Topics on survival analysis

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December 27, 2021

Outline

- 1. Introduction
- 2. Definition
- 3. Statistical Methods
- 4. Related Topics
- 5. Applications

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Introduction¹

- ightharpoonup The goal of survival analysis is to analyze and model data where the outcome is the time until an event of interest occurs, T.
- ▶ T can always be transformed into an occurrence of the event within a specified period.
- ► The advantages and disadvantages of using either survival analysis methods or other rudimentary regression methods should be weighed.

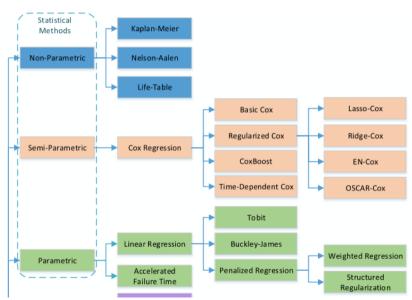
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^{1.} The following contents are primarily based on Wang, Li, and Reddy (2019, ACM Computing Surveys).

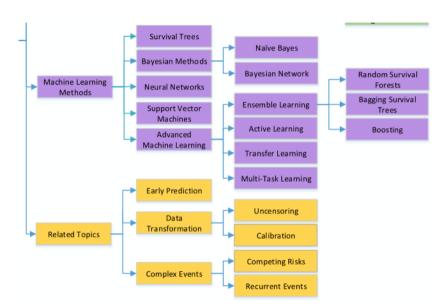
Pros and Cons

- Survival analysis can deal with
 - Censoring
 - Time-dependent covariates
 - Competing risks
- ► However, the costs are that
 - Assumptions for causal inference are strong
 - Models are complicated and not so intuitive
 - Machine learning methods are difficult to apply

Topics



Topics



Outline

1. Introduction

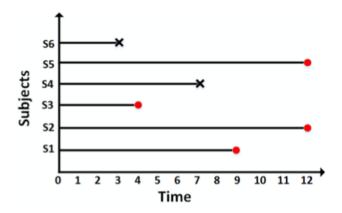
2. Definition

3. Statistical Methods

4. Related Topic

5. Applications

Survival Data and Censoring



X marks: event occurrence Red dots: censored

Problem Statement

- ▶ Data: a triplet (X_i, y_i, δ_i)
 - $i \in \{1 \dots N\}$: individual
 - $X_i \in \mathbb{R}^p$: independent variables
 - δ_i : 0 if censored and 1 if otherwise
 - y_i : T_i if $\delta_i = 1$ and C_i if otherwise
 - \blacksquare T_i : survival time
 - $lackbox{ } C_i$: censored time
- lackbox Goal: estimate the effect of some elements of X on T or predict T with X

Survival and Hazard Function

- ▶ Survival function: $S(t) = Pr(T \ge t)$
- ▶ Cumulative distribution function: F(t) = 1 S(t)
- ► Density function:

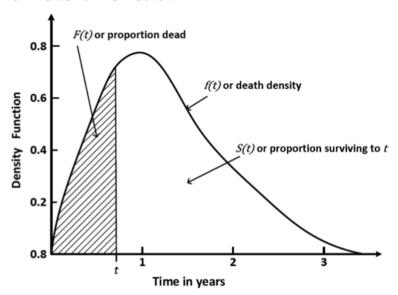
$$f(t) = \frac{d}{dt}F(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t)}{\Delta t}$$

Hazard function:

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t | T \ge t)}{\Delta t} = \frac{f(t)}{S(t)}$$

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Survival and Hazard Function



▶ Consider a simple nonparametric estimator of f(t):

$$\hat{f}(t) = \frac{1}{N} \sum_{i=1}^{N} I(t \le T_i < t + \Delta t)$$

$$\rightarrow \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t, \delta = 1)}{\Delta t}$$

Unfortunately,

$$\Pr(t \le T < t + \Delta t, \delta = 1) < \Pr(t \le T < t + \Delta t)$$

unless $\delta_i = 1$ for all i, no censoring.

