

An Alternating Minimization Algorithm with Trajectory for Direct Exoplanet Detection

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Exoplanet Imaging



Exoplanet Imaging





Exoplanet Imaging

The image is a collage of various news snippets and images related to exoplanet imaging:

- euronews.next**: Headline: "Signs of life? James Webb reveals more about exoplanet K2-18 b's atmosphere". Below the headline is a small image of a red star with a small blue planet.
- BBC NEWS**: Headline: "Exoplanet discovered around neighbouring star". Below the headline is a small image of a brownish-orange planet.
- The Brussels Times**: Headline: "Names chosen for Belgium's official exoplanet and star". Below the headline is a large image of a grey exoplanet.
- HD 49674 b**: A caption below the large image reads "A Neptune-like gas giant planet".

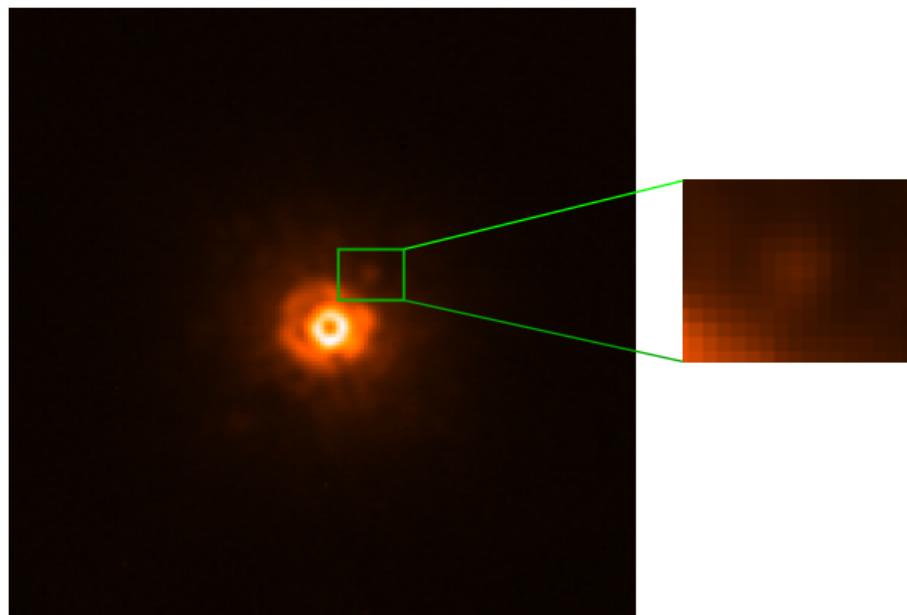
Exoplanet Imaging

A real image



Exoplanet Imaging

A real image of exoplanet



Direct Imaging



Credit: <https://exoplanets.nasa.gov/>

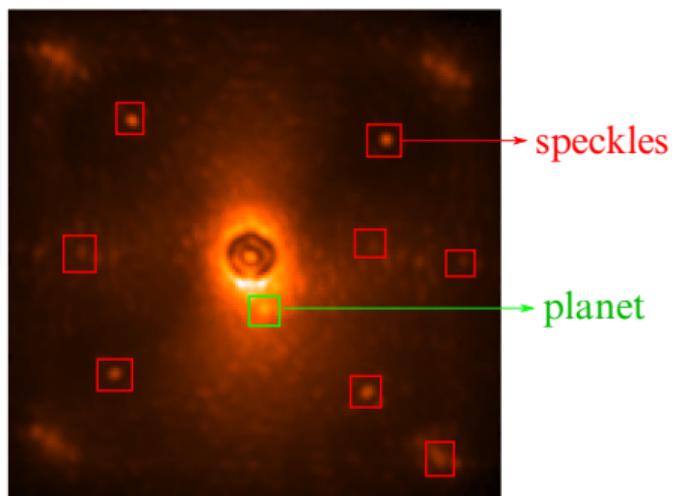
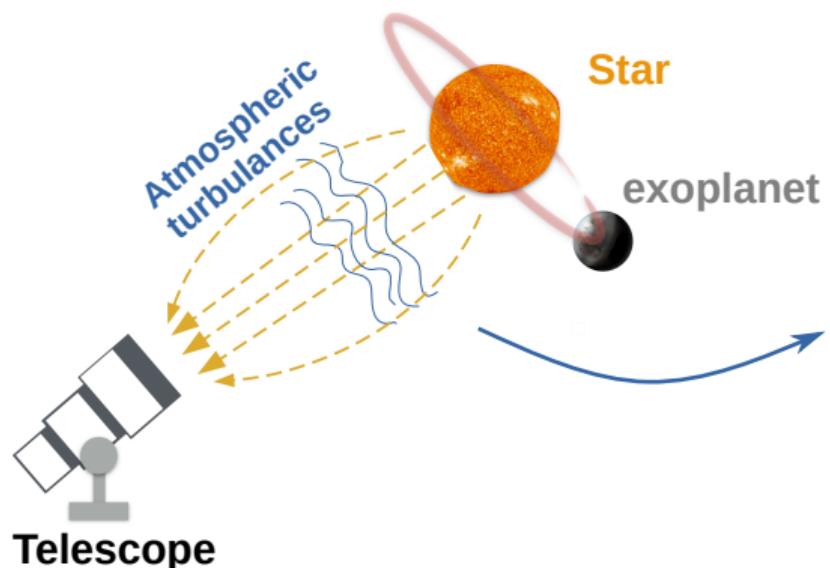
Direct Imaging



