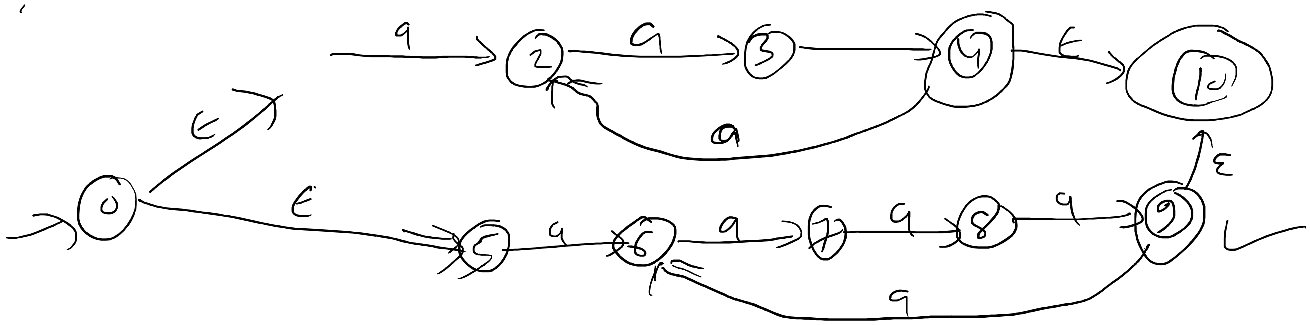
1^e

Monday, 30 November, 2020 12:20 PM

ε-NFA to DFA Conversion

https://www.javatpoint.com/automata-conversion-from-nfa-with-null-to-dfa

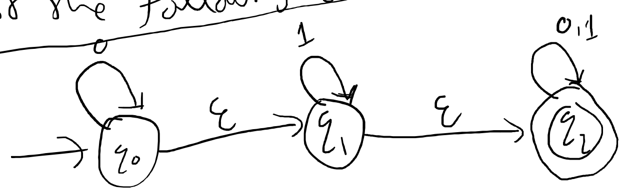


$L_1 = \{aaa, aaaaaa, ----\}$

$L = L_1 \cup L_2$

$L_2 = \{aaaa, aaaaaaaa, ---\}$

Q: Convert the following ε-NFA to DFA.



States	Input 0	1	ε
$\to q_0$	q_0	–	q_1
q_1	$\emptyset$	q_1	q_2
$* q_2$	q_2	q_2	–

$\epsilon\text{-closure}(q_0) = \epsilon^{c}(q_0) = \{q_0, q_1, q_2\}$

$\epsilon\text{-closure}(q_1) = \epsilon^{c}(q_1) = \{q_1, q_2\}$

$\epsilon\text{-closure}(q_2) = \epsilon^{c}(q_2) = \{q_2\}$

States	Input 0	1
$\to$ (A)	A	B
(B)	C	B

$q_0' = \epsilon^{c}(q_0) = [q_0, q_1, q_2]$
$= A$

$\epsilon^{c}(\delta(\{q_0, q_1, q_2\}, 0))$

$\delta'(A, 0) = \delta'([q_0, q_1, q_2], 0) =$

$= \epsilon^{c}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$\rightarrow$ A		
B	C	B
C	C	C

$M' = (\{A, B, C\}, \{0, 1\}, \delta', A, \{A, B, C\})$
$\quad\quad\; Q \quad\quad\quad\; \Sigma' \quad\quad\quad\quad\quad\quad F$

$\delta'(C, 0) = \delta'([q_2], 0)$
$= \varepsilon^c(\delta(\{q_2\}, 0))$
$= \varepsilon^c(q_2) = [q_2] = C$

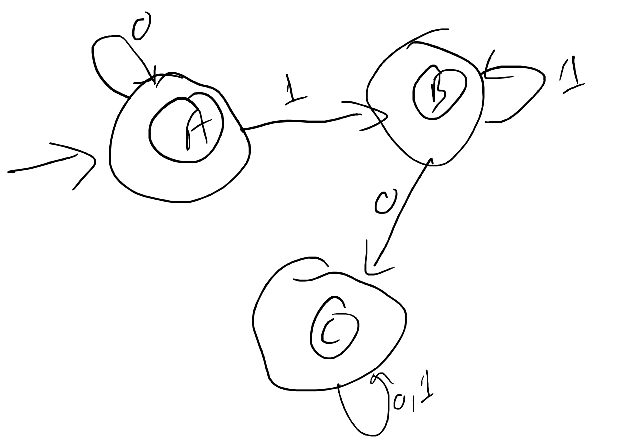
$\delta'(C, 1) = \delta'([q_2], 1)$
$= \varepsilon^c(\delta(\{q_2\}, 1))$
$= \varepsilon^c(q_2) = [q_2] = C$

$= \varepsilon^c(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$
$= \varepsilon^c(\{q_0\} \cup \phi \cup \{q_2\})$
$= \varepsilon^c(\{q_0, q_2\})$
$= [\varepsilon^c(q_0) \cup \varepsilon^c(q_2)]$
$= [\{q_0, q_1, q_2\} \cup \{q_2\}] = [q_0, q_1, q_2]$
$= A$

$\delta'(A, 1) = \delta'([q_0, q_1, q_2], 1)$
$= \varepsilon^c(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$
$= \varepsilon^c(\phi \cup \{q_1\} \cup \{q_2\})$
$= \varepsilon^c(\{q_1, q_2\})$
$= \varepsilon^c(q_1) \cup \varepsilon^c(q_2) = \{q_1, q_2\} \cup \{q_2\}$
$= [q_1, q_2] = B$

$\delta'(B, 0) = \delta'([q_1, q_2], 0)$
$= \varepsilon^c(\delta(\{q_1, q_2\}, 0))$
$= \varepsilon^c(\phi \cup \{q_2\})$
$= [q_2] = C$

$\delta'(B, 1) = \delta'([q_1, q_2], 1)$
$= \varepsilon^c(\delta(\{q_1, q_2\}, 1))$
$= \varepsilon^c(\{q_1\} \cup \{q_2\})$
$= \varepsilon^c(\{q_1, q_2\})$
$= [\varepsilon^c(q_1) \cup \varepsilon^c(q_2)]$
$= [q_1, q_2] = B$



Minimization of Finite Automation

Equivalence of two states: Any two states q_1, q_2 are said to be equivalent states if $\delta(q_1, a)$ and $\delta(q_2, a)$ are either finals states or both are non-final states.

non-final states.

