Monday, 21 December, 2020 11:48 AM

Module-3

Removing ambiguity from a grammar

- Elimination of Left Recursion
- Left factoring

Elimination of Left Recursion:

$A \Rightarrow \alpha A$

$A \rightarrow A\alpha \mid \beta$

$w = \beta\alpha\alpha\alpha\alpha$

$A \Rightarrow A\alpha \Rightarrow A\alpha\alpha \Rightarrow A\alpha\alpha\alpha \Rightarrow A\alpha\alpha\alpha\alpha \Rightarrow \beta\alpha\alpha\alpha\alpha$

$A \rightarrow A\alpha / \beta$

$A \rightarrow \beta A'$

$A' \rightarrow \alpha A' \mid \epsilon$

$w = \beta\alpha\alpha\alpha\alpha$

$A \Rightarrow \beta A' \Rightarrow \beta\alpha A' \Rightarrow \beta\alpha\alpha A' \Rightarrow \beta\alpha\alpha\alpha A' \Rightarrow \beta\alpha\alpha\alpha\alpha A'$

$A \Rightarrow \beta A' \Rightarrow$

$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_n \mid \beta_1 \mid \beta_2 \mid \dots \mid \beta_n$

$A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \dots \mid \beta_n A'$

$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \dots \mid \alpha_n A' \mid \epsilon$

$A \rightarrow A\alpha / \beta$

$\beta_2 \quad w = a + a * b$

$A = S$

$S \rightarrow S+S \mid S*S \mid a \mid b \qquad \alpha_1 = +S, \quad \alpha_2 = *S$

$\beta_1 = a, \quad \beta_2 = b$

$A \rightarrow \beta_1 A' \mid \beta_2 A'$

$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \epsilon$

$S \rightarrow aS' \mid bS'$

$S' \rightarrow +SS' \mid *SS' \mid \epsilon$

$w = a + a * b$

$S \Rightarrow S+S \Rightarrow a+S \Rightarrow a+S*S \Rightarrow a+a*S \Rightarrow a+a*b$

$S \Rightarrow S*S \Rightarrow S+S*S \Rightarrow a+S*S \Rightarrow a+a*S \Rightarrow a+a*b$

$a+a*b$

$E \rightarrow E+E \mid E-E \mid E*E \mid E/E \mid (E) \mid id$ ambiguous

$E \rightarrow E+T \mid E-T \mid T$

$T \rightarrow T*F \mid T/F \mid F$

$F \rightarrow (E) \mid id$

Unambiguous

$id + id * id$

(1)

$F \to (E) \mid id$ Unambiguous

(1)

(2)

$id + id * id$

$id * (id + id)$ $2 + 3 * 4$

$A \to A\alpha_1 \mid A\alpha_2 \mid \beta$

$2 * (3+4)$ $(3+4) - 2 = 5$

$3 + (4-2) = 5$

$/ * \Rightarrow +/-$

Production	Rules	Converted Production
$E \to E+T \mid E-T \mid T$	$\alpha_1 = +T, \alpha_2 = -T$ $\beta = T$	$E \to TE'$ $E' \to +TE' \mid -TE' \mid \epsilon$
$T \to T*F \mid T/F \mid F$	$\alpha_1 = *F, \alpha_2 = /F$ $\beta = F$	$T \to FT'$ $T' \to *FT' \mid /FT' \mid \epsilon$
$F \to (E) \mid id$	this not left recursive	non change required