Credit: <https://exoplanets.nasa.gov/>

- ▶ firefly → exoplanet
- ▶ lighthouse → star

Direct Imaging



Angular Differential Imaging

Problem Setup & Goal

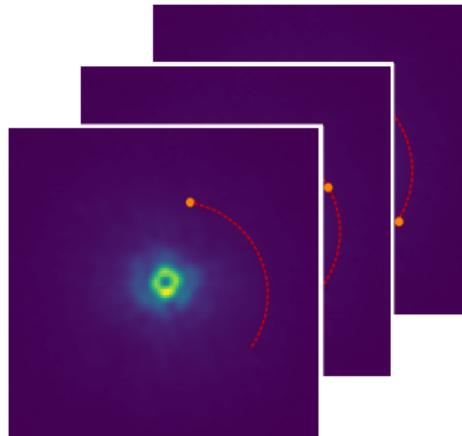
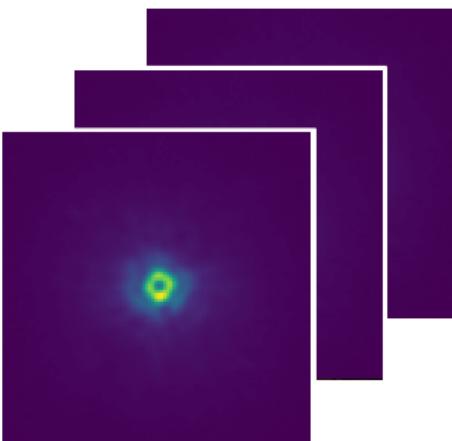


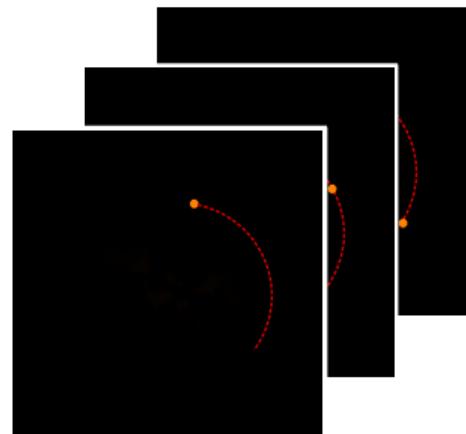
Image sequence

=



Background
(star+speckles)

+



Foreground
(planet)

Background: (Annular) PCA^{1,2}

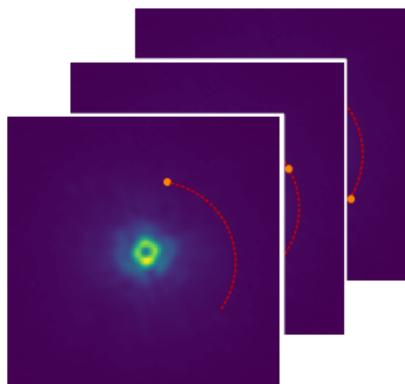
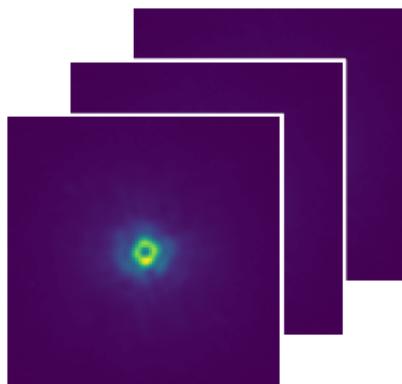


