Øving 3

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$\boxed{1}$

$$
\begin{aligned}
&\neg\forall x(P(x) \Rightarrow \exists y Q(x,y)) \\
&\neg\forall x(\exists y P(x) \Rightarrow Q(x,y)) \\
&\exists x \forall y \neg P(x) \Rightarrow Q(x,y) \\
&\exists x \forall y P(x) \vee Q(x,y)
\end{aligned}
$$

$\boxed{2}$ $\mathcal{P}(A) = \{\emptyset, \{\{a\}\}, \{\{b,c\}\}, \{\{c,d,\{e,f\}\}\}, \{\{a\},\{b,c\}\}, \{\{a\},\{c,d,\{e,f\}\}\},$
$\{\{b,c\},\{c,d,\{e,f\}\}\}, \{\{a\},\{b,c\},\{c,d,\{e,f\}\}\}\}$

$\boxed{3}$ $A = \{1,2,3,4,5\}, B = \{3,4,5,6,7\}$

(a) $A \cap B = \{3,4,5\}$

(b) $A \triangle B = \{1,2,6,7\}$
$C := \{1,2\}, \quad D := \{6,7\}$
$C \cup D = A \triangle B$

$\boxed{4}$ Prove: $\overline{X - Y} = \overline{X} \cup Y$

$$X - Y := \{x : x \in X \wedge x \notin Y\} = X \cap \overline{Y}$$

$$
\begin{aligned}
\overline{X - Y} &= \overline{X \cap \overline{Y}} \\
&= \overline{X} \cup \overline{\overline{Y}} \quad \textit{(De morgan)} \\
&= \overline{X} \cup Y \quad \square
\end{aligned}
$$

$\boxed{5}$ (a) $(\overline{X} \cup Y) \cap (X \cup Y)$, logisk riktig når $X = \emptyset$

(b) $(\overline{X} \cap Y) \cup (X \cap Y)$, logisk riktig når $X = \emptyset$

$\boxed{6}$

$$\overline{Y} - X = \overline{Y} \equiv X \subseteq Y \equiv X \cap Y = X$$
$$\Updownarrow$$
$$X = \emptyset$$

$\boxed{7}$

$$|\mathcal{P}(A) - \{\{x\} : x \in A\}|, \quad \{\{x\} : x \in A\} \in \mathcal{P}(A)$$

$$|\mathcal{P}(A)| = 2^n$$
$$|\mathcal{P}(A) - \{\{x\} : x \in A\}| = 2^n - 1$$

$\boxed{8}$ (a)

$$|\mathcal{P}(A \times B)| = 2^{m \cdot n}, \quad |A| = m, |B| = n$$
$$|\mathcal{P}(A)| \cdot |\mathcal{P}(B)| = 2^m \cdot 2^n = 2^{m+n}$$

For $m, n \in \{0, 2\}$ er $|\mathcal{P}(A \times B)| = |\mathcal{P}(A)| \cdot |\mathcal{P}(B)|$

For $m, n = 1$ er $|\mathcal{P}(A \times B)| < |\mathcal{P}(A)| \cdot |\mathcal{P}(B)|$

For $m, n < 0$ og $m, n > 2$ er $|\mathcal{P}(A \times B)| > |\mathcal{P}(A)| \cdot |\mathcal{P}(B)|$

(b) Prove: $(A - B) \cap (B - A) = \emptyset$

$$\begin{aligned}(A - B) \cap (B - A) &= (A \cap \overline{B}) \cap (\overline{A} \cap B)\\ &= A \cap \overline{B} \cap \overline{A} \cap B\\ &= A \cap \overline{A} \cap B \cap \overline{B}\\ &= \emptyset \cap \emptyset\\ &= \emptyset \quad \square\end{aligned}$$