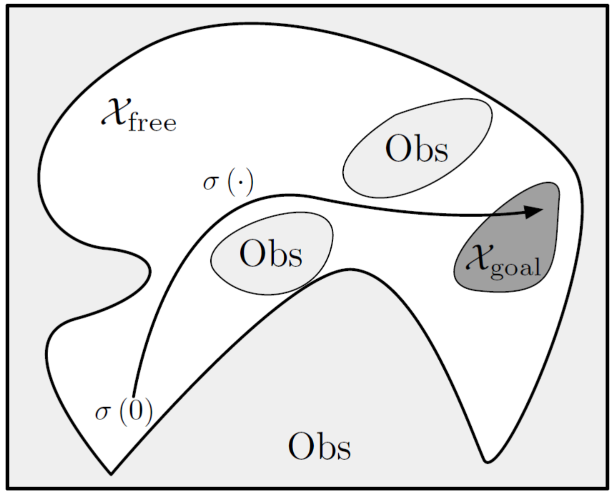
Principles of Robot Autonomy I

Motion planning II: sampling-based methods

Motion planning

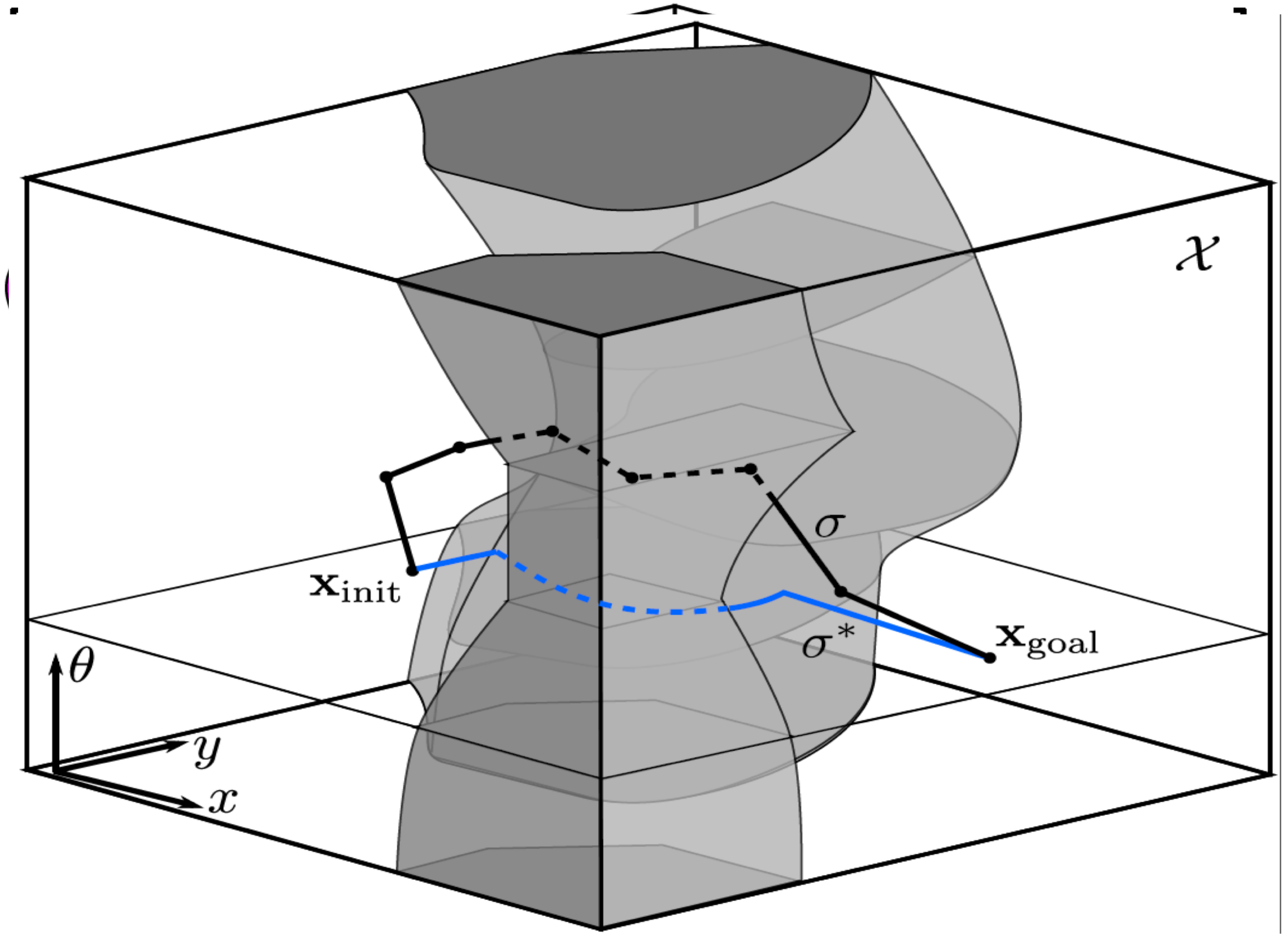
Compute sequence of actions that drives a robot from an initial condition to a terminal condition while avoiding obstacles, respecting motion constraints, and possibly optimizing a cost function



- Aim
  - Learn about sampling-based motion planning algorithms

- Readings:
  - S. LaValle. Planning Algorithms. Chapter 5.