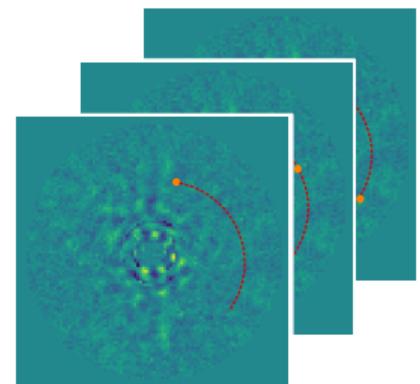
Image sequence
 M

=

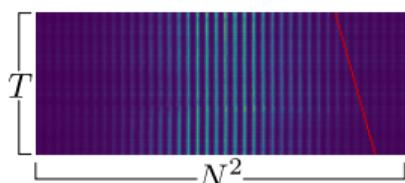


Low rank
 L

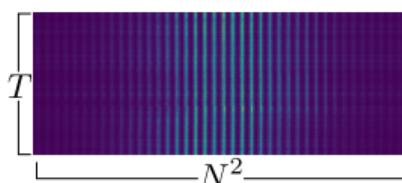
+



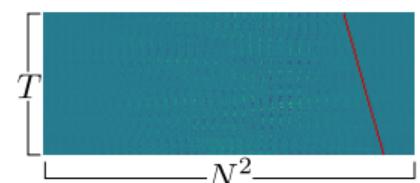
Foreground
 R



=

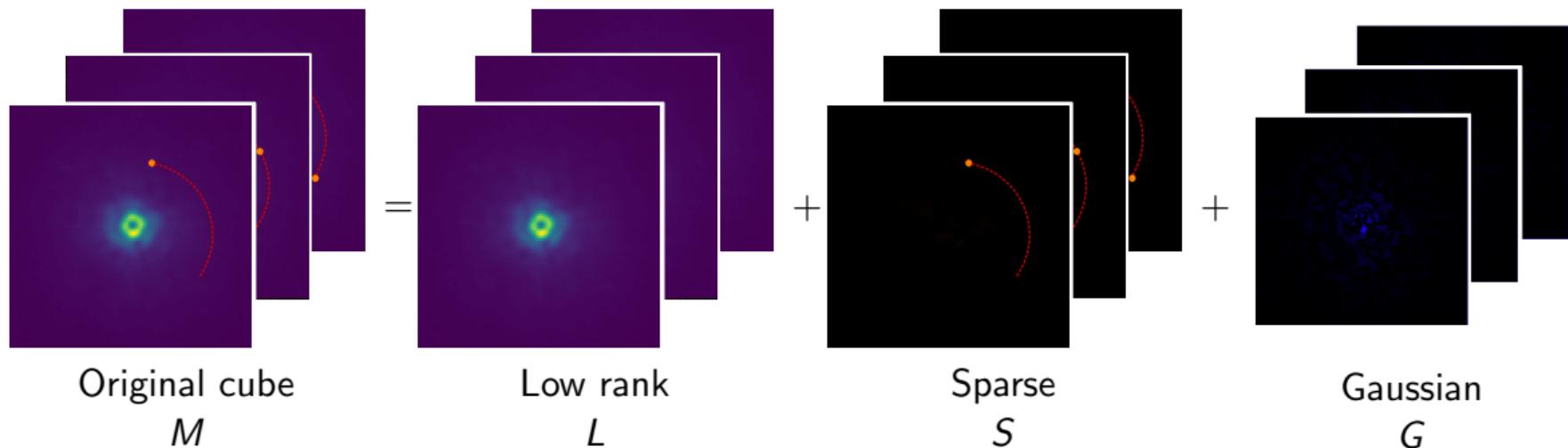


+



¹Amara and Quanz, 2012

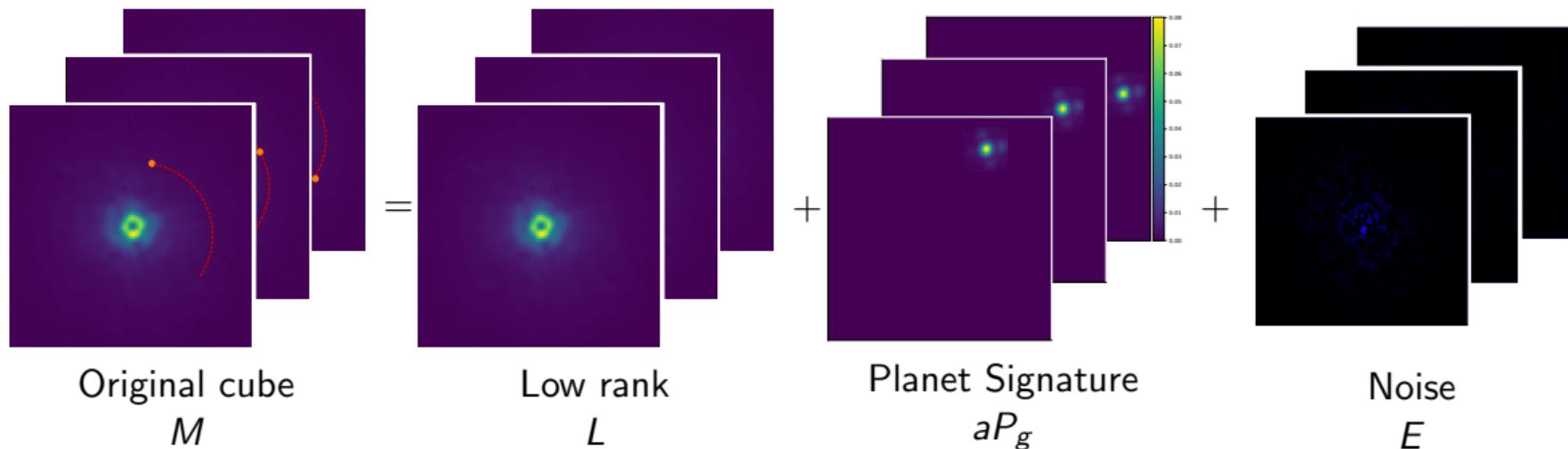
²Soummer, et al., 2012

State of art: LLSG³

$$\text{rank}(L) \leq k, \quad \text{card}(S) \leq s$$

³Gomez Gonzalez, et al., 2016

Alternating Minimization Algorithm with Trajectory (AMAT)

Original cube
 M Low rank
 L Planet Signature
 aP_g Noise
 E

AMAT

$$\text{rank}(L) \leq k, \quad P_g \in \Lambda$$

$$\begin{aligned} & \min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\| \\ \text{s.t. } & \text{rank}(L) \leq k \end{aligned}$$

AMAT

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\| \quad \text{s.t.} \quad \text{rank}(L) \leq k$$

- ▶ Solve (1) by SVD or L1-LRA⁴
- ▶ Solve (2) by

$$a_i = \frac{\langle P_g, M - L_i \rangle}{\|P_g\|_F^2}$$

or minimum of the points
 $(M - L_i)/P_g$

⁴Gillis and Vavasis, 2018

AMAT

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\| \quad \text{s.t.} \quad \text{rank}(L) \leq k$$

