<u>Revision of CAT_II Syllabus</u>

Topics:

1. Rules of Inferences
2. Fundamentals of graphs
3. Trees

1a. Rules of Inferences for propositions

Rule of Inference
$\begin{array}{ll} & p \\ & p \rightarrow q \\ \hline \therefore & q \end{array}$
$\begin{array}{ll} & \neg q \\ & p \rightarrow q \\ \hline \therefore & \neg p \end{array}$
$\begin{array}{ll} & p \rightarrow q \\ & q \rightarrow r \\ \hline \therefore & p \rightarrow r \end{array}$
$\begin{array}{ll} & p \vee q \\ & \neg p \\ \hline \therefore & q \end{array}$
$\begin{array}{ll} & p \\ \hline \therefore & p \vee q \end{array}$
$\begin{array}{ll} & p \wedge q \\ \hline \therefore & p \end{array}$
$\begin{array}{ll} & p \\ & q \\ \hline \therefore & p \wedge q \end{array}$
$\begin{array}{ll} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$

1b.

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\dfrac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

Examples:

i. "Each of five roommates, Melissa, Aaron, Ralph, Vanish and Keshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year."

ii. Use resolution to show that the hypotheses "Jasmine is skiing or it is not snowing" and "It is snowing or Bart is playing hockey" imply that "Jasmine is skiing or Bart is playing hockey."

2. Fundamentals of Graphs:

2a. Define

i. Graph
ii. simple graph
iii. connected graph
iv. disconnected graph
v. complete graph
vi. degree of vertex
vii. parallel edges
viii. adjacent edges
ix. adjacent vertex
x. self -loops
xi. subgraphs

2b. Theorems based question

- **i.** Find the sum of the degrees of the vertices of each graph and verify that it equals twice the number of edges in the graph.
- **ii.** Can a simple graph exist with 15 vertices each of degree five?

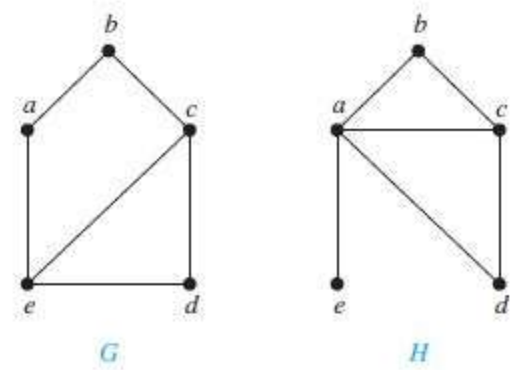
2c. Isomorphism of Graphs

Necessary Conditions but not sufficient:

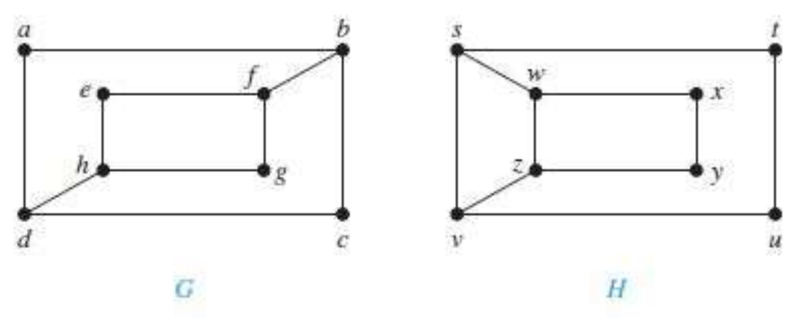
- **a)** same number of vertices
- **b)** same number of edges
- **c)** the no. of vertices of same degree should be same.

Important Note: Sometimes it is not hard to show that two graphs are not isomorphic. In particular, we can show that two graphs are not isomorphic if we can find a property only one of the two graphs have, but that is preserved by isomorphism.