K-equivalence of states: states q_1, q_2 are K-equivalent if

$$\delta(q_1, \underbrace{x_1 x_2 \cdots x_k}\, x_{k+1} \cdots x_n)$$
$$\delta(q_2, \underbrace{x_1 x_2 \cdots x_k}\, x_{k+1} \cdots x_n)$$

are either final or non final.

e.g. q_1 & q_2 are 2-equivalent $\Sigma = \{0, 1\}$

$\delta(q_1, 00)$ or $\delta(q_1, 01)$ or $\delta(q_1, 10)$ or $\delta(q_1, 11)$

$\delta(q_2, 00)$ or $\delta(q_2, 01)$ or $\delta(q_2, 10)$ or $\delta(q_2, 11)$

Equivalence classes: $\underbrace{\Pi_0, \Pi_1, \Pi_2, \cdots \Pi_k, \Pi_{k+1} \cdots}$

$$\Pi_k = \{\{q_2 q_5\}, \{q_1, q_3, q_7\}, \{q_4 q_6\}, \cdots\}$$

q_1, q_7

Stopping criterion: $\Pi_k = \Pi_{k+1}$

Final set of minimal states

$\delta(q_1, a)$ — q_1 & q_2 are k-equivalent $\Rightarrow$

$\delta(q_1, a)$ — q_2 & q_1 " "

$q_1 R q_2 \Rightarrow q_2 R q_1$

Partition method: Consider a set S which is divided into n disjoint subsets $\underbrace{S_1, S_2, \cdots, S_n}$ such that

$A \cap B = \phi$

1) $S_1 \cup S_2 \cup S_3 \cup \cdots \cup S_n = S$

$A \cap B = \phi$

1) $S_1 \cup S_2 \cup S_3 \cup - - - \cup S_n = S$

2) $S_i \cap S_j = \phi \quad \forall\, i \neq j$ ✓ $\quad S_i \cap S_i = S_i$

$A \cap A = A$

$Q = \{q_0, q_1, - \quad -, q_7\}$

$\pi_0 = \{Q_1^0, Q_2^0\}$

$Q_1^0 = F, \quad Q_2^0 = Q - F = Q - Q_1^0$

$\pi_0 = \{\{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}\}$

q_0 is 1-equivalent to q_4

q_0 ,, ,, ,, to q_6

$\pi_1 = \{Q_1^1, Q_2^1, Q_3^1, - - \}$

$Q_1^1 = \{q_2\}, \quad Q_2^1 = \{q_0, q_4, q_6\}$

$Q_3^1 = \{q_1, q_7\}, \quad Q_4^1 = \{q_3, q_5\}$

q_0 is not 1-equivalent to q_1

q_0 ,, ,, ,, q_3

q_0 ,, ,, ,, q_5

$\boxed{\pi_1 = \{\{q_2\}, \{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}}$

$\delta(q, \epsilon) = q$

$\pi_2 = \{Q_1^2, Q_2^2, Q_3^2, - - - \}$

$Q_1^2 = \{q_2\}, \quad Q_2^2 = \{q_0, q_4\}, \quad Q_3^2 = \{q_6\}$

$Q_4^2 = \{q_1, q_7\}, \quad Q_5^2 = \{q_3, q_5\}$

q_4 is 2-equivalent to q_0

q_6 is not 2-equivalent to q_0

$\boxed{\pi_2 = \{\{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}}$

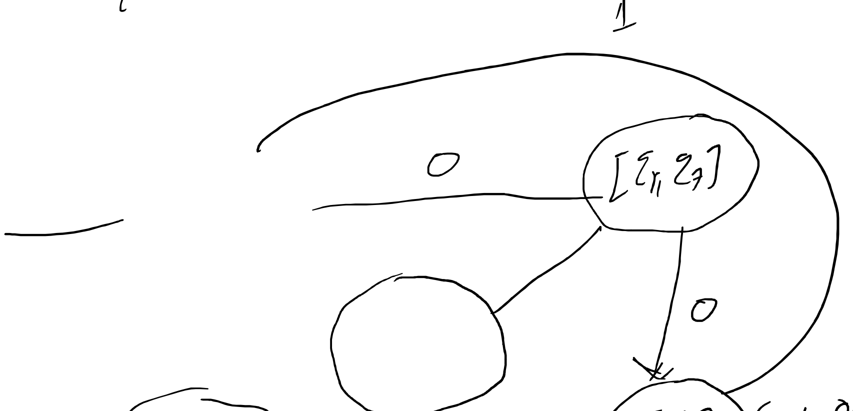
$\pi_3 = \{Q_1^3, Q_2^3, Q_3^3, Q_4^3, - - - \}$

$Q_1^3 = \{q_2\}, \quad Q_2^3 = \{q_0, q_4\}, \quad Q_3^3 = \{q_6\}, \quad Q_4^3 = \{q_1, q_7\}, \quad Q_5^3 = \{q_3, q_5\}$

$\boxed{\pi_3 = \{\{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}}$

$\Pi_3 = \{\{q_2\}$

$\Pi_2 = \Pi_3$



$M' = (\{[q_0, q_4], [q_1, q_7], [q_2], [q_3, q_5], [q_6]\}, \{0, 1\}, \delta', [q_0, q_4], \{[q_2]\})$