$\underline{2+3+4}$

$2 + 3 - 4$

$+ \doteq -$

$+ \doteq +$

$- \doteq -$

Associativity

$(2+3)+4$ or $(3*4)*5$ $4/2 = 2$

$(2+3)-4$ $(8/4)/2$ $8/2 = 4$

$+, -, *, /$ are left associative

are right associative

$3 \uparrow (4 \uparrow 2)$ $(3^4)^2$ $3^{(4^2)} = 3^{16}$

$a := b := c := d$

$E \to E+T \mid E-T \mid T$

$T \to T*F \mid T/F \mid F$

$F \to G \uparrow F \mid G$

$G \to (E) \mid id$

$E \to E*T \mid T$

$T \to T-F \mid F$

$F \to G \uparrow F \mid G$

$G \to (E) \mid id$

(i) $() > \uparrow > - > *$

(ii) Left associative $= *, -$

Right associative $= \uparrow$

$E \to E - E$

$-$ is neither left associative nor right associative

Left Factoring:

$$A \to \alpha\beta_1 \mid \alpha\beta_2 \mid \alpha\beta_3 \mid \cdots \mid \alpha\beta_n$$

$$A \to \alpha A'$$
$$A' \to \beta_1 \mid \beta_2 \mid \beta_3 \mid \cdots \mid \beta_n \leftarrow \text{Left factored}$$

Ex

$S \to aAB \mid aA \mid a\epsilon$
$A \to Bc \mid B$
$B \to b$

$A = S,\ \alpha = a,\ \beta_1 = AB,\ \beta_2 = A,\ \beta_3 = \epsilon$

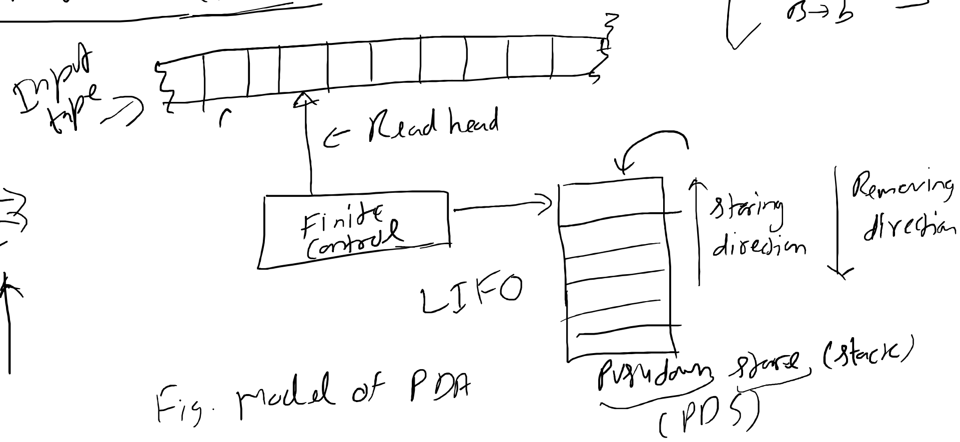
$\Rightarrow$

$S \to aS'$
$S' \to AB \mid A \mid \epsilon \Rightarrow S' \to AS'' \mid \epsilon$, $S'' \to B \mid \epsilon$
$A \to BA'$
$A' \to c \mid \epsilon$
$B \to b$

Left factored:

$S \to aS'$
$S' \to AS'' \mid \epsilon$
$S'' \to B \mid \epsilon$
$A \to BA'$
$A' \to c \mid \epsilon$
$B \to b$

Pushdown Automata (PDA):



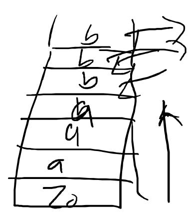


Fig. model of PDA

Formal Definition:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$
or
$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \phi)$$

$\Sigma = \{a, b\}$
$\Sigma^* = ?$

Q – Non empty finite set of states
Σ – " " " of input symbols
Γ – " " " " pushdown symbols
δ – Transition function $\delta: Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \to Q \times \Gamma^*$ $\Gamma = \{X, Z_0\}$
$q_0 \in Q$ – Initial state
$Z_0 \in \Gamma$ – Initial pushdown symbol
$F \subseteq Q$ – Set of final states

$\delta: Q \times \Sigma \to Q$ (DFA)
$\delta: Q \times \Sigma \to 2^Q$ (NFA)
$\delta: Q \times (\Sigma \cup \{\epsilon\}) \to 2^Q$ (ϵ-NFA)

