MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2022

SET 2

Deadline: Deadline: Monday 31.01.2022, 11:59 pm

Exercise 1. *Lewis, Zax: Exercise 12.1*

Exercise 2. *Consider the following predicates in the universe of real numbers:*

(1) F(x,y) : x is smaller then y
(2) G(x,y) : x is equal to y

Translate the following statements into an English sentence and then comment on whether they should be true or false.

$a)\forall x \exists y F(x,y)$

$b)\exists x \forall y G(x,y)$

$c)\forall x \forall y \forall z F(x,y) \wedge F(y,z) \Rightarrow F(x,z)$

$d)\forall x \forall y \forall z [F(x,y) \vee G(x,y)] \wedge [F(y,z) \vee G(y,z)] \wedge G(x,z) \Rightarrow G(x,y)$

Compare the statements in a) and b) to statements a) and c) of exercise 3 in Set 1. What is the difference?

Exercise 3. *Which of the following four quantificational formulas are logically equivalent? Justify your answer.*

$a)\neg[\forall x \exists y F(x,y) \Rightarrow F(y,x)]$

$b)\exists x \forall y \neg F(x,y) \wedge F(y,x)$

$c)\exists x \forall y F(x,y) \wedge \neg F(y,x)$

$c)\exists y \forall x \neg F(x,y) \wedge F(y,x)$

Exercise 4. *Translate the following English sentences into predicate logic by defining predicates in an appropriate universe and forming statements with them:*

a) All apples are either red or green.
b) All fruits are red or green or not apples
c) It does not exist an apple that is neither red nor green

Exercise 5. *Lewis, Zax: Exercise 12.3 a),b),c)*

Exercise 6. *Lewis, Zax: Exercise 12.6*
Hint: There might not exist a model (the statement is not satisfiable). In this case explain why.

Date: January 24, 2022.