Øving 2

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Ønsker retting

1 $L(x, y)$: *x loves y*

(a) $\exists x \forall y L(x, y)$

(b) $\exists x \forall y \neg L(x, y)$

(c) $\neg \exists x \forall y L(x, y)$

2 (a) For all x there exists a y such that x is smaller then y.

(b) There exists an x for all y such that x is equal to y.

(c) For all x and all y and all z x is smaller then y and y is smaller then z implies that x is smaller then z.

(d) For all x and all y and all z z is smaller then or equal to y and y is smaller then or equal to z and x is equal to z implies that x is equal to y.

Det er ingen logisk forskjell på (a) og (b) og (a) og (c) i oppgave 3 i øving 1.

3 Evaluerer utsagnene

(a)
$$\begin{aligned} &\neg(\forall x \exists y F(x, y) \Rightarrow F(y, x)) \\ &\neg(\neg \forall x \exists y F(x, y) \vee F(y, x)) && \textit{(Material Implication)} \\ &\forall x \exists y F(x, y) \wedge \neg F(y, x) && \textit{(De Morgan \& Double Negation)} \end{aligned}$$

(d)
$$\exists y \forall a \neg F(x, y) \wedge F(y, x)$$

$$\forall x \exists y F(x, y) \wedge \neg F(y, x) \equiv \exists y \forall a \neg F(x, y) \wedge F(y, x) \blacksquare$$

4 $F(x)$: *x is red or green,* x : *is an apple,* y : *is a fruit,* $x \subseteq y$

(a) $\forall x F(x)$

(b) $\forall y F(y) \vee \neg x$

(c) $\neg \exists x \neg F(x)$

5 $\exists x \exists y \exists z (P(x, y) \wedge P(z, y) \wedge (x, z) \wedge \neg P(x, x))$

(a) True, $x \leqslant z \leqslant y$

(b) False

(c) True

6 (a) $\forall x \exists y \exists z P(x, y, z) \wedge \forall u, v (P(u, v) \Leftrightarrow \neg P(v, v, u))$
For a model to be true for the statement, all three terms must be true.

(b) $\forall x \exists y \forall z ((P(x, y) \Leftrightarrow P(y, z) \wedge P(x, y) \Rightarrow \neg P(y, x)))$
For a model to be true for the statement, $P(x, y)$ and $P(y, z)$ must be true, and $P(y, x)$ must be false.