- ▶ Thus, we cannot get a consistent estimator for f(t).
- \blacktriangleright How about h(t)?
- ightharpoonup R(t): set of individuals considered to be "at risk" at time t
- ightharpoonup r(t): number of individuals considered to be "at risk" at time t, |R(t)|

Then, a simple nonparametric estimator of h(t) is

$$\hat{h}(t) = \frac{1}{r(t)} \sum_{i \in R(t)} I(t \le T_i < t + \Delta t)$$

$$\rightarrow \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t, \delta = 1 | T \ge t, \delta = 1)}{\Delta t}$$

What is the condition to be

$$\begin{split} & \Pr(t \leq T < t + \Delta t, \delta = 1 | T \geq t, \delta = 1) \\ = & \Pr(t \leq T < t + \Delta t | T \geq t)? \end{split}$$

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If $T \perp \!\!\! \perp \delta$, then

$$\Pr(t \le T < t + \Delta t, \delta = 1 | T \ge t, \delta = 1)$$

$$= \frac{\Pr(t \le T < t + \Delta t, \delta = 1)}{\Pr(T \ge t, \delta = 1)}$$

$$= \frac{\Pr(t \le T < t + \Delta t) \Pr(\delta = 1)}{\Pr(T \ge t) \Pr(\delta = 1)}$$

$$= \Pr(t \le T < t + \Delta t | T \ge t).$$

Thus, we can get a consistent estimator for h(t).

From h(t) to f(t)

As

$$h(t) = \frac{f(t)}{1 - F(t)} = -\frac{d \log\{1 - F(t)\}}{dt}$$

$$\Leftrightarrow F(t) = 1 - \exp\left\{-\int_0^t h(\tau)d\tau\right\}$$

$$\Leftrightarrow f(t) = h(t) \exp\left\{-\int_0^t h(\tau)d\tau\right\},$$

we can calculate f(t) from h(t).

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Nonparametric Models: Kaplan-Meier Curve

Let event times be $T_1 < T_2 < \cdots < T_K$ for N individuals and consider a specific event time T_i :

- $ightharpoonup d(T_i)$: observed events
- $ightharpoonup c(T_j)$: censored individuals between T_j and T_{j+1}
- ▶ then, $r(T_i) = r(T_{i-1}) d(T_{i-1}) c(T_{i-1})$

Kaplan-Meier Curve

ightharpoonup Conditional probability of surviving beyond T_j :

$$p(T_j) = \frac{r(T_j) - d(T_j)}{r(T_j)}$$

▶ The estimator of $S(t) = Pr(T \ge t)$ is given as

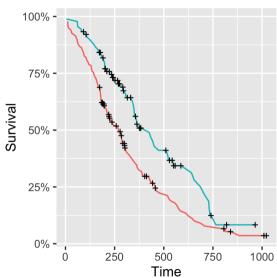
$$\hat{S}(t) = \prod_{j:T_j < t} p(T_j)$$

▶ Nonparametric: no functional form specification

Kaplan-Meier Curve—Covariates

- ▶ The effect of *X* can be quantified by stratification.
- ightharpoonup Time-dependent covariates X(t) can be incorporated as well.
- ightharpoonup Set of individuals considered to be "at risk" at time t will be R(t, X(t)).

Kaplan-Meier Curve



Semiparametric Models: Cox Model

- ► Semiparametric models can give more efficient estimates than nonparametric models
- ▶ Hazard function $h(t, X_i)$ follows the proportional hazards assumption:

$$h(t, X_i) = h_0(t) \exp(X_i \beta)$$

Proportional hazards:

$$\frac{h(t, X_i)}{h(t, X_j)} = \frac{h_0(t) \exp(X_i \beta)}{h_0(t) \exp(X_j \beta)} = \frac{\exp(X_i \beta)}{\exp(X_j \beta)}$$

