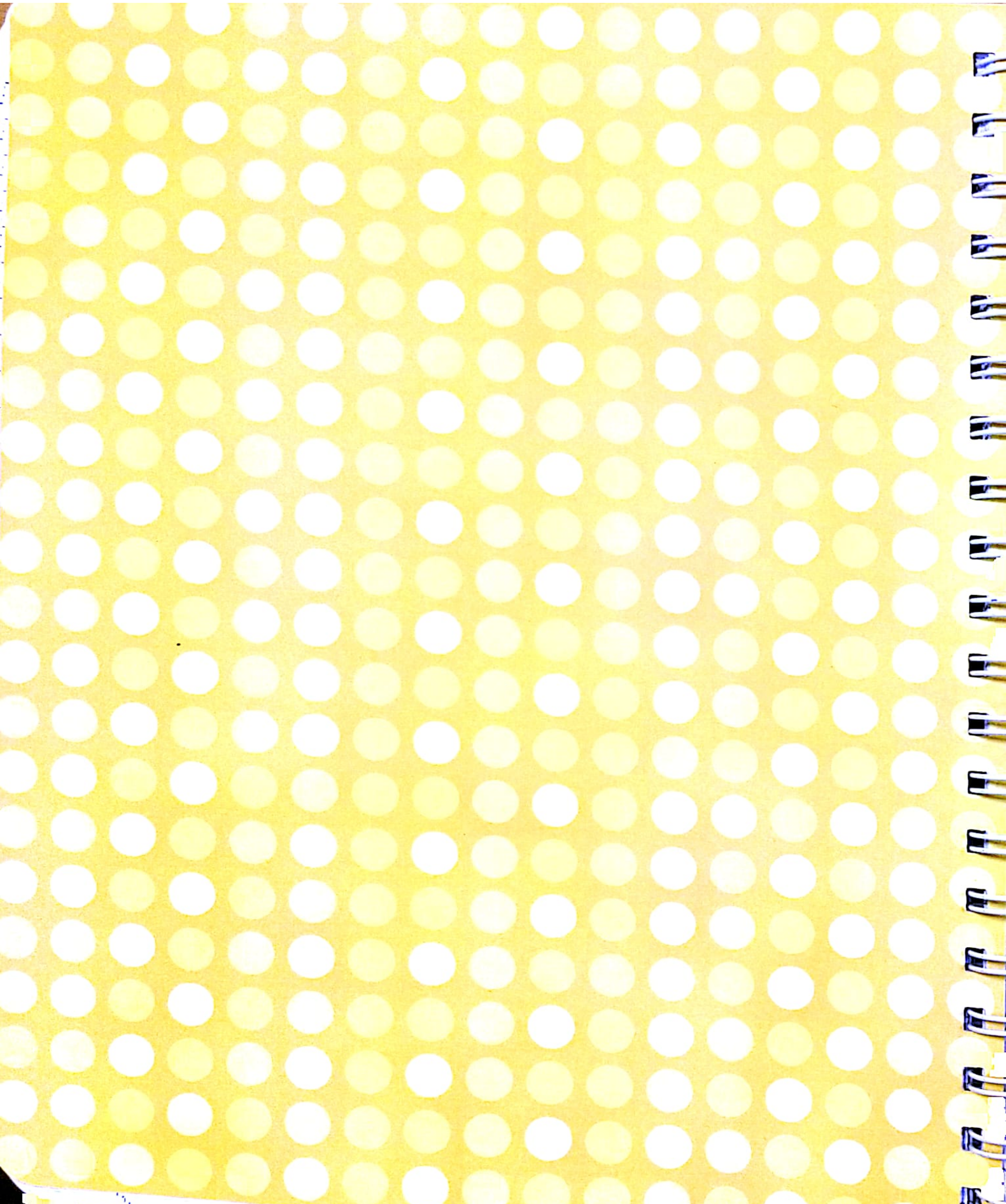

__/__/____

When computation is done using the principles of quantum mechanics, it is called quantum computing.

Linear algebra: A Hilbert space H is a normal vector space over the complex numbers c.

Dirac's bra-ket notation: z^* - complex conjugate

In Dirac's notation, conjugate transpose of $|\psi\rangle$ is $\langle\psi|$

The inner product $\langle\psi|\phi\rangle$ is defined to be scalar obtained by multiplying the conjugate transpose $\langle\psi| = (\bar{\psi}_1 \ldots \bar{\psi}_n)$ with $|\phi\rangle$

$$\langle\psi|\phi\rangle = (\bar{\psi}_1, \bar{\psi}_2 \cdots \psi_n) \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} = \sum_{i=1}^{n} \bar{\psi}_i \phi_i$$

$$|\psi\rangle = \begin{bmatrix} 2 \\ 6i \end{bmatrix} \qquad |\phi\rangle = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\langle\psi|\phi\rangle = [2 \ \ -6i] \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 6 - 24i$$

