\max_{\beta} \prod_{i=1}^{N} \left\{ \frac{\exp(X_i\beta)}{\sum_{j \in R_i} \exp(X_j\beta)} \right\}^{\delta_i}$$

Time-Dependent Cox Model

- Cox model can handle time-dependent covariates
- Hazard function:

$$h(t, X_i(t)) = h_0(t) \exp(X_i(t)\beta)$$

Time-Dependent Cox Model

Parametric Models: Parametric Hazard Function

▶ Parametric baseline hazard function, i.e, $h_0(t; \lambda)$
▶ Hazard function can be estimated more efficiently than semiparametric models

Parametric Hazard Function

▶ Hazard function:
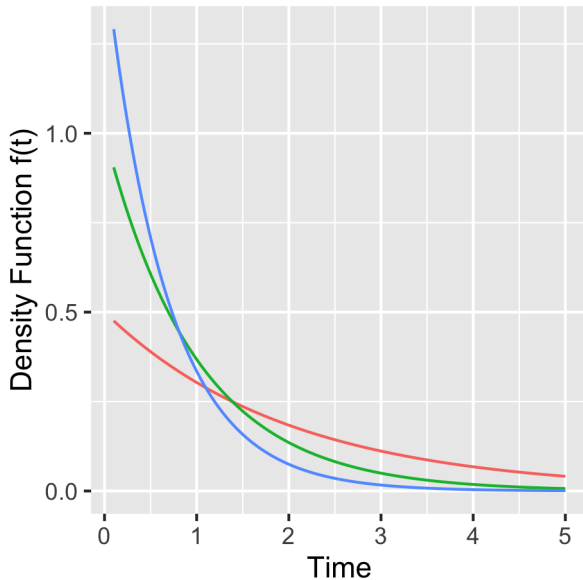
$$h(t, X_i) = h_0(t; \lambda) \exp(X_i \beta)$$

▶ Likelihood:

$$\prod_{i:\delta_i=1} f(T_i; \beta, \lambda) \prod_{i:\delta_i=0} S(C_i; \beta, \lambda)$$

Parametric Hazard Function

Distribution	PDF $f(t)$	Survival $S(t)$	Hazard $h(t)$
Exponential	$\lambda exp(-\lambda t)$	$exp(-\lambda t)$	λ
Weibull	$\lambda k t^{k-1} exp(-\lambda t^k)$	$exp(-\lambda t^k)$	$\lambda k t^{k-1}$
Logistic	$\dfrac{e^{-(t-\mu)/\sigma}}{\sigma(1+e^{-(t-\mu)/\sigma})^2}$	$\dfrac{e^{-(t-\mu)/\sigma}}{1+e^{-(t-\mu)/\sigma}}$	$\dfrac{1}{\sigma(1+e^{-(t-\mu)/\sigma})}$
Log-logistic	$\dfrac{\lambda k t^{k-1}}{(1+\lambda t^k)^2}$	$\dfrac{1}{1+\lambda t^k}$	$\dfrac{\lambda k t^{k-1}}{1+\lambda t^k}$
Normal	$\dfrac{1}{\sqrt{2\pi}\sigma} exp(-\dfrac{(t-\mu)^2}{2\sigma^2})$	$1-\Phi(\dfrac{t-\mu}{\sigma})$	$\dfrac{1}{\sqrt{2\pi}\sigma(1-\Phi((t-\mu)/\sigma))} exp(-\dfrac{(t-\mu)^2}{2\sigma^2})$
Log-normal	$\dfrac{1}{\sqrt{2\pi}\sigma t} exp(-\dfrac{(log(t)-\mu)^2}{2\sigma^2})$	$1-\Phi(\dfrac{log(t)-\mu}{\sigma})$	$\dfrac{\frac{1}{\sqrt{2\pi}\sigma t} exp(-(log(t)-\mu)^2/2\sigma^2)}{1-\Phi(\frac{log(t)-\mu}{\sigma})}$