Alternating Minimization

$$L_i = \arg \min_{L \in \mathbb{R}^{t \times n}} \|M - L - a_{i-1}P_g\| \quad (1)$$

$$a_i = \arg \min_{a \in \mathbb{R}} \|M - L_i - aP_g\| \quad (2)$$

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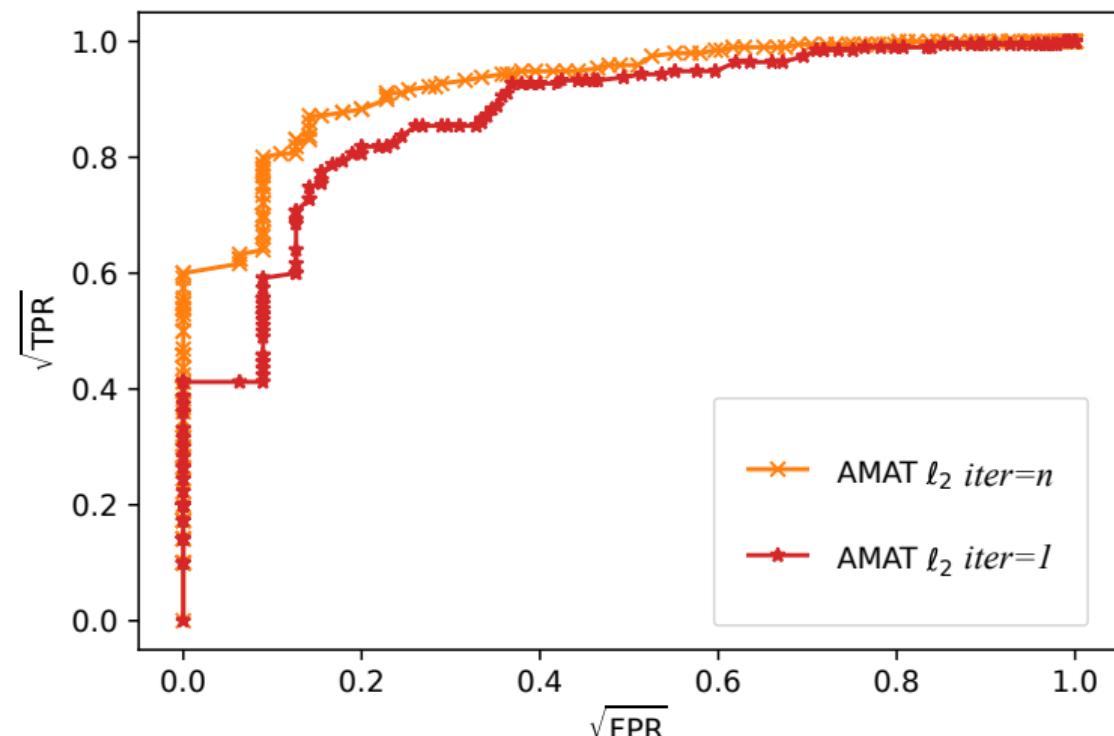
or minimum of the points
 $(M - L_i)/P_g$

⁴Gillis and Vavasis, 2018

Numerical Experiments

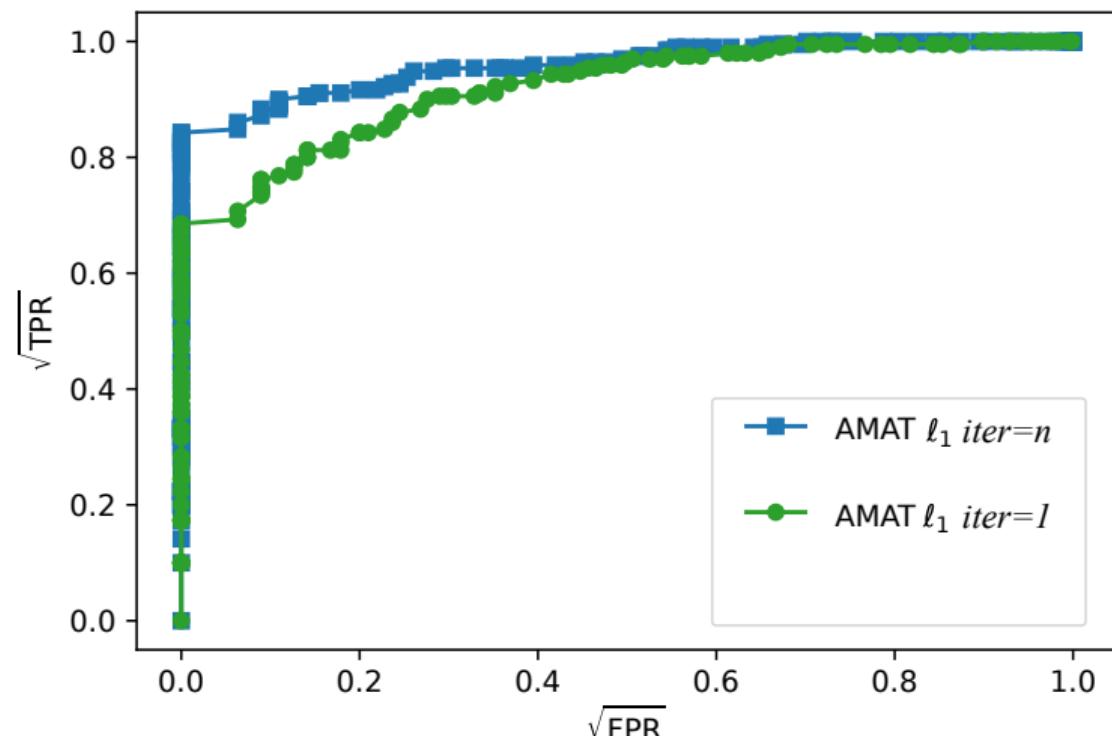
Numerical Experiments - ROC Curves

- ▶ Synthetic planets are injected.
- ▶ $\sqrt{\text{TPR}}$ & $\sqrt{\text{FPR}}$ are used instead of TPR & FPR.



