1. a) x er minst like gammel som y

b) x er venner med y

c) x og y er (hel)søsken

---

2. a)

1		$\forall x(P(x) \wedge \neg P(x))$	AS
2		$P(a) \wedge \neg P(a)$	$\forall$E 1
3		$P(a)$	$\wedge$E 2
4		$\neg P(a)$	$\wedge$E 2
5		$\bot$	$\neg$E 3,4
6		$\neg\forall x(P(x) \wedge \neg P(x))$	$\neg$I 1-5

b)

1		$\forall x(R(x,b) \to \neg R(x,b)) \wedge \exists x R(x,b)$	AS
2		$\forall x(R(x,b) \to \neg R(x,b))$	$\wedge$E 1
3		$\exists x R(x,b)$	$\wedge$E 1
4		$R(a,b)$	AS
5		$R(a,b) \to \neg R(a,b)$	$\forall$E 2
6		$\neg R(a,b)$	$\to$E 4,5
7		$\bot$	$\neg$E 4,6
8		$\bot$	$\exists$E 3, 4-7
9		$\neg(\forall x(R(x,b) \to \neg R(x,b)) \wedge \exists x R(x,b))$	$\neg$I 1-8

c)

1		$\forall x(F(x) \to G(x)) \wedge \exists x(\neg G(x) \wedge \forall y F(y))$	AS
2		$\forall x(F(x) \to G(x))$	$\wedge$E 1
3		$\exists x(\neg G(x) \wedge \forall y F(y))$	$\wedge$E 1
4		$\neg G(a) \wedge \forall y F(y)$	AS
5		$\neg G(a)$	$\wedge$E 4
6		$\forall y F(y)$	$\wedge$E 4
7		$F(a) \to G(a)$	$\forall$E 2
8		$F(a)$	$\forall$E 6
9		$G(a)$	$\to$E 7,8
10		$\bot$	$\neg$E 5,8
11		$\bot$	$\exists$E 3, 4-10
12		$\neg(\forall x(F(x) \to G(x)) \wedge \exists x(\neg G(x) \wedge \forall y F(y)))$	$\neg$I 1-11

2. d)

1	$\exists x \forall y \exists z \forall w (R(x,z) \wedge \neg R(y,w))$	AS
2	$\forall y \exists z \forall w (R(a,z) \wedge \neg R(y,w))$	AS
3	$\exists z \forall w (R(a,z) \wedge \neg R(a,w))$	$\forall E$ 2
4	$\forall w (R(a,b) \wedge \neg R(a,w))$	AS
5	$R(a,b) \wedge \neg R(a,b)$	$\forall E$ 4
6	$R(a,b)$	$\wedge E$ 5
7	$\neg R(a,b)$	$\wedge E$ 5
8	$\bot$	$\neg E$ 6,7
9	$\bot$	$\exists E$ 3,4-8
10	$\bot$	$\exists E$ 1,2-9
11	$\neg(\exists x \forall y \exists z \forall w (R(x,z) \wedge \neg R(y,w)))$	$\neg I$ 1-10

3. a)

1	$a = a$	$=I$
2	$\exists x (x = x)$	$\exists I$ 1

b)

1	$P(a)$	AS
2	$P(a)$	R 1
3	$P(a) \rightarrow P(a)$	$\rightarrow I$ 1-2
4	$\forall x (P(x) \rightarrow P(x))$	$\forall I$ 3

c)

1	$\exists x (P(x) \wedge \neg P(x))$	AS
2	$P(a) \wedge \neg P(a)$	AS
3	$P(a)$	$\wedge E$ 2
4	$\neg P(a)$	$\wedge E$ 2
5	$\bot$	$\neg E$ 3,4
6	$\bot$	$\exists E$ 1,2-5
7	$\neg \exists x (P(x) \wedge \neg P(x))$	$\neg I$ 1-6

d)

1	$\forall x \forall y R(x,y)$	AS
2	$\forall y R(a,y)$	$\forall E$ 1
3	$R(a,a)$	$\forall E$ 2
4	$\forall x (R(x,x))$	$\forall I$ 3
5	$\forall x \forall y R(x,y) \rightarrow \forall x R(x,x)$	$\rightarrow I$ 1-4

c)