Exponential distribution

Parametric Models: Accelerated Failure Time Model

▶ Event time is directly parameterized:
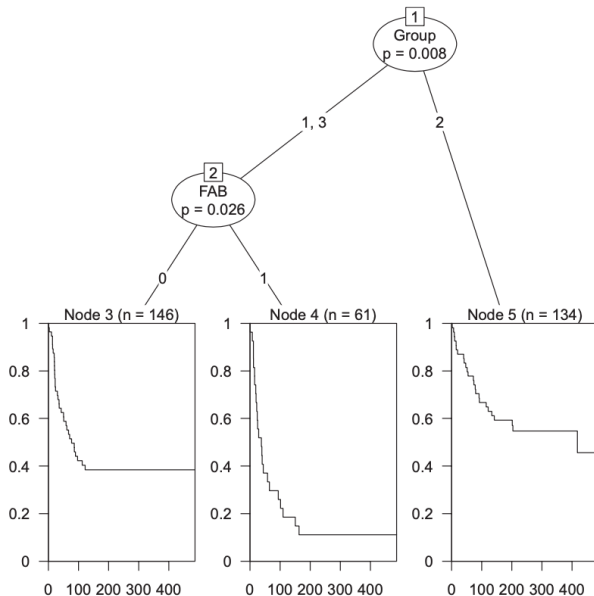
$$\log(T_i) = X_i\beta + \sigma\epsilon$$

▶ Likelihood:

$$\prod_{i:\delta_i=1} f(T_i; \beta, \sigma) \prod_{i:\delta_i=0} S(C_i; \beta, \sigma)$$

Machine Learning Methods

- ▶ Available methods:
  - Regularized Cox models
  - Survival trees
  - Bayesian methods
  - Neural networks
  - Support vector machines
- ▶ Machine learning methods primarily aim to build prediction models
- ▶ Causal inference framework is still in its infancy

Survival Trees

Outline

Performance Measures: C-index

▶ Compare the rankings of observed and predicted survival times

▶ Concordance probability:

$$c = \Pr(\hat{y}_i > \hat{y}_j | y_i > y_j)$$

C-index

▶ Estimator of C-index for Cox models:
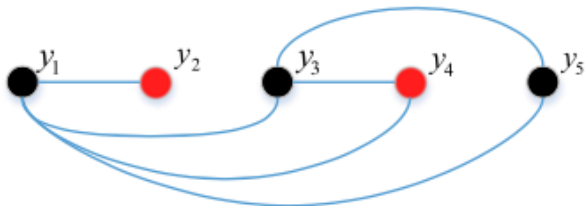
$$\hat{c} = \frac{1}{num} \sum_{i:\delta_i=1} \sum_{j:y_i<y_j} I(X_i\hat{\beta} > X_j\hat{\beta})$$

num: number of all comparable pairs

▶ If y_i is binary, C-index = AUC

C-index

$$y_1 < y_2 < y_3 < y_4 < y_5$$

Black: event observed
Red: censored
Edges: possible ranking comparisons

Performance Measures: Brier Score

▶ Compare the observed and predicted event occurrence before time t

▶ $z_i(t)$: indicator of event before t

▶ $\hat{z}_i(t)$: predicted probability of event before t

▶ Brier score at t:

$$BS(t) = \frac{1}{N} \sum_{i=1}^{N} \{\hat{z}_i(t) - z_i(t)\}$$

▶ Censored information can be incorporated

Competing Risks

▶ $D_i \in \{1 \ldots K\}$: i's cause of event
▶ Cause-specific density:

$$f_k(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t, D = k)}{\Delta t}$$

▶ Cumulative incidence function (CIF):

$$I_k(t) = \int_0^t f_k(\tau) d\tau = \Pr\{T \le t, D = k\}$$

▶ Can we estimate the effect of X on CIF?

Competing Risks

▶ Consider cause-specific hazard function:

$$h_k^{cs}(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t, D = k | T \ge t)}{\Delta t}$$

▶ $h_k^{cs}(t)$ can be estimated using the Cox model:

$$h_k^{cs}(t, X_i) = h_{k0}^{cs}(t) \exp(X_i \beta_k^{cs})$$

▶ Effect of X on CIF cannot be inferred from β_k^{cs}

Competing Risks

▶ Alternative is subdistribution hazard function[2]:

$$h_k^{sd}(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \leq T < t + \Delta t, D = k | A)}{\Delta t},$$

where $A = \{T \geq t \text{ or } (T < t, D \neq k)\}$

▶ $h_k^{sd}(t)$ can be estimated using the Cox model:

$$h_k^{sd}(t, X_i) = h_{k0}^{sd}(t) \exp(X_i \beta_k^{sd})$$

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[2]Fine and Gray (1999, JASA)

Competing Risks

▶ Effect of X on CIF can be inferred from β_k^{sd} as

$$\log(-\log(1 - I_k(t, X)))$$
$$= \log(-\log(1 - I_{k0}(t))) + X\beta_k^{sd}$$

▶ When the probability of an event is low, then the logistic link function and the complementary log-log link function are very similar.

