Mathematical Prediction and Validation of DIY Stethoscope Frequency Response

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1 Introduction

Cardiovascular diseases are the number 1 cause of death globally, taking an estimated 17.9 million lives each year. On top of this worrisome statistic, covid-19 has had an unexpected impact - in addition to lung damage, patients are also developing heart problems and dying of cardiac arrest, and more doctors and experts have come to believe that the virus can directly infect the heart muscle.

The need to diagnose potential heart issues is now higher than ever, but with social distancing rules and busy doctors offices, many patients are skipping their visits and not getting the preventive care they need. This part of the project plans to create and demonstrate the framework for approximating the performance of home made stethoscopes such as the one shown in Figure 1.

Figure 1: Physical Device

2 Calculations

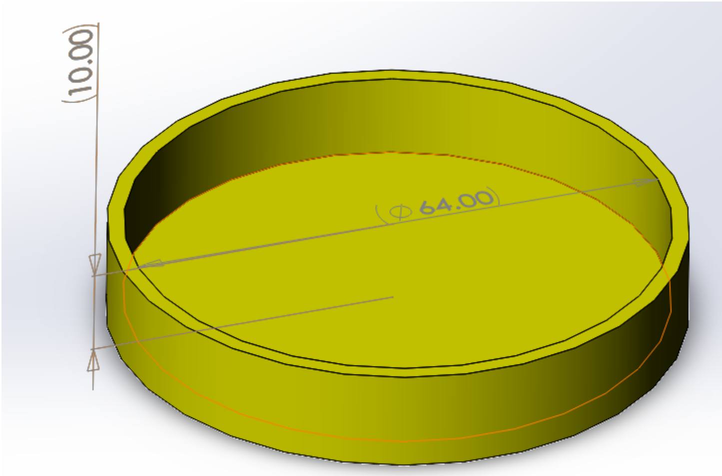

Figure 2: Internal Dimensions of the Lid

To achieve an accurate model, we first must determine the internal depth (h) and internal radius of the lid (r).

$$h = 10 * 10^{-}3$$

$$r = 32 * 10^{-}3$$

Now, we calculate the contact area and and volume of any given cylinder using the following equations.

$$A = \pi r^2 m^2$$

$$V = Ahm^3$$

If we assume the air is 25 deg C, the following will be true for air:

$$\rho = 1.1839 kg/m^3$$

$$c = 346.13 m/s$$

From "NUSSBAUMER, Maximilian et al. "A theory for stethoscope acoustics."(2019)." The equivalent stiffness of a simple stethoscope system such as ours can be modeled using

$$k = \rho * c * A^2/V = 131.8230$$

We plug in and solve for the natural frequency, w0, using the equation.

$$w_0 = \sqrt{k/m}$$

However, we can normalize the amplitude (x) to the mass component as the oscillating mass will not change significantly from stethoscope to stethoscope.

$$w_0 = \sqrt{k} = \ 11.4814$$

Now, using the equation for the displacement response for a driven, damped mass-spring system. Where x is the amplitude of displacement F0 is the forcing function m is the total mass attached to the oscillating system and B is the damping rate

$$x = \frac{(F_0/m)}{(\sqrt{(w^2 - w_0^2)^2 + (2 * B * w)^2})}$$

Next, we again, normalize x about m since it does not change from design to design.

$$x = \frac{(F_0)}{(\sqrt{(w^2 - w_0^2)^2 + (2 * B * w)^2})}$$

Next since the forcing frequency (the heart beat)is not affected by the stethoscope, we can remove it by dividing x by F0.

$$x = \frac{1}{(\sqrt{(w^2 - w_0^2)^2 + (2 * B * w)^2})}$$

Now, we assume the system is only moderately dampened by the friction of the bag, the plastic membrane, and air resistance and approximate the damping ratio to be.

$$B = 0.8$$

Leaving us with a simplified equation only in terms of the frequency w and the known value w0.

$$x = \frac{1}{(\sqrt{(w^2 - w_0^2)^2 + (2 * (0.8) * w)^2})}$$

$$= \frac{1}{((w^2 - 4638110471092567/35184372088832)^2 + \sqrt{(64 * w^2)/25})}$$

In MATLAB, we then establish that is graphed in the frequency range f=0hz to f=500hz and we can visualize the approximate frequency response by plotting the normalized amplitude(x) about the frequency f.

