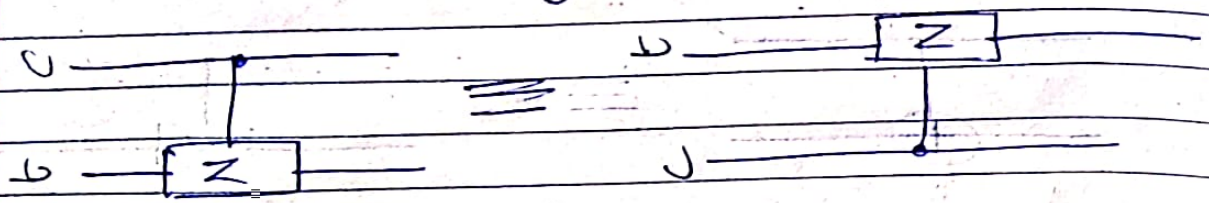
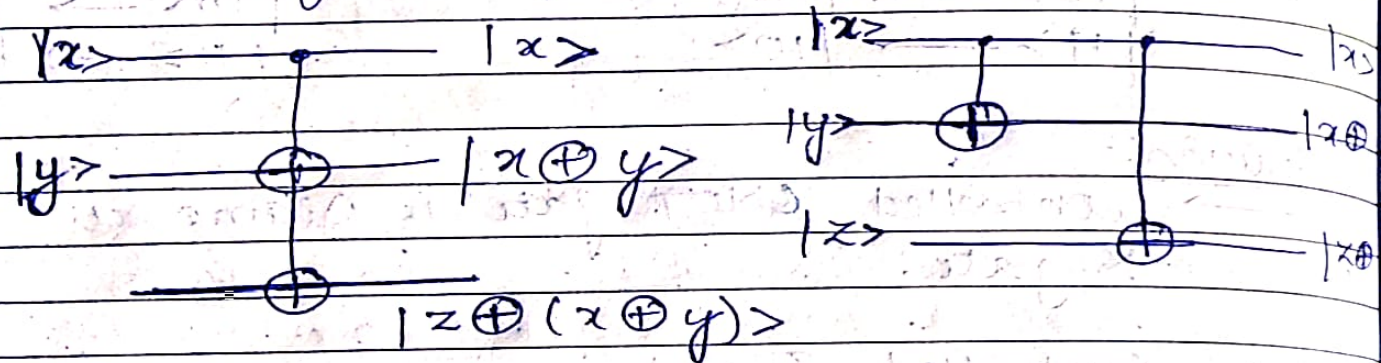
Edge

A → 0
B → 1

(15) Is the following circuits are equivalent



HW. (2) Is the following circuits are equivalent.



~~Controlled~~. They are equal (≡).

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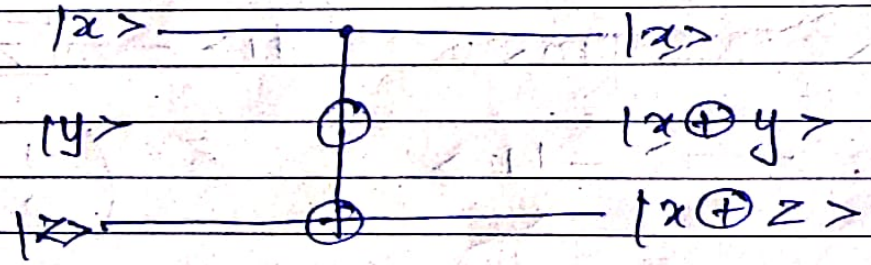
⇒ Controlled - NOT gate with multiple targets. When the control bit qubit is 1, then all the qubits marked with ⊕ are flipped, otherwise nothing happens.



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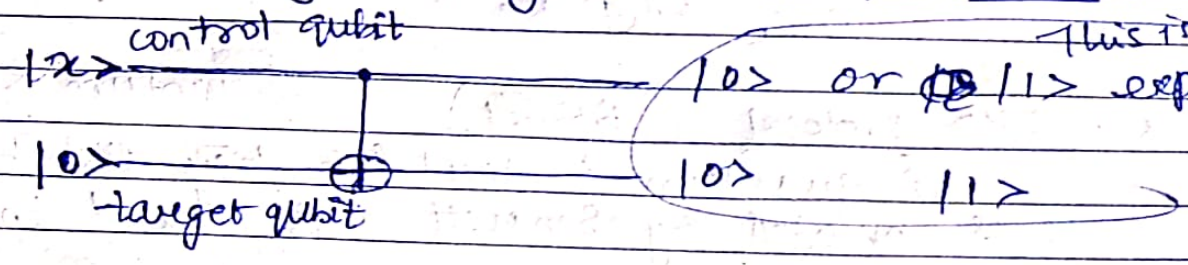
No ~~controlling~~ cloning with simple circuit.



