1. a)

1	$\forall x\,(F(x) \vee G(x))$	PR
2	$\neg G(a)$	PR
3	$F(a) \vee G(a)$	$\forall E$ 1
4	$F(a)$	DS 2,3

b)

1	$\forall x(F(x) \rightarrow G(x))$	PR
2	$F(a)$	PR
3	$F(a) \rightarrow G(a)$	$\forall E$ 1
4	$G(a)$	$\rightarrow E$ 2,3
5	$\exists x\, G(x)$	$\exists I$ 4

c)

1	$\exists x \neg R(x,a)$	PR
2	$\quad \neg R(b,a)$	AS
3	$\quad \exists y \neg R(y,a)$	$\exists I$ 2
4	$\exists y \neg R(y,a)$	$\exists E$ 1, 2-3
5	$\neg \forall y R(y,a)$	CQ 4
6	$\exists z \neg \forall y R(y,z)$	$\exists I$ 5

c)

1	$\exists x\,(F(x) \wedge G(x))$	PR
2	$\quad F(a) \wedge G(a)$	AS
3	$\quad F(a)$	$\wedge E$ 2
4	$\quad G(a)$	$\wedge E$ 2
5	$\quad \exists x\, F(x)$	$\exists I$ 3
6	$\quad \exists x\, G(x)$	$\exists I$ 4
7	$\exists x\, F(x)$	$\exists E$ 1, 2-5
8	$\exists x\, G(x)$	$\exists E$ 1, 2-6
9	$\exists x\, F(x) \wedge \exists x\, G(x)$	$\wedge I$ 7,8

d)

1	$\forall x\,(F(x) \leftrightarrow G(x))$	PR
2	$\exists x\, F(x)$	PR
3	$\quad F(a)$	AS
4	$\quad F(a) \leftrightarrow G(a)$	$\forall E$ 1
5	$\quad G(a)$	$\leftrightarrow E$ 3,4
6	$\quad \exists x\, G(x)$	$\exists I$ 5
7	$\exists x\, G(x)$	$\exists E$ 2,3-6

f)

1	$\exists x\,(F(a) \wedge \forall y R(x,y))$	PR
2	$\quad F(a) \wedge \forall y R(a,y)$	AS
3	$\quad \forall y R(a,y)$	$\wedge E$ 2
4	$\quad R(a,a)$	$\forall E$ 3
5	$\quad \exists z R(z,z)$	$\exists I$ 4
6	$\exists z R(z,z)$	$\exists E$ 1, 2-5

2. a)

1	$\neg\exists x\,(G(x)\wedge F(x))$	PR
2	$\exists x\,(G(x)\wedge H(x))$	PR
3	$\forall x\,\neg(G(x)\wedge F(x))$	CQ 1
4	$G(a)\wedge H(a)$	AS
5	$G(a)$	$\wedge$E 4
6	$\neg(G(a)\wedge F(a)$	$\forall$E 3
7	$\neg G(a)\vee\neg F(a)$	DM 6
8	$\neg F(a)$	DS 5,7
9	$H(a)$	$\wedge$E 4
10	$H(a)\wedge\neg F(a)$	$\wedge$I 8,9
11	$\exists x\,(H(x)\wedge\neg F(x))$	$\exists$I 10
12	$\exists x\,(H(x)\wedge\neg F(x))$	$\exists$E 2,4-11

b)

1	$\exists x\,(G(x)\wedge\neg F(x))$	PR
2	$\forall x\,(G(x)\rightarrow H(x))$	PR
3	$G(a)\wedge\neg F(a)$	AS
4	$G(a)$	$\wedge$E 3
5	$\neg F(a)$	$\wedge$E 3
6	$G(a)\rightarrow H(a)$	$\forall$E 2
7	$H(a)$	$\rightarrow$E 4,6
8	$H(a)\wedge\neg F(a)$	$\wedge$I 5,7
9	$\exists x\,(H(x)\wedge\neg F(x))$	$\exists$I 8
10	$\exists x\,(H(x)\wedge\neg F(x))$	$\exists$E 1,3-9

c)

1	$\forall x\,(F(x)\rightarrow G(x))$	PR
2	$\neg\exists x\,(G(x)\wedge H(x))$	PR
3	$\exists x\,(H(x)\wedge F(x))$	AS
4	$H(a)\wedge F(a)$	AS
5	$H(a)$	$\wedge$E 4
6	$F(a)$	$\wedge$E 4
7	$F(a)\rightarrow G(a)$	$\forall$E 1
8	$G(a)$	$\rightarrow$E 6,7
9	$G(a)\wedge H(a)$	$\wedge$I 5,8
10	$\exists x\,(G(x)\wedge H(x))$	$\exists$I 9
11	$\bot$	$\neg$E 2,10
12	$\bot$	$\exists$E 3,4-11
13	$\neg\exists x\,(H(x)\wedge F(x))$	$\neg$I 3-12

d)