$\Sigma = \{0, 1\}$

Q- Design a PDA to accept $L = \{0^n 1^n \mid n \geq 1\}$ ($n \geq 0$) by null store (stack) / by empty pushdown (store)

ϵ_1

Q. Design a PDA to accept $L = \{0^n 1^n \mid n \geq 1\}$ by null store / by empty stack (store) $(n \geq 0)$

$L = \{\epsilon, 01, 0011, 000111, \ldots\}$

$n \geq 0 \rightarrow \delta(q_0, \epsilon, Z_0) = \{(q_f, Z_0)\}$ or $\{(q_0, \epsilon)\}$

$R_1 \quad \delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$

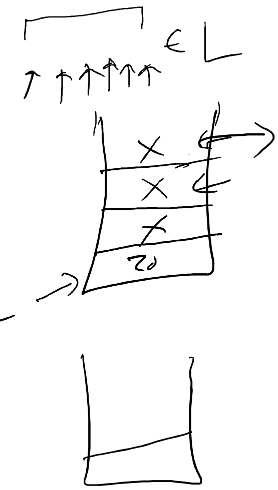
$R_2 \quad \delta(q_0, 0, X) = \{(q_0, XX)\}$

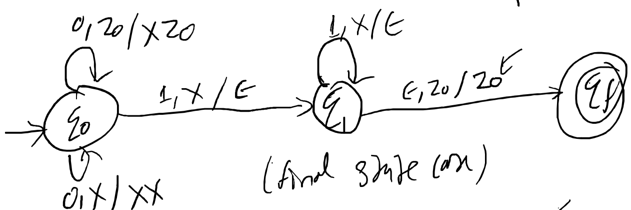
$R_3 \rightarrow \delta(q_0, 1, X) = \{(q_1, \epsilon)\}$

$R_4 \rightarrow \delta(q_1, 1, X) = \{(q_1, \epsilon)\}$

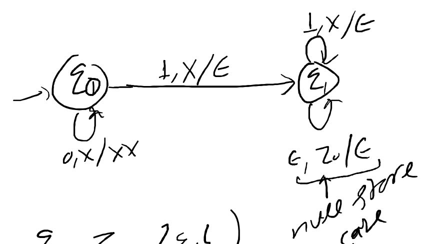
$R_5 \quad \delta(q_1, \epsilon, Z_0) = \{(q_f, Z_0)\}$ or $\{(q_f, \epsilon)\}$

$\{(q_1, \epsilon)\}$ null store case





(final state case)



null store case

$M = (\{q_0, q_1, q_f\}, \{0,1\}, \{Z_0, X\}, \delta, q_0, Z_0, \{q_f\})$
$\quad Q \quad \Sigma \quad \Gamma$

Initial ID $\rightarrow (q_0, 0011, Z_0) \vdash (q_0, 011, XZ_0)$ (By Rule R_1)

$\vdash (q_0, 11, XXZ_0)$ (By Rule R_2)

$\vdash (q_1, 1, XZ_0)$ (By Rule R_3)

$\vdash (q_1, \epsilon, Z_0)$ (By Rule R_4)

$\vdash (q_f, Z_0)$ or (q_f, ϵ) (By Rule R_5)

final state case — success

$\vdash (q_1, \epsilon)$ (null store) (success)

$(q_0, 011, Z_0) \vdash (q_0, 11, XZ_0) \vdash (q_1, 1, Z_0)$ halt in non final state (unsuccessful termination)

Q. Design a PDA to accept $L = \{a^n b^{2n} \mid n \geq 1\} = \{abb, aabbbb, aaabbbbbb, \ldots\}$ $n \geq 0$

$\delta(q_0, \epsilon, Z_0) = \{(q_f, Z_0)\}$

$R_1 \quad \delta(q_0, a, Z_0) = \{(q_0, XZ_0)\}$

aabbbb

$\delta(q_0, \epsilon, Z_0) = \{(q_f, Z_0)\} \leftarrow$

$R_1 \quad \delta(q_0, a, Z_0) = \{(q_0, XZ_0)\}$

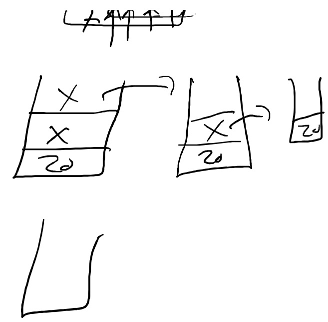
$R_2 \quad \delta(q_0, a, X) = \{(q_0, XX)\}$