This is for

If qubit $|x\rangle$ is known quantum ~~table~~ state either $|0\rangle$ or $|1\rangle$ then it is always possible to copy.

If qubit is an unknown quantum state is

$$\text{if } |x\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

then at the output we will get entangled state

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Thus copying is not possible here.

Till Here

→ Quantum Information Applications.

We will discuss 3 protocols.

(i) Quantum key distribution (QKD)
(ii) Superdense coding.
(iii) Quantum ~~teleportion~~ teleportation.

* The one-time pad

The one-time pad is defined by

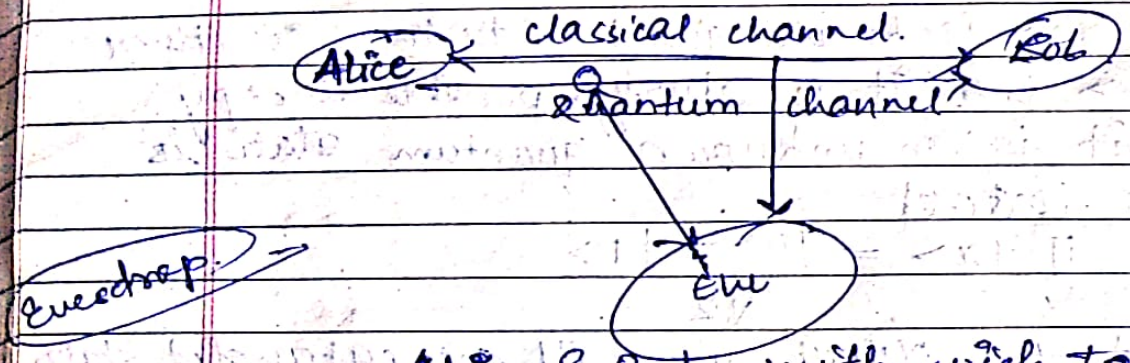
$$C_i = m_i \oplus K_i, \quad i = 1, 2, 3, \ldots$$

where m_i are plaintext digits
K_i are ~~energy~~ encryption key digits.
C_i are ciphertext digits.
$\oplus$ XOR operation.

Q204

Decryption is defined by
$m_i = C_i \oplus K_i$.

* QKD protocol (BB84) (Quantum Key Distribution)
A quantum protocol for key distribution was invented by Bennett & Brassard in 1984



Alice & Bob ~~with~~ wish to agree on common key not known to Eve.

Requirements for BB84

- Alice and Bob share a public authenticated classical channel
- Alice can ~~pubblically~~ publicly send qubits to Bob.

$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|0\rangle$	$\|1\rangle$
X	H	X	H	H	X

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(1) $\frac{3}{\sqrt{5}}|0\rangle + \frac{1}{\sqrt{5}}|1\rangle$

$$\frac{\left(\frac{3}{\sqrt{5}}\right)}{\sqrt{\left(\frac{3}{\sqrt{5}}\right)^2+\left(\frac{1}{\sqrt{5}}\right)^2}} \qquad \frac{\frac{1}{\sqrt{5}}}{\sqrt{\frac{10}{5}}}$$

$$= \frac{\frac{3}{\sqrt{5}}}{\sqrt{\frac{9}{5}+\frac{1}{5}}} \qquad = \frac{1/\sqrt{5}}{\sqrt{10}/\sqrt{5}}$$

$$= \frac{\frac{3}{\sqrt{5}}}{\frac{\sqrt{10}}{\sqrt{5}}} \qquad = \frac{1}{\sqrt{10}}$$

$$= \frac{3/\sqrt{5}}{\sqrt{10}/\sqrt{5}}$$

$$= \frac{3}{\sqrt{10}}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\left|\frac{9}{10}\right| + \left|\frac{1}{10}\right| = 1$$

$$\frac{10}{10} = 1$$

Now becomes valid state.