▶ Thus, if this is the case, β_k^{sd} can be interpreted as odds ratios for the CIF.

Competing Risks with Time-Dependent Covariates[3]

- ▶ Individuals with time-dependent covariates can be incorporated in a competing risk model.
- ▶ Split their observation periods into periods where the covariates do not vary with time.
- ▶ Consider left truncation as well as right censoring in the estimation.

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[3]Geskus (2011, Biometrics)

Competing Risks with Time-Dependent Covariates

▶ "Weights" of individuals who experienced competing events need to be adjusted for left truncation and right censoring.

▶ Each i has weight $w_i(T_j)$ at T_j given by:

$$w_i(T_j) = \begin{cases} 1 & \text{if "at risk" at } T_j \\ \frac{\Pr(C \geq T_j)\Pr(L < T_j)}{\Pr(C \geq T_j')\Pr(L < T_j')} & \text{if } B \\ 0 & \text{otherwise,} \end{cases}$$

where L is left entry time and
$B = \{i \text{ had competing events at } T_j' < T_j\}$.

Competing Risks with Time-Dependent Covariates: Estimation Procedure

▶ Split each observation period into periods where the covariates do not vary with time.
  ■ Some rows will have left entry time > 0.
▶ Attach weights to the observations.
  ■ Individuals who experienced competing events will have many time-varying weights.
▶ Apply survival analysis program that can handle left truncation in addition to right censoring.

Multi-State Models: Continuous-Time Markov Model[4]

▶ Survival analysis models usually deal with transition from one state to another.

▶ In some cases, modeling transitions between multiple states may be valuable.

▶ Continuous-time Markov model can be used for this purpose.

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[4]Kalbfleisch and Lawless (1985, JASA)

Continuous-Time Markov Model

- Continuous-time Markov model can be viewed as an extension of a parametric Cox model.
- $S_i(t) \in \{1 \dots R\}$: i's state at time t
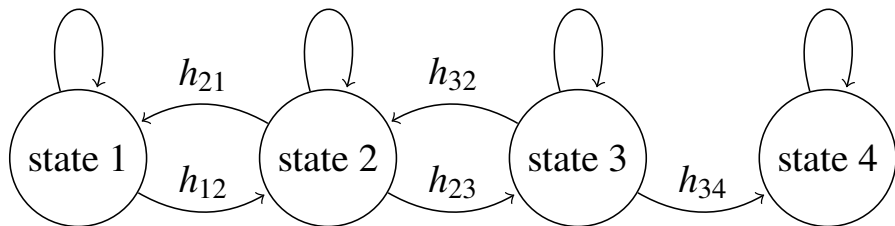
Continuous-Time Markov Model

▶ Hazard (instantaneous risk) of moving from state r to state r' is assumued to be:

$$h_{rr'}(t) = \lim_{\Delta t \to 0} \frac{\Pr(S(t + \Delta t) = r | S(t) = r')}{\Delta t}$$

▶ $h_{rr'}(t)$ is estimated using the same parameterization as the parametric Cox model:

$$h_{rr'}(t, X_i(t)) = h_{rr'0}(t; \lambda_{rr'}) \exp(X_i(t)\beta_{rr'})$$

Continuous-Time Markov Model

Outline

Kaito et al. (2020, Blood Adv.)