$R_3 \quad \delta(q_0, b, X) = \{(q_1, X)\} \leftarrow$ neutral case

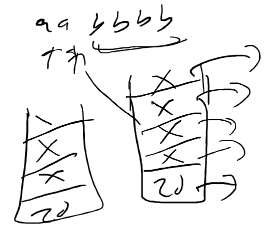
$R_4 \quad \delta(q_1, b, X) = \{(q_0, \epsilon)\}$

$R_5 \quad \delta(q_0, \epsilon, Z_0) = \underbrace{\{(q_f, Z_0)\} \text{ or } \{(q_f, \epsilon)\}}_{\text{final state case}}$

$= \underbrace{\{(q_0, \epsilon)\}}_{\text{null state case}}$



OR



$\delta(q_0, a, Z_0) = \{(q_0, XXZ_0)\}$

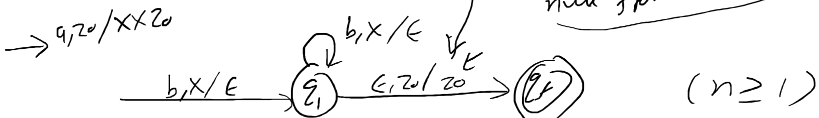
$\delta(q_0, a, X) = \{(q_0, XXX)\}$

$\delta(q_0, b, X) = \{(q_1, \epsilon)\} \leftarrow$

$\delta(q_1, b, X) = \{(q_1, \epsilon)\} \leftarrow$

$R_5: \quad \delta(q_1, \epsilon, Z_0) = \underbrace{\{(q_f, Z_0)\} \text{ or } \{(q_f, \epsilon)\}}_{\text{final state case}}$

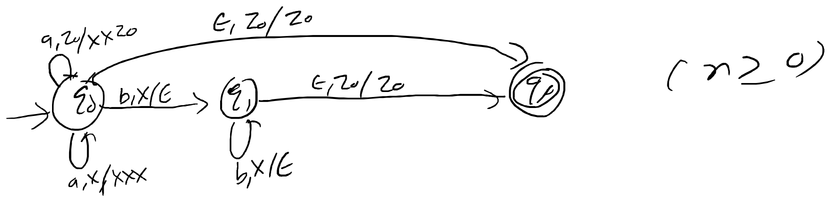
$= \underbrace{\{(q_1, \epsilon)\}}_{\text{null state case}}$



$(n \geq 1)$

$M = (\underbrace{\{q_0, q_1, q_f\}}_{Q}, \underbrace{\{a, b\}}_{\Sigma}, \underbrace{\{Z_0, X\}}_{\Gamma}, \delta, q_0, Z_0, \{q_f\})$

$M = (\{q_0, q_1\}, \{a, b\}, \{Z_0, X\}, \delta, q_0, Z_0, \phi)$



$(n \geq 0)$

Q: Design a PDA $L = \{a^{2n}b^n \mid n \geq 1\} = \{aab, aaaabb, aaaaaabbb, \ldots\}$ $(n \geq 0)$

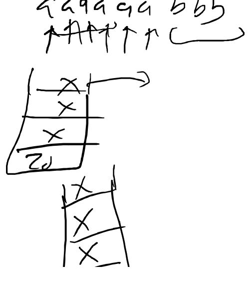
$\delta(q_0, \epsilon, Z_0) = \{(q_f, Z_0)\} \leftarrow$

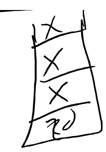
$R_1: \quad \delta(q_0, a, Z_0) = \{(q_1, Z_0)\} \leftarrow$ neutral case