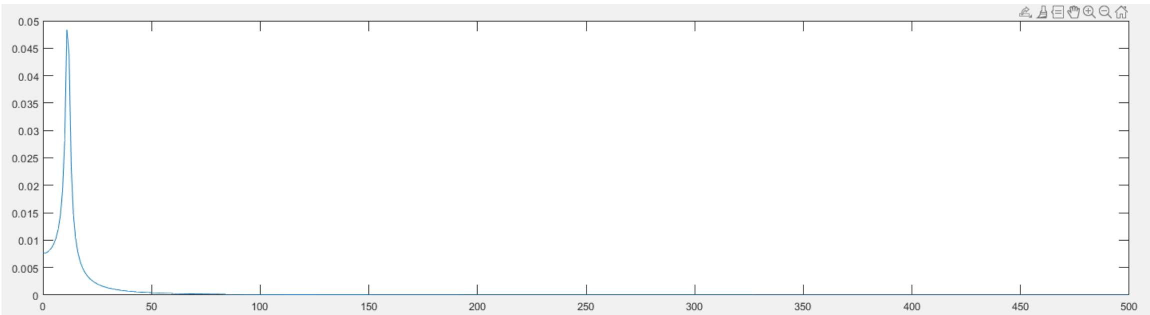

Figure 3: Theoretical frequency response found with MATLAB

Our predictions indicate the the stethoscope will behave in the desired manner, responding aggressively to the lower frequencies of the heart while suppressing the high frequency noise.

3 Design Validation

Initially, 10 second audio clip was recorded from the device and a FFT was completed to plot the signal amplitude about the frequency and can be directly.

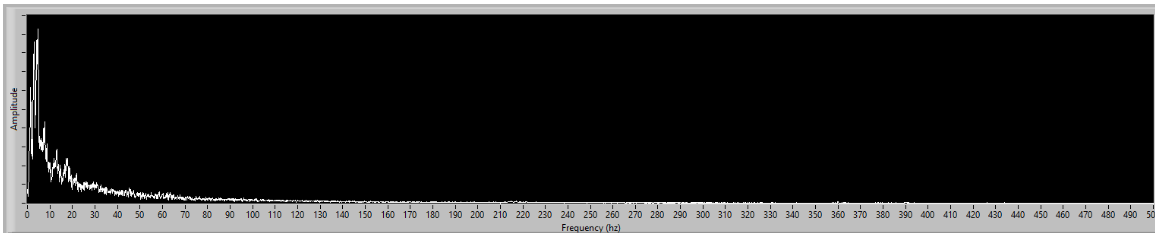

Figure 4: Empirical frequency response (calculated using FFT)

This chart confirms our predictive mathematical model. The system's response to low frequencies appears to be relatively high while higher frequencies are attenuated greatly. To investigate further we took a longer two minute recording and received the following result when preforming a FFT.

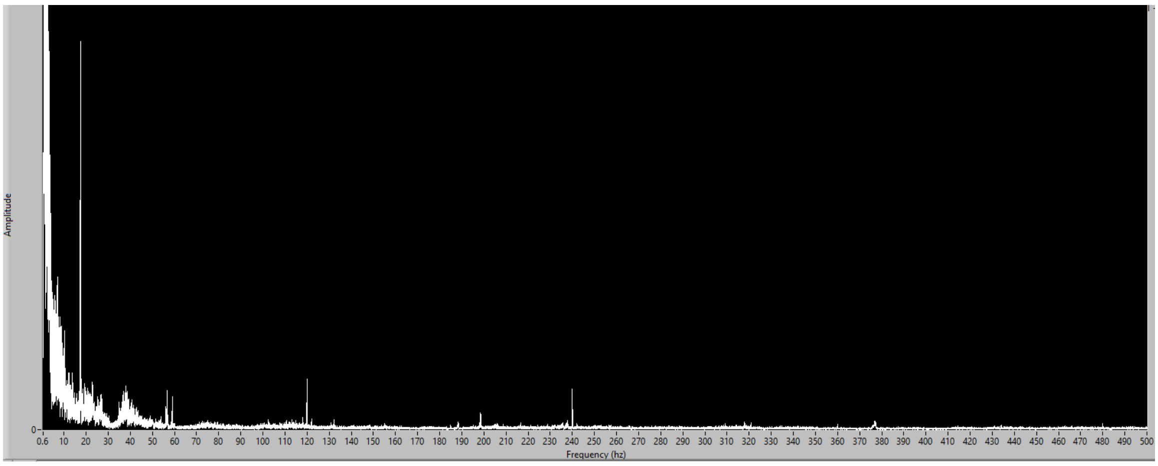

Figure 5: Empirical frequency response (calculated using FFT)

While there is still some discrepancy in the peak frequency location, it is now much closer to our mathematical prediction. We can see that our dampening factor surprisingly may have been estimated too low and the actual response curve appears to be steeper than the mathematical model predicted. Nonetheless, the numerical estimation provided a general idea of the response curve shape and has been sufficient for our purposes.

4 References

[1] E. O. Brigham and R. E. Morrow, ”The fast Fourier transform,” in IEEE Spectrum, vol. 4, no. 12, pp. 63-70, Dec. 1967, doi: 10.1109/MSPEC.1967.5217220.

[2]NUSSBAUMER, Maximilian et al. “A theory for stethoscope acoustics.” (2019).