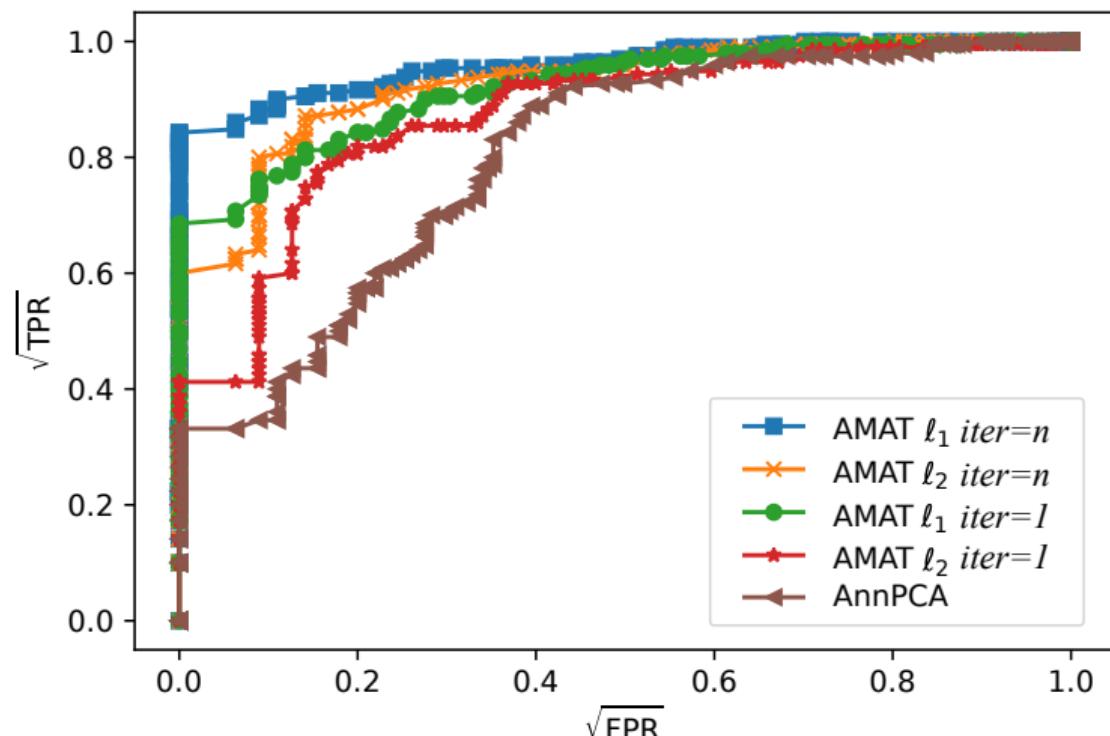
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Numerical Experiments - ROC Curves

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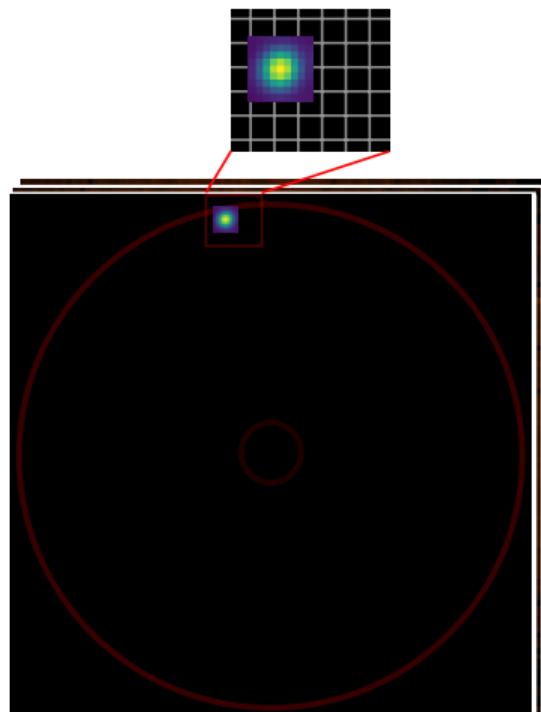


Thank you for your attention!
Any questions?