Heterogeneous impact of cytomegalovirus reactivation on nonrelapse mortality in hematopoietic stem cell transplantation

Satoshi Kaito,[1,*] Yujiro Nakajima,[2,3,*] Konan Hara,[1,4] Takashi Toya,[1] Tetsuya Nishida,[5] Naoyuki Uchida,[6] Junichi Mukae,[1] Takahiro Fukuda,[7] Yukiyasu Ozawa,[8] Masatsugu Tanaka,[9] Kazuhiro Ikegame,[10] Yuta Katayama,[11] Takuro Kuriyama,[12] Junya Kanda,[13] Yoshiko Atsuta,[14,15] Masao Ogata,[16] Ayumi Taguchi,[17] and Kazuteru Ohashi[1]

Introduction

▶ Investigated the heterogeneous impact of CMV reactivation on NRM in hematopoietic stem cell transplantation.

▶ Used time-dependent Cox model considering competing risks.

▶ Heterogeneous impact was quantified by interaction terms.

Heterogeneous Treatment Effect

▶ Treatment effects may vary across the levels of baseline characteristics.

▶ Subgroup analysis is one way to investigate heterogeneous treatment effects (HTEs).

▶ However, subgroups can be very small if there are a lot of baseline characteristics.

▶ Statistical tests of interactions between the treatment and baseline characteristics can be a solution.
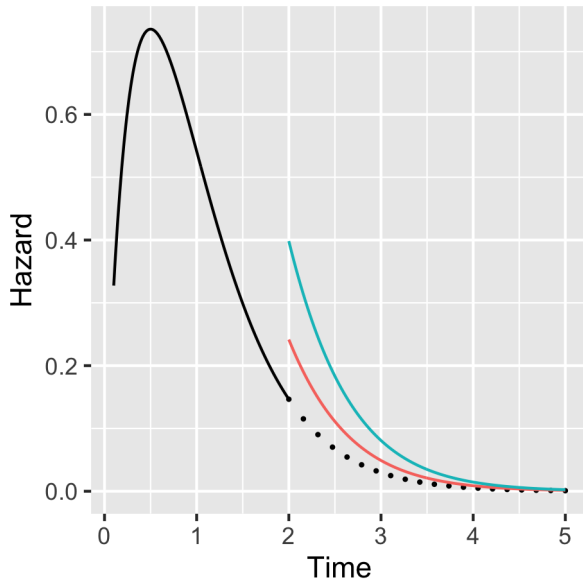
Heterogeneous Treatment Effect

- D: treatment
- X: baseline characteristics
- Statistical tests of interactions:

$$y = X\beta + D * X\gamma$$

- Significant γ indicates the HTE.
- In this study,

$$\text{NRM} = X\beta + \text{CMV} * X\gamma.$$

Heterogeneous Treatment Effect

Scoring model for CMV reactivation

▶ Cumulative incidence of CMV reactivation was evaluated considering relapse and NRM as competing risks.

▶ Scoring model for CMV reactivation was developed and assessed by landmark analysis at day 100.

Scoring model for CMV reactivation

Significant factors (HR > 1):

- ▶ Recipient positive/donor negative CMV serology
- ▶ Recipient positive/donor positive CMV serology
- ▶ TCD *in vivo*
- ▶ HLA disparity
- ▶ ≥ 50 years
- ▶ Transplant from an unrelated donor
- ▶ TBI
- ▶ Older transplant year
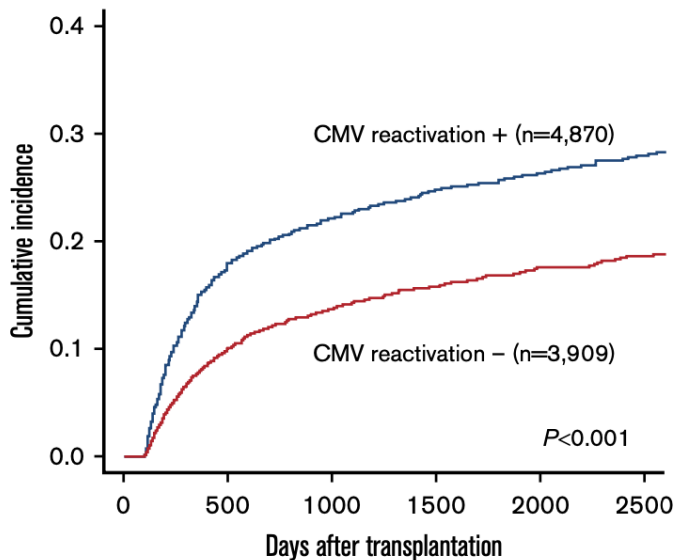
Scoring model for CMV reactivation