1	$\neg(\exists x P(x) \vee \forall x \neg P(x))$	AS
2	$\neg \exists x P(x) \wedge \neg \forall x \neg P(x)$	DeM 1
3	$\neg \exists x P(x)$	$\wedge E$ 2
4	$\neg \forall x \neg P(x)$	$\wedge E$ 2
5	$\forall x \neg P(x)$	CQ 3
6	$\bot$	$\neg E$ 4,5
7	$\exists x P(x) \vee \forall x \neg P(x)$	IP 1-6

d)

1	$\exists x R(x,x)$	AS
2	$R(a,a)$	AS
3	$\exists y R(a,y)$	$\exists I$ 2
4	$\exists x \exists y R(x,y)$	$\exists I$ 3
5	$\exists x \exists y R(x,y)$	$\exists E$ 1,2-4
6	$\exists x R(x,x) \rightarrow \exists x \exists y R(x,y)$	$\rightarrow I$ 1-5

3 g)

1	$\exists x\, P(x)$	AS
2	$P(a)$	AS
3	$P(b)$	AS
4	$P(a)$	R 2
5	$P(b) \to P(a)$	$\to$I 3-4
6	$\forall y\,(P(y) \to P(a))$	$\forall$I 5
7	$\exists x \forall y\,(P(y) \to P(x))$	$\exists$I 6
8	$\exists x \forall y\,(P(y) \to P(x))$	$\exists$E 1,2-7
9	$\neg \exists x\, P(x)$	AS
10	$\forall x \neg P(x)$	CQ 9
11	$\neg P(a)$	$\forall$E 10
12	$P(a)$	AS
13	$\bot$	$\neg$E 11,12
14	$P(b)$	X 13
15	$P(a) \to P(b)$	$\to$I 12-14
16	$\forall y\,(P(y) \to P(b))$	$\forall$I 15
17	$\exists x \forall y\,(P(y) \to P(x))$	$\exists$I 16
18	$\exists x \forall y\,(P(y) \to P(x))$	LEM 1-8, 9-17

h)

1	$P(a)$	AS
2	$P(a)$	R 1
3	$P(a) \to P(a)$	$\to$I 1-2
4	$\exists y\,(P(y) \to P(a))$	$\exists$I 3
5	$\forall x \exists y\,(P(y) \to P(x))$	$\forall$I 4

3 i)

1	∀x P(x)		AS
2	| P(a)		AS
3	| P(b)		∀E 1
4	P(a) → P(b)		→I 2-3
5	∀y (P(a) → P(y))		∀I 4
6	∃x∀y (P(x) → P(y))		∃I 5
7	¬ ∀x P(x)		AS
8	∃x ¬P(x)		CQ 7
9	| ¬P(a)		AS
10	| | P(a)		AS
11	| | ⊥		¬E 9,10
12	| | P(b)		X 12
13	| P(a) → P(b)		→I 10-12
14	| ∀y (P(a) → P(y))		∀I 13
15	| ∃x∀y (P(x) → P(y))		∃I 14
16	∃x∀y (P(x) → P(y))		∃E 8, 9-15
17	∃x∀y (P(x) → P(y))		LEM 1-6, 7-16

4 a)

1	$\exists x (\forall y (P(y) \to x = y) \wedge P(x)$	PR
2	$\forall y (P(y) \to a = y) \wedge P(a)$	AS
3	$\forall y (P(y) \to a = y)$	$\wedge$E2
4	$P(a)$	$\wedge$E2
5	$P(b)$	AS
6	$P(b) \to a = b$	$\forall$E 3
7	$a = b$	$\to$E 6
8	$a = b$	AS
9	$P(b)$	=E 4, 8
10	$P(b) \leftrightarrow a = b$	$\leftrightarrow$I 5-7, 8-9
11	$\forall y (P(y) \leftrightarrow a = y)$	$\forall$I 10
12	$\exists x \forall y (P(y) \leftrightarrow x = y)$	$\exists$I 11
13	$\exists x \forall y (P(y) \leftrightarrow x = y)$	$\exists$E 1, 2-12

