

# Øving 3

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*Ønsker retting*

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$$\begin{aligned} & \neg \forall x (P(x) \Rightarrow \exists y Q(x, y)) \\ & \neg \forall x (\exists y P(x) \Rightarrow Q(x, y)) \\ & \exists x \forall y \neg P(x) \Rightarrow Q(x, y) \\ & \exists x \forall y P(x) \vee Q(x, y) \end{aligned}$$

2  $\mathcal{P}(A) = \{\emptyset, \{\{a\}\}, \{\{b, c\}\}, \{\{c, d, \{e, f\}\}\}, \{\{a\}, \{b, c\}\}, \{\{a\}, \{c, d, \{e, f\}\}\},$   
 $\{\{b, c\}, \{c, d, \{e, f\}\}\}, \{\{a\}, \{b, c\}, \{c, d, \{e, f\}\}\}\}$

3  $A = \{1, 2, 3, 4, 5\}, B = \{3, 4, 5, 6, 7\}$

(a)  $A \cap B = \{3, 4, 5\}$   
(b)  $A \Delta B = \{1, 2, 6, 7\}$   
 $C := \{1, 2\}, \quad D := \{6, 7\}$   
 $C \cup D = A \Delta B$

4 Prove:  $\overline{X - Y} = \overline{X} \cup Y$

$$X - Y := \{x : x \in X \wedge x \notin Y\} = X \cap \overline{Y}$$

$$\begin{aligned} \overline{X - Y} &= \overline{X \cap \overline{Y}} \\ &= \overline{X} \cup \overline{\overline{Y}} \quad (De\ Morgan) \\ &= \overline{X} \cup Y \quad \square \end{aligned}$$

5 (a)  $(\overline{X} \cup Y) \cap (X \cup Y)$ , logisk riktig når  $X = \emptyset$   
(b)  $(\overline{X} \cap Y) \cup (X \cap Y)$ , logisk riktig når  $X = \emptyset$

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$$\begin{aligned}\overline{Y} - X = \overline{Y} &\equiv X \subseteq Y \equiv X \cap Y = X \\ &\Downarrow \\ X &= \emptyset\end{aligned}$$

7

$$|\mathcal{P}(A) - \{\{x\} : x \in A\}|, \quad \{\{x\} : x \in A\} \in \mathcal{P}(A)$$

$$\begin{aligned}|\mathcal{P}(A)| &= 2^n \\ |\mathcal{P}(A) - \{\{x\} : x \in A\}| &= 2^n - 1\end{aligned}$$

8 (a)

$$\begin{aligned}|\mathcal{P}(A \times B)| &= 2^{m \cdot n}, \quad |A| = m, |B| = n \\ |\mathcal{P}(A)| \cdot |\mathcal{P}(B)| &= 2^m \cdot 2^n = 2^{m+n}\end{aligned}$$

$$\begin{aligned}\text{For } m, n \in \{0, 2\} \text{ er } |\mathcal{P}(A \times B)| &= |\mathcal{P}(A)| \cdot |\mathcal{P}(B)| \\ \text{For } m, n = 1 \text{ er } |\mathcal{P}(A \times B)| &< |\mathcal{P}(A)| \cdot |\mathcal{P}(B)| \\ \text{For } m, n < 0 \text{ og } m, n > 2 \text{ er } |\mathcal{P}(A \times B)| &> |\mathcal{P}(A)| \cdot |\mathcal{P}(B)|\end{aligned}$$

(b) Prove:  $(A - B) \cap (B - A) = \emptyset$

$$\begin{aligned}(A - B) \cap (B - A) &= (A \cap \overline{B}) \cap (\overline{A} \cap B) \\ &= A \cap \overline{B} \cap \overline{A} \cap B \\ &= A \cap \overline{A} \cap B \cap \overline{B} \\ &= \emptyset \cap \emptyset \\ &= \emptyset \quad \square\end{aligned}$$