Cox Model

- Semiparametric: baseline hazard function $h_0(t)$ can be left unspecified in the estimation of β
- \blacktriangleright $h_0(t)$ can be estimated nonparametrically
- ightharpoonup Likelihood of event time T_i :

$$\frac{h(T_i, X_i)\Delta t}{\sum_{j \in R(T_i)} h(T_i, X_j)\Delta t}$$

Maximum partial likelihood estimator:

$$\hat{\beta} = \arg\max_{\beta} \prod_{i=1}^{N} \left\{ \frac{\exp(X_i \beta)}{\sum_{j \in R_i} \exp(X_j \beta)} \right\}^{\delta_i}$$

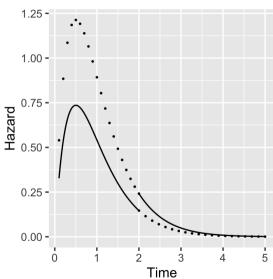
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Time-Dependent Cox Model

- ► Cox model can handle time-dependent covariates
- Hazard function:

$$h(t, X_i(t)) = h_0(t) \exp(X_i(t)\beta)$$

Time-Dependent Cox Model



Parametric Models: Parametric Hazard Function

- ▶ Parametric baseline hazard function, i.e, $h_0(t; \lambda)$
- ▶ Hazard function can be estimated more efficiently than semiparametric models
- Hazard function:

$$h(t, X_i) = h_0(t; \lambda) \exp(X_i \beta)$$

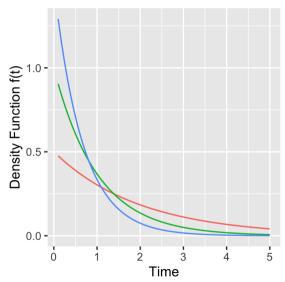
Likelihood:

$$\prod_{i:\delta_i=1} f(T_i; \beta, \lambda) \prod_{i:\delta_i=0} S(C_i; \beta, \lambda)$$

Parametric Hazard Function

Distribution	PDF $f(t)$	Survival S(t)	Hazard $h(t)$
Exponential	$\lambda exp(-\lambda t)$	$exp(-\lambda t)$	λ
Weibull	$\lambda k t^{k-1} exp(-\lambda t^k)$	$exp(-\lambda t^k)$	$\lambda k t^{k-1}$
Logistic	$rac{e^{-(t-\mu)/\sigma}}{\sigma(1+e^{-(t-\mu)/\sigma})^2}$	$\frac{e^{-(t-\mu)/\sigma}}{1+e^{-(t-\mu)/\sigma}}$	$\frac{1}{\sigma(1+e^{-(t-\mu)/\sigma})}$
Log-logistic	$\frac{\lambda k t^{k-1}}{(1+\lambda t^k)^2}$	$\frac{1}{1+\lambda t^k}$	$\frac{\lambda k t^{k-1}}{1+\lambda t^k}$
Normal	$\frac{1}{\sqrt{2\pi}\sigma}exp(-\frac{(t-\mu)^2}{2\sigma^2})$	$1 - \Phi(\frac{t-\mu)}{\sigma})$	$\frac{1}{\sqrt{2\pi}\sigma(1-\Phi((t-\mu)/\sigma))}exp(-\frac{(t-\mu)^2}{2\sigma^2})$
Log-normal	$\frac{1}{\sqrt{2\pi}\sigma t}exp(-\frac{(log(t)-\mu)^2}{2\sigma^2})$	$1 - \Phi(\frac{\log(t) - \mu}{\sigma})$	$\frac{\frac{1}{\sqrt{2\pi}\sigma t}exp(-(log(t)-\mu)^2/2\sigma^2)}{1-\Phi(\frac{log(t)-\mu}{\sigma})}$