(3) $|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$

$$= \frac{1}{2}\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}$$

$$= \frac{1}{2}\left(\begin{pmatrix}1\\-1\end{pmatrix} \otimes \begin{pmatrix}1\\1\end{pmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\right)$$

$$= \frac{1}{2}\left((|0\rangle - |1\rangle) \otimes (|0\rangle + |1\rangle)\right)$$

FOC. ASSLW.
TO. 5.5 + 4 = 12.5 / 20.
OE. LAS.
NLP AML.

(5.) $0: \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$

$|00\rangle$ with $\frac{1}{2}$

$|01\rangle$ with $\frac{1}{2}$

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 = \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$ //

$|V\rangle = -\sqrt{\frac{2}{3}}|10\rangle + \frac{1}{\sqrt{3}}|11\rangle$

$|10\rangle = \frac{2}{3}$ $\quad |11\rangle = \frac{1}{3}$.

$\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$ //.

(6.) $V|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$

$\psi = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad V|\psi\rangle|0\rangle =$

$V\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}|0\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(V|0\rangle|0\rangle + V|1\rangle|0\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ ... (1).

B) $B|\psi\rangle|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |11\rangle)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle$

$= \frac{1}{2}\left(|00\rangle + |10\rangle\right) + |10\rangle + |11\rangle$

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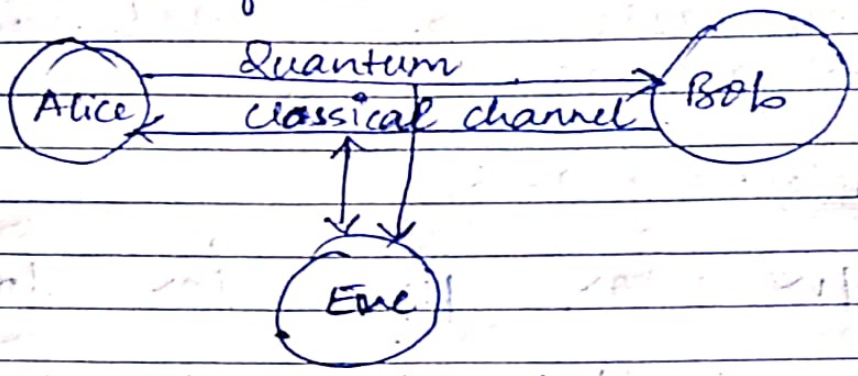
Continuation of QKD :-



Alice & Bob wish to agree on common key not known to Eve.

Assumptions :-

(1.) Alice and Bob share public authenticated classical channel.
(2.) Alice can publicly send qubits to Bob.

The Basic BB84 protocol :-

(1.) for i = 1 to A
- Alice randomly chooses a qubit |0> OR |1> and randomly applies basis H or X & sends it to Bob (over quantum channel)

(2.) Bob recieves a random sequence of qubits.

(3) For each qubit, Bob randomly applies basis H or X and measure the qubit in chosen basis.

(4) Bob announces (over the classical channel), which basis he used for each measurement.

(5) Alice tell Bob which measurements were made in the correct basis.

IBM. Internet.

(6) The qubits which were measured in the wrong basis are rejected, while rest form a shared key.

Example :- $n = 6$.

	1	2	3	4	5	6
Alice	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|0\rangle$	$\|1\rangle$
	X	H	X	X	H	H
	$\|0\rangle$	$\|+\rangle$	$\|0\rangle$	$\|1\rangle$	$\|+\rangle$	$\|->$
						public
	H	H	X	H	X	H
						public
Bob.	$\|+\rangle$	$\|0\rangle$	$\|1\rangle$	$\|-\rangle$	$\|+\rangle$	$\|1\rangle$
Alice		✓	✓			✓
						(public)

Shared key : $|0\rangle\ |1\rangle\ |1\rangle$.

X = 1 0 0
K = 0 1 1
$X_i \oplus K_i = C_i$ 1 1 0 + 0 1 1 = 1 0 0.
1 ⊕ 0 = 1
0 ⊕ 1 = 1
0 ⊕ 1 = 1
$C_i \oplus K_i$ = 1 0 0.