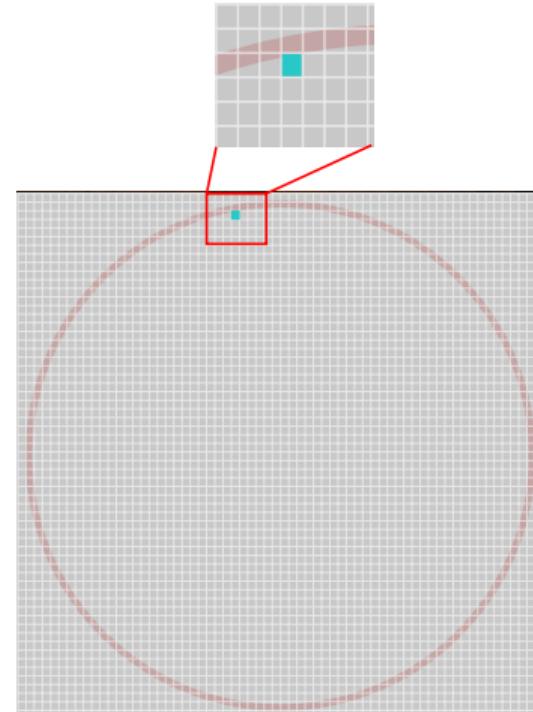
hazan.daglayan@uclouvain.be
GitHub: [hazandaglayan/AMAT](https://github.com/hazandaglayan)



