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1234

Permutations And Combinations

Permutation:

permutation as different ways of arranging some or all the members of a set in a specific order. It implies all the possible arrangement or rearrangement of the given set, into distinguishable order.

For example, All possible permutation created with letters x, y, z –

- By taking all three at a time are xyz, xzy, yxz, yzx, zxy, zyx.

Total number of possible permutations of n things, taken r at a time, can be calculated as:

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

Combination :

The combination as the different ways, of selecting a group, by taking some or all the members of a set, without the following order.

For example, All possible combinations chosen with letter m, n, o –

- When three out of three letters are to be selected, then the only combination is mno
- When two out of three letters are to be selected, then the possible combinations are mn, no, om.

$$^{n}C_{r} = \frac{n!}{(r!)(n-r)!}$$

Illustration 1: In a class, there are 15 boys and 10 girls. The teacher wants to select one boy and one girl, so as to represent the class in a function. In how many ways can the teacher make the selection?

Sol: Here the teacher has to perform two jobs:

i. Selecting a boy among 15 boys
ii. Selecting a girl among 10 girls.

The first task can be done in 15 ways and the second in 10 ways. By the fundamental principle of multiplication, the total number of ways is: $15 \times 10 = 150$.

Illustration 2: In a class, there are 15 boys and 10 girls. The teacher wants to select a boy or a girl so as to represent the class in a function. In how many ways can the teacher make the selection?

Sol: Here the teacher has to perform either of the following two jobs.

i. Selecting a boy among 15 boys or
ii. Selecting a girl among 10 girls.

The first task can be accomplished in 15 ways and the second in 10 ways. By fundamental principle of addition, either of the two jobs can be accomplished in: $15 + 10 = 25$ ways. Hence, the teacher can make the selection of either a boy or a girl in 25 ways.

Illustration 3: In how many ways can 3 non- identical rings be worn in 5 fingers ?

Sol. Such questions in which things are to be distributed amongst someone or something, NOTE that the things to be distributed always comes in the power.

So total ways of distributing the rings amongst the 5 fingers will be 5^3.The total ways thus come out to be 125.

OR , we can say that there are 5 ways for the first ring, 5 for the second and 5 for the 3rd ring so the total cases would be 5*5*5=125 cases.

Illustration 4: In how many ways can 4 letters be posted in 3 letter boxes?

Sol: Since each letter can be posted in any one of the three letter boxes, so a letter can be posted in 3 ways. So, the total number of ways in which all four letters can be posted = 34 ways.

- **Circular Permutations:** Permutation of n distinct objects along a circle can be done in (n – 1)! ways.

Permutation along a circle- clockwise and anticlockwise arrangements, are considered alike.

The number of permutations of n distinct objects- clockwise and anticlockwise arrangements, is similar = ((n-1)!/2)

- Combinations & Combination Formulas: Each of the different selections made by taking some or all of the number of objects, irrespective of their arrangements, is called a combination.

Illustration 5: How many four letter words, with or without meaning, can be formed using the letters of the word 'FATHERLY' using each letter exactly once (having essentially 'F' as one of the letters)?

i. Number of four letter words beginning with 'F' = 8-1P4-1 = 7P3
ii. Number of four letter words having 'F' as 2nd letter = 8-1P4-1= 7P3
iii. Number of four letter words having 'F' as 3rd letter = 8-1P4-1= 7P3
iv. Number of four letter words having 'F' as last letter = 8-1P4-1= 7P3

Total number of words = 7P3 + 7P3 +7P3 + 7P3 = 4. 7P3

Permutation of ‘n’ distinct objects taken ‘r’ at time where a particular object is never taken is n-1Pr . Here, one particular object (out of n given objects) is never taken. So, find the no. of ways in which r places can be filled with (n – 1) distinct objects, the no. of arrangement is n-1Pr.

Permutation of ‘n’ different objects, taking ‘r’ at a time, in which two specified objects always occur together is 2! (r – 1) n-2Pr-2 Here, if we leave out two specified objects, then the number of permutations of the remaining (n – 2) objects, taking (r – 2) at a time is n-2Pr-2 . Now, consider two specified objects temporarily as a single object and add to each of these n-2Pr-2 permutations which can be done in (r – 1) ways, the number of permutations becomes (r – 1) n-2Pr-2 . But the two specific things can be put together in 2! Ways, the required number of permutations is 2! (r – 1) n-2Pr-2.