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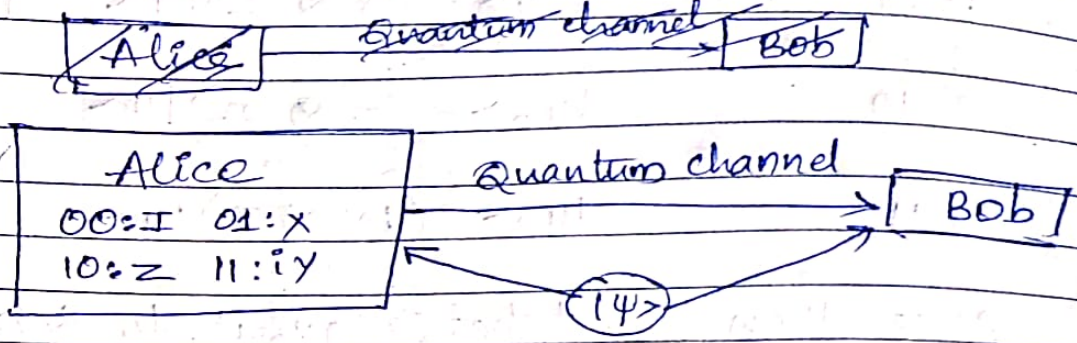
Superdense Coding:-



communication from Alice to Bob

In supeedense coding Alice wants to transmit Bob 2 classical bits by sending one qubit over the quantum channel. Suppose Alice & Bob initially share a pair of qubits in the entangled state.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right).$$

Alice Bob Alice Bob

Alice initially in possession of 1st qubit while Bob has possession of the second qubit.

Alice: Alice wants to send Bob a binary number $x \in \{00, 01, 10, 11\}$ Depending on the number, Alice performs the transformation $\{I, X, Z, iy\}$ on her qubit of the entangled state $|\psi\rangle$

The resultant new state is:

x	transformation	Resulting
0 = 00	$\|\psi_0\rangle = (I \otimes I)\|\psi\rangle$	$\|\psi_0\rangle = \frac{1}{\sqrt{2}}(\|00$
1 = 01	$\|\psi_1\rangle = (X \otimes I)\|\psi\rangle$	$\|\psi_1\rangle = \frac{1}{\sqrt{2}}(\|1$
2 = 10	$\|\psi_2\rangle = (Z \otimes I)\|\psi\rangle$	$\|\psi_2\rangle = \frac{1}{\sqrt{2}}($
3 = 11	$\|\psi_3\rangle = (iY \otimes I)\|\psi\rangle$	$\|\psi_3\rangle = \frac{1}{\sqrt{2}}($

Alice then send her qubit to Bob

Bob: To decode the information Bob applies a CNOT to 2 qubits of the entangled pair and then applies the Hadamard transformation H to the first qubit.

Received state	output of CNOT
$\|\psi_0\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle + \|10\rangle)$
$\|\psi_1\rangle$	$\frac{1}{\sqrt{2}}(\|11\rangle + \|01\rangle)$
$\|\psi_2\rangle$	$1/\sqrt{2}(\|00\rangle - \|10\rangle)$
$\|\psi_3\rangle$	$1/\sqrt{2}(-\|11\rangle + \|01\rangle)$

$|\psi_0\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle) \otimes |0\rangle$

$|\psi_1\rangle = 1/\sqrt{2}(|1\rangle + |0\rangle) \otimes |1\rangle$

$|\psi_2\rangle = 1/\sqrt{2}(|0\rangle - |1\rangle) \otimes |0\rangle$

$|\psi_3\rangle = 1/\sqrt{2}(-|1\rangle + |0\rangle) \otimes |1\rangle$

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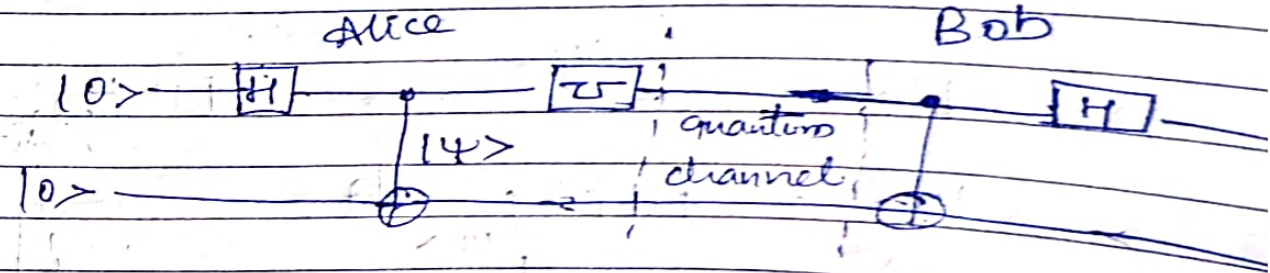
	output of H :
$\rangle + \lvert 11\rangle)$	$\lvert 00\rangle$
$0\rangle + \lvert 01\rangle)$	$\lvert 01\rangle$
$\lvert 00\rangle - \lvert 11\rangle)$	$\lvert 10\rangle$
$-\lvert 10\rangle + \lvert 01\rangle)$	$\lvert 11\rangle$

