

MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2022

SET 6

Deadline: Monday 28.02.2022, 23:59

Exercise 1. *Lewis, Zax: Exercise 6.3a.*

Exercise 2. *Lewis, Zax: Exercise 6.7.*

Exercise 3. *Use induction to show that: if n is a positive integer, then $\sum_{m=1}^n m = 1+2+3+\dots+n = \frac{n(n+1)}{2}$.*

Exercise 4. *Lewis, Zax: Exercise 3.7.*

Exercise 5. a) *Find a formula for $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)}$ by examining the values of this expression for small values of n .*

b) *Use induction to prove the formula you conjectured in part a.*

Exercise 6. *What is wrong with this "proof"?*

"Theorem" For every positive integer n , if x and y are positive integers with $\max(x, y) = n$, then $x = y$.

Basis Step: Suppose that $n = 1$. If $\max(x, y) = 1$ and x and y are both positive integers we have $x = y$.

Inductive Step: Let k be a positive integer. Assume that whenever $\max(x, y) = k$ and x and y are positive integers, then $x = y$. Now let $\max(x, y) = k + 1$, where x and y are positive integers. Then $\max(x - 1, y - 1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step.

Exercise 7. *Use induction to prove that $3^n < n!$ if n is an integer greater than 6.*

Exercise 8. *Use induction to prove that 6 divides $n^3 - n$ whenever n is a nonnegative integer.*