Permutation of objects (not all distinct)**:** we will discuss the permutations of a given number of objects when not all objects are different. The number of mutually distinguishable permutations of ‘n’ things, taken all at a time, of which p are of one kind, q are of second kind, such that p + q = n is (n!/p!q!)

Question: 01

How many even numbers of four digits can be formed with the digits 0, 1, 2, 3, 4, 5, 6 and 7; no digit being used more than once?

A. 400
B. 420
C. 750
D. 210

Answer:C

Explanation:

Lets do some basic combination math

Here no. of digits = 8

Case – I where ‘0’ occurs at unit place

Unit place can be filled in 1 way (0) ; Ten’s place can be filled in 7 ways

Hundred’s place can be filled in 6 ways ; Thousand’s place can be filled in 5 ways ;

Using fundamental principle of multiplication the required no. = $1 \times 7 \times 6 \times 5 = 210$;

Case – II When 0 does not occur at unit place

Unit place can be filled in 3 ways (2, 4, 6); Thousand place can be filled in 6 ways; (one of the six digits other than zero); Hundred place can be filled in 6 ways; Ten’s place can be filled in 5 ways. Required number of ways = $3 \times 6 \times 6 \times 5 = 540$; Total number of numbers $= 210 + 540 = 750$

Question: 02

How many numbers of four digits greater than 2,400 can be formed with digits 0, 1, 2, 3, 4, 5 & 6; no digit being repeated in any number?

A. 140
B. 480
C. 540
D. 1120

Answer:C

Explanation:

Case – I When 2 occurs at thousand's place

Thousand's place can be filled up by 2 in 1 way. Hundred's place can be filled up by any of the four digits i.e. 4, 5 and 6 in 3 ways; Ten's place can be filled in 5 ways ; unit's place can be filled in 4 ways; using fundamental principle of multiplication, the required number = $1 \times 3 \times 5 \times 4 = 60$.

Case – II When thousand's place can be occupied by any of the digits out of 3, 4, 5 and 6; thousand's place can be filled in 4 ways; Hundred's place can be filled in 6 ways; Ten's place can be filled in 5 ways. Unit's place can be filled in 4 ways. Hence, the total number of ways are: $4 \times 6 \times 5 \times 4 = 480$. So, the required numbers = $60 + 480 = 540$.

Question: 03

There are 20 books of which 4 are single volume and the other are books of 8, 2 and 6 volumes respectively. In how many ways can all these books be arranged on a shelf so that volumes of the same book are not separated?

A. 2!3!4!5!
B. 3!4!5!6!
C. 4!5!6!7!
D. 8! 7! 6! 2!

Answer:D

Explanation:

Here volumes of the same book are not to be separated i.e. all the volumes of the same book are to be kept together.

Regarding all volumes of the same book as one book, we have only $4 + 1 + 1 + 1 = 7$ books.

These 7 books can be arranged in $7!$ ways. Volumes of book having 6 volumes can be arranged in $6!$ Ways.

Volumes of book having 8 volumes can be arranged in $8!$ ways.

Volumes of book having 2 volumes can be arranged in $2!$ ways.

$\therefore$ Required number $= 7!\,8!\,6!\,2!$.

Question: 04

A round table conference is to be held among 25 delegates from 25 countries. In how many ways can they be seated if two particular delegates are always to sit together?

A. 23!
B. 2! ×23!
C. 3! ×23!
D. None of these

Answer: B

Explanation:

Treating 2 particular delegates who are to sit together as one person, we have only 24 persons. These 24 persons can be seated at a round table in 23! Ways. But 2 particular persons can be arranged among themselves in 2! ways. ∴ Required no $=2! \times 23!$

Question: 05

Given 5 line segments of lengths 2, 3, 4, 5, 6 and 7 units. Then the number of triangles that can be formed by joining these lines is

A. 6C3
B. 6C3 - 7
C. 6C3 - 5
D. 6C3 - 1

Answer:B

Explanation:

We know that in any triangle the sum of two sides is always greater than the third side. ∴ the triangle will not be formed if we select segments of lengths (2, 3, 5) , (2, 3, 6), (2,3,7), (3,4,7),(2,4,7), (2,5,7) and (2, 4, 6). Hence number of triangles formed = 6C3 - 7

Question: 06

Find the number of divisors of 43200.

