MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2022

SET 4

Deadline: Deadline: Monday 14.02.2022, 11:59 pm

Exercise 1. *Consider the set* $X = \{a, b, c\}$.

a) What is the power set $\mathcal{P}(X)$*?*

b) Show that for any set Y*, the relation defined by set inclusion* $R = \{\langle A, B\rangle | A \subseteq B\}$ *defines a partial ordering on* $\mathcal{P}(Y)$.

c) Draw the Hasse diagram for $\mathcal{P}(X)$ *with the partial ordering given by set inclusion like in b).*

Exercise 2. *Consider the set* $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

a) Define the relation R *on* X *that relates every number in* X *to those that have the same number of divisors as it.*

b) Show that R *is an equivalence relation.*

c) Find the partition of X *corresponding to* R.

Exercise 3. *Let* $p \in \mathbb{Z}$*. Define the following relation*

$$R_p = \{\langle x, y\rangle | \exists n \in \mathbb{Z} : x = y + n \cdot p\} \subset \mathbb{Z} \times \mathbb{Z}.$$

a) Show that R *is an equivalence relation.*

b) Characterize the equivalence classes of R_p*. How many are there for a fixed* p*?.*

Exercise 4. *a) Let* X *be a set and let* $R_1 \subseteq X \times X$ *and* $R_2 \subseteq X \times X$ *be two equivalence relations on* X*. Show that* $R_1 \cap R_2$ *also defines an equivalence relation.*

b) Let the cardinality of X *be finite. Define* $R \subseteq X \times X$ *to be any relation on* X*. We make a list with all equivalence relations that contain* R*:*

$$\{R_1, R_2, \ldots, R_n\} = \{R_i \subseteq X \times X | (R \subseteq R_i) \wedge (R_i \text{ is an equivalence relation})\}.$$

Note that this list is finite because X *has finite cardinality. Prove that* $\tilde{R} := R_1 \cap R_2 \cap \ldots \cap R_n$ *is the smallest equivalence relation on* X *that contains* R *i.e that for all equivalence relations* $A \subset X \times X$ *such that* $R \subseteq A$ *it follows that* $\tilde{R} \subseteq A$*. We call* $\tilde{R}$ *the equivalence relation generated by* R.

c) What is the equivalence relation generated by the relation from exercise 1 a)?

Date: February 4, 2022.