Trajectories

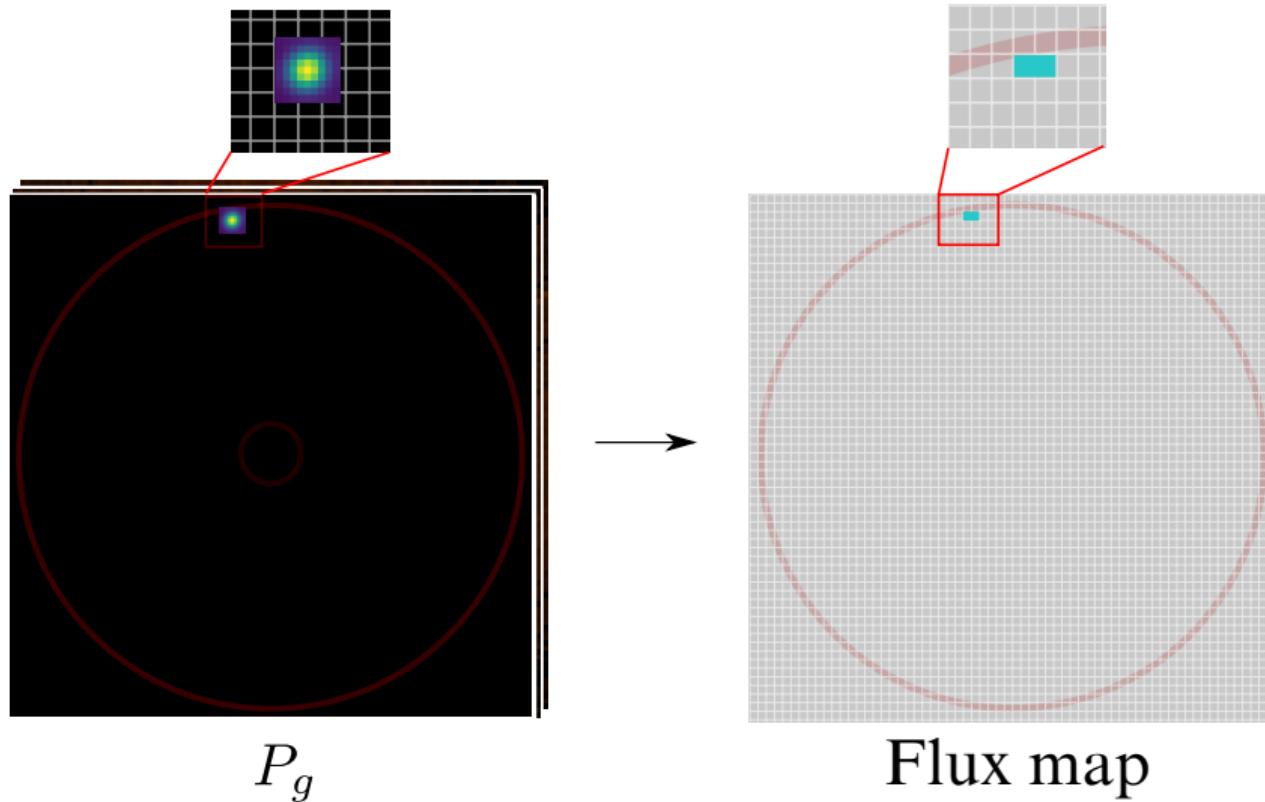


P_g

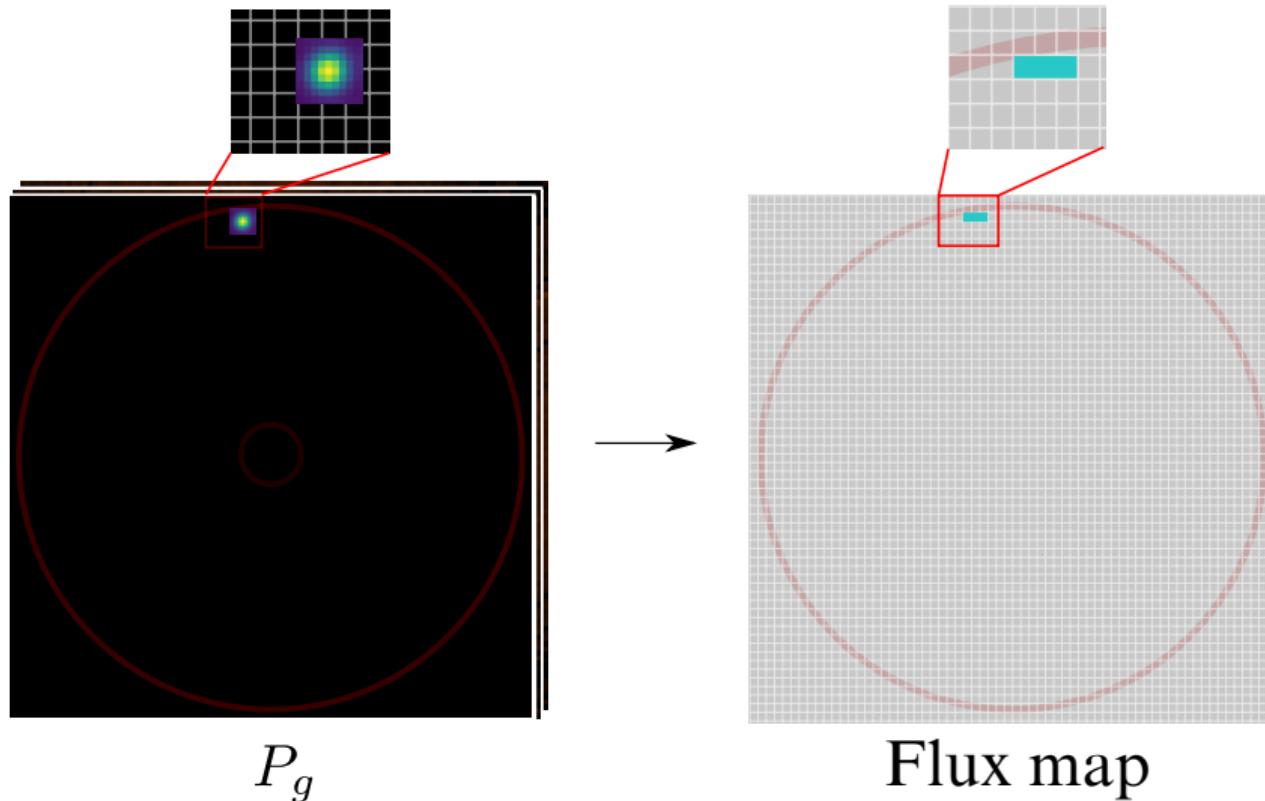


Flux map

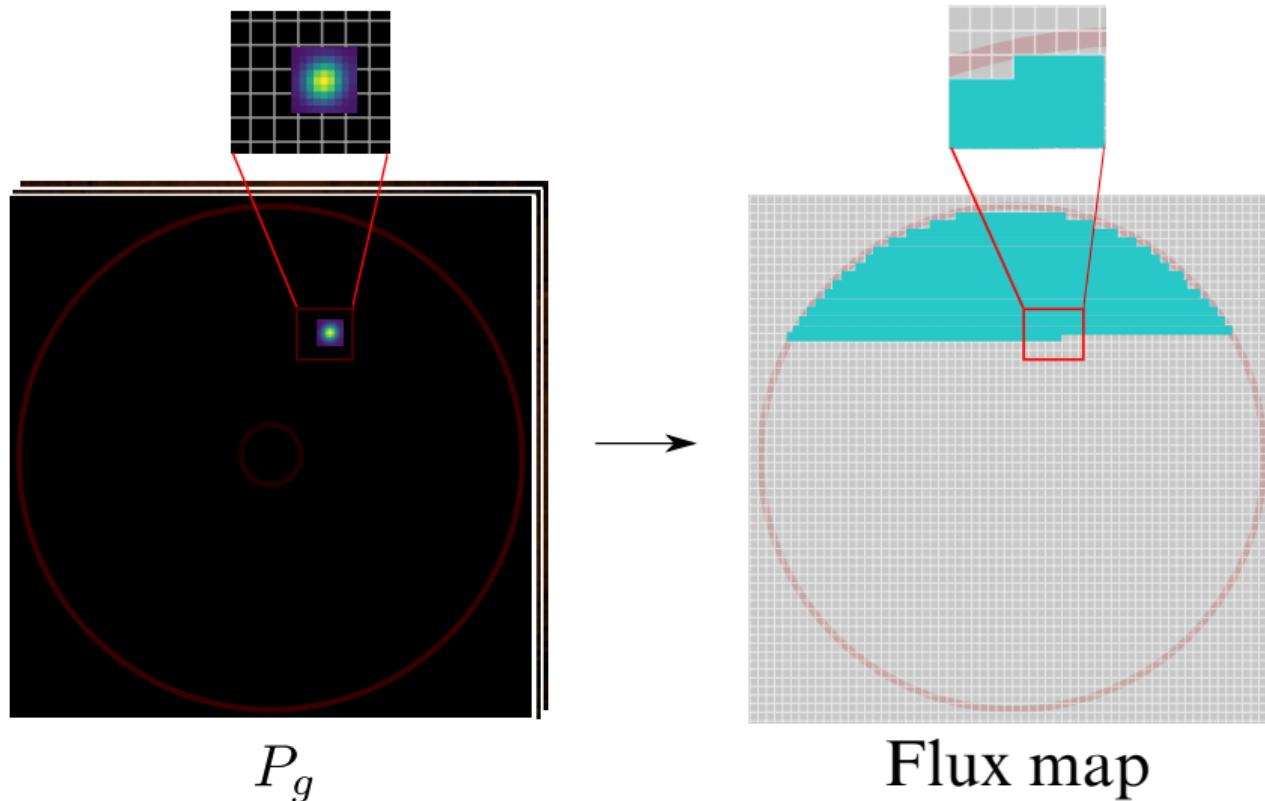
Trajectories



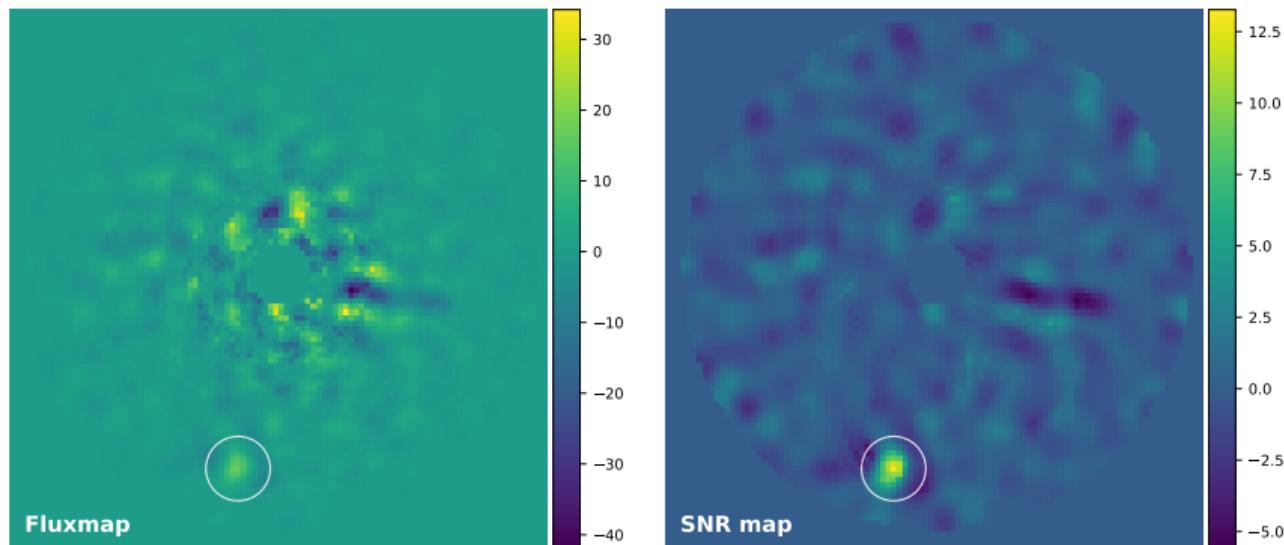
Trajectories