Exponential distribution



Parametric Models: Accelerated Failure Time Model

► Event time is directly parameterized:

$$\log(T_i) = X_i \beta + \sigma \epsilon$$

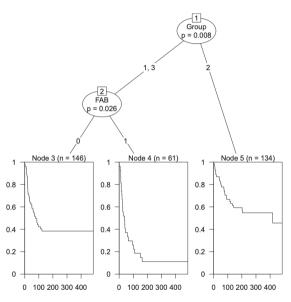
Likelihood:

$$\prod_{i:\delta_i=1} f(T_i; \beta, \sigma) \prod_{i:\delta_i=0} S(C_i; \beta, \sigma)$$

Machine Learning Methods

- Available methods:
 - Regularized Cox models
 - Survival trees
 - Bayesian methods
 - Neural networks
 - Support vector machines
- Machine learning methods primarily aim to build prediction models
- Causal inference framework is still in its infancy

Survival Trees



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Performance Measures: C-index

- Compare the rankings of observed and predicted survival times
- ► Concordance probability:

$$c = \Pr(\hat{y}_i > \hat{y}_j | y_i > y_j)$$

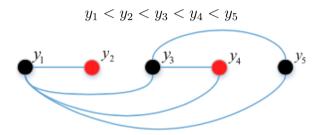
Estimator of C-index for Cox models:

$$\hat{c} = \frac{1}{num} \sum_{i:\delta_i = 1} \sum_{j:y_i < y_j} I(X_i \hat{\beta} > X_j \hat{\beta})$$

num: number of all comparable pairs

▶ If y_i is binary, C-index = AUC

C-index



Black: event observed

Red: censored

Edges: possible ranking comparisons

Performance Measures: Brier Score

- \triangleright Compare the observed and predicted event occurrence before time t
- $ightharpoonup z_i(t)$: indicator of event before t
- $ightharpoonup \hat{z}_i(t)$: predicted probability of event before t
- ▶ Brier score at *t*:

$$BS(t) = \frac{1}{N} \sum_{i=1}^{N} {\{\hat{z}_i(t) - z_i(t)\}}$$

Censored information can be incorporated

- ▶ $D_i \in \{1 \dots K\}$: *i*'s cause of event
- Cause-specific density:

$$f_k(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t, D = k)}{\Delta t}$$

Cumulative incidence function (CIF):

$$I_k(t) = \int_0^t f_k(\tau)d\tau = \Pr\{T \le t, D = k\}$$

► Can we estimate the effect of X on CIF?

Consider cause-specific hazard function:

$$h_k^{cs}(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t, D = k | T \ge t)}{\Delta t}$$

 $ightharpoonup h_k^{cs}(t)$ can be estimated using the Cox model:

$$h_k^{cs}(t, X_i) = h_{k0}^{cs}(t) \exp(X_i \beta_k^{cs})$$

lacktriangle Effect of X on CIF cannot be inferred from eta_k^{cs}

► Alternative is subdistribution hazard function²:

$$h_k^{sd}(t) = \lim_{\Delta t \to 0} \frac{\Pr(t \le T < t + \Delta t, D = k|A)}{\Delta t},$$

where $A = \{T \ge t \text{ or } (T < t, D \ne k)\}$

▶ $h_k^{sd}(t)$ can be estimated using the Cox model:

$$h_k^{sd}(t, X_i) = h_{k0}^{sd}(t) \exp(X_i \beta_k^{sd})$$

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▶ Effect of X on CIF can be inferred from β_k^{sd} as

$$\log(-\log(1 - I_k(t, X))) = \log(-\log(1 - I_{k0}(t))) + X\beta_k^{sd}$$

- ▶ When the probability of an event is low, then the logistic link function and the complementary log-log link function are very similar.
- ightharpoonup Thus, if this is the case, β_k^{sd} can be interpreted as odds ratios for the CIF.

Competing Risks with Time-Dependent Covariates³

- ▶ Individuals with time-dependent covariates can be incorporated in a competing risk model.
- ▶ Split their observation periods into periods where the covariates do not vary with time.
- ▶ Consider left truncation as well as right censoring in the estimation.