$\Pi_0 = \{Q_1^0, Q_2^0\}$, $Q_1^0 = F$, $Q_2^0 = Q - Q_1^0$

$\Pi_0 = \{\underbrace{\{q_3, q_4\}}_{Q_1^0}, \underbrace{\{q_0, q_1, q_2, q_5, q_6, q_7\}}_{Q_2^0}\}$

q_3 is 1-equivalent to q_4
$\therefore$ q_0 is not 1-equivalent to q_1, q_2, q_5 and q_7 but it is 1-equivalent to q_6

$\Pi_1 = \{Q_1^1, Q_2^1, Q_3^1, ---\}$

$Q_1^1 = \{q_3, q_4\}$, $Q_2^1 = \{q_0, q_6\}$,

$Q_3^1 = \{q_1, q_2\}$, $Q_4^1 = \{q_5, q_7\}$

$$\boxed{\Pi_1 = \{\{q_3, q_4\}, \{q_0, q_6\}, \{q_1, q_2\}, \{q_5, q_7\}\}}$$

q_3 is 2-equivalent to q_4
q_0 is not 2-equivalent to q_6

$\Pi_2 = \{Q_1^2, Q_2^2, Q_3^2, ---\}$

$Q_1^2 = \{q_3, q_4\}$, $Q_2^2 = \{q_0\}$, $Q_3^2 = \{q_6\}$, $Q_4^2 = \{q_1, q_2\}$

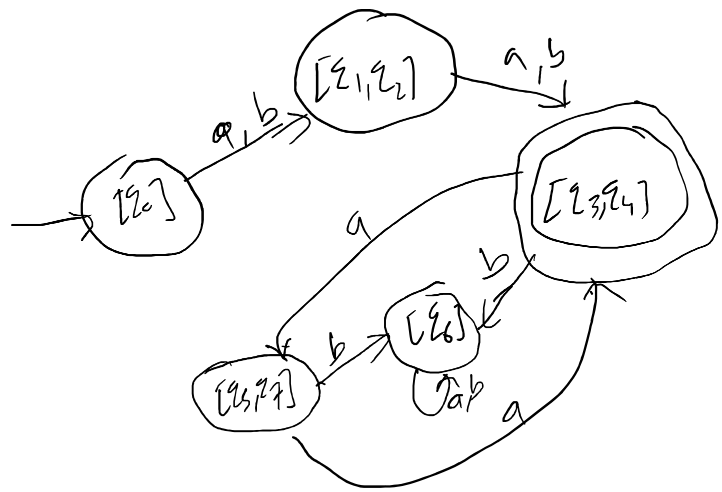
$Q_5^2 = \{q_5, q_7\}$

$$\boxed{\Pi_2 = \{\{q_3, q_4\}, \{q_0\}, \{q_6\}, \{q_1, q_2\}, \{q_5, q_7\}\}}$$

$\Pi_2 = \{ \ldots$

$$\Pi_3 = \{ \underbrace{\{q_3, q_4\}}_{Q_1^3}, \underbrace{\{q_0\}}_{Q_2^3}, \underbrace{\{q_6\}}_{Q_3^3}, \underbrace{\{q_1, q_2\}}_{Q_4^3}, \underbrace{\{q_5, q_7\}}_{Q_5^3} \}$$

$\Pi_2 = \Pi_3$



Regular Expressions :

(i) A terminal symbol (i.e. element of Σ), ε, ϕ are regular expressions

(ii) If R_1 and R_2 are regular expressions (RE) then $R_1 + R_2$ is a RE

(iii) If R_1 & R_2 are REs $\Rightarrow$ $R_1 R_2$ is also a RE

(iv) If R is RE $\Rightarrow$ R^* is also a RE
$R^* = \{\varepsilon, R, RR, RRR, RRRR, \ldots\}$

(v) If R is RE $\Rightarrow$ (R) is also a RE

(vi) REs are obtained by using above rules once or several times repeatedly (recursively)

$L \subseteq \Sigma^*$ is a language

... a regular set.

$L \subseteq \Sigma^*$ is a language

A regular expression represents a regular set.

AND

A regular set (regular language) is represented by a RE.

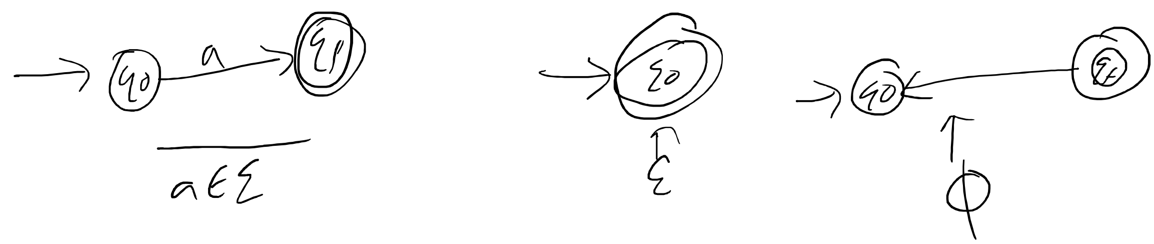
If R is RE $\Rightarrow L(R)$ is a regular set represented by R.