$R_2: \quad \delta(q_1, a, Z_0) = \{(q_0, XZ_0)\}$

$R_3: \rightarrow \delta(q_0, a, X) = \{(q_1, X)\}$

$\delta(q_1, a, X) = \{(q_0, XX)\}$



$R_3: \rightarrow \delta(q_0, a, X) = \{(q_1, X)\}$ 

$R_4: \rightarrow \delta(q_1, a, X) = \{(q_0, XX)\}$

$R_5: \delta(q_0, b, X) = \{(q_2, \epsilon)\}$

$R_6: \quad q_2, b, X) = \{(q_2, \epsilon)\}$

$R_7: \delta(q_2, \epsilon, Z_0) = \{(q_f, Z_0)\}$ or $\{(q_f, \epsilon)\}$

final state case

Tuple = ?

$= \{(q_2, \epsilon)\}$

null store case

$w \in \{a,b\}^+$.

Q Design a PDA to accept $L = \{wcw^R \mid w \in \{a,b\}^+\}$

$L = \{\epsilon, aca, bcb, aacaa, abcba, bacab, bbcbb, \ldots\}$

$Rev(abcba) = abcba$

$R_1 \quad \delta(q_0, c, Z_0) = \{(q_1, Z_0)\}$

$R_2 \quad \delta(q_0, a, Z_0) = \{(q_0, XZ_0)\}$

$\delta(q_0, b, Z_0) = \{(q_0, YZ_0)\}$

$R_3 \quad \delta(q_0, a, X) = \{(q_0, XX)\}$

$R_4 \quad \delta(q_0, b, X) = \{(q_0, YX)\}$

$w \quad R_5 \quad \delta(q_0, a, Y) = \{(q_0, XY)\}$

$R_6 \quad \delta(q_0, b, Y) = \{(q_0, YY)\}$

$R_7 \quad \delta(q_0, c, X) = \{(q_1, X)\}$

$R_8 \quad \delta(q_0, c, Y) = \{(q_1, Y)\}$

$R_9 \quad \delta(q_1, a, X) = \{(q_1, \epsilon)\}$

$R_{10} \quad \delta(q_1, b, Y) = \{(q_1, \epsilon)\}$

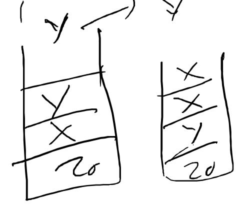
$R_{11} \quad \delta(q_1, \epsilon, Z_0) = \{(q_f, Z_0)\}$

R_{12}

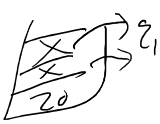
$w = aa, ab, ba, bb$

$bbcbba \in L$

$baac \; aab$



$aacaa \epsilon$



1) $L = \{a^n b^n c^m \mid n, m \geq 1\}$ $(m \geq 0)$

2) $L = \{a^m b^n c^n \mid m, n \geq 1\}$ $(m \geq 0)$

$\delta(q_0, a, Z_0) = \{(q_1, Z_0)\}$

$\delta(q_1, a, Z_0) = \{(q_1, Z_0)\}$

$\delta(q_1, b, Z_0) = \{(q_1, bZ_0)\}$

$\delta(q_1, b, b) = \{(q_1, bb)\}$

$\delta(q_1, c, b) = \{(q_2, \epsilon)\}$

$\delta(q_2, c, b) = \{(q_2, \epsilon)\}$

$\delta(q_2, \epsilon, Z_0) = \{(q_f, Z_0)\}$

3) $L = \{a^n b^m c^n \mid m, n \geq 1\}$ $(m \geq 0)$

4) $L = \{a^m b^n c^{m+n} \mid m, n \geq 1\}$

5) $L = \{a^m b^n c^n d^m \mid m, n \geq 1\}$

6) $L =$ set of all strings of a's & b's with equal a's & b's.

6) L = set of all strings of a's & b's ...