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Competing Risks with Time-Dependent Covariates

- "Weights" of individuals who experienced competing events need to be adjusted for left truncation and right censoring.
- ▶ Each *i* has weight $w_i(T_i)$ at T_i given by:

$$w_i(T_j) = \begin{cases} 1 & \text{if "at risk" at } T_j \\ \frac{\Pr(C \geq T_j) \Pr(L < T_j)}{\Pr(C \geq T_j') \Pr(L < T_j')} & \text{if } B \\ 0 & \text{otherwise,} \end{cases}$$

where L is left entry time and $B = \{i \text{ had competing events at } T'_i < T_j\}.$

Competing Risks with Time-Dependent Covariates: Estimation Procedure

- ▶ Split each observation period into periods where the covariates do not vary with time.
 - Some rows will have left entry time > 0.
- Attach weights to the observations.
 - Individuals who experienced competing events will have many time-varying weights.
- Apply survival analysis program that can handle left truncation in addition to right censoring.

Multi-State Models: Continuous-Time Markov Model⁴

- Survival analysis models usually deal with transition from one state to another.
- ▶ In some cases, modeling transitions between multiple states may be valuable.
- ► Continuous-time Markov model can be used for this purpose.

4. Kalbfleisch and Lawless (1985, JASA)

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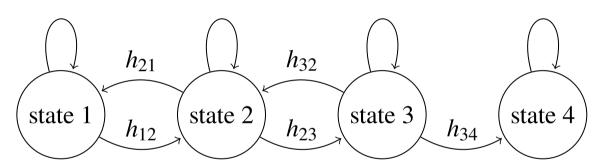
- Continuous-time Markov model can be viewed as an extension of a parametric Cox model.
- $ightharpoonup S_i(t) \in \{1 \dots R\}: i$'s state at time t

 \blacktriangleright Hazard (instantaneous risk) of moving from state r to state r' is assumued to be:

$$h_{rr'}(t) = \lim_{\Delta t \to 0} \frac{\Pr(S(t + \Delta t) = r | S(t) = r')}{\Delta t}$$

 $ightharpoonup h_{rr'}(t)$ is estimated using the same parameterization as the parametric Cox model:

$$h_{rr'}(t, X_i(t)) = h_{rr'0}(t; \lambda_{rr'}) \exp(X_i(t)\beta_{rr'})$$



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REGULAR ARTICLE



Heterogeneous impact of cytomegalovirus reactivation on nonrelapse mortality in hematopoietic stem cell transplantation

Satoshi Kaito,^{1,*} Yujiro Nakajima,^{2,3,*} Konan Hara,^{1,4} Takashi Toya,¹ Tetsuya Nishida,⁵ Naoyuki Uchida,⁶ Junichi Mukae,¹ Takahiro Fukuda,⁷ Yukiyasu Ozawa,⁸ Masatsugu Tanaka,⁹ Kazuhiro Ikegame,¹⁰ Yuta Katayama,¹¹ Takuro Kuriyama,¹² Junya Kanda,¹³ Yoshiko Atsuta,^{14,15} Masao Ogata,¹⁶ Ayumi Taguchi,¹⁷ and Kazuteru Ohashi¹

Introduction

- ► Investigated the heterogeneous impact of CMV reactivation on NRM in hematopoietic stem cell transplantation.
- Used time-dependent Cox model considering competing risks.
- ▶ Heterogeneous impact was quantified by interaction terms.

Heterogeneous Treatment Effect

- ▶ Treatment effects may vary across the levels of baseline characteristics.
- Subgroup analysis is one way to investigate heterogeneous treatment effects (HTEs).
- However, subgroups can be very small if there are a lot of baseline characteristics.
- Statistical tests of interactions between the treatment and baseline characteristics can be a solution.