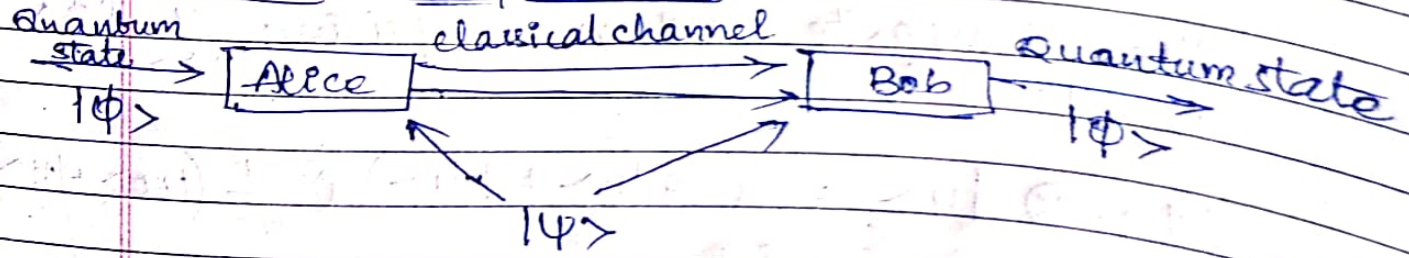
The number x is either 00 or 01 if the measurement of 1st qubit results in $\lvert 0\rangle$, while it either $\lvert 10\rangle$ or $\lvert 11\rangle$ if it is $\lvert 1\rangle$.



$U = \{I \text{ or } X \text{ or } Z \text{ or } iy\}$

Quantum circuit implementation of superdense coding.

Quantum Teleportation:



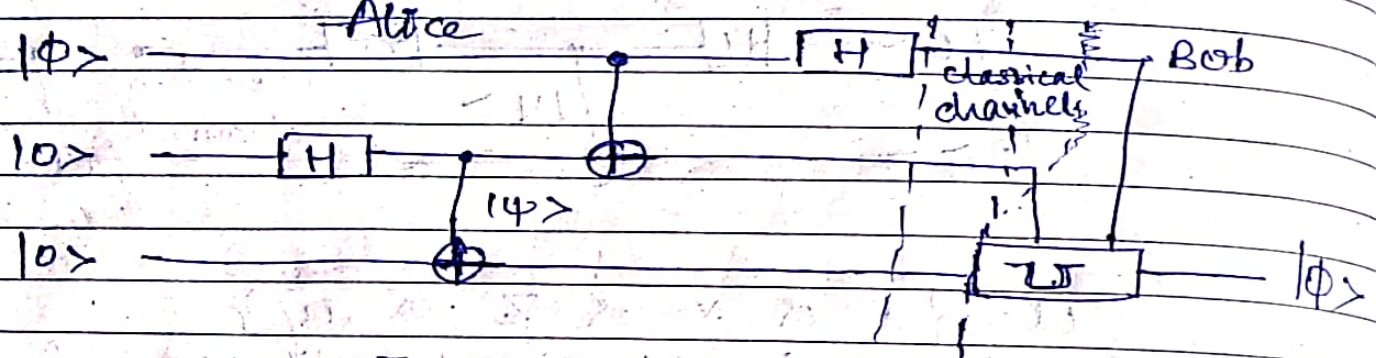
Quantum teleportation is the opposite of superdense coding in that it uses 2 classical bits to transmit single qubit quantum state $|\phi\rangle = a|0\rangle + b|1\rangle$

Alice: Alice has a qubit $|\phi\rangle = a|0\rangle + b|1\rangle$, wants to send to Bob using 2 classical bits.

Assume that Alice & Bob share one qubit of an entangled state.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

(A B) (A B)



$U = I$ or X or Z or iY

Quantum circuit Implementation Quantum teleportation.