A. 48
B. 60
C. 72
D. 84

Answer:D

Explanation:

If a number N = ax . by . cz then no of divisors = (x+1)(y+1)(z+1)

Here 43,200 = 26 × 33 × 52

∴ Number of Divisors = (6 + 1)(3 + 1)(2 + 1) = 84

Question: 07

A gentleman has 5 friends to invite. In how many ways can he send invitation cards to them if he has four servants to carry the cards?

A. 16
B. 64
C. 1024
D. 1458

Answer:B

Explanation: 07

Here each card can be carried by any of the four servants.∴ Required number = $4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$.

Question: 08

In how many ways can 5 boys and 5 girls can be seated in a row so that boys and girls are placed alternately?

A. 5!

B. 5!×2!

C. 2×5! × 5!

D. None of these

Answer:C

Explanation: 08

The 5 boys and 5 girls can be seated in $5!5!$ ways. There are further two cases when the arrangement starts with boy or girl. Thus the required number of arrangements is $2 \times 5! \times 5!$

Question: 09

Find the sum of all the 4 digit numbers that can be formed with the digits 3, 4, 4 and 2.

A. 43339
B. 43999
C. 43329
D. None of these

Answer:C

Explanation:

Here each of the digits 2 and 3 will occur at unit, tens, hundred and thousand place (3P3/2!) = 3 times. Digit 4 will occur at each place = 6 times;

∴ Sum of digits at unit, tens,

hundred and thousand place = $3 \times 3 + 6 \times 4 + 3 \times 2 = 39$.

Sum of numbers formed =

$= 39 \times 103 + 39 \times 102 + 39 \times 101 + 39 \times 100 = 43329$

Question: 10

The number of straight lines that can be drawn out of 12 points of which 8 are collinear is

A. 39
B. 29
C. 49
D. 59

Answer:A

Explanation:

The required number of lines= 12C2 – 8C2 + 1 = 1 + 66 – 28 = 39

Question: 11

A box contains three white balls, four black balls and three red balls. The number of ways in which three balls can be drawn from the box so that at least one of the balls is black is

A. 50
B. 100
C. 150
D. 200

Answer:B

Explanation: 11

The required number of ways

(a) 1 black and 2 others = 4C1.6C2 = 4 × 15 = 60

(b) 2 black and 1 other = 4C2.6C1 = 6 × 6 = 36

(c) All the three black = 4C3 = 4

Total =60 + 36 + 4 = 100

Question: 12

In how many ways can you rearrange the word JUMBLE such that the rearranged word starts with a vowel?

A. 120
B. 240
C. 360
D. 60

Answer:B

Explanation:

JUMBLE is a six-lettered word. Since the rearranged word has to start with a vowel, the first letter can be either U or E. The balance 5 letters can be arranged in 5P5 or 5! ways. Total number of words = $2 \times 5! = 240$.

Question: 13

In an examination, a candidate is required to pass all five different subjects. The number of ways he can fail is:

A. 32
B. 31
C. 30
D. 29

Answer:B

Explanation:

The candidate will fail if he fails either in 1 or 2 or 3 or 4 or 5 subjects,

∴ Required number of ways 5C1 + 5C2 + 5C3 + 5C4 + 5C5 = 31

Question: 14

Nine chairs are numbered 1 to 9. Three women and four men wish to occupy one chair each. First the women chose the chairs from amongst the chair marked 1 to 5; and then the men select the chairs from amongst the remaining. The number of possible arrangements is

A. 5C3 × 4C2
B. 5C2 × 4P3
C. 5C3 × 6C4
D. None of these

Answer:C

Explanation:

Women can select 3 chairs from chairs numbered 1 to 5 in 5C3 ways and remaining 6 chairs can be selected by 4 men in 6C4 ways. Hence the required number of ways = 5C3 × 6C4.

Question: 15

In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

A. 810
B. 1440
C. 2880
D. 50400

Answer:D

Explanation:

In the word 'CORPORATION', we treat the vowels OOAIO as one letter

Thus, we have CRPRTN (OOAIO)

This has 7 (6 + 1) letters of which R occurs 2 times and the rest are different.

Number of ways arranging these letters = 7!/2! =2520

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged in 5!/3! = 20 ways

Required number of ways = (2520 x 20) = 50400.

THANK YOU