Øving 1

Håvard Solberg Nybøe

MA0301 – 24. januar 2022

Ønsker retting

Exsercise 1

(a)

p	q	r	$q \vee r$	$p \Rightarrow (q \vee r)$
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

(b)

p	q	r	$p \Rightarrow q$	$r \Rightarrow (p \Rightarrow q)$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

(c)

p	q	r	$p \oplus r$	$q \Rightarrow \neg r$	$(q \Rightarrow \neg r) \vee (p \oplus r)$	$p \Rightarrow (q \Rightarrow \neg r) \vee (p \oplus r)$
0	0	0	0	1	1	1
0	0	1	1	1	1	1
0	1	0	0	1	1	1
0	1	1	1	0	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	0	0	0

Exsercise 2

(a) $(p \lor q) \lor (p \Rightarrow q)$
Hvis p og/eller q er sann, er $p \lor q$ sann. Hvis begge er usanne er $p \Rightarrow q$ sann.
Utsagnet er en tautologi. ■

(b) $(p \Rightarrow (q \land \neg q)) \land p$
Anta at p er sann. Skal utsagnet være sant, må $p \Rightarrow (q \land \neg q)$ være sant.
$(q \land \neg q) \equiv \mathrm{F}$, $p \Rightarrow \mathrm{F} \equiv \mathrm{F}$
Utsagnet er en kontradiksjon. ■

(c) $(p \Rightarrow (q \land \neg q)) \land p \Rightarrow r$
Gitt resultatet i forrige oppgave er $(p \Rightarrow (q \land \neg q)) \land p$ usant.
Utsagnet er en tautologi. ■

Exsercise 3

(a) $p \land q \Rightarrow r$
a is smaller than b and b is smaller than c implies that a is smaller than c.
Logisk riktig antagelse.

(b) $p \land q \Rightarrow u$
a is smaller than b and b is smaller than c implies that a is equal to c.
Logisk feil antagelse da a må være mindre enn c.

(c) $(p \lor s) \land (q \lor t) \land u \Rightarrow s$
a smaller or equal to b and b smaller or equal to c and a equal to c implies that a is equal to b.
Logisk riktig da alle vil ha samme verdi.

Exsercise 4

Gitt at q er sann (T), og at

$$(q \Rightarrow ((p \lor \neg r) \land s)) \land (s \Rightarrow (r \land q))$$

er en tautologi.
Tester med r satt til sann.

$$\begin{gathered}(q \Rightarrow ((p \lor \neg r) \land s)) \land (s \Rightarrow (r \land q)) \\ (q \Rightarrow ((p \lor \mathrm{F}) \land s)) \land (s \Rightarrow \mathrm{T}), \quad s \equiv \mathrm{T}, p \equiv \mathrm{T} \\ (q \Rightarrow (\mathrm{T} \land \mathrm{T})) \land \mathrm{T}\end{gathered}$$

p, r og s er sann. ■

Exsercise 5

(a) Negate: $(p \wedge q) \Rightarrow (\neg r \vee \neg s)$

$$\begin{aligned}
&\neg((p \wedge q) \Rightarrow (\neg r \vee \neg s)) && \\
&\neg(\neg(p \wedge q) \vee (\neg r \vee \neg s)) && \textit{(Material implication)} \\
&\neg(\neg(p \wedge q)) \wedge \neg(\neg r \vee \neg s) && \textit{(De Morgan)} \\
&(p \wedge q) \wedge (r \wedge s) && \textit{(De Morgan \& Double Negation)} \\
&p \wedge q \wedge r \wedge s && \textit{(Associativity)}
\end{aligned}$$

(b) Negate: $p \Rightarrow (r \oplus s)$

$$\begin{aligned}
&\neg(p \Rightarrow (r \oplus s)) && \\
&\neg(\neg p \vee (r \oplus s)) && \textit{(Material implication)} \\
&p \wedge \neg(r \oplus s) && \textit{(De Morgan \& Double Negation)} \\
&p \wedge \neg((r \vee s) \wedge (\neg r \vee \neg s)) && \textit{(Replacement)} \\
&p \wedge \neg(r \vee s) \vee \neg(\neg r \vee \neg s) && \textit{(De Morgan)} \\
&p \wedge (\neg r \wedge \neg s) \vee (r \wedge s) && \textit{(De Morgan)}
\end{aligned}$$

Exsercise 6

AL1 (V.S.)

α	β	γ	$(\alpha \vee \beta)$	$(\alpha \vee \beta) \vee \gamma$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

AL1 (H.S.)

α	β	γ	$(\beta \vee \gamma)$	$\alpha \vee (\beta \vee \gamma)$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

$$(\alpha \vee \beta) \vee \gamma \equiv \alpha \vee (\beta \vee \gamma) \blacksquare$$

AL2 (V.S.)

α	β	γ	$(\alpha \wedge \beta)$	$(\alpha \wedge \beta) \wedge \gamma$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

AL2 (H.S.)

α	β	γ	$(\beta \wedge \gamma)$	$\alpha \wedge (\beta \wedge \gamma)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$(\alpha \wedge \beta) \wedge \gamma \equiv \alpha \wedge (\beta \wedge \gamma) \blacksquare$$

Exsercise 7

(a)

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

Utsagnet $p \Rightarrow (p \vee q)$ er en tautologi. $\blacksquare$

(b)

p	q	$p \vee q$	$\neg(p \Rightarrow (p \vee q))$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	0

Utsagnet $p \Rightarrow (p \vee q)$ er tilfredstillbar. $\blacksquare$

(c)

p	q	$p \Rightarrow q$	$p \Rightarrow (p \Rightarrow q)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

Utsagnet $p \Rightarrow (p \Rightarrow q)$ er tilfredstillbar. ■

Exsercise 8

1. $\neg p \Rightarrow (q \Leftrightarrow r)$
2. $r \Rightarrow \neg p$
3. $\neg r \oplus (p \wedge q)$
4. $p \Rightarrow (r \wedge q)$
5. $\neg q \oplus r$