R_1 $\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$

R_2 $\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, c, a) = \{(q_2, \epsilon)\}$

$\delta(q_0, b, a) = \{(q_1, a)\}$] neutral case

$\delta(q_1, b, a) = \{(q_1, a)\}$

$\delta(q_1, c, a) = \{(q_2, \epsilon)\}$

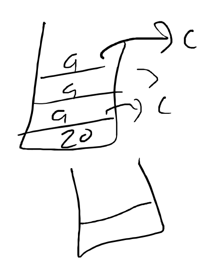
$\delta(q_2, c, a) = \{(q_2, \epsilon)\}$

$\delta(q_2, \epsilon, Z_0) = \{(q_f, Z_0)\}$

$\{(q_2, \epsilon)\}$ null store case

Tuple ?
diagram ?
validation ?

aaabbccc



6) R_1 $\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$

R_2 $\delta(q_0, b, Z_0) = \{(q_0, bZ_0)\}$

R_3 $\delta(q_0, a, a) = \{(q_0, aa)\}$

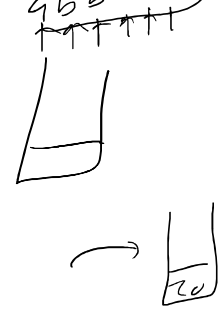
R_4 $\delta(q_0, b, b) = \{(q_0, bb)\}$

R_5 $\delta(q_0, a, b) = \{(q_0, \epsilon)\}$

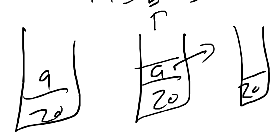
$\delta(q_0, b, a) = \{(q_0, \epsilon)\}$

R_7 $\delta(q_0, \epsilon, Z_0) = \{(q_f, Z_0)\}$

abbaab

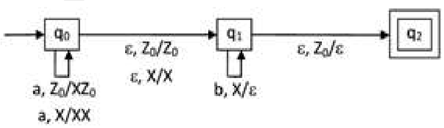


aabbab



PDA to CFG Conversion

$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{X, Z_0\}, \delta, q_0, Z_0, \{q_2\})$
Q Σ Γ F



R_1: $\delta(q_0, a, Z_0) = \{(q_0, XZ_0)\}$

R_2: $\delta(q_0, a, X) = \{(q_0, XX)\}$

R_3: $\delta(q_0, \epsilon, Z_0) = \{(q_1, Z_0)\}$

R_4: $\delta(q_0, \epsilon, X) = \{(q_1, X)\}$

R_5: $\delta(q_1, b, X) = \{(q_1, \epsilon)\}$

R_6: $\delta(q_1, \epsilon, Z_0) = \{(q_2, \epsilon)\}$

$3+9+9+3+3+1+1$
27

$G = (V_N, \Sigma, P, S)$

Step I Construction of set of variables V'_N:

$[q, z, q']$ ← A variable

S, $[q_0, \overset{A}{z_0}, q_0]$, $[q_0, \overset{B}{X}, q_0]$, $[q_0, \overset{C}{z_0}, q_1]$, $[q_0, X, q_1]$, $[q_0, z_0, q_2]$, $[q_0, X, q_2]$

$[q_1, z_0, q_0]$, $[q_1, X, q_0]$, $[q_1, z_0, q_1]$, $[q_1, X, q_1]$, $[q_1, z_0, q_2]$, $[q_1, X, q_2]$

$[q_2, z_0, q_0]$, $[q_2, X, q_0]$, $[q_2, z_0, q_1]$, $[q_2, X, q_1]$, $[q_2, z_0, q_2]$, $[q_2, X, q_2]$

Step II Construction of set of productions P

$S \to [q_0, z_0, q']$

P_1: $S \to [q_0, z_0, q_0]$

P_2: $S \to [q_0, z_0, q_1]$

P_3: $S \to [q_0, z_0, q_2]$

$A \to \alpha \in$ (CFG)