Tensor product of $|\psi\rangle$ & $|\phi\rangle$

$$|\psi\rangle|\phi\rangle = \begin{bmatrix} 2 \\ 6i \end{bmatrix} \otimes \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 18i \\ 24i \end{bmatrix}$$

ψ^+ - Hermition conjugate (adjoint) of matrix ψ

$$\psi = \begin{bmatrix} 1 & 6i \\ 3i & 2+4i \end{bmatrix} \qquad \psi^+ = \begin{bmatrix} 1 & -3i \\ -6i & 2-4i \end{bmatrix} \qquad \left[\psi^+ = (\bar{\psi})^T\right.$$

Note

Norm: $\|\psi\| = \sqrt{\langle\psi|\psi\rangle}$

$$|\psi\rangle = \begin{bmatrix} 1 \\ 2 \\ -i \end{bmatrix} \qquad \||\psi\rangle\| = \frac{1}{\sqrt{6}}(1, 2, -i) = \left(\tfrac{1}{\sqrt{6}}, \tfrac{2}{\sqrt{6}}, \tfrac{-i}{\sqrt{6}}\right)$$

__/__/__

Linearly dependent vectors example:

$|\psi_1\rangle = \begin{bmatrix} 1 \\ 2 \\ -9 \end{bmatrix}$ $|\psi_2\rangle = \begin{bmatrix} -2 \\ -4 \\ 18 \end{bmatrix}$ Linearly dependent in C^3

$a_1|\psi_1\rangle + a_2|\psi_2\rangle = 0 \Rightarrow a_1 = 2, a_2 = 1$

a_1, a_2 both $= 0 \Rightarrow$ Linearly independent

$|\psi_1\rangle = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $|\psi_2\rangle = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$ $|\psi_3\rangle = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ are linearly independent

Basis: The set of vectors $|\psi_1\rangle, |\psi_2\rangle \ldots |\psi_n\rangle$ is said to basis for C^n, then every vector $|v\rangle$ can be expressed as a linear combination of $|\psi_1\rangle, |\psi_2\rangle \ldots |\psi_n\rangle$

$|v\rangle = a_1|\psi_1\rangle + a_2|\psi_2\rangle + \ldots a_n|\psi_n\rangle$ where $a_1, a_2 \ldots a_n$ are scalars

Basis for C^n contains n linearly independent vectors.

Orthonormal basis: A basis is said to be orthonormal if each vector has norm 1 and each pair of vectors are orthogonal.

Ex: $|\psi_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|\psi_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are orthonormal basis in C^2.

$\| |\psi_1\rangle \| = \| |\psi_2\rangle \| = 0$ and $\langle \psi_1 \psi_2 \rangle = 0$ (mutually orthogonal)

3-qubit entangled state (GHZ state)

The state $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ is 3-qubit entangled state.

It is called Greenberger-Horne-Zeilinger (GHZ) state.

Hermitian matrix: A complex matrix A is said to be Hermitian if $A^\dagger = A$ ($A^\dagger$ is conjugate transpose $(\bar{A})^T$)

Unitary matrix: A complex matrix is said to be unitary if $A^\dagger A = I$ or $A^\dagger = A^{-1}$.

Eigen values and eigen vectors: Let A be any complex square matrix. A scalar λ is called an Eigen value of A if there exists a non zero column vector $|v\rangle \in C^n$ such that $A|v\rangle = \lambda|v\rangle$, $\lambda \in C$

__/__/____

The vector $|v\rangle$ is called an eigen vector of A corresponding to λ.

Quantum computation: A quantum computer is built from quantum circuits containing wires and elementary quantum gates to carry around and manipulate quantum information.

Logical gates: Other than AND, OR, NOT, NAND, NOR

① Single qubit quantum gate: $|0\rangle \to |1\rangle$ $|1\rangle \to |0\rangle$

Quantum NOT gate: Matrix $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ represents quantum NOT gate

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$X(\alpha|0\rangle + \beta|1\rangle) = X \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

$$\alpha|0\rangle + \beta|1\rangle \text{ --- } \boxed{X} \text{ --- } \beta|0\rangle + \alpha|1\rangle$$

X is unitary $\quad X^{\dagger}X = I$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

② Z-gate (phaseflip gate): $|0\rangle \to |0\rangle$ $\quad |1\rangle \to -|1\rangle$

Z-gate is defined by the matrix $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$$

$$Z(\alpha|0\rangle + \beta|1\rangle) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$$

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \boxed{Z} \longrightarrow \alpha|0\rangle - \beta|1\rangle$

③ Hadamard gate (H-gate) : $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \boxed{H} \longrightarrow \alpha\frac{(|0\rangle + |1\rangle)}{\sqrt{2}} + \beta\frac{(|0\rangle - |1\rangle)}{\sqrt{2}}$