Significant factors (HR < 1):

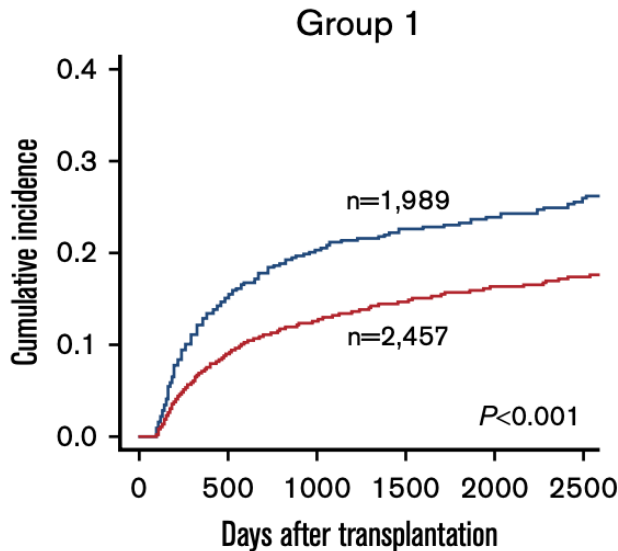- ▶ Tacrolimus-based GVHD prophylaxis regimen
- ▶ CB

Scoring model for CMV reactivation
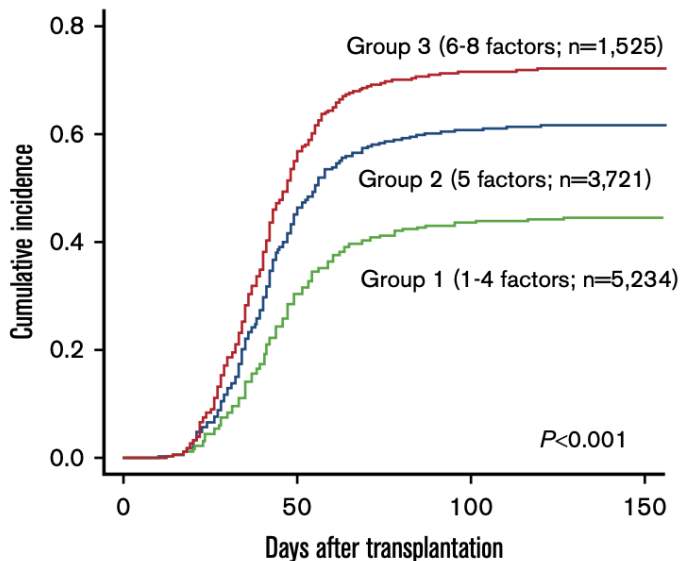
Impact of CMV reactivation on NRM

Impact of CMV reactivation on NRM

Group 1

Impact of CMV reactivation on NRM

Group 2

n=1,970

n=1,136

P<0.001

Days after transplantation

Impact of CMV reactivation on NRM

Group 3
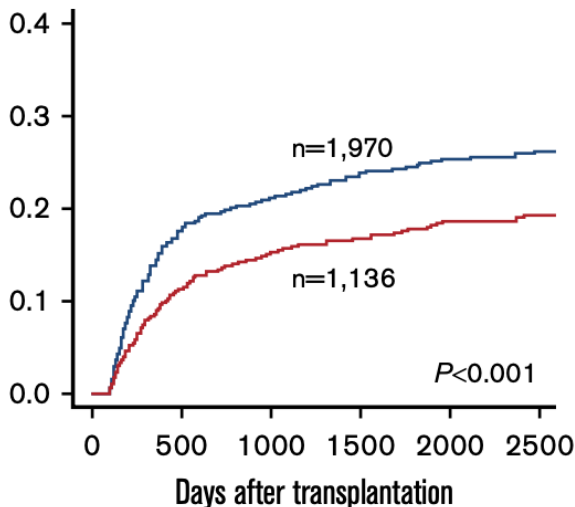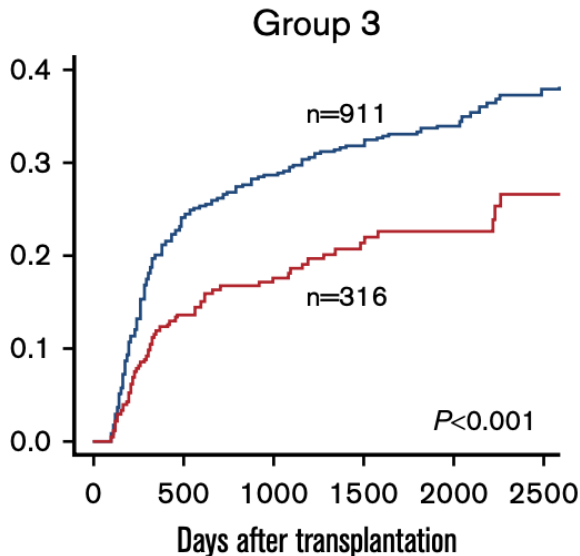
Heterogeneous impact