R_1: $\delta(q_0, a, z_0) = \{(q_0, \underbrace{Xz_0})\}$

$= m^n$

m = no of states

n = no of pushdown symbols in RHS of δ

P_4: $[q_0, z_0, q_0] \to a\, [q_0, X, q_0]\, [q_0, z_0, q_0]$

$[q_0, z_0, q_0] \to a\, [q_0, X, q_1]\, [q_1, z_0, q_0]$

$[q_0, z_0, q_0] \to a\, [q_0, X, q_2]\, [q_2, z_0, q_0]$

$[q_0, z_0, q_1] \to a\, [q_0, X, q_0]\, [q_0, z_0, q_1]$

$[q_0, z_0, q_1] \to a\, [q_0, X, q_1]\, [q_1, z_0, q_1]$

$[q_0, z_0, q_1] \to a\, [q_0, X, q_2]\, [q_2, z_0, q_1]$

$[q_0, z_0, q_2] \to a\, [q_0, X, q_0]\, [q_0, z_0, q_2]$

$[q_0, z_0, q_2] \to a\, [q_0, X, q_1]\, [q_1, z_0, q_2]$

P_{12}: $[q_0, z_0, q_2] \to a\, [q_0, X, q_2]\, [q_2, z_0, q_2]$

q_0, q_1, q_2

q_0, q_1

$A \to \alpha$ (CFG) (GNF)

R_2: $\delta(q_0, a, X) = \{(q_0, XX)\}$ $\quad 3^2 = 9$

$[q_0, X, q_0] \to a\, [q_0, X, q_0]\, [q_0, X, q_0]$

$[q_0, X, q_0] \to a\, [q_0, X, q_1]\, [q_1, X, q_0]$

$[q_0, X, q_0] \to a\, [q_0, X, q_2]\, [q_2, X, q_0]$

$q_0 \to q_0 \to q_0$

$q_0 \to q_1 \to q_0$

$q_0 \to q_2 \to q_0$

q_0, q_1, q_2

$[q_0, X, q_1] \to a\, [q_0, X, q_0]\, [q_0, X, q_1]$

$[q_0, X, q_1] \to a\, [q_0, X, q_1]\, [q_1, X, q_1]$

$[q_0, X, q_1] \to a\, [q_0, X, q_2]\, [q_0, X, q_1]$

$[q_0, X, q_2] \to a\, [q_0, X, q_0]\, [q_0, X, q_2]$

$[q_0, X, q_2] \to a\, [q_0, X, q_1]\, [q_1, X, q_2]$

P_{21}: $[q_0, X, q_2] \to a\, [q_0, X, q_2]\, [q_2, X, q_2]$

$R_3 \quad \delta(q_0, \epsilon, z_0) = \{(q_1, z_0)\}$ $\quad 3^1 = 3$

$P_{22} \quad [q_0, z_0, q_0] \rightarrow [q_1, z_0, q_0]$

$P_{23} \quad [q_0, z_0, q_1] \rightarrow [q_1, z_0, q_1]$

$P_{24} \quad [q_0, z_0, q_2] \rightarrow [q_1, z_0, q_2]$

$R_4: \delta(q_0, \epsilon, X) = \{(q_1, X)\}$

$P_{25} \quad [q_0, X, q_0] \rightarrow [q_1, X, q_0]$

$P_{26} \quad [q_0, X, q_1] \rightarrow [q_1, X, q_1]$

$P_{27} \quad [q_0, X, q_2] \rightarrow [q_1, X, q_2]$

$R_5 \quad \delta(q_1, b, X) = \{(q_1, \epsilon)\}$ $\quad 3^0 = 1$

$[q_1, X, q_1] \rightarrow b\,[q_1, \epsilon, q_1]$ $\quad (\because [q, \epsilon, q] \rightarrow \epsilon)$