The composite system comprising of $|\phi\rangle$ & $|\psi\rangle$ is

$$|\psi_0\rangle = |\phi\rangle \otimes |\psi\rangle = (a|0\rangle + b|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle)$$

(A A B)

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Alice controls the 1st 2 qubits & Bob controls 3rd qubit

Alice applies CNOT to $|\psi_0\rangle$

$$|\psi_1\rangle = (CNOT \otimes I)\,|\psi_0\rangle$$
$$= (CNOT \otimes I)\left(\frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle)\right)$$
$$= \frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|110\rangle + b|101\rangle)$$

Alice now applies H gate to her qubit

$$|\psi_2\rangle = (H \otimes I \otimes I)\,|\psi_1\rangle$$
$$= (H \otimes I \otimes I)\,\frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|110\rangle + b|101\rangle)$$
$$= \frac{1}{2}\left(a(|000\rangle + |100\rangle + |011\rangle + |111\rangle) + b(|010\rangle - |110\rangle + |001\rangle - |101\rangle)\right)$$

$$= \frac{1}{2}\left(|00\rangle(a|0\rangle + b|1\rangle) + |01\rangle(a|1\rangle + b|0\rangle) + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|1\rangle - b|0\rangle)\right)$$

Bob: when Bob receives 2 classical bits from Alice, he uses them to select a transformation for her qubit.

bits	state	Decoding
00	$a\|0\rangle + b\|1\rangle$	I
01	$a\|1\rangle + b\|0\rangle$	X
10	$a\|0\rangle - b\|1\rangle$	Z
11	$a\|1\rangle - b\|0\rangle$	iy

$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$iy = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle$

$|\phi\rangle = a|0\rangle + b|1\rangle$

$|\psi_0\rangle = \frac{1}{2}\left[|00\rangle(a|0\rangle - \right.$

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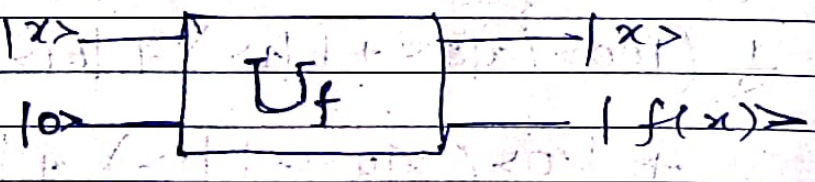
Quantum Parallelism

Given an input x, a typical quantum computer computes in such a way as

$$U_f : |x\rangle|0\rangle \longrightarrow |x\rangle|f(x)\rangle$$



Suppose U_f acts on the input which is a superposition of many states. Then o/p is also a superposition of all the results.

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$$U_f : \sum_x |x\rangle |0\rangle \longrightarrow \sum_x |x\rangle |f(x)\rangle$$

when the input is a superposition of n states, U_f computes n values $f(x_k)$, $1 \le k \le n$ simultaneously. This feature is called Quantum parallelism.

$$|0\rangle^n \xrightarrow{H^n}$$

$$|0000\ldots0\rangle \xrightarrow{H^n}$$

$$|0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|0\rangle \otimes |0\rangle \xrightarrow{H^2} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$\underbrace{|0\rangle \otimes |0\rangle \otimes |0\rangle \ldots |0\rangle}_{n\text{-qubit}} \xrightarrow{H^n} \frac{1}{\sqrt{2^n}}(|000\ldots0\rangle + |00\ldots1\rangle + \ldots + |11\ldots1\rangle)$$

$$= \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle$$

Simple Quantum Algorithms

1.) Deutsch Algorithm.

Let $f : \{0,1\} \longrightarrow \{0,1\}$ be a binary function. Note that there are

four ~~parallel~~ possible values of f namely

$f_1: 0 \to 0, 1 \to 0$, $f_2: 0 \to 1, 1 \to 1$,
$f_3: 0 \to 0, 1 \to 1$, $f_4: 0 \to 1, 1 \to 0$.

The first 2 cases, f_1 and f_2 are called constant while the rest f_3 and f_4 are balanced.