Heterogeneous Treatment Effect

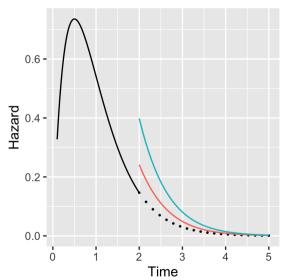
- ▶ *D*: treatment
- ▶ X: baseline characteristics
- Statistical tests of interactions:

$$y = X\beta + D * X\gamma$$

- ightharpoonup Significant γ indicates the HTE.
- In this study,

$$\mathsf{NRM} = X\beta + \mathsf{CMV} * X\gamma.$$

Heterogeneous Treatment Effect



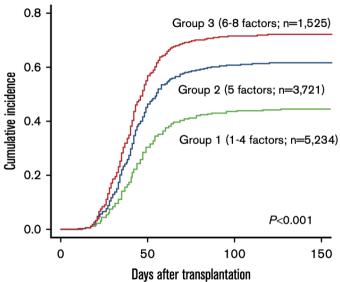
- Cumulative incidence of CMV reactivation was evaluated considering relapse and NRM as competing risks.
- ► Scoring model for CMV reactivation was developed and assessed by landmark analysis at day 100.

Significant factors (HR > 1):

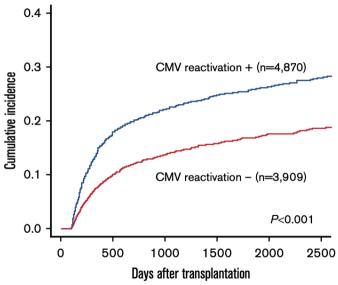
- Recipient positive/donor negative CMV serology
- ► Recipient positive/donor positive CMV serology
- ► TCD in vivo
- HLA disparity
- $ightharpoonup \geq 50$ years
- Transplant from an unrelated donor
- ► TBI
- Older transplant year

Significant factors (HR < 1):

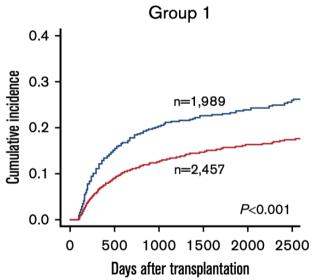
- ► Tacrolimus-based GVHD prophylaxis regimen
- ► CB

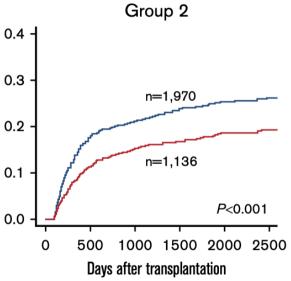


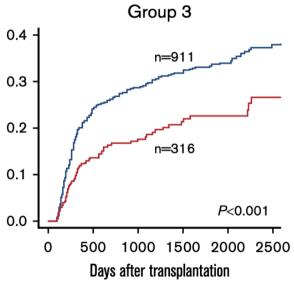
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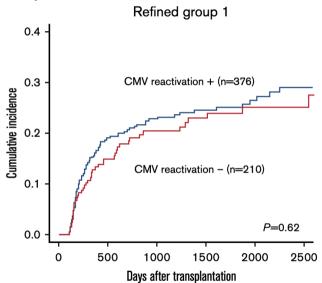
- Cumulative incidence of NRM was evaluated considering relapse as a competing risk and CMV reactivation as a time-dependent covariate.
- Interaction terms between CMV reactivation and baseline characteristics were included in the model.
- ► Scoring model for the heterogeneous impact of CMV reactivation on NRM was developed and assessed by landmark analysis at day 100.

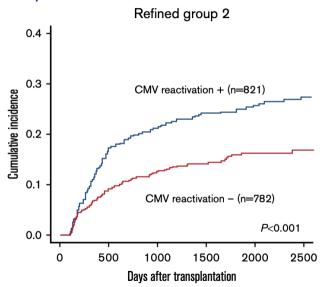
Significant factors (HR > 1):

CML

Significant factors (HR < 1):

- Poor PS
- Transplantation from HLA-mismatched donors
- High disease risk





Taguchi et al. (2020, Cancers)