$P_{28}: [q_1, X, q_1] \rightarrow b$

$R_6: \delta(q_1, \epsilon, z_0) = \{(q_2, \epsilon)\}$

$P_{29} \quad [q_1, z_0, q_2] \rightarrow \epsilon\,[q_2, \epsilon, q_2] =$

$P_{29}: \quad [q_1, z_0, q_2] \rightarrow \epsilon$

1. Convert the following Push Down Automata to Context Free Grammar

   $M = (\{q_0,q_1\},\{a,b\}\{z_0,za\},\delta,q_0,z_0,\varphi)$ where δ is given by

   $\delta(q_0,a,z_0) = (q_0, z_az_0)$
   $\delta(q_0,a, z_a) = (q_0,z_az_a)$
   $\delta(q_0,b, z_a) = (q_1,\epsilon)$
   $\delta(q_1,b, z_a) = (q_1,\epsilon)$
   $\delta(q_1, \epsilon, z_0) = (q_1,\epsilon)$

$2 + 4 + 4 + 1 + 1 + 1$ $\quad 13$

7 tuple of PDA $\rightarrow$ 4 tuple of CFG

$G = (V_H, \Sigma, P, S)$

$\Sigma = \{a, b\}$

Step I Construction of set of variables V_H

$[q, z, q']$

$V_H = \{ S, [q_0, z_0, q_0], \overset{B}{[q_0, z_a, q_0]}, \overset{C}{[q_0, z_0, q_1]}, \overset{D}{[q_0, z_a, q_1]},$
$\quad \underset{E}{[q_1, z_0, q_0]}, \underset{F}{[q_1, z_a, q_0]}, \underset{G}{[q_1, z_0, q_1]}, \underset{H}{[q_1, z_a, q_1]} \}$

Step II: Construction of set of production P

$S \rightarrow [q_0, z_0, q'] \quad \forall\, q' \in Q$

$P_1: S \rightarrow [q_0, z_0, q_0]$

$P_2: S \rightarrow [q_0, z_0, q_1]$

P_1: $S \to [q_0, z_0, q_0]$

P_2: $S \to [q_0, z_0, q_1]$

$n^m = 2^2 =$

$\delta(q_0, a, z_0) = (q_0, z_a z_0)$

$[q_0, z_0, q_0] \to a\, [q_0, z_a, q_0]\, [q_0, z_0, q_0]$ $\quad n = 2,\ m = 2$

q_0, q_1

$[q_0, z_0, q_0] \to a\, [q_1, z_a, q_1]\, [q_1, z_0, q_0]$

$[q_0, z_0, q_1] \to a\, [q_0, z_a, q_0]\, [q_0, z_0, q_1]$

$[q_0, z_0, q_1] \to a\, [q_0, z_a, q_1]\, [q_1, z_0, q_1]$ $\quad 2^3 = 8$

$\delta(q_0, a, z_a) = (q_0, z_a z_a)$ $\quad 2^2 = 4$

q_0, q_1

P_7 $[q_0, z_a, q_0] \to a\, [q_0, z_a, q_0]\, [q_0, z_a, q_0]$

P_8 $[q_0, z_a, q_0] \to a\, [q_0, z_a, q_1]\, [q_1, z_a, q_0]$

P_9 $[q_0, z_a, q_1] \to a\, [q_0, z_a, q_0]\, [q_0, z_a, q_1]$

P_{10} $[q_0, z_a, q_1] \to a\, [q_0, z_a, q_1]\, [q_1, z_a, q_1]$

$\delta(q_0, b, z_a) = (q_1, \epsilon)$ $\quad 2^0 = 1$ $\quad [\because [q, \epsilon, q] \to \epsilon]$

$[q_0, z_a, q_1] \to b\, [q_1, \epsilon, q_1]$

$b\epsilon = b$

P_{11} $[q_0, z_a, q_1] \to b$

$2^0 = 1$

$\delta(q_1, b, z_a) = (q_1, \epsilon)$

$[q_1, z_a, q_1] \to b\, [q_1, \epsilon, q_1]$

P_{12}: $[q_1, z_a, q_1] \to b$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$[q_1, z_0, q_1] \to \epsilon\, [q_1, \epsilon, q_1]$

P_{13}: $[q_1, z_0, q_1] \to \epsilon$