Tutorial-2

1. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$ **d)** $\exists x\, Q(x)$ **e)** $\forall x\, Q(x)$ **f)** $\exists x \neg Q(x)$ **g)** $\forall x \neg Q(x)$

2. Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x\,(x^3 = -1)$ **b)** $\exists x\,(x^4 < x^2)$ **c)** $\forall x\,((-x)^2 = x^2)$ **d)** $\forall x(2x > x)$

3. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$ **b)** $\forall x P(x)$ **c)** $\exists x \neg P(x)$ **d)** $\forall x \neg P(x)$

4. Express the statements "Some student in this class has visited Mexico" and "Every student in this class has visited either Canada or Mexico" using predicates and quantifiers.

5. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

a) $\forall x(x^2 = x)$ **b)** $\forall x(x^2 = 2)$ **c)** $\forall x(|x| > 0)$