Article

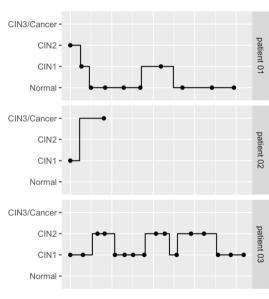
Multistate Markov Model to Predict the Prognosis of High-Risk Human Papillomavirus-Related Cervical Lesions

Ayumi Taguchi ^{1,2}, Konan Hara ^{3,4}, Jun Tomio ⁵, Kei Kawana ^{6,*}, Tomoki Tanaka ¹, Satoshi Baba ¹, Akira Kawata ¹, Satoko Eguchi ¹, Tetsushi Tsuruga ¹, Mayuyo Mori ¹, Katsuyuki Adachi ¹, Takeshi Nagamatsu ¹, Katsutoshi Oda ¹, Toshiharu Yasugi ^{1,2}, Yutaka Osuga ¹ and Tomoyuki Fujii ¹

Introduction

- Investigated the prognosis of hrHPV-related cervical lesions.
- ► CIN has a natural history of bidirectional transition between different states.
- Cox models assuming a unidirectional disease progression may oversimplify CIN fate.
- ▶ Application of continuous-time Markov model to this situation may be of interest.

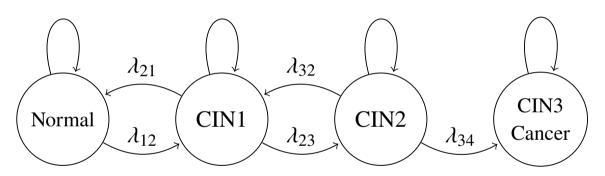
Samples



Summary of transitions

	HPV Category	Diagnosis at t th Visit				
Diagnosis at (t-1) th Visit		Normal	CIN1	CIN2	CIN3	Cancer
Normal	HPV 16	206 (84.4)	13 (5.3)	21 (8.6)	4 (1.6)	0 (0.0)
	HPV 18	89 (81.6)	12 (11.0)	8 (7.3)	0 (0.0)	0 (0.0)
	HPV 52	277 (75.8)	54 (14.7)	32 (8.7)	2 (0.5)	0 (0.0)
	HPV 58	230 (80.4)	33 (11.5)	21 (7.3)	2 (0.6)	0 (0.0)
	Other hrHPVs	611 (86.1)	72 (10.1)	23 (3.2)	3 (0.4)	0 (0.0)
	No hrHPVs	1289 (90.2)	109 (7.6)	26 (1.8)	3 (0.2)	1 (0.0)
CIN1	HPV 16	29 (28.9)	34 (34.0)	35 (35.0)	2 (2.0)	0 (0.0)
	HPV 18	18 (38.2)	19 (40.4)	8 (17.0)	2 (4.2)	0 (0.0)
	HPV 52	80 (35.0)	90 (39.4)	53 (23.2)	5 (2.1)	0 (0.0)
	HPV 58	51 (31.6)	68 (42.2)	40 (24.8)	2 (1.2)	0 (0.0)
	Other hrHPVs	132 (40.7)	143 (44.1)	45 (13.8)	4 (1.2)	0 (0.0)
	No hrHPVs	203 (54.5)	132 (35.4)	34 (9.1)	3 (0.8)	0 (0.0)
CIN2	HPV 16	31 (12.1)	37 (14.4)	147 (57.4)	40 (15.6)	1 (0.3)
	HPV 18	10 (12.9)	8 (10.3)	51 (66.2)	8 (10.3)	0 (0.0)
	HPV 52	41 (13.8)	53 (17.9)	168 (56.9)	33 (11.1)	0 (0.0)
	HPV 58	32 (10.2)	45 (14.4)	210 (67.5)	24 (7.7)	0 (0.0)
	Other hrHPVs	49 (16.7)	52 (17.8)	166 (56.8)	25 (8.5)	0 (0.0)
	No hrHPVs	58 (27.2)	31 (14.5)	114 (53.5)	10 (4.6)	0 (0.0)

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▶ Parameterization of the hazard function:

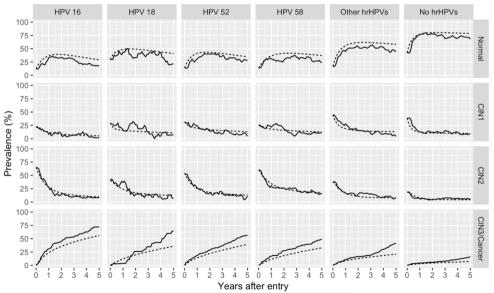
$$\lambda_{rr'}(t, \mathsf{HPV}) = \lambda_{rr'0}^{\mathsf{HPV}}$$

- ▶ Hazards were assumed to be constant across time.
- Hazards were estimated for each HPV category.
- ▶ It was challenging to find a parameterization where the estimation converges.

Predicted 2-year transition probabilities

Current State		State after 2 Years							
Current State	HPV Category	Normal	CIN1	CIN2	CIN3/Cancer				
Normal	HPV 16	0.598 (0.506-0.684)	0.099 (0.074-0.128)	0.169 (0.127-0.215)	0.132 (0.090-0.183)				
	HPV 18	0.610 (0.479-0.719)	0.156 (0.109-0.215)	0.156 (0.093-0.230)	0.076 (0.033-0.148)				
	HPV 52	0.533 (0.474-0.593)	0.189 (0.162-0.219)	0.180 (0.146-0.216)	0.096 (0.070-0.130)				
	HPV 58	0.559 (0.484-0.627)	0.171 (0.140-0.205)	0.206 (0.162-0.255)	0.062 (0.041-0.089)				
	Other hrHPVs	0.723 (0.676-0.766)	0.155 (0.132-0.182)	0.085 (0.066-0.108)	0.034 (0.023-0.050)				
	No hrHPVs	0.838 (0.814-0.861)	0.105 (0.090-0.121)	0.042 (0.032-0.054)	0.012 (0.007-0.020)				
CIN1	HPV 16	0.434 (0.349-0.512)	0.089 (0.067–0.115)	0.175 (0.134-0.223)	0.300 (0.225-0.378)				
	HPV 18	0.535 (0.396-0.652)	0.146 (0.100-0.207)	0.172 (0.102-0.257)	0.146 (0.069-0.267)				
	HPV 52	0.473 (0.413-0.529)	0.178 (0.152-0.208)	0.183 (0.150-0.221)	0.164 (0.122-0.219)				
	HPV 58	0.469 (0.399-0.535)	0.165 (0.135-0.197)	0.239 (0.192-0.291)	0.126 (0.084-0.181)				
	Other hrHPVs	0.656 (0.606-0.702)	0.156 (0.133-0.181)	0.102 (0.079-0.128)	0.084 (0.058-0.119)				
	No hrHPVs	0.808 (0.781-0.835)	0.107 (0.091–0.123)	0.049 (0.038-0.065)	0.034 (0.021–0.054)				
CIN2	HPV 16	0.335 (0.266-0.404)	0.079 (0.059-0.101)	0.165 (0.121-0.218)	0.418 (0.330-0.512)				
	HPV 18	0.373 (0.245-0.501)	0.119 (0.074-0.178)	0.186 (0.099-0.302)	0.320 (0.178-0.507)				
	HPV 52	0.381 (0.324-0.434)	0.156 (0.129-0.184)	0.175 (0.138-0.216)	0.286 (0.220-0.367)				
	HPV 58	0.356 (0.291-0.419)	0.150 (0.122–0.181)	0.260 (0.209-0.319)	0.232 (0.167–0.307)				
	Other hrHPVs	0.518 (0.453-0.571)	0.146 (0.122-0.169)	0.117 (0.089–0.148)	0.218 (0.159–0.299)				
	No hrHPVs	0.706 (0.643-0.749)	0.106 (0.090-0.123)	0.063 (0.045-0.089)	0.124 (0.079-0.191)				

Observed and simulated prevalence



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