Configuration space



Motion planning algorithms

- Key point: motion planning problem described in the real-world, but it really lives in an another space - the configuration (C-)space!

- Two main approaches to *continuous* motion planning:
  - *Combinatorial planning:* constructs structures in the C-space that discretely and completely capture all information needed to perform planning
  - *Sampling-based planning:* uses collision detection algorithms to probe and incrementally search the C-space for a solution, rather than completely characterizing all of the C_{free} structure

Sampling-based motion planning

Limitations of combinatorial approaches stimulated the development of sampling-based approaches

- Abandon the idea of explicitly characterizing C_{free} and C_{obs}
- Instead, capture the structure of C by random sampling
- Use a black-box component (collision checker) to determine which random configurations lie in C_{free}
- Use such a probing scheme to build a roadmap and then plan a path

Reference: LaValle, S. M. Motion planning. 2011.

Sampling-based motion planning

Pros:

- Conceptually simple
- Relatively-easy to implement
- Flexible: one algorithm applies to a variety of robots and problems
- Beyond the geometric case: can cope with complex differential constraints, uncertainty, etc.

(Mild) cons:

- Unclear how many samples should be generated to retrieve a solution
- Can not determine whether a solution does not exist

Outline

- The geometric case
- The kinodynamic case
- De-randomizing sampling-based planners

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Review of sampling-based methods

Traditionally, two major approaches:

- Probabilistic Roadmap (PRM): graph-based
  - Multi-query planner, i.e., designed to solve multiple path queries on the same scenario
  - Original version: [Kavraki et al., ‘96]
  - “Lazy” version: [Bohlin & Kavraki, ‘00]
  - Dynamic version: [Jaillet & T. Simeon, ‘04]
  - Asymptotically optimal version: [Karaman & Frazzoli, '11]
- Rapidly-exploring Random Trees (RRT): tree-based
  - Single-query planner
  - Original version: [LaValle & Kuner, ‘01]
  - RDT: [LaValle, ‘06]
  - SRT: [Plaku et al., ‘05]
  - Asymptotically optimal version [Karaman & Frazzoli, '11]

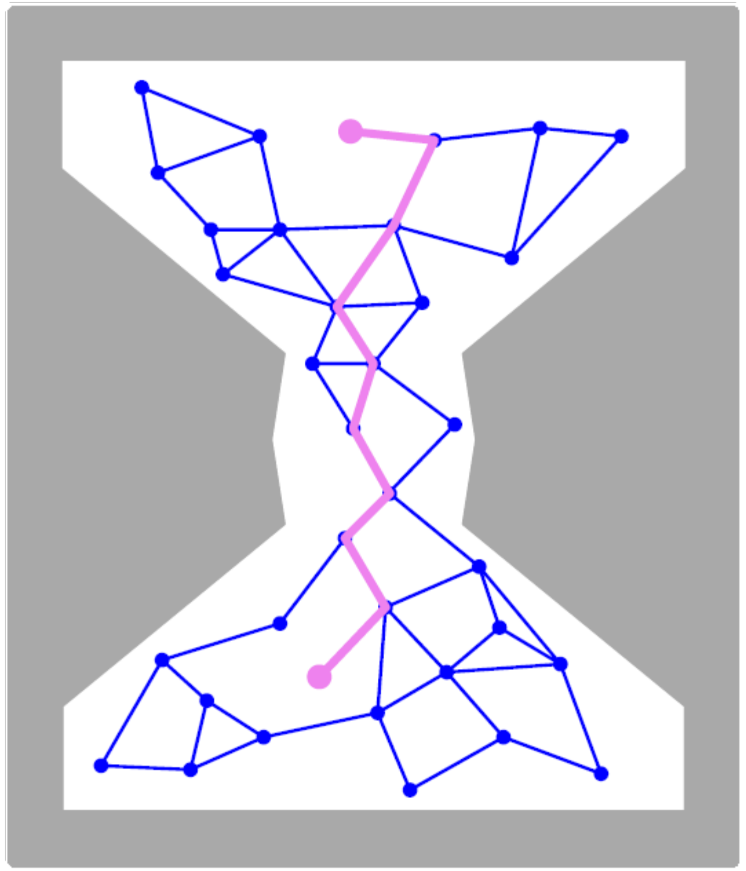
Probabilistic roadmaps (PRM)

A multi-query planner, which generates a roadmap (graph) G, embedded in the free space

Preprocessing step:

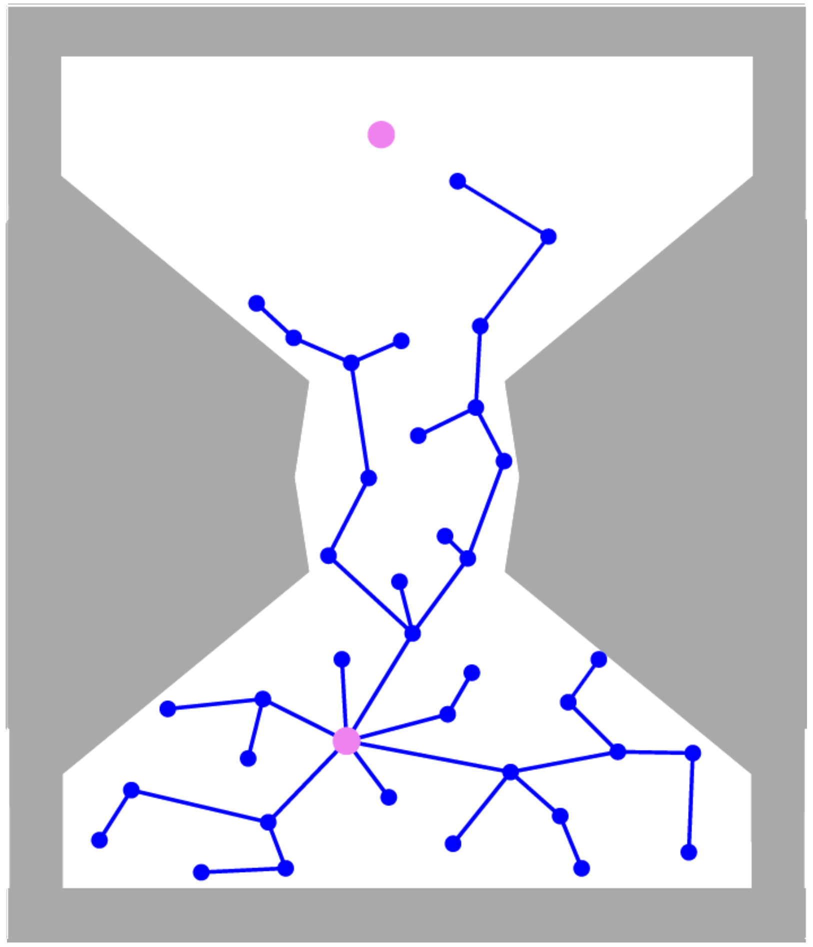
1. Sample a collection of n configurations X_n; discard configurations leading to collisions
2. Draw an edge between each pair of samples $x, x' \in X_n$ such that $\|x - x'\| \leq r$ *and* straight-line path between x and x' is collision free

Given a query $s, t \in C_{free}$, connect them to G and find a path on the roadmap



Rapidly-exploring random trees (RRT)

A single-query planner, which grows a tree T , rooted at the start configuration s, embedded in C_{free}



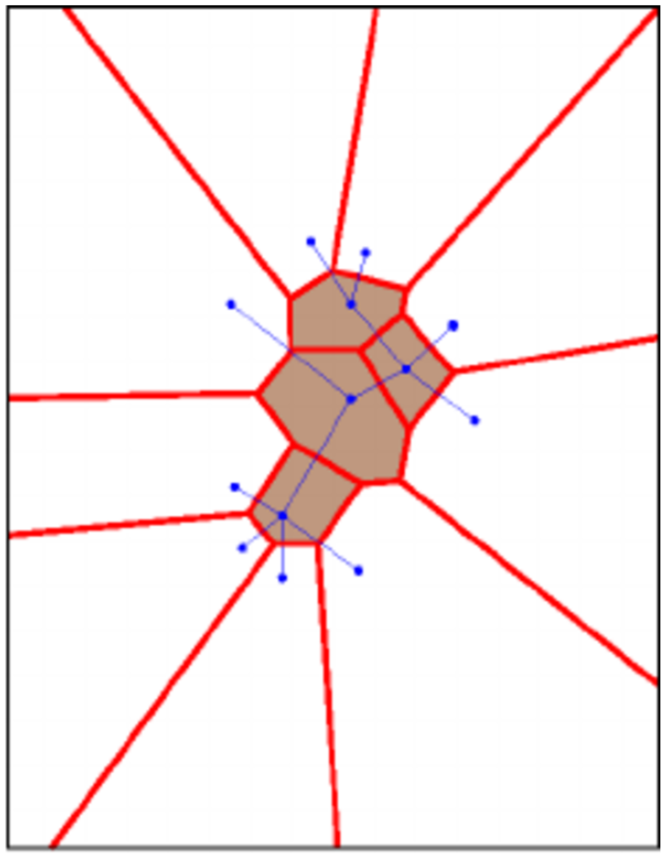
Algorithm works in n iterations:

1. Sample configuration x_{rand}
2. Find nearest vertex x_{near} in T to x_{rand}
3. Generate configuration x_{new} in direction of x_{rand} from x_{near}, such that $\overline{x_{near}x_{new}} \subset C_{free}$
4. Update tree: $T = T \cup \{x_{new}, (x_{near}, x_{new})\}$

Every once in a while, set x_{rand} to be the target vertex t; terminate when $x_{new} = T$

Rapidly-exploring random trees (RRT)

- RRT is known to work quite well in practice
- Its performance can be attributed to its Voronoi bias:
  - Consider a Voronoi diagram with respect to the vertices of the tree
  - For each vertex, its Voronoi cell consists of all points that are closer to that vertex than to any other
  - Vertices on the frontier of the tree have larger Voronoi cells – hence sampling in those regions is more likely



Theoretical guarantees: probabilistic completeness

Question: how large should the number of samples n be? We can say something about the asymptotic behavior:

Kavraki et al. 96: PRM, with $r = \text{const}$, will eventually (as $n \rightarrow \infty$) find a solution if one exists

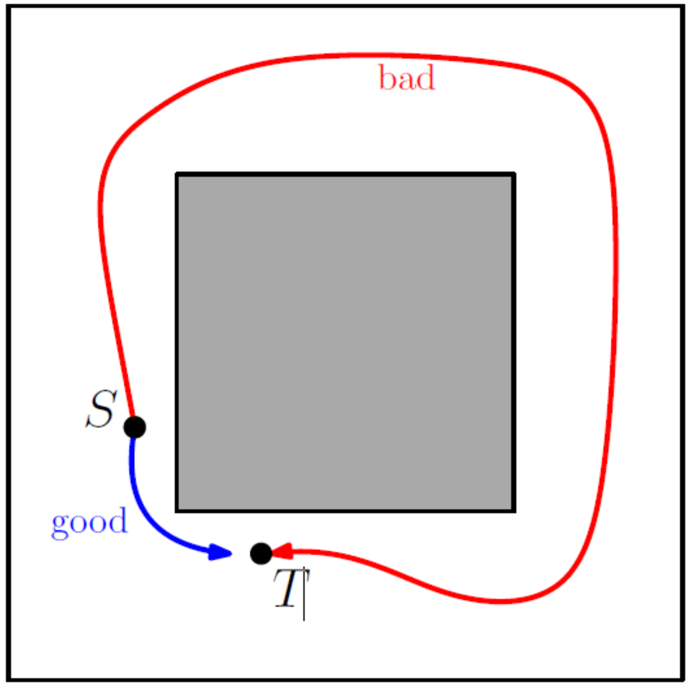
LaValle, 98; Kleinbort et al., 18: RRT will eventually (as $n \rightarrow \infty$) find a solution if one exists

* Unless stated otherwise, the configuration space is assumed to be the d-dimensional Euclidean unit hypercube $[0,1]^d$, with $2 \leq d \leq \infty$

Theoretical guarantees: quality

Question: what can be said about the quality of the returned solution for PRM and RRT, in terms of length, energy, etc.?

Nechushtan et al. (2011) and Karaman and Frazzoli (2011) proved that RRT can produce arbitrarily-bad paths with non-negligible probability: for example, RRT would prefer to take the long (red) way



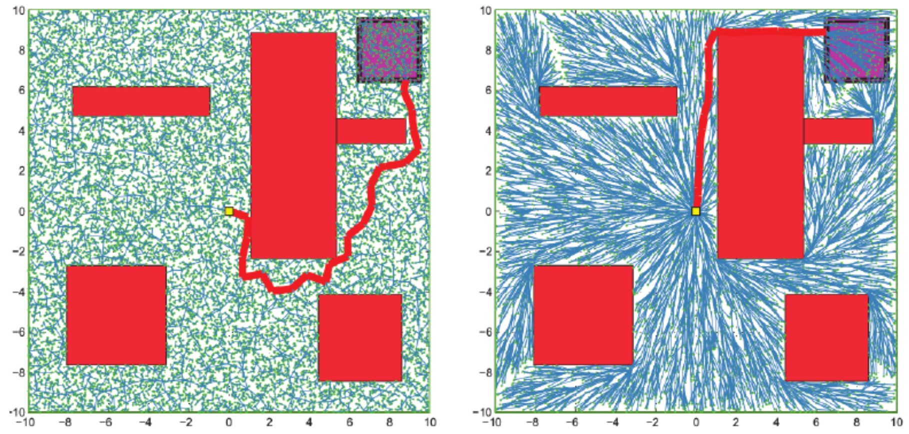
Theoretical guarantees: quality

Karaman and Frazzoli in 2011 provided the first rigorous study of optimality in sampling-based planners:

Theorem: The cost of the solution returned by PRM converges, as $n \to \infty$, to the optimum, when $r_n = \gamma \left(\frac{\log n}{n}\right)^{\frac{1}{d}}$, where γ only depends on d