▶ Cumulative incidence of NRM was evaluated considering relapse as a competing risk and CMV reactivation as a time-dependent covariate.

▶ Interaction terms between CMV reactivation and baseline characteristics were included in the model.

▶ Scoring model for the heterogeneous impact of CMV reactivation on NRM was developed and assessed by landmark analysis at day 100.

Heterogeneous impact

Significant factors (HR > 1):

- ▶ CML

Significant factors (HR < 1):

- ▶ Poor PS
- ▶ Transplantation from HLA-mismatched donors
- ▶ High disease risk

Heterogeneous impact

Refined group 1

CMV reactivation + (n=376)

CMV reactivation − (n=210)

P=0.62

Cumulative incidence

Days after transplantation

Heterogeneous impact

Refined group 2

CMV reactivation + (n=821)

CMV reactivation − (n=782)

P<0.001

Cumulative incidence

Days after transplantation

Taguchi et al. (2020, Cancers)

cancers
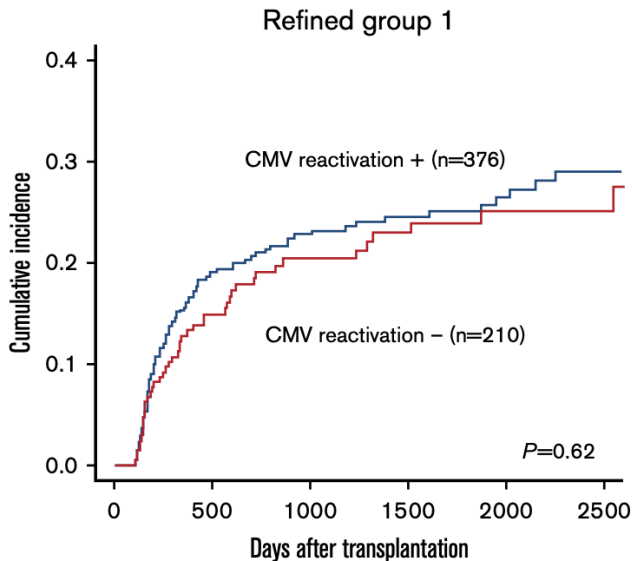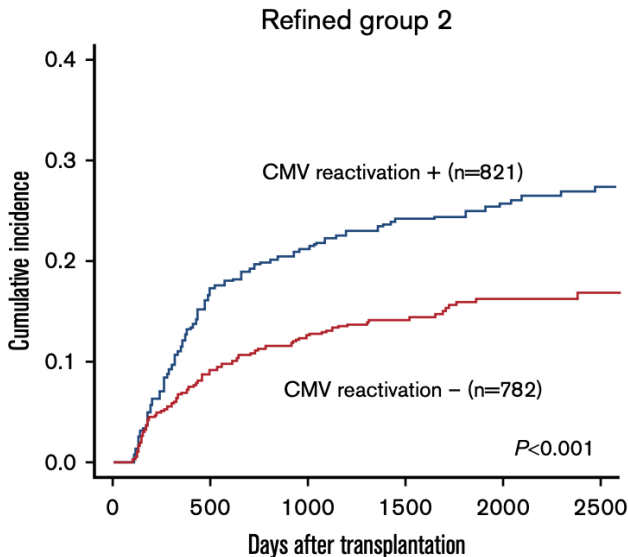
Article