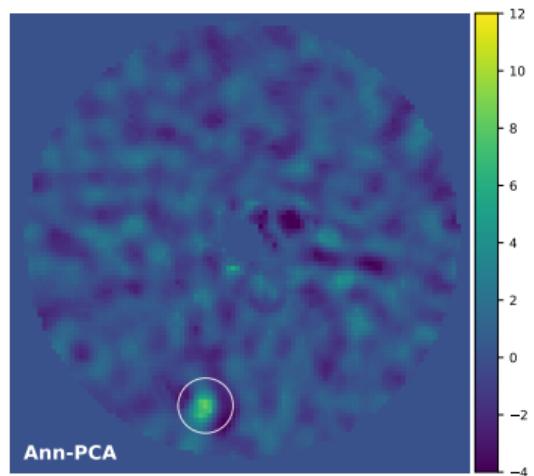
Trajectories



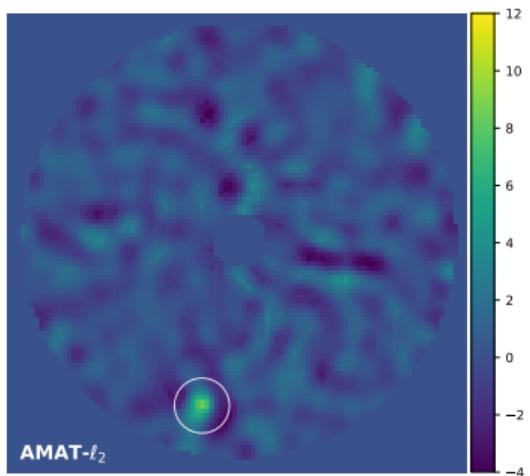
From Fluxmap to SNR



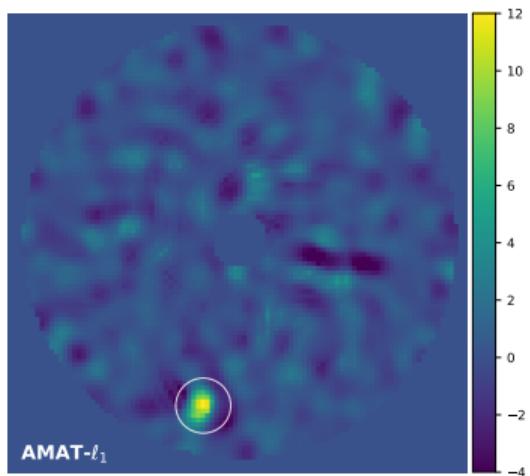
Detection Maps Comparison



Ann-PCA



AMAT- l_2



AMAT- l_1