En lat students guide til MA0001

av Håvard Solberg Nybøe

Sist oppdatert - 24. november 2021

Innhold

Pensum

Under er en oversikt over pensum for MA0001 høsten 2021.

Tema	Kapittel
Introduction	Chapters 1.1.1 - 1.1.4
Exponentials & Logarithms	Chapter 1.1.5
Functions	Chapter 1.2
Inverse Functions	Chapter 1.2.6
Limits	Chapters 3.1, 3.3
Continuity	Chapter 3.2
Differentiation	Chapters 4.1, 4.2, 4.3
Taylor Polynommials	Chapter 7.6 (p.373-379)
Implicit Differentiation	Chapter 4.4.2
L'Hospital's rule	Chapters 5.5
Extrema, Monotonicity & Graphs	Chapters 5.1, 5.2, 5.3
Derivative of the Inverse Function	Chapter 4.7.1
Integration & The Fundamental Theorem of Calculus	Chapters 5.8, 6.1, 6.2
Leibniz's Rule	Chapter 6.2
Integration with Substitution	Chapter 7.1
Integration by Parts	Chapter 7.2
Areas & Improper Integrals	Chapters 6.3.1, 7.4

Disclaimer

Innholdet i dette dokumentet er hentet fra læreboka til MA0001. Dokumentet er kun laget for eget bruk.

Dokumentet ble originalt laget til mitt eget bruk som repetisjon til eksamen høsten 2021, men kan godt benyttes av andre late studenter som ikke gidder å lese til eksamen. Jeg eier så og si ingen ting av innholdet i dokumentet da alt er ~~stjålet~~ hentet fra læreboka, emnesiden eller Wikipedia.

Introduction

This section reviews some of the concepts and techniques from algebra and trigonometry that are frequently used in calculus. The problems at the end of the section will help you reacquaint yourself with this material.

The Real Numbers

The **real numbers** can most easily be visualized on the **real-number line**, on which numbers are ordered so that if $a < b$, then a is to the left of b. Sets (collections) of real numbers are typically denoted by the capital letters A, B, C, etc. To describe the set A , we write

$$A = \{x : condition\}$$

where "condition" tells us which numbers are in the set A. The most important sets in calculus are **intervals**.

The location of the number 0 on the real-number line is called the **origin**, and we can measure the distance of the number x to the origin. For instance, -5 is 5 units to the left of the origin. A convenient notation for measuring distances from the origin on the real-number line is the absolute value of a real number.

The absolute value of a real number a , denoted by $|a|$, is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Lines in the Plane

A **line** in the plane is a straight line that passes through two points. A line is represented by the equation

$$y = mx + b$$

where m is the slope of the line and b is the y-intercept of the line.

Equation of the Circle

A **circle** is the set of all points at a given distance, called the **radius**, from a given point, called the **center**. If r is the distance from (x_0, y_0) to (x, y), then, using the Pythagorean theorem, we find that

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

If $r = 1$ and $(x_0, y_0) = (0, 0)$, the circle is called the **unit circle**.

Trigonometry

To convert between degree and radian measure, we use the formula

$$\text{rad}(a) = \frac{a}{180}\pi$$
$$\text{deg}(a) = \frac{a}{\pi}180$$

where a is the angle measured in degrees or radians.

There are four trigonometric functions that you should be familiar with: sine, cosine, tangent, and secant; the other two, cotangent and cosecant, are rarely used. The six are defined on a unit circle and are abbreviated as sin, cos, tan, sec, cot, and csc, respectively. Recall that a positive angle is measured counter-clockwise from the positive x -axis, whereas a negative angle is measured clockwise. The six trigonometric functions are defined as follows:

$$\sin(a) = \frac{y}{1} \qquad \csc(a) = \frac{1}{\sin(a)}$$
$$\cos(a) = \frac{x}{1} \qquad \sec(a) = \frac{1}{\cos(a)}$$
$$\tan(a) = \frac{y}{x} \qquad \cot(a) = \frac{1}{\tan(a)}$$

There are a number of frequently used trigonometric identities. First, since $\tan(\theta) = \frac{y}{x}$ with $y = \sin(\theta)$ and $x = \cos(\theta)$, it follows that

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

Exponentials & Logarithms

An **exponential** is an expression of the form

$$a^r$$

where a is called the **base** and r is called the **exponent**.

Unless r is an integer or unless r is a rational number of the form p/q where p is an integer and q is an odd integer, we will assume that a is positive. We summarize some of the properties of an exponential as follows:

$$a^r \cdot a^5 = a^{r+5} \qquad (ab)^r = a^r \cdot b^r$$
$$\frac{a^r}{a^5} = a^{r-5} \qquad \left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$
$$a^{-r} = \frac{1}{a^r} \qquad (a^{-r})^5 = a^{r \cdot 5}$$

A **logarithm** is the inverse function to the exponential and is expressed in the form

$$\log_b(a)$$

where b is called the **base** and a is called the **argument**.

The logarithm of a **product** is the sum of the logarithms of the numbers being multiplied; the logarithm of the **ratio of two numbers** is the difference of the logarithms. The logarithm of the **p-th power** of a number is p times the logarithm of the number itself; the logarithm of a **p-th root** is the logarithm of the number divided by p. The following table lists these identities with examples

$$\log_b(xy) = \log_b(x) + \log_b(y)$$
$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$
$$\log_b(x^p) = p\log_b(x)$$
$$\log_b\left(\sqrt[p]{x}\right) = \frac{\log_b(x)}{p}$$

Functions

hei

What Is a Function?

Polynomial Functions

Rational Functions

Power Functions

Exponential Functions

Inverse Functions

Logarithmic Functions

Trigonometric Functions

Limits & Continuity

Limits

What is a Limit?

Limit Laws

Continuity

What Is Continuity?

Combinations of Continuous Functions

Limits at Infinity

Differentiation

Taylor Polynommials

Implicit Differentiation

L'Hospital's rule

Extrema, Monotonicity & Graphs

Derivative of the Inverse Function

Integration & The Fundamental Theorem of Calculus

Leibniz's Rule

Integration with Substitution

Integration by Parts

Areas & Improper Integrals