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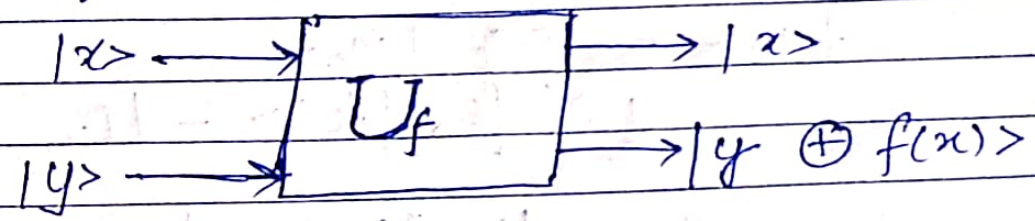
ORACLE

Oracle is basically a black-box computation of a function.



Black box

$U_f : |x, y\rangle \longrightarrow |x, y \oplus f(x)\rangle$

If we set $y = 0$, output is $f(x)$

If we set $y = 1$, output is $\overline{f(x)}$.

This is nothing but CNOT-gate with control bit $f(x)$, the target bit, y is flipped if and only if $f(x) = 1$, otherwise no change.

Deutsch Algorithm

Input : one qubit $|0\rangle$ or $|1\rangle$

Output : function is constant or balanced.

(i) Constant : $f(0) = f(1) = \begin{cases} 0 \\ 1 \end{cases}$

(ii) Balanced : $f(0) \neq f(1)$

ie $f(0) = 0$, $f(1) = 1$

or $f(0) = 1$, $f(1) = 0$

$|0\rangle$ & $|1\rangle$ correspond to classical bits

$$|\psi_0\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

Apply oracle U_f to $|\psi_0\rangle$

$$|\psi_1\rangle = U_f |\psi_0\rangle$$
$$= \frac{1}{2}(|0, f(0)\rangle - |0, 1 \oplus f(0)\rangle + |1, f(1)\rangle - |1, 1 \oplus f(1)\rangle)$$
$$= \frac{1}{2}(|0, f(0)\rangle - |0, \overline{f(0)}\rangle + |1, f(1)\rangle - |1, \overline{f(1)}\rangle)$$

Apply Hadamard gate on the first qubit

$$|\psi_2\rangle = (H \otimes I)(|\psi_1\rangle)$$
$$= \frac{1}{2\sqrt{2}}[(|0\rangle + |1\rangle)(|f(0)\rangle - |\overline{f(0)}\rangle) + (|0\rangle - |1\rangle)(|f(1)\rangle - |\overline{f(1)}\rangle)]$$

If f is constant, for which $f(0) = f(1)$ then

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle(|f(0)\rangle - |\overline{f(0)}\rangle)$$

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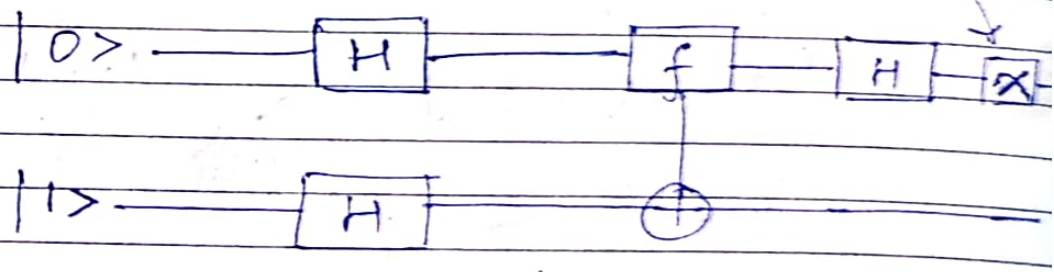
function is constant if measurement of 1st qubit is $|0\rangle$.

If f is Balanced for which $|f(\bar{0})\rangle = |f(1)\rangle$.

$$|\psi_2\rangle = \frac{1}{2\sqrt{2}}\Big[(|0\rangle + |1\rangle)(|f(0)\rangle - |f(1)\rangle) + (|0\rangle - |1\rangle)(|f(1)\rangle - |f(0)\rangle)\Big]$$

$$= \frac{1}{\sqrt{2}}|1\rangle(|f(0)\rangle - |f(\bar{0})\rangle)$$

$\Rightarrow$ function is balanced if measurement of 1st qubit is $|1\rangle$



Circuit implementation.