- KF11 also introduced an asymptotically optimal variant of RRT called RRT* (right)
- Result was later updated to [Solovey et al. 2019]:

$$r_n = \gamma \left(\frac{\log n}{n}\right)^{\frac{1}{d+1}}$$



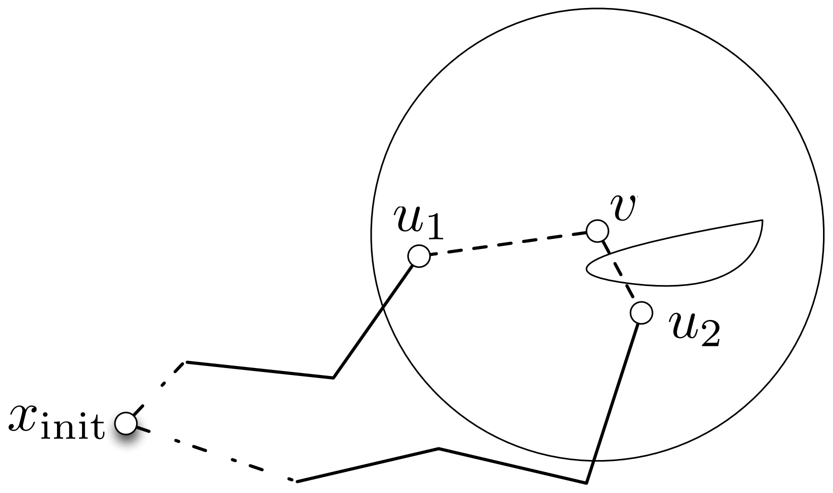
Observations

- PRM-like motion planning algorithms
  - For a give number of nodes n, they find “good” paths
  - …however, require many costly collision checks

- RRT-like motion planning algorithms
  - Finds a feasible path quickly
  - …however the quality of that path is, in general, poor
  - “traps” itself by disallowing new better paths to emerge - RRT* (partially) offsets this behavior

Fast Marching Tree Algorithm (FMT*)

- Key idea: run dynamic programming on sampled nodes, skipping any step in which the attempted connection causes a collision
  - lazy DP operator:
    $$c(v) = \min_{u:\|u-v\|<r_n} \mathrm{Cost}(u,v) + c(u)$$
- Laziness introduces "suboptimal" connections, but such connections are vanishingly rare and FMT* is asymptotically optimal
- Ratio of # of collision-checks for FMT* versus PRM* goes to zero!



Reference: Janson et al. Fast Marching Tree: A Fast Marching Sampling-Based Method for Optimal Motion Planning in Many Dimensions. 2015

Sampling-based planning: summary

- Sampling-based planners transform the difficult global problem into a large set of local and easy problems
- A key ingredient is collision detection, which is conceptually easy, as it can be solved in the workspace (2D or 3D)
- Local planning (edge validation) is typically performed by dense sampling of path and collision detection
- Another key ingredient is nearest-neighbor search: given a query point find its nearest neighbor(s) within a set of points -- also well studied theoretically and practically

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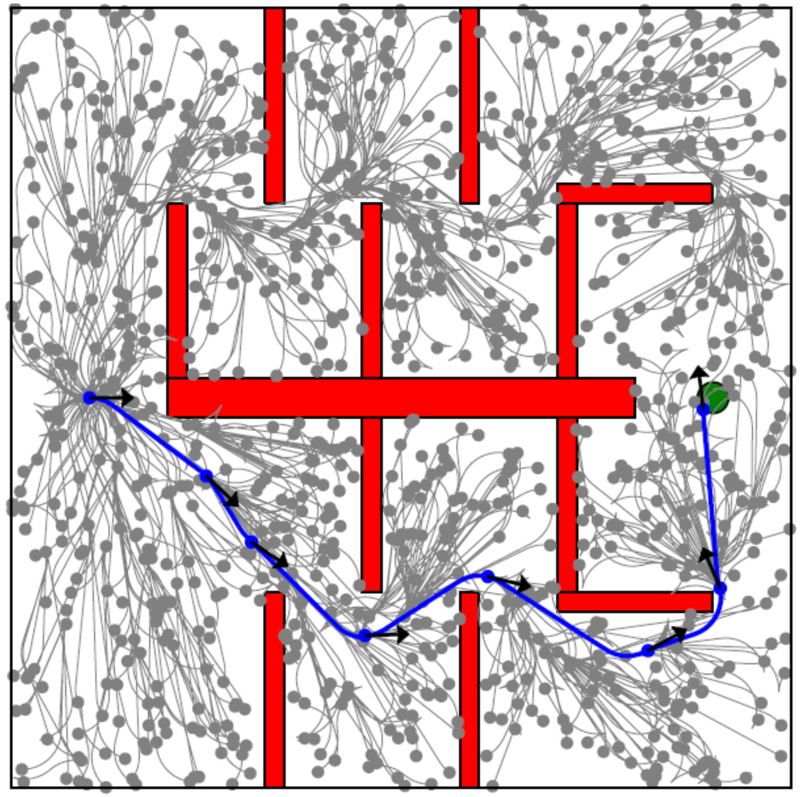
Kinodynamic planning