4 b)

1	$\exists x \forall y (P(y) \leftrightarrow x = y)$	PR
2	$\forall y (P(y) \leftrightarrow a = y)$	AS
3	$P(b) \leftrightarrow a = b$	$\forall$E 2
4	$P(b)$	AS
5	$a = b$	$\leftrightarrow$E 4, 3
6	$P(b) \rightarrow a = b$	$\rightarrow$I 4-5
7	$\forall y (P(y) \rightarrow a = y)$	$\forall$I 6
8	$P(a) \leftrightarrow a = a$	$\forall$E 2
9	$a = a$	=I
10	$P(a)$	$\leftrightarrow$E 8, 9
11	$\forall y (P(y) \rightarrow a = y) \wedge P(a)$	$\wedge$I 7, 10
12	$\exists x (\forall y (P(y) \rightarrow x = y) \wedge P(x))$	$\exists$I 11
13	$\exists x (\forall y (P(y) \rightarrow x = y) \wedge P(x))$	$\exists$E 1, 2-12

4. c)

1	$\forall x \forall y (R(x,y) \to \neg R(y,x))$	PR
2	$R(a,b) \wedge R(b,a)$	AS
3	$R(a,b)$	$\wedge$E 2
4	$R(b,a)$	$\wedge$E 2
5	$\forall y (R(a,y) \to \neg R(y,a))$	$\forall$E 1
6	$R(a,b) \to \neg R(b,a)$	$\forall$E 5
7	$\neg R(b,a)$	$\to$E 3,6
8	$\bot$	$\neg$E 4,7
9	$a = b$	X 8
10	$(R(a,b) \wedge R(b,a)) \to a = b$	$\to$I 2-9
11	$\forall y((R(a,y) \wedge R(y,a)) \to a = y)$	$\forall$I 10
12	$\forall x \forall y ((R(x,y) \wedge R(y,x)) \to x = y)$	$\forall$I 11

4. d)

1	$\forall x \forall y (\forall z (R(z,x) \leftrightarrow R(z,y)) \rightarrow x = y)$	PR
2	$\exists x \forall y \neg R(y,x)$	PR
3	$\quad \forall y \neg R(y,a)$	AS
4	$\quad \forall y (\forall z (R(z,a) \leftrightarrow R(z,y)) \rightarrow a = y)$	$\forall E$ 1
5	$\quad \forall z (R(z,a) \leftrightarrow R(z,b)) \rightarrow a = b$	$\forall E$ 4
6	$\quad\quad \forall z \neg R(z,b)$	AS
7	$\quad\quad\quad R(c,a)$	AS
8	$\quad\quad\quad \neg R(c,a)$	$\forall E$ 3
9	$\quad\quad\quad \bot$	$\neg E$ 7,8
10	$\quad\quad\quad R(c,b)$	X 9
11	$\quad\quad\quad R(c,b)$	AS
12	$\quad\quad\quad \neg R(c,b)$	$\forall E$ 6
13	$\quad\quad\quad \bot$	$\neg E$ 11,12
14	$\quad\quad\quad R(c,a)$	X 13
15	$\quad\quad R(c,a) \leftrightarrow R(c,b)$	$\leftrightarrow I$ 7-10, 11-14
16	$\quad\quad \forall z (R(z,a) \leftrightarrow R(z,b))$	$\forall I$ 15
17	$\quad\quad a = b$	$\rightarrow E$ 5
18	$\quad\quad\quad a = b$	AS
19	$\quad\quad\quad \forall z \neg R(z,b)$	$=E$ 3, 18
20	$\quad \forall z \neg R(z,b) \leftrightarrow a = b$	$\leftrightarrow I$ 6-17, 18-19
21	$\quad \forall y (\forall z \neg R(z,y) \leftrightarrow a = y)$	$\forall I$ 20
22	$\quad \exists x \forall y (\forall z \neg R(z,y) \leftrightarrow x = y)$	$\exists I$ 21
23	$\exists x \forall y (\forall z \neg R(z,y) \leftrightarrow x = y)$	$\exists E$ 2, 3-22