Ex. 1

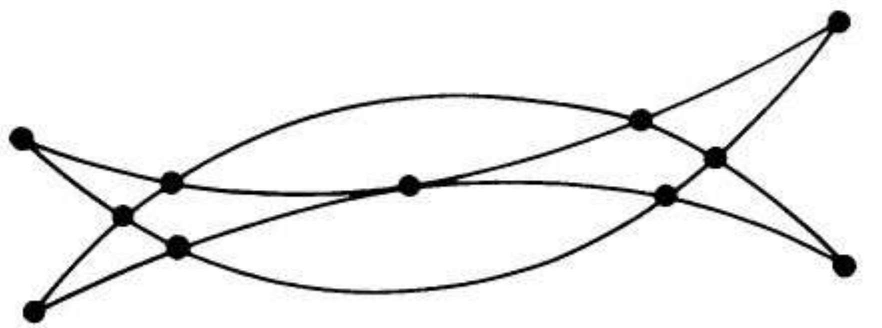


Ex. 2

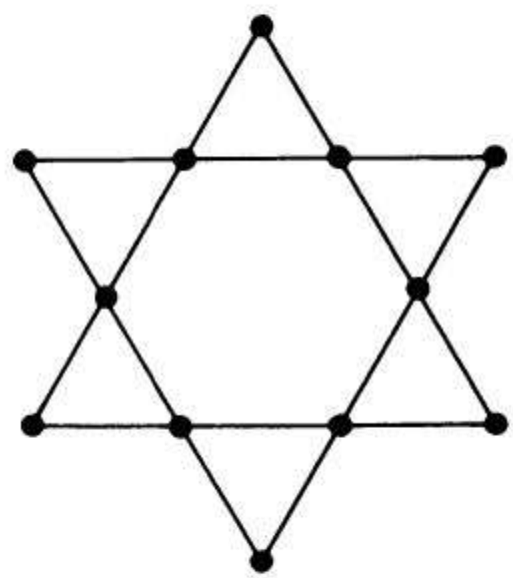


2d. Walk, path**,** circuit

A given connected graph G is an Euler graph if and only if all vertices of G are even degree.

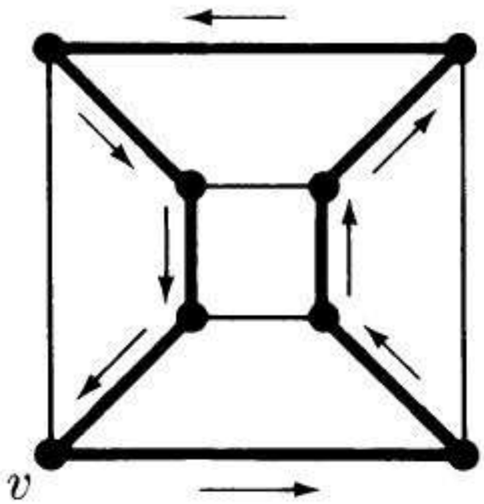


(a)

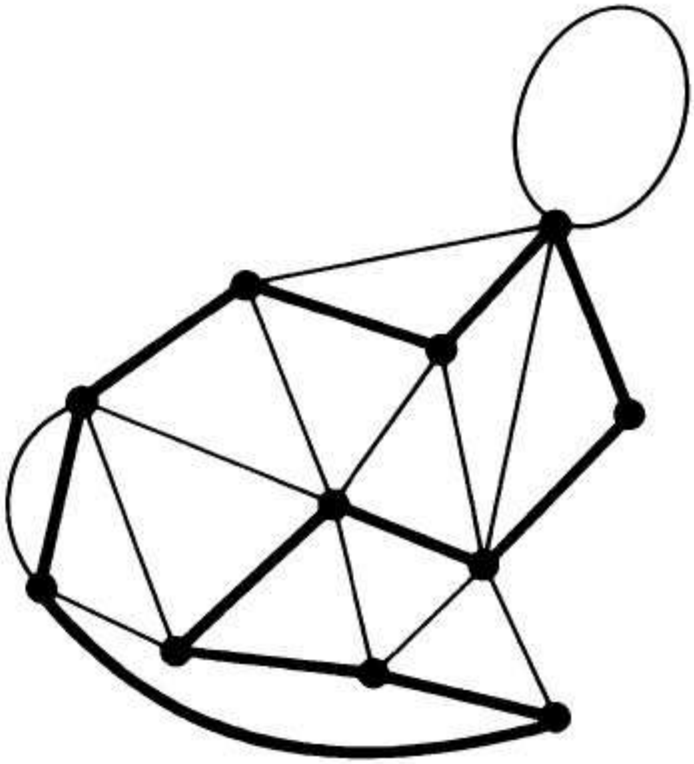


A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.

An Euler line of a connected graph was characterized by the property of being a closed walk that traverses *every edge* of the graph (exactly once). A *Hamiltonian circuit* in a connected graph is defined as a closed walk that traverses *every vertex* of G exactly once, except of course the starting vertex,



(a)



(b)

3. Trees

3a. Trees

3b. properties of trees

3c. distance and centers in tree

3d. rooted and binary trees (with properties of binary trees)

3e. spanning trees

3f. spanning trees in a weighted graph.