The Quantum fourier Transform (QFT)

Definition:- Given a general qubit state

$$|\psi\rangle = \sum_{j=0}^{N-1} a_j |j\rangle = \begin{pmatrix} a_0 \\ \vdots \\ a_{N-1} \end{pmatrix}$$

we define QFT of this state as

$$F|\psi\rangle \sum_{k=0}^{N-1} b_k |k\rangle$$

where $b_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} a_j e^{\frac{2\pi i j k}{N}}$, $i = \sqrt{-1}$

Example of a QFT for 2-qubit

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + \{a_{10}|10\rangle + a_{11}|11\rangle$$

which has $N = 4$

we have

$$b_0 = \frac{1}{2}\sum_{j=0}^{3} a_j = \frac{1}{2}(a_{00} + a_{01} + a_{10} + a_{11})$$

$$b_1 = \frac{1}{2}\sum_{j=0}^{3} a_j e^{\frac{2\pi i j}{N}}$$

$$= \frac{1}{2}\left(a_{00} + a_{01}^{\frac{\pi i}{2}} + a_{10}^{\pi i} + a_{11}^{\frac{3\pi i}{2}}\right.$$

$$b_2 = \frac{1}{2}\sum_{j=0}^{3} a_j e^{\pi i j} = \frac{1}{2}\left(a_{00} + a_{01} e^{i\pi} + a_{10}\right.$$

$$\left. a_n e^{3\pi i}\right\}$$

$$b_3 = \frac{1}{2}\sum_{j=0}^{3} a_j e^{\frac{2\pi i 3 j}{2^2}} = \frac{1}{2}\left(a_{00} + a_{01} e^{\frac{3\pi i}{2}} + a_{10} e^{3\pi i} + \cdots\right)$$

$$F = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & 1 & \omega^2 \\ 1 & \omega^3 & \omega^2 & \omega \end{pmatrix}$$

$$FF^{-1} = I$$

F is unitary.

$$F = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

(1.) find QFT for $N=4$ of the function.

$$|f\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$$F|f\rangle = \frac{1}{4}\underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & -i \end{pmatrix}}_{4\times 4}\underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}_{4\times 1}$$

$$= \frac{1}{4}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_{4\times 1}$$

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Similarly if $|f\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

then

$$F|f\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

(2.) Find QFT for $M=4$ of the following $|g\rangle = |00\rangle$

$$F|g\rangle = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

(3.) Find QFT for $M=4$ of the formation $|h\rangle = |01\rangle$

$$F|h\rangle = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix}$$

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The inverse QFT.

We have the matrix for 2-qubit QFT,

$$F = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

Since F is unitary, then $F^{\dagger}F = I$

$$\left(F^{\dagger} = (\overline{F})^{T}\right)$$

$N = 4 \Rightarrow F^{-1} = F^{\dagger}$

$$F^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}$$

(1) Find Inverse QFT, for $N = 4$, of the function $|f\rangle = |00\rangle$

$$\Rightarrow F^{-1}|00\rangle = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

Santosh

* Quantum Error Correcting Code:

Suppose we transmit a series of ~~0's~~ 0's & 1's through a noisy classical channel. Each bit is assumed to flip independently with a probability p. How does classical error correction work?

The simplest example of classical error ~~correction~~ correcting code in a repetition code.

We replace the bit we wish to protect by 3 copies of the bit.

$$0 \longrightarrow (000)$$
$$1 \longrightarrow (111)$$

For example, when 000 is sent through this channel,

It will be recieved as 000 with probability $(1-p)^3$

It will be recieved as 100, 010, or 001 with probability $3p(1-p)^2$

It will be recieved as 011, 101 or 110 with probability: $3p^2(1-p)$

It will be recieved as 111 with probability p^3.

Note that the summation of all the probabilities is 1 as it should be

By taking the majority vote, we correctly reproduces the desired result 0 with probab

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$P_0 \to 0.1$
$P_0 \to P_1$

$$P_0 = (1-P)^3 + 3P(1-P)^2$$
$$= (1-P)^2 (1+2P) //$$

The probability of fail is
$$P_1 = 3P^2(1-P) + P^3 = (3-2P)P^2 //$$

Bit-Flip QECC

Suppose Alice wants to send a qubit $|\psi\rangle = a|0\rangle + b|1\rangle$ to Bob through a noisy quantum channel.

Let p be the probability with which a qubit is flipped ($|0\rangle \leftrightarrow |1\rangle$) and we assume there are no other types of errors in the channel.

$$|\psi\rangle \longrightarrow |\psi'\rangle = X|\psi\rangle = a|1\rangle + b|0\rangle.$$