Multistate Markov Model to Predict the Prognosis of High-Risk Human Papillomavirus-Related Cervical Lesions

Ayumi Taguchi [1,2], Konan Hara [3,4], Jun Tomio [5], Kei Kawana [6,*], Tomoki Tanaka [1], Satoshi Baba [1], Akira Kawata [1], Satoko Eguchi [1], Tetsushi Tsuruga [1], Mayuyo Mori [1], Katsuyuki Adachi [1], Takeshi Nagamatsu [1], Katsutoshi Oda [1], Toshiharu Yasugi [1,2], Yutaka Osuga [1] and Tomoyuki Fujii [1]

Introduction

▶ Investigated the prognosis of hrHPV-related cervical lesions.

▶ CIN has a natural history of bidirectional transition between different states.

▶ Cox models assuming a unidirectional disease progression may oversimplify CIN fate.

▶ Application of continuous-time Markov model to this situation may be of interest.

Samples
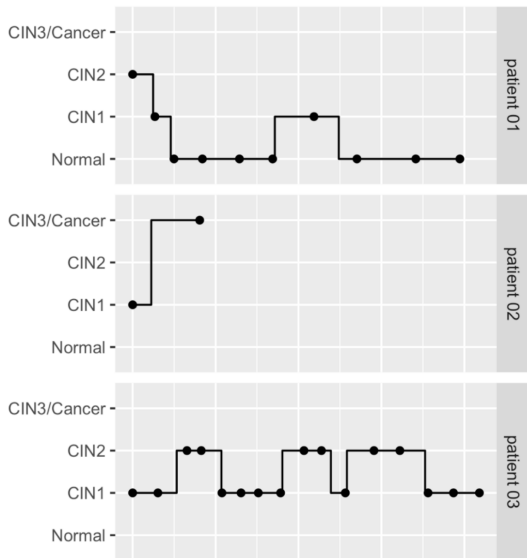
Summary of transitions

| Diagnosis at (t-1)th Visit	HPV Category	Diagnosis at tth Visit				
		Normal	CIN1	CIN2	CIN3	Cancer
Normal	HPV 16	206 (84.4)	13 (5.3)	21 (8.6)	4 (1.6)	0 (0.0)
	HPV 18	89 (81.6)	12 (11.0)	8 (7.3)	0 (0.0)	0 (0.0)
	HPV 52	277 (75.8)	54 (14.7)	32 (8.7)	2 (0.5)	0 (0.0)
	HPV 58	230 (80.4)	33 (11.5)	21 (7.3)	2 (0.6)	0 (0.0)
	Other hrHPVs	611 (86.1)	72 (10.1)	23 (3.2)	3 (0.4)	0 (0.0)
	No hrHPVs	1289 (90.2)	109 (7.6)	26 (1.8)	3 (0.2)	1 (0.0)
CIN1	HPV 16	29 (28.9)	34 (34.0)	35 (35.0)	2 (2.0)	0 (0.0)
	HPV 18	18 (38.2)	19 (40.4)	8 (17.0)	2 (4.2)	0 (0.0)
	HPV 52	80 (35.0)	90 (39.4)	53 (23.2)	5 (2.1)	0 (0.0)
	HPV 58	51 (31.6)	68 (42.2)	40 (24.8)	2 (1.2)	0 (0.0)
	Other hrHPVs	132 (40.7)	143 (44.1)	45 (13.8)	4 (1.2)	0 (0.0)
	No hrHPVs	203 (54.5)	132 (35.4)	34 (9.1)	3 (0.8)	0 (0.0)
CIN2	HPV 16	31 (12.1)	37 (14.4)	147 (57.4)	40 (15.6)	1 (0.3)
	HPV 18	10 (12.9)	8 (10.3)	51 (66.2)	8 (10.3)	0 (0.0)
	HPV 52	41 (13.8)	53 (17.9)	168 (56.9)	33 (11.1)	0 (0.0)
	HPV 58	32 (10.2)	45 (14.4)	210 (67.5)	24 (7.7)	0 (0.0)
	Other hrHPVs	49 (16.7)	52 (17.8)	166 (56.8)	25 (8.5)	0 (0.0)
	No hrHPVs	58 (27.2)	31 (14.5)	114 (53.5)	10 (4.6)	0 (0.0)