1	$\exists x\,(F(x)\wedge G(x))$	PR
2	$\forall x\,(G(x)\rightarrow H(x))$	PR
3	$F(a)\wedge G(a)$	AS
4	$F(a)$	$\wedge$E 3
5	$G(a)$	$\wedge$E 3
6	$G(a)\rightarrow H(a)$	$\forall$E 2
7	$H(a)$	$\rightarrow$E 5,6
8	$H(a)\wedge F(a)$	$\wedge$I 4,7
9	$\exists x\,(H(x)\wedge F(x))$	$\exists$I 8
10	$\exists x\,(H(x)\wedge F(x))$	$\exists$E 1,3-9

2. e)

1	$\neg\exists x(F(x) \wedge G(x))$	PR
2	$\exists x(G(x) \wedge H(x))$	PR
3	$G(a) \wedge H(a)$	AS
4	$G(a)$	$\wedge$E 3
5	$H(a)$	$\wedge$E 3
6	$\forall x \neg(F(x) \wedge G(x))$	CQ 1
7	$\neg(F(a) \wedge G(a))$	$\forall$E 6
8	$\neg F(a) \vee \neg G(a)$	DM 7
9	$\neg F(a)$	DS 4,8
10	$H(a) \wedge \neg F(a)$	$\wedge$I 5,9
11	$\exists x(H(x) \wedge \neg F(x))$	$\exists$I 10
12	$\exists x(H(x) \wedge \neg F(x))$	$\exists$E 2, 3-11

3. a)

1		$\exists x (F(t) \to G(x))$	PR
2	|	$F(t)$	AS
3	| |	$F(t) \to G(a)$	AS
4	| |	$G(a)$	→E 2,3
5	| |	$\exists x G(x)$	∃I 4
6	|	$\exists x G(x)$	∃E 1, 3-5
7		$F(t) \to \exists x G(x)$	→I 2-6

b)

1		$F(t) \to \exists x G(x)$	PR
2	|	$F(t)$	AS
3	|	$\exists x G(x)$	→E 1,2
4	| |	$G(a)$	AS
5	| | |	$F(t)$	AS
6	| | |	$G(a)$	R 4
7	| |	$F(t) \to G(a)$	→I 5-6
8	| |	$\exists x (F(t) \to G(a))$	∃I 7
9	|	$\exists x (F(t) \to G(x))$	∃E 3,4-8
10	|	$\neg F(t)$	AS
11	| |	$F(t)$	AS
12	| |	$\bot$	¬E 10,11
13	| |	$G(a)$	X 12
14	|	$F(t) \to G(a)$	→I 11-13
15	|	$\exists x (F(t) \to G(x))$	∃I 14
16		$\exists x (F(t) \to G(x))$	LEM 2-9,10-15

c)

1		$\exists x \forall y (R(x,y) \leftrightarrow \neg R(y,y))$	PR
2	|	$\forall y (R(a,y) \leftrightarrow \neg R(y,y))$	AS
3	|	$R(a,a) \leftrightarrow \neg R(a,a)$	∀E 2
4	| |	$R(a,a)$	AS
5	| |	$\neg R(a,a)$	↔E 3,4
6	| |	$\bot$	¬E 4,5
7	| |	$\neg R(a,a)$	AS
8	| |	$R(a,a)$	↔E 3,7
9	| |	$\bot$	¬E 7,8
10	|	$\bot$	LEM 4-6,7-9
11	|	$\exists y S(y)$	X 10
12		$\exists y S(y)$	∃E 1,2-11

3. d)

1	$\forall x \exists y R(x,y)$	PR
2	$\exists y R(a,y)$	$\forall E$ 1
3	$R(a,b)$	AS
4	$\exists z R(b,z)$	$\forall E$ 1
5	$R(b,c)$	AS
6	$R(a,b) \to \neg R(b,c)$	AS
7	$\neg R(b,c)$	$\to E$ 6
8	$\bot$	$\neg E$ 5,7
9	$\neg(R(a,b) \to \neg R(b,c))$	$\neg I$ 6-8
10	$\exists z \neg(R(a,b) \to \neg R(b,z))$	$\exists I$ 9
11	$\exists z \neg(R(a,b) \to \neg R(b,z))$	$\exists E$ 4,5-10
12	$\neg \forall z (R(a,b) \to \neg R(b,z))$	CQ 11
13	$\exists y \neg \forall z (R(a,y) \to \neg R(y,z))$	$\exists I$ 12
14	$\exists y \neg \forall z (R(a,y) \to \neg R(y,z))$	$\exists E$ 2,3-13

3. e)