$L(R_1+R_2) \Rightarrow L(R_1) \cup L(R_2)$ $\qquad L_1 = L(R_1)$

$L(R_1R_2) \Rightarrow L(R_1)\cdot L(R_2) \Rightarrow L_1 \cdot L_2$ $\qquad L_2 = L(R_2)$

$L(R^*) \Rightarrow L(R)^*$ $\qquad L(R_1R_2) = L_1 \cdot L_2$

Finite automata for $a \in \Sigma$, ε, ϕ



If R_1 & R_2 are REs $\Rightarrow R_1+R_2$ is a RE

Example If $\underset{R_1}{a}$ and $\underset{R_2}{b}$ are REs $\Rightarrow a+b$

$a \Leftrightarrow \{a\}$
RE $\quad$ RS

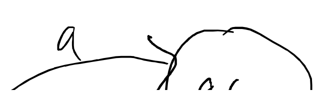
$L(R)$ is a regular set represented by $R_{R_1+R_2}$

a is a regular expression and $L(a) = \{a\}$

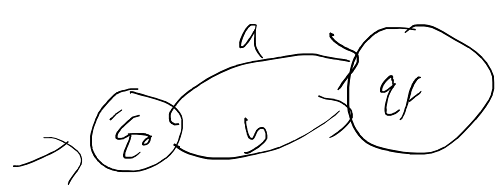
(1) $L(R_1+R_2) = L(R_1) \cup L(R_2) = \{a\} \cup \{b\} = \{a,b\}$

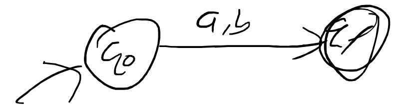
$L(R_1+R_2) = L(R_2+R_1)$ $\qquad \boxed{L(a+b) = \{a,b\}}$

$L(R_2)$ $\qquad L(R_1) \cup L(R_2)$ $\hookrightarrow L(R_1+R_2)$



$L(R_1+R_2)$





2) If R_1 and R_2 are REs $\Rightarrow R_1R_2$ is a RE

Ex If a and b are REs $\Rightarrow ab$ is a RE

$R_1 = a$

$\Downarrow$

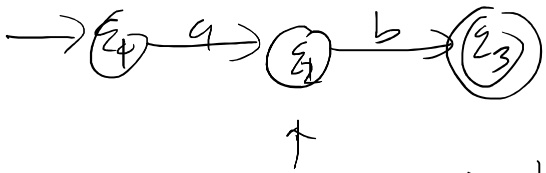
$L_1 = L(R_1) = \{a\}$

$R_2 = b$

$\Downarrow$

$L_2 = L(R_2) = \{b\}$

$L_1L_2 = \{uv \mid u \in L_1, v \in L_2\}$



$L_1 = L(R_1)$

$L(R_1R_2) = L(R_1) \cdot L(R_2) = L_1 \cdot L_2 \quad L_2 = L(R_2)$

$L(ab) = \{a\}\{b\} = \{ab\}$ $\boxed{L(ab) = \{ab\}}$

$R_1R_2 \neq R_2R_1$

$L(R_1R_2) \neq L(R_2R_1)$

$L(\underbrace{a+ab+abb}_{R}) = \{\underbrace{a, ab, abb}\}$

3) If R is RE $\Rightarrow R^*$ is a RE

$\Rightarrow L(R^*) = \{\varepsilon, R, RR, RRR, \text{———}\}$

Ex If a is RE $\Rightarrow a^*$ is a RE

$L(a^*) = \{\varepsilon, a, aa, aaa, \text{— —}\}$ } Regular set

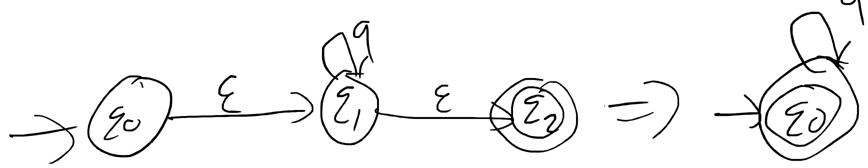
$= \{a^n \mid n \geq 0\}$

$\varepsilon a \varepsilon = a$

aa

$L(a^*) = \{\varepsilon, a, \ldots\}$

$= \{a^n \mid n \geq 0\}$



$\varepsilon a \varepsilon = a$

$\varepsilon a a \varepsilon = aa$

$\boxed{\varepsilon x = x \varepsilon = x}$

4) If R is a RE $\Rightarrow$ (R) is also a RE

$(1)^* = 1^*$

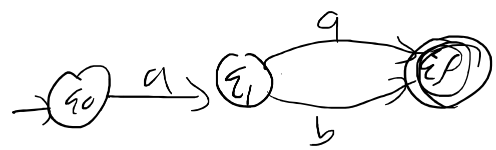
$(xy)^3 \neq xy^3$

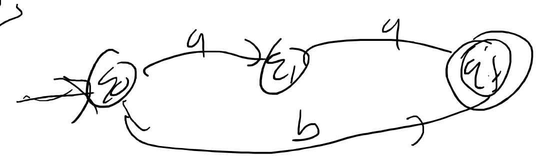
x^3y^3

Ex $a(a+b)$

$\Downarrow$

$aa+b$





$(11)^* \neq 11^*$

$\{\varepsilon, 11, 1111, 111111, \ldots\}$

$L(a(a+b)) = \{aa, ab\}$

$L(aa+b) = \{aa, b\}$

$1\varepsilon = 1$

$1\{\varepsilon, 1, 11, 111, \ldots\}$

$\{1, 11, 111, 1111, \ldots\}$

$RR^* = R^+ = R^*R$

$= \{\varepsilon, R, RR, \ldots\}R$

$11^* = 1^+$

$\boxed{R^+ = RR^* = R^*R}$

$\boxed{x\varepsilon = \varepsilon x = x}$

$RR^* = R\{\varepsilon, R, RR, \ldots\}$

$= \{R\varepsilon, RR, RRR, \ldots\}$

$\boxed{RR^* = \{R, RR, RRR, \ldots\}}$

$R^*R = \{$

Equivalence of Regular set and Regular Expressions

Equivalence of Regular set and Regular Expressions

$a \Leftrightarrow \{a\}$

$b \Leftrightarrow \{b\}$

$a+b \Leftrightarrow \{a,b\}$

$ab \Leftrightarrow \{ab\}$

$a^* \Leftrightarrow \{\varepsilon, a, aa, aaa, \ldots\} = \{a^n \mid n \geq 0\}$