Continuous-time Markov model
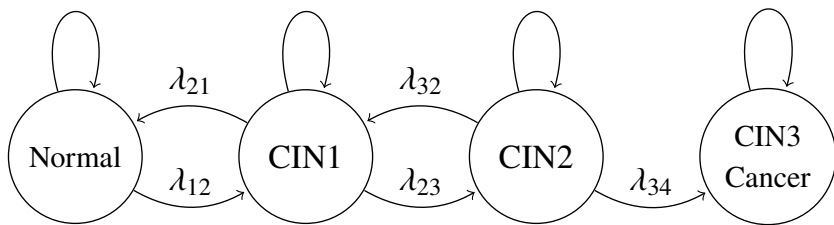
Continuous-Time Markov Model

▶ Parameterization of the hazard function:

$$\lambda_{rr'}(t, \mathsf{HPV}) = \lambda_{rr'0}^{\mathsf{HPV}}$$

▶ Hazards were assumed to be constant across time.

▶ Hazards were estimated for each HPV category.

▶ It was challenging to find a parameterization where the estimation converges.

Predicted 2-year transition probabilities

Current State	HPV Category	State after 2 Years			
		Normal	CIN1	CIN2	CIN3/Cancer
Normal	HPV 16	0.598 (0.506–0.684)	0.099 (0.074–0.128)	0.169 (0.127–0.215)	0.132 (0.090–0.183)
	HPV 18	0.610 (0.479–0.719)	0.156 (0.109–0.215)	0.156 (0.093–0.230)	0.076 (0.033–0.148)
	HPV 52	0.533 (0.474–0.593)	0.189 (0.162–0.219)	0.180 (0.146–0.216)	0.096 (0.070–0.130)
	HPV 58	0.559 (0.484–0.627)	0.171 (0.140–0.205)	0.206 (0.162–0.255)	0.062 (0.041–0.089)
	Other hrHPVs	0.723 (0.676–0.766)	0.155 (0.132–0.182)	0.085 (0.066–0.108)	0.034 (0.023–0.050)
	No hrHPVs	0.838 (0.814–0.861)	0.105 (0.090–0.121)	0.042 (0.032–0.054)	0.012 (0.007–0.020)
CIN1	HPV 16	0.434 (0.349–0.512)	0.089 (0.067–0.115)	0.175 (0.134–0.223)	0.300 (0.225–0.378)
	HPV 18	0.535 (0.396–0.652)	0.146 (0.100–0.207)	0.172 (0.102–0.257)	0.146 (0.069–0.267)
	HPV 52	0.473 (0.413–0.529)	0.178 (0.152–0.208)	0.183 (0.150–0.221)	0.164 (0.122–0.219)
	HPV 58	0.469 (0.399–0.535)	0.165 (0.135–0.197)	0.239 (0.192–0.291)	0.126 (0.084–0.181)
	Other hrHPVs	0.656 (0.606–0.702)	0.156 (0.133–0.181)	0.102 (0.079–0.128)	0.084 (0.058–0.119)
	No hrHPVs	0.808 (0.781–0.835)	0.107 (0.091–0.123)	0.049 (0.038–0.065)	0.034 (0.021–0.054)
CIN2	HPV 16	0.335 (0.266–0.404)	0.079 (0.059–0.101)	0.165 (0.121–0.218)	0.418 (0.330–0.512)
	HPV 18	0.373 (0.245–0.501)	0.119 (0.074–0.178)	0.186 (0.099–0.302)	0.320 (0.178–0.507)
	HPV 52	0.381 (0.324–0.434)	0.156 (0.129–0.184)	0.175 (0.138–0.216)	0.286 (0.220–0.367)
	HPV 58	0.356 (0.291–0.419)	0.150 (0.122–0.181)	0.260 (0.209–0.319)	0.232 (0.167–0.307)
	Other hrHPVs	0.518 (0.453–0.571)	0.146 (0.122–0.169)	0.117 (0.089–0.148)	0.218 (0.159–0.299)
	No hrHPVs	0.706 (0.643–0.749)	0.106 (0.090–0.123)	0.063 (0.045–0.089)	0.124 (0.079–0.191)

Observed and simulated prevalence