Kinodynamic motion planning problem: in addition to obstacle avoidance, paths are subject to differential constraints

- The robot operates in the state space X
- To move the robot applies control $u \in U$
- Motion needs to satisfy the system's constraints:
  $\dot{x} = f(x, u)$ for $x \in X, u \in U$

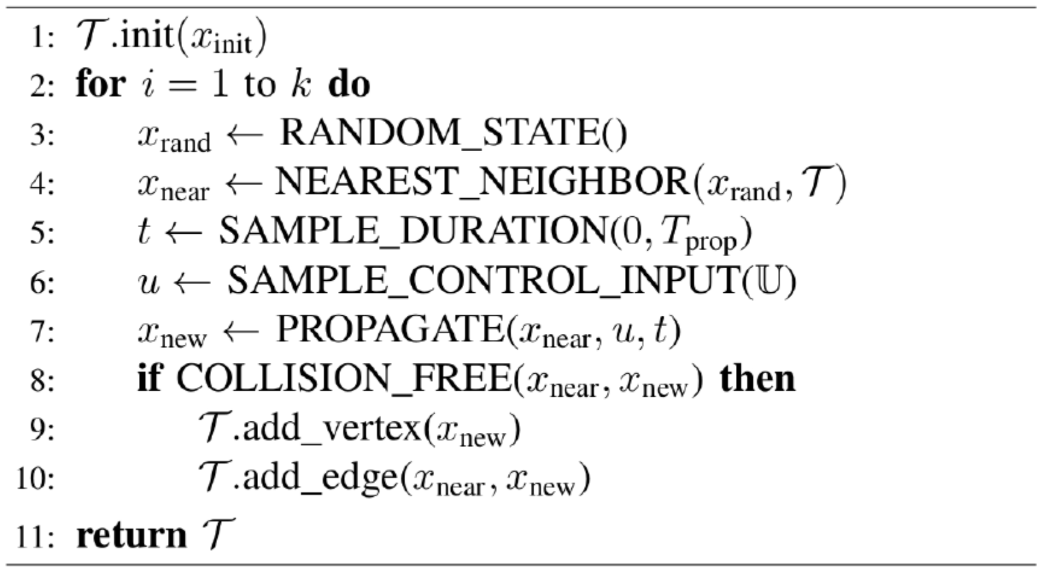


Reference: Schmerling and Pavone. Kinodynamic Planning. 2019

Forward-propagation-based algorithms

RRT can be extended to kinodynamic case in a relatively easy way:

1. Draw a random state and find its nearest neighbor x_{near}
2. Sample a random control $u \in U$ and random duration t
3. Forward propagate the control u for t time from x_{near}



```
$\mathcal{T}$.init($x_{\text{init}}$)
for $i = 1$ to $k$ do
    $x_{\text{rand}} \leftarrow$ RANDOM_STATE()
    $x_{\text{near}} \leftarrow$ NEAREST_NEIGHBOR($x_{\text{rand}}, \mathcal{T}$)
    $t \leftarrow$ SAMPLE_DURATION($0, T_{\text{prop}}$)
    $u \leftarrow$ SAMPLE_CONTROL_INPUT($\mathbb{U}$)
    $x_{\text{new}} \leftarrow$ PROPAGATE($x_{\text{near}}, u, t$)
    if COLLISION_FREE($x_{\text{near}}, x_{\text{new}}$) then
        $\mathcal{T}$.add_vertex($x_{\text{new}}$)
        $\mathcal{T}$.add_edge($x_{\text{near}}, x_{\text{new}}$)
return $\mathcal{T}$
```