1	$\forall y \exists x (R(y,x) \vee Q(y,x))$	PR
2	$\neg \forall y (\exists x R(y,x) \vee \exists x Q(y,x))$	AS
3	$\exists y \neg (\exists x R(y,x) \vee \exists x Q(y,x))$	CQ 2
4	$\neg (\exists x R(a,x) \vee \exists x Q(a,x))$	AS
5	$\neg \exists x R(a,x) \wedge \neg \exists x Q(a,x)$	DM 4
6	$\neg \exists x R(a,x)$	$\wedge$E 5
7	$\neg \exists x Q(a,x)$	$\wedge$E 5
8	$\exists x (R(a,x) \vee Q(a,x))$	$\forall$E 1
9	$R(a,b) \vee Q(a,b)$	AS
10	$R(a,b)$	AS
11	$\exists x R(a,x)$	$\exists$I 10
12	$\perp$	$\neg$E 6, 11
13	$Q(a,b)$	AS
14	$\exists x Q(a,x)$	$\exists$I 13
15	$\perp$	$\neg$E 7, 14
16	$\perp$	$\vee$E 9, 10-12, 13-15
17	$\perp$	$\exists$E 8, 9-16
18	$\perp$	$\exists$E 3, 4-17
19	$\forall y (\exists x R(y,x) \vee \exists x Q(y,x))$	IP 2-18

3. d) $A: \exists x F(x)$

$B: \exists y G(y)$

1	$(A \wedge B) \vee (\neg A \wedge \neg B)$	PR
2	A	AS
3	$\neg A \wedge \neg B$	AS
4	$\neg A$	$\wedge$E 3
5	$\perp$	$\neg$E 2, 4
6	$\neg(\neg A \wedge \neg B)$	$\neg$I 3-5
7	$A \wedge B$	DS 1, 6
8	B	$\wedge$E 7
9	B	AS
10	$\neg A \wedge \neg B$	AS
11	$\neg B$	$\wedge$E 10
12	$\perp$	$\neg$E 9, 11
13	$\neg(\neg A \wedge \neg B)$	$\neg$I 10-12
14	$A \wedge B$	DS 1, 13
15	A	$\wedge$E 14
16	$A \leftrightarrow B$	$\leftrightarrow$I 2-8, 9-15

3. g)

1	$\forall x \forall y (F(x) \to (G(y) \to \neg R(x,y,x)))$	PR
2	$\forall y (F(a) \to (G(y) \to \neg R(a,y,a)))$	$\forall$E 1
3	$F(a) \to (G(b) \to \neg R(a,b,a))$	$\forall$E 2
4	$R(a,b,a)$	AS
5	$G(b)$	AS
6	$F(a)$	AS
7	$G(b) \to \neg R(a,b,a)$	$\to$E 3,6
8	$\neg R(a,b,a)$	$\to$E 5,7
9	$\bot$	$\neg$E 4,8
10	$\neg F(a)$	$\neg$I 6-9
11	$G(b) \to \neg F(a)$	$\to$I 5-10
12	$R(a,b,a) \to (G(b) \to \neg F(a))$	$\to$I 4-11
13	$\forall y (R(a,y,a) \to (G(y) \to \neg F(a))$	$\forall$I 12
14	$\forall x \forall y (R(x,y,x) \to (G(y) \to \neg F(x)))$	$\forall$I 13

3. b)

1	$\forall x\,(F(x) \to \exists y\,(R(y,x) \wedge F(y)))$	PR
2	$\forall x \forall y \forall z\,(R(y,x) \to (R(z,y) \to \neg F(z)))$	PR
3	$\forall x F(x)$	AS
4	$F(a) \to \exists y\,(R(y,a) \wedge F(y))$	$\forall$E 1
5	$F(a)$	$\forall$E 3
6	$\exists y\,(R(y,a) \wedge F(y))$	$\to$E 4,5
7	$R(b,a) \wedge F(b)$	AS
8	$R(b,a)$	$\wedge$E 7
9	$\forall y \forall z\,(R(y,a) \to (R(z,y) \to \neg F(z)))$	$\forall$E 2
10	$\forall z\,(R(b,a) \to (R(z,b) \to \neg F(z)))$	$\forall$E 9
11	$F(b)$	$\wedge$E 7
12	$F(b) \to \exists y\,(R(y,b) \wedge F(y))$	$\forall$E 1
13	$\exists y\,(R(y,b) \wedge F(y))$	$\to$E 11,12
14	$R(c,b) \wedge F(c)$	AS
15	$R(b,a) \to (R(c,b) \to \neg F(c))$	$\forall$E 10
16	$R(c,b) \to \neg F(c)$	$\to$E 8,15
17	$R(c,b)$	$\wedge$E 14
18	$\neg F(c)$	$\to$E 16,17
19	$F(c)$	$\wedge$E 14
20	$\bot$	$\neg$E 18,19
21	$\bot$	$\exists$E 13, 14-20
22	$\bot$	$\exists$E 6, 7-21
23	$\neg \forall x F(x)$	$\neg$I 3-22