$a^+ = aa^* = a^*a \Leftrightarrow \{a^n \mid n \geq 1\}$

$a + ab + abb + b^* \Leftrightarrow \{a, ab, abb, b^*\}$

$R = a^*$

$\{\varepsilon, a, aa, aaa, \ldots\} = a^*$
$\varepsilon + a + aa + aaa + \ldots$

$\varepsilon x = x\varepsilon = x$

Q. Find RE for the RS $\{11, 1111, 111111, \ldots\} = 11\{\varepsilon, 11, 1111, \ldots\}$

Let $a = 11$ Let $a = 11$

$\{a, aa, aaa, \ldots\} \Leftrightarrow aa^* = 11(11)^*$

$\cancel{1111^*}$

$= a\{\varepsilon, a, aa, aaa, \ldots\}$
$= aa^* = 11(11)^*$

Q. Find REs for the following regular sets over $\{a, b\}$

(i) set of strings with length exactly 2.

(ii) " " " even length

(iii) " " " odd "

(iv) " " " at least two a's

(v) " " " exact 2 a's

(vi) " " " at most two a's

(vii) " " " aa

(viii) same no of a's and b's

(ix) length of strings of multiple of 3

$x = \varepsilon x = x\varepsilon$

$ab + abb = ab\varepsilon + abb$
$= ab(\varepsilon + b)$

$\underbrace{2\times 3} + \underbrace{3 \times 5}$

$\varepsilon ab + bab$
$(\varepsilon + b)ab$

(i) $\{aa, ab, ba, bb\} \Leftrightarrow aa + ab + ba + bb = a(a+b) + b(a+b)$

Sol^n $L = \{aa, ab, ba, bb\} \Leftrightarrow aa + ab + ba + bb = a(a+b) + b(a+b)$

$= (a+b)(a+b)$

(i) $\Sigma = \{a, b\}$

(ii) $L = \{\varepsilon, aa, ab, ba, bb, aaaa, aaab, \ldots\} \Leftrightarrow ((a+b)(a+b))^*$

(i) exact 2 length $= \{aa, ab, ba, bb\} \Leftrightarrow (a+b)(a+b)$

(ii) even " $= \{\varepsilon, aa, ab, ba, bb, \ldots\} \Leftrightarrow ((a+b)(a+b))^*$

(iii) odd " $= \{a, b, aaa, aab, \ldots\} \Leftrightarrow (a+b)((a+b)(a+b))^*$

(iv) at least 2 a's $= \{aa, abab, aaa, aab, \ldots\}$

$\Downarrow$ $(a+b)^* a (a+b)^* a (a+b)^*$

(v) at most 2 a's $\Leftrightarrow b^* + b^* a b^* + b^* a b^* a b^*$

$b^* aa b^*$

(vi) Containing aa $\Leftrightarrow (a+b)^* aa (a+b)^*$

(vii) Length of strings is multiple of 3 $\Leftrightarrow ((a+b)(a+b)(a+b))^* \Leftrightarrow$ includes 0 length

$((a+b)(a+b)(a+b))((a+b)(a+b)(a+b))^*$

(viii) Set of all strings starts and ends with same symbol

$L = \{aa, bb, aaa, aba, bab, bbb, \ldots\}$

a, b $\quad a + b + a(a+b)^* a + b(a+b)^* b$

$(a+b)^* \Leftrightarrow \{a, b\}^* \Leftrightarrow \Sigma^*$

$\Sigma = \{a, b\}$

(ix) Starting & ends with different symbols

$a(a+b)^* b + b(a+b)^* a$

(x) Set of strings which does not include two consecutive a's

$(b^* a b^*)^*$

Identities for Regular Expression:

Identities for Regular Expression:

(i) $R+\phi = \phi+R = R$

(ii) $R\phi = \phi R = \phi$

(iii) $\varepsilon R = R\varepsilon = R$

(iv) $\varepsilon^* = \varepsilon$, $\phi^* = \varepsilon$

(v) $R+R = R$

(vi) $R^*R^* = R^*$

(vii) $RR^* = R^*R = (R^+)$

(viii) $(R^*)^* = R^*$

(ix) $\varepsilon + \underline{RR^*} = R^* = \varepsilon + \underline{R^*R}$

(x) $(PQ)^*P = P(QP)^*$

(xi) $(P+Q)^* = (P^*+Q^*)^* = (P^*Q^*)^*$

(xii) $(P+Q)R = PR+QR$
and
$R(P+Q) = RP+RQ$

$RR^* = R^*R \rightarrow \{\varepsilon, R, RR, \ldots\}R$ $\rightarrow \{R, RR, RRR, \ldots\}$

$R\{\varepsilon, R, RR, \ldots\}$
$= \{R, RR, RRR, \ldots\}$

$L(R^*) = \{\varepsilon, R, RR, \ldots\}$

$L(\varepsilon^*) = \{\varepsilon, \varepsilon, \varepsilon\varepsilon, \varepsilon\varepsilon\varepsilon, \ldots\}$

$L(\phi^*) = \{\varepsilon, \phi, \phi\phi, \phi\phi\phi, \ldots\}$

$S = Z^+ = \{0, 1, 2, 3, \ldots\}$

$a, e \in S$ $\quad a \otimes e = e \otimes a = a$

$a+0 = 0+a = a$
$a \times 1 = 1 \times a = a$

$\{\varepsilon, PQ, PQPQ, \ldots\}P$

$\{\varepsilon P, PQP, PQPQP, \ldots\}$

$\{P, PQP, PQPQP, \ldots\}$

$P\{\varepsilon, QP, QPQP, QPQPQP, \ldots\}$

$P(QP)^*$

$QPPQPPQ$

$P^*Q^*P^*Q^*P^*Q^*$

$\varepsilon\ Q\ PP\ Q\ PP\ Q$

Algebraic Laws for Regular Expressions:

1. ~~Commutative Law~~

$R_1+R_2 = R_2+R_1$ (always)

But $R_1R_2 \neq R_2R_1$ (in general)

2. Associative ~~Law~~

$R_1+(R_2+R_3) = (R_1+R_2)+R_3$

$R_1(R_2R_3) = (R_1R_2)R_3$

3. Identity Law

$\underline{2\times 3} + \underline{3\times 5} = 3(2+5)$

$ab \neq ba$

$\varepsilon ab + bab$
$(\varepsilon+b)ab$

$x = x\varepsilon = \varepsilon x$

$RR^* = RR^*$

$R_1R_2 \quad R_1R_2$

3. Identity Law

$\phi + R = R + \phi = R$

$\varepsilon R = R\varepsilon = R$

$R_1R_2 = \cancel{RR^*} = R^+$

4. ~~Annihilator for Regular Expression~~:

$R\phi = \phi R = \phi$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. Distributive Law

$R(P+Q) = RP + RQ$ (Left distributivity)

$(P+Q)R = PR + QR$ (Right distributivity)

Q. Write a RE to represent L = set of strings in which every 0 must
(a) be followed by at least two 1's.

$\boxed{(1 + 011)^*}$ ✓ $\quad (a+b)^*$

1^*, $(011)^*$, $(11)^*$, $(11011)^*$ ✓

$(1011)^*$, $(0111)^*$ ✓

(b)

$R = \underbrace{1^*(011)^*)^*}_{P^*} = (1+011)^*$ ✓

$(P^*Q^*)^* = (P+Q)^* = (P^*+Q^*)^*$

$R = \varepsilon + PP^*$, where $P = 1^*(011)^*$

$= P^* = (1^*(011)^*)^* = (1+011)^*$

$\varepsilon + PP^* = P^*$ $\quad (P^* Q^*)^*$ $\quad P = 1, Q = 011$

$R = R\varepsilon = \varepsilon R$

(1 + 00*1) + (1 + 00*1)(0 + 10*1)* (0 + 10*1) = 0*1(0 + 10*1)*

(1 + 00*1) + (1 + 00*1)(0 + 10*1)* (0 + 10*1) = 0*1(0 + 10*1)*

$L.H.S = (1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1)$ $\quad [R = R\varepsilon]$

$[RP+RQ = R(P+Q)]$

$= (1+00^*1)\varepsilon +$ ''

$= (1+00^*1)[\varepsilon + \underbrace{(0+10^*1)^*}_{P^*}\underbrace{(0+10^*1)}_{P}]$ $\quad [PR+QR = (P+Q)R]$

$R = \varepsilon R$ $\quad = (\varepsilon 1+00^*1)[\qquad ''\qquad]$ $\quad P = \underline{0+10^*1}$

$= (\varepsilon+00^*)1[\varepsilon + P^*P]$

$= (\varepsilon+QQ^*)1[\varepsilon+P^*P]$

$= Q^*1P^*$ $\quad$ where $Q = 0,\ P = 0+10^*1$ $\quad [\because \varepsilon + RR^* = R^* = \varepsilon + R^*R]$

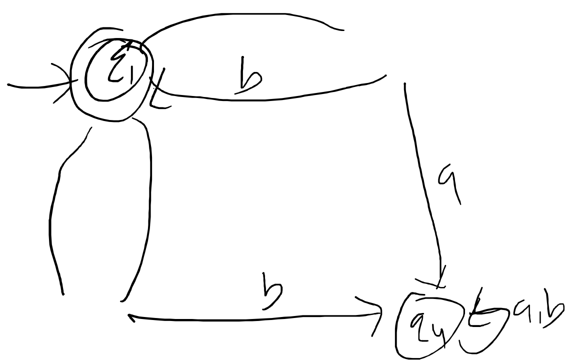
$= \boxed{0^*1(0+10^*1)^*}$

Finite Automation to RE Conversion:

__Arden's theorem:__ If P, Q are any regular expression $(P \neq \varepsilon)$ then an equation of the form $R = Q + RP$ has unique solution

$$\boxed{R = QP^*}$$

Q//



$q_1 = q_2 b + q_3 a + \varepsilon$ —— (1)

$\epsilon_1 = \epsilon_2 b + \epsilon_3 a + \epsilon$ —— (1)

$\epsilon_2 = \epsilon_1 a$ —— —— (2)

$\epsilon_3 = \epsilon_1 b$ —— —— (3)

$\epsilon_4 = \epsilon_2 a + \epsilon_3 b + \epsilon_4 (a+b)$ —— —— (4)

Substitute ϵ_2 & ϵ_3 from (2), (3) in Eq (1)

$\epsilon_1 = \overset{R\,P}{\epsilon_1 ab} + \overset{R\,Q}{\epsilon_1 ba} + \epsilon$ [$RP + RQ = R(P+Q)$]

$= \epsilon_1 (ab+ba) + \epsilon$ [∵ $R_1 + R_2 = R_2 + R_1$]

$\boxed{\epsilon_1 = \epsilon + \epsilon_1 (ab+ba)}$

$R = Q + RP \Rightarrow R = QP^*$

$\epsilon_1 = \epsilon (ab+ba)^*$ [$\epsilon R = R$]

$\boxed{\epsilon_1 = (ab+ba)^*}$

$(ab)^*$, $(ba)^*$

$(abab)^*$ $(baba)^*$

$(abba)^*$ $(baab)^*$

$L = \{ \epsilon, ab, ba, abba, baab, \_\times\_ \times \}$

aabb×

bbaa + aaaa

abab

baba

abab

aba (prefix)

baba

bab

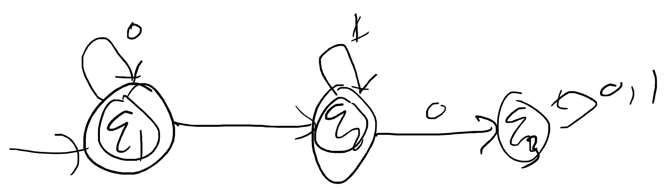
ba

b

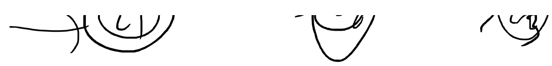
Prefixes of abab: abab, aba, ab, a, ϵ

" " baba: baba, bab, ba, b, ϵ

Q/



—(1 1)



$$\varepsilon_1 = \varepsilon_1 0 + \varepsilon \quad \text{———} (1)$$

$$\varepsilon_2 = \varepsilon_1 1 + \varepsilon_2 1 \quad \text{———} (2)$$

$$\varepsilon_3 = \varepsilon_2 0 + \varepsilon_3 (0+1) \quad \text{———} (3)$$

$[R_1 + R_2 = R_2 + R_1]$

$$\varepsilon_1 = \varepsilon + \varepsilon_1 0$$

$$R = Q + RP \Rightarrow R = QP^* \quad \text{[Arden's Theorem]}$$

$$\boxed{\varepsilon_1 = \varepsilon 0^*}$$

$[\because \varepsilon R = R\varepsilon = R]$

[Substitute ε_1 in Eq (2)]

$$\varepsilon_2 = 0^* 1 + \varepsilon_2 1$$

$$R = Q + RP$$

$$\Downarrow$$

$$R = QP^*$$

$$\Downarrow$$

$$\boxed{\varepsilon_2 = 0^* 1 1^*}$$

$$\varepsilon_1 + \varepsilon_2 = \underset{\varepsilon_1}{0^*} + \underset{\varepsilon_2}{0^* 1 1^*}$$

$$= 0^* \varepsilon + 0^* \underline{1 1^*} \qquad [R = R\varepsilon]$$

$[RP + RQ = R(P+Q)]$

$$= 0^* (\varepsilon + 1 1^*)$$

$$= 0^* (1^*)$$

$[\varepsilon + RR^* = R^* = \varepsilon + R^* R]$

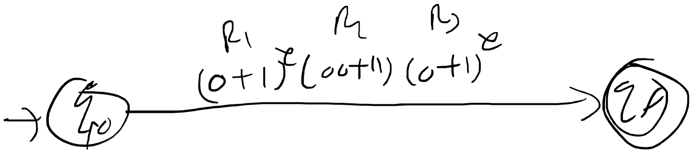
$$\boxed{\varepsilon_1 + \varepsilon_2 = 0^* 1^*}$$

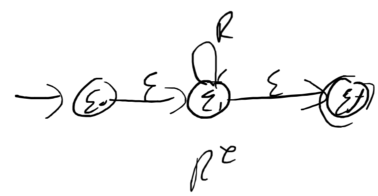
Regular Expression to Finite Automation Conversion

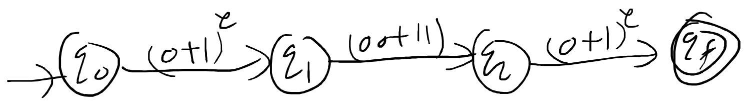
$$RE = \underbrace{(0+1)^e}_{R_1}\underbrace{(00+11)}_{R_2}\underbrace{(0+1)^e}_{R_3}$$

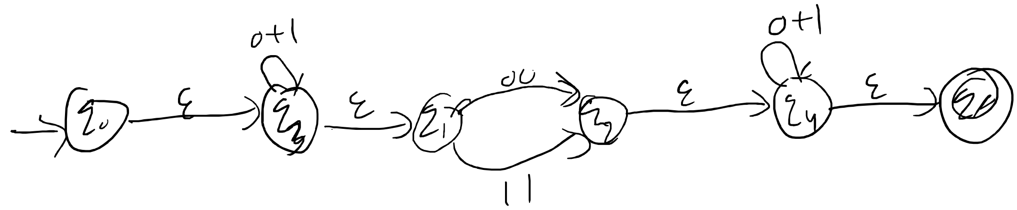
a+b ✓ a^e ✓

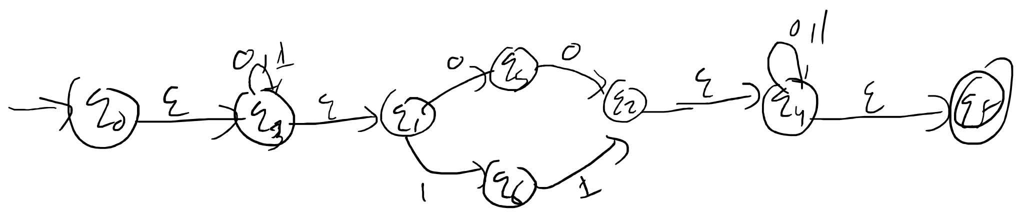
ab ✓

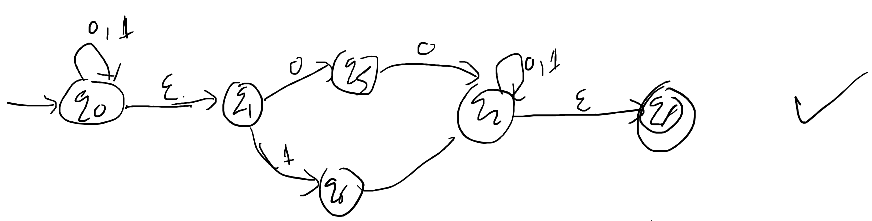


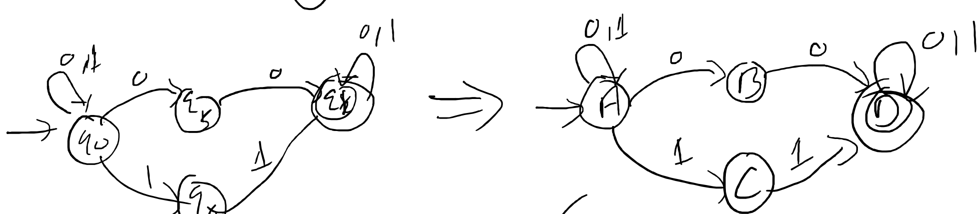












NFA

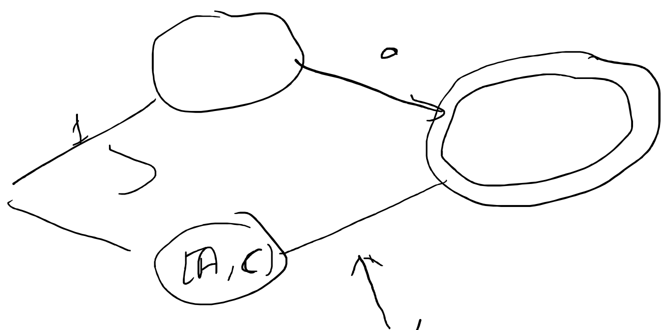
Convert it into DFA

Transition table for NFA

States	Input 0	Input 1
	A,B	A,C
	D	-
	-	D
	D	D

Convert it into DFA

States	Input 0	Input 1
→[A]	[A,B]	[A,C]
[A,B]	[A,B,D]	[A,C]
[A,C]	[A,B]	[A,C,D]
* [A,B,D]	[A,B,D]	[A,C,D]
* [A,C,D]	[A,B,D]	[A,C,D]



$M' = (\{[A], [A,B], [A,C], [A,B,D]\}, \{0,1\}, \delta', [A], \{[A,B,D]\})$

Pumping Lemma for Regular Sets

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automation with n states. Let L be the regular set accepted by M. Let $w \in L$ and $|w| \geq m$. If $m \geq n$ then there exist x, y, z such that $w = xyz$, $y \neq \varepsilon$, $|xy| \leq n$ and $xy^iz \in L \ \forall\ i \geq 0$.

Equivalence of Finite Automation (Comparison method)

Equivalence of two Regular Expressions

$R_1 \equiv R_2$

$L(R_1) = L(R_2)$

Simplify R_1 and R_2 using identity rules

Convert R_1 & R_2 into FA & prove M_1, M_2 are equivalent.

Simplify R_1 then R_2 using identities ...

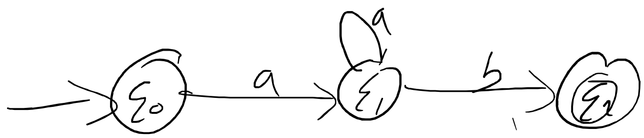
Convert R_1 & R_2 into FA & prove M_1, M_2 are equivalent.

$|xy| = |ab| = 2$ $\qquad |w| \geq n$

$A \subseteq B$

$A \subseteq B$

$B \subseteq A$



$n = 3$

$|w| \geq 3$

$\underbrace{a}_{x}\underbrace{aa}_{y}\underbrace{b}_{z}$ $\qquad |\underbrace{aaab}_{xyz}| = 4 = m$

$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2$ $\qquad |xyz| \geq n$

$w \in L \Rightarrow |w| \geq n \Rightarrow w = xyz,\ y \neq \varepsilon\ (|y| > 0),\ |xy| \leq n$

$$\boxed{xy^iz \in L \ \forall i \geq 0}$$

$w = xyz \longrightarrow xy^0z = xz \in L \quad (i = 0)$

$xyz \in L \quad (i = 1)$

$xy^2z = xyyz \in L \quad (i = 2)$

Q. Show that $L = \{a^{i^2} \mid i \geq 1\}$ is not regular.

$L = \{a, aaaa, aaaaaaaaa, \ldots\}$

Proof: Step I: Let L be a regular language and n be the no of states in finite automation $M = (Q, \Sigma, \delta, q_0, F)$ accepting L.

Step II: Let $w = a^{n^2} \in L$, $|w| = n^2 \geq n$

$w = xyz$, $|xy| \leq n$, $|y| > 0$ $(y \neq \varepsilon)$

$|w| = n^2 \Rightarrow |xyz| = n^2 \Rightarrow |x| + |y| + |z| = n^2$

Step III Find suitable i such that $xy^iz \notin L$ to get contradiction

Step III Find suitable i such that $xy^iz \notin L$ to get contradiction of assumption in step I.

Consider string $\underline{xy^2z}$ (i.e. $i=2$)

$$|xy^2z| = |x| + 2|y| + |z|$$
$$= |x| + |y| + |z| + |y|$$
$$|xy^2z| = n^2 + |y|$$

Now as $1 \leq |y| \leq n$ [As $|xy| \leq n$, $|y| > 0$]

$$\therefore n^2+1 \leq |xy^2z| \leq n^2+n$$
$$n^2 < |xy^2z| < n^2+n+n+1$$
$$n^2 < |xy^2z| < (n+1)^2$$

Since $|xy^2z|$ lies b/w perfect squares of two consecutive integers n and $n+1$ but it does not hold equality with any of them.

Therefore, $|xy^2z|$ is not a perfect square.

So, $xy^2z \notin L$ i.e. $xy^iz \notin L$ for $i=2$

This is contradiction to assumption in step I. Thus, L is not a regular language.