$H\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

X gate , Z-gate H-gate

$|0\rangle - |1\rangle$ $|0\rangle - |0\rangle$ $|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|1\rangle - |0\rangle$ $|1\rangle - -|1\rangle$ $|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$X^\dagger X = I$ $Z^\dagger Z = I$ $H^\dagger H = I$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Y-gate $|0\rangle \to i|1\rangle$ $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$|1\rangle \to -i|0\rangle$

$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$

$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \boxed{Y} \longrightarrow i(\alpha|1\rangle - \beta|0\rangle)$

$Y^\dagger Y = I$

__/__/__

Phase shift gate :

$|0\rangle \to |0\rangle$

$|1\rangle \to e^{i\theta}|1\rangle$

$e^{i\theta} = \cos\theta + i\sin\theta$

$$R_\theta = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

$$R_\theta|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$R_\theta|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e^{i\theta} \end{bmatrix} = e^{i\theta}|1\rangle$$

when $\theta = \pi$ we get z gate.

1) write the following quantum gate in braket notation

X, Z, H, Y, R_θ

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$= |1\rangle\langle 0| + |0\rangle\langle 1|$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$= |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|01\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Two qubit controlled gates:

Controlled NOT gate: It flips the second bit if the first bit is 1, and does nothing otherwise.

control bit, target bit

$$C_{NOT}: |00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

In matrix form this is $C_{NOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$$C_{NOT} C_{NOT}^{\dagger} = I$$

C_{NOT} is a generalization of XOR gate because $|AB\rangle \rightarrow |A B \oplus A\rangle$

Tensor product of 2 single qubit states:

C_{NOT} takes the unentangled qubit state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$ to the unentangled state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$$C_{NOT}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle\right) = C_{NOT}\left(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\right)$$

$$= C_{NOT}\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

No cloning principle: This states that we cannot copy or clone an unknown qubit.

Let U be a unitary transformation that clones

$U(|a0\rangle) = |aa\rangle$ for all quantum states $|a\rangle$

Let $|a\rangle$ and $|b\rangle$ be 2 orthonormal quantum states.

Consider $|c\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$

$$U|c0\rangle = U\left(\frac{1}{\sqrt{2}}(|a0\rangle + |b0\rangle)\right)$$

$$= \frac{1}{\sqrt{2}}(|aa\rangle + |bb\rangle)$$

$$U(|c0\rangle) = |cc\rangle$$

$$= |c\rangle \otimes |c\rangle$$

$$= \frac{1}{2}(|aa\rangle + |ab\rangle + |ba\rangle + |bb\rangle)$$

$$\neq \frac{1}{\sqrt{2}}(|aa\rangle + |bb\rangle)$$

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

SWAP Gate: Exchanges bit values

$|00\rangle \to |00\rangle$ $\quad |10\rangle \to |01\rangle$ $\quad |01\rangle \to |10\rangle$ $\quad |11\rangle \to |11\rangle$

Matrix: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Tensor product for matrices: $|v\rangle = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $\quad |w\rangle = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$$|v\rangle \otimes |w\rangle = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \otimes \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} v_1 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ v_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} v_1 w_1 \\ v_1 w_2 \\ v_2 w_1 \\ v_2 w_2 \end{bmatrix}$$

Suppose we have 2 matrices:

$$M = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \qquad N = \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix}$$

Tensor product of M and N is defined as

$$M \otimes N = \begin{bmatrix} m_1 \cdot \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} & m_2 \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} \\ m_3 \cdot \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} & m_4 \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} m_1n_1 & m_1n_2 & m_2n_1 & m_2n_2 \\ m_1n_3 & m_1n_4 & m_2n_3 & m_2n_4 \\ m_3n_1 & m_3n_2 & m_4n_1 & m_4n_2 \\ m_3n_3 & m_3n_4 & m_4n_3 & m_4n_4 \end{bmatrix}$$

Find tensor product of x-gate and z-gate: $X \otimes Z = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

Scanned by CamScanner

Quantum measurement :

Quantum state : The collection of all relevant physical properties of quantum system (For ex. position, momentum, spin, polarization) is known as the state of the system.

Physical support	Name	Info support	$\|0\rangle$	$\|1\rangle$
Photon	Polarization	Polarization	Vertical	Horizontal
Electron	Electronic spin	spin	Up ↑	Down ↓

For example, if we use the energy of an electron as our qubits $|0\rangle$ and $|1\rangle$, we could say that the ground state (lowest energy) in our qubit $|0\rangle$ and and excited state (higher energy) in our qubit $|1\rangle$.

We can define state $|+\rangle$ and $|-\rangle$ with the vectors.

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \qquad |-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left[\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}\right] = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left[\begin{bmatrix}1\\0\end{bmatrix}-\begin{bmatrix}0\\1\end{bmatrix}\right] = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$