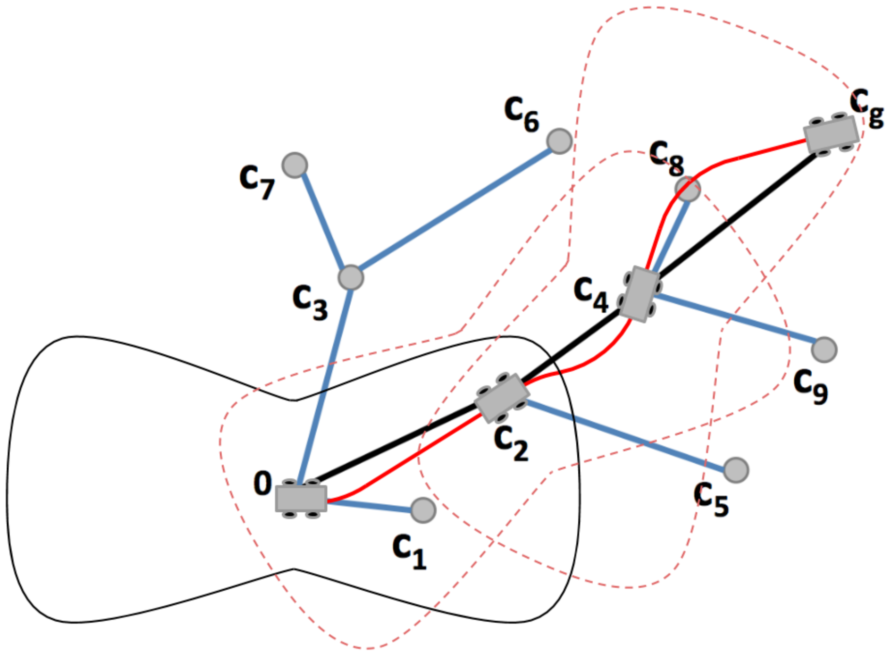
Reference: Kleinbort et al. Probabilistic completeness of RRT for geometric and kinodynamic planning with forward propagation. 2018.

Steering-based algorithms

When efficient online *steering* subroutines exist, kinodynamic planning algorithms may take advantage of this domain knowledge

1. Connect samples by using an optimal trajectory (steering problem)
2. Use reachable sets to find nearest neighbors

Reference: E. Schmerling et al. Optimal Sampling-Based Motion Planning under Differential Constraints: the Driftless Case. 2015



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Should probabilistic planners be probabilistic?

Key question: would theoretical guarantees and practical performance still hold if these algorithms were to be derandomized, i.e., run on deterministic samples?

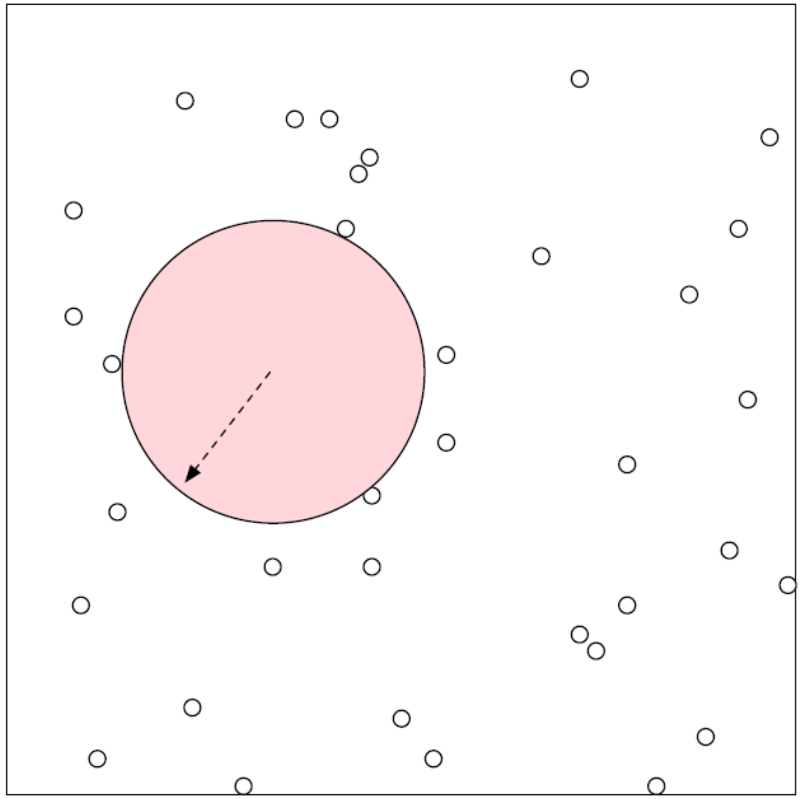
Important question as derandomization would:

- Ease certification process
- Ease use of offline computation
- Potentially simplify a number of operations (e.g., NN search)

Designing “good” sequences

ℓ_2-dispersion: For a finite set S of points contained in $X \subset \mathbb{R}^d$, its ℓ_2-dispersion $D(S)$ is defined as

$$D(S) \coloneqq \sup_{x \in X} \min_{s \in S} \|s - x\|_2$$

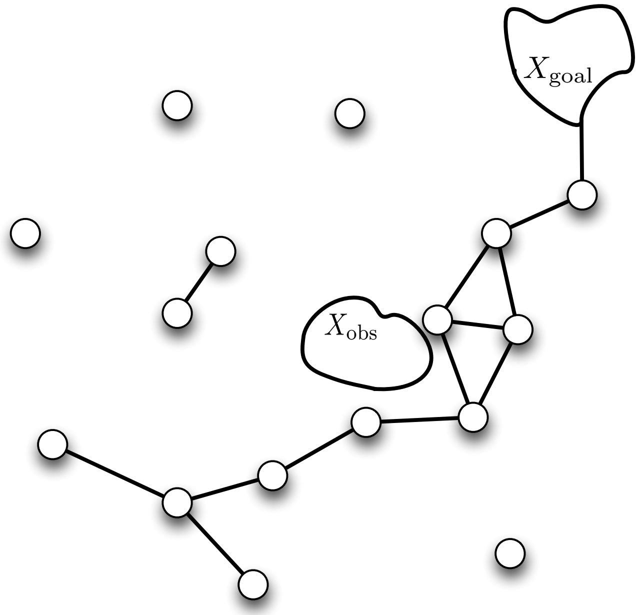


Key facts:

- There exist deterministic sequences with $D(S)$ of order $O\left(n^{-1/d}\right)$, referred to as **low-dispersion** sequences
- Sequences minimizing ℓ_2-dispersion only known for $d = 2$

Optimality of deterministic planning

```
$V \leftarrow \{x_{\text{init}}\} \cup \texttt{SampleFree}(n)$; $E \leftarrow \emptyset$
for all $v \in V$ do
    $X_{\text{near}} \leftarrow \texttt{Near}(V \backslash \{v\}, v, r_n)$
    for $x \in X_{\text{near}}$ do
        if $\texttt{CollisionFree}(v, x)$ then
            $E \leftarrow E \cup \{(v, x)\} \cup \{(x, v)\}$
        end if
    end for
end for
return $\texttt{ShortestPath}(x_{\text{init}}, V, E)$
```

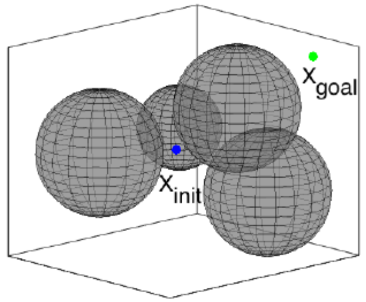


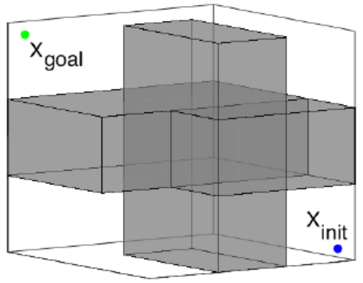
Optimality: Let c_n denote the arc length of the path returned with n samples. Then if

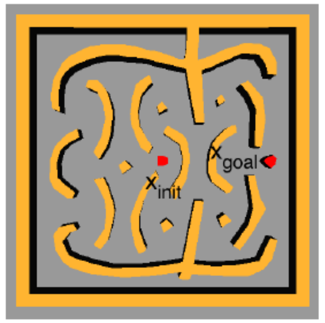
1. Samples set S has dispersion $D(S) \leq \gamma n^{-1/d}$ for some $\gamma > 0$,
2. $n^{1/d} r_n \to \infty$,

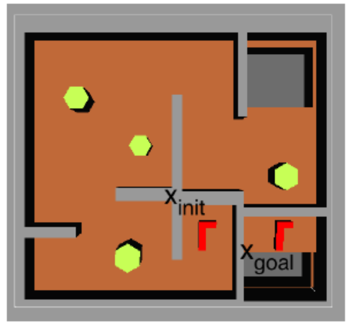
then $\lim_{n\to\infty} c_n = c^*$, where c^* is the cost of an optimal path

Numerical results









		Halton			Lattice		
Dim	Obstacles	90% Success	Medium	High	90% Success	Medium	High
2	Rectangular	38%	118%	80%	15%	56%	80%
3	Rectangular	36%	88%	94%	19%	80%	87%
2	Rect Maze	13%	98%	99%	13%	100%	99%
2	Sphere	16%	93%	99%	7%	93%	99%
3	Sphere	36%	97%	100%	8%	97%	99%
4	Sphere	100%	97%	97%	100%	97%	100%
2	Recursive Maze	33%	100%	100%	18%	100%	100%
3	Recursive Maze	22%	95%	99%	22%	96%	98%
4	Recursive Maze	56%	95%	98%	56%	100%	100%
5	Recursive Maze	45%	97%	96%	60%	95%	96%
6	Recursive Maze	56%	95%	97%	75%	94%	96%
8	Recursive Maze	56%	98%	99%	75%	99%	99%
8	Chain	67%	112%	91%	7%	76%	87%
3	SE(2)	81%	96%	100%	81%	101%	101%
6	SE(3)	32%	96%	93%	42%	94%	95%

Main takeaways

- Asymptotic optimality can be achieved with deterministic sequences and with a smaller connection radius
- Deterministic convergence rates: instrumental to the certification of sampling-based planners
- Computational and space complexity: under some assumptions, arbitrarily close to theoretical lower bound
- Deterministic sequences appear to provide superior performance

Reference: Janson et al. Deterministic Sampling-Based Motion Planning: Optimality, Complexity, and Performance. 2018

Next time